Steel and Composite Structures, *Vol. 18, No. 2 (2015) 443-465*
DOI: http://dx.doi.org/10.12989/scs.2015.18.2.443

Thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory

Wafa Tebboune [1], Kouider Halim Benrahou [1],
Mohammed Sid Ahmed Houari [2] and Abdelouahed Tounsi [*1,2]

[1] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[2] *Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de Génie Civil, Algeria*

(*Received May 28, 2014, Revised July 06, 2014, Accepted July 27, 2014*)

Abstract. In this paper, an efficient and simple trigonometric shear deformation theory is presented for thermal buckling analysis of functionally graded plates. It is assumed that the plate is in contact with elastic foundation during deformation. The theory accounts for sinusoidal distribution of transverse shear stress, and satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without using shear correction factor. Unlike the conventional trigonometric shear deformation theory, the proposed sinusoidal shear deformation theory contains only four unknowns. It is assumed that the mechanical and thermal non-homogeneous properties of functionally graded plate vary smoothly by distribution of power law across the plate thickness. Using the non-linear strain-displacement relations, the equilibrium and stability equations of plates made of functionally graded materials are derived. The boundary conditions for the plate are assumed to be simply supported on all edges. The elastic foundation is modelled by two-parameters Pasternak model, which is obtained by adding a shear layer to the Winkler model. The effects of thermal loading types and variations of power of functionally graded material, aspect ratio, and thickness ratio on the critical buckling temperature of functionally graded plates are investigated and discussed.

Keywords: functionally graded materials; buckling; plate theory; elastic foundations

1. Introduction

In recent years, functionally graded materials (FGMs) have gained such popularity as high thermal resistance materials with low thermal stresses that structural components exposed to high-temperature environments such as aircraft structures are made of FGMs. They are a new generation of composite structures first introduced by a group of Japanese scientists in 1984 (Yamanouchi *et al*. 1990, Koizumi 1993). Typically, FGMs are made of a ceramic and a metal in such a way that the ceramic can resist the severe thermal loading from the high temperature

*Corresponding author, Professor, E-mail: tou_abdel@yahoo.com

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

environment, whereas the metal serves to decrease the large tensile stress occurring on the ceramic surface at the earlier stage of cooling. Since the material properties of FGMs vary continuously from one interface to the other, this results in eliminating interface problems of composite materials and achieving smooth stress distribution. The FGM is widely used in many structural applications such as aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, and chemical.

Rectangular thick plates made of FGMs are often employed as a part of engineering structures. In addition, to describe the interaction between plate and foundation, various kinds of foundation models have been proposed. The simplest one is Winkler or one-parameter model which regards the foundation as a series of separated spring without coupling effects between each other. Hence, in this model the properties of the soil are described only by one parameter (K_w), that represents the stiffness of the vertical springs (Avramidis and Morfidis 2006). However, Winkler's model is unable to take into account the continuity or cohesion of the soil. Also, the assumption that there is no interaction between adjacent springs results in ignoring the influence of the soil on either side of the beam. To overcome this weakness, many two-parameter elastic foundation models have been proposed, such as Pasternak's elastic foundations (Pasternak 1954). The Pasternak or two-parameter mode is widely used to describe the mechanical behavior of structure-foundation interactions. Zenkour (2009) presented a thermoelastic bending analysis of a functionally graded (FG) plate subjected to a transverse uniform load and resting on a two-parameter elastic foundation using the refined sinusoidal shear deformation plate theory. Benyoucef *et al.* (2010) investigated the bending response of FG thick plates resting on Pasternak's elastic foundations using a hyperbolic shear deformation plate theory. Ait Atmane *et al.* (2010) studied the free vibration behaviour of FG plates resting on Winkler-Pasternak elastic foundations. Cheng and Kitipornchai (1999) proposed a membrane analogy to derive exact explicit eigenvalues for compression buckling, hydrothermal buckling, and vibration of FG plates on a Winkler-Pasternak foundation based on the first-order shear deformation theory (FSDT). The same membrane analogy was later applied to the analyses of FG plates and shells based on a third-order plate theory (Cheng and Batra 2000, Reddy and Cheng 2002). Effect of the Pasternak elastic foundation on mechanical post-buckling of moderately thick FG plates is discussed by Yang *et al.* (2005a). In their study, four sides of plate are assumed to be clamped and formulation is based on the FSDT. They obtained the post-buckling equilibrium paths based on a two-dimensional differential quadrature approach combined with the perturbation technique. Thermo-mechanical post-buckling response of FG plates based on an analytical solution is presented by Woo *et al.* (2005). They used third order shear deformation plate theory (Reddy's displacement field) and von Karman type of large deflections to obtain the coupled partial differential equations and used a mixed series solution to solve them. Their study includes four types of boundary conditions for plate. Zenkour and Sobhy (2010) investigated the thermal buckling behavior of various types of FG sandwich plates. Zenkour and Sobhy (2011) studied the thermal buckling of FG plate resting on one parameter elastic foundation or two-parameter ones using the trigonometric shear deformation plate theory. Ameur *et al.* (2011) proposed a new trigonometric shear deformation plate theory to study the bending response of FG plate resting on Pasternak elastic foundation. Zenkour and Sobhy (2012) examined the static response of simply supported FG viscoelastic sandwich plates subjected to transverse uniform loads. Zenkour and Sobhy (2013) have studied the dynamic bending response of thermoelastic FG plates resting on elastic foundations. Tounsi and his co-workers (Abdelaziz *et al.* 2011, El Meiche *et al.* 2011, Hadji *et al.* 2011, Benachour *et al.* 2011, Bourada *et al.* 2012, Kaci *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Fekrar *et al.* 2012, Bouderba *et* *al*. 2013, Tounsi *et al*. 2013, Kettaf *et al*. 2013, Bachir Bouiadjra *et al*. 2013, Houari *et al*. 2013, Bessaim *et al*. 2013, Belabed *et al*. 2014, Fekrar *et al*. 2014, Bousahla *et al*. 2014, Hebali *et al*. 2014, Zidi *et al*. 2014, Klouche Djedid *et al*. 2014, Draiche *et al*. 2014, Sadoune *et al*. 2014, Nedri *et al*. 2014, Khalfi *et al*. 2014) studied the mechanical behaviors of composite and FG plates using new refined shear deformation theories. Sobhy (2013) studied the free vibration and the buckling behaviours of exponentially graded sandwich plates resting on Pasternak elastic foundation. Yaghoobi and Torabi (2013a) developed an exact solution for thermal buckling of FG plates supported by elastic foundations and various boundary conditions are considered. Yaghoobi and Yaghoobi (2013) studied sandwich plates with FG face sheets resting on elastic foundation. Yaghoobi and Torabi (2013b) investigated the post-buckling and nonlinear free vibration responses of geometrically imperfect FG beams resting on nonlinear elastic foundation. Recently, Yaghoobi and Fereidoon (2014) presented a refined nth-order shear deformation theory for the mechanical and thermal buckling responses of FG plates resting on elastic foundation.

Due to the importance and wide engineering applications of FGMs such as aerospace, nuclear, civil, automotive, the thermal buckling behavior of these materials has been addressed by many investigators. Indeed, with the development of new industries and modern processes, many machines and structures experience extreme thermal environments, resulting in various types of thermal loads (Noda *et al*. 2003). This situation has created a need for a text that is focused on the analysis of thermal buckling.

The purpose of this study is to extend the new trigonometric shear deformation plate theory (Ameur *et al*. 2011) for the thermal buckling analysis of FG plates on elastic foundation. This theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. Unlike the conventional trigonometric shear deformation theory (Zenkour and Sobhy 2011), the proposed trigonometric shear deformation theory contains only four unknowns. The plate is graded in the thickness direction assuming a power law distribution of the constituent materials. The temperatures are assumed to be uniform, linear and non-linear distribution through the thickness. The elastic foundation is modelled as two-parameter Pasternak foundation. The results are compared and validated with the results of previous works which are available in the literature.

2. Theoretical formulations

Consider a rectangular plate made of FGMs of thickness h, length a, and width b, referred to the rectangular Cartesian coordinates (x, y, z), as shown in Fig. 1.

Unlike the other theories, the number of unknown functions involved in the present efficient and simple trigonometric shear deformation theory is only four, as opposed to five in case of the conventional trigonometric shear deformation theory (Zenkour and Sobhy 2011) or of the other shear deformation theories (Reddy 1984, Karama *et al*. 2003). The theory presented is variationally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions.

2.1 Basic assumptions

The assumptions of the present theory are as follows:

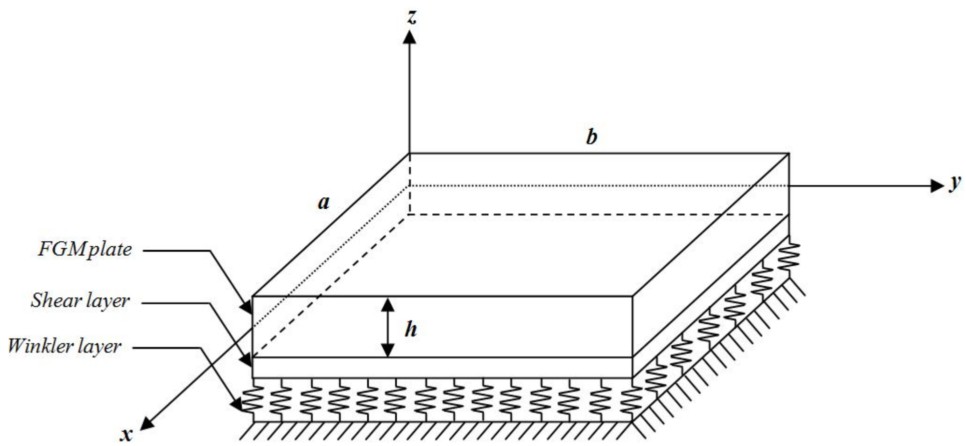


Fig. 1 Coordinate system and geometry for rectangular FG plates on Pasternak elastic foundation

(1) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.
(2) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x and y only.

$$w(x, y, z) = w_b(x, y) + w_s(x, y) \tag{1}$$

(3) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.
(4) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

$$u = u_0 + u_b + u_s, \quad v = v_0 + v_b + v_s \tag{2}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial x}, \quad v_b = -z\frac{\partial w_b}{\partial y} \tag{3}$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the sinusoidal variations of shear strains γ_{xz}, γ_{yz} and consequently the variation of shear stresses τ_{xz} and τ_{yz} through the thickness of the plate becomes also nonlinear in nature in such a way that shear stresses τ_{xz}, τ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as (Ameur *et al.* 2011)

$$u_s = -f(z)\frac{\partial w_s}{\partial x}, \quad v_s = -f(z)\frac{\partial w_s}{\partial y} \tag{4}$$

where

$$f(z) = \left(z - \frac{h}{\pi}\sin\frac{\pi z}{h}\right) \tag{5}$$

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)-(5) as

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{6a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \tag{6b}$$

$$w(x,y,z) = w_b(x,y) + w_s(x,y) \tag{6c}$$

The non-linear von Karman strain-displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{7}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)^2, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2} \tag{8a}$$

$$\varepsilon_y^0 = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)^2, \quad k_y^b = -\frac{\partial^2 w_b}{\partial y^2}, \quad k_y^s = -\frac{\partial^2 w_s}{\partial y^2} \tag{8b}$$

$$\gamma_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right), \quad k_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y}, \quad k_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y} \tag{8c}$$

$$\gamma_{yz}^s = \frac{\partial w_s}{\partial y}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial x}, \quad g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{8d}$$

2.3 Constitutive equations

The material properties of FG plate are assumed to vary continuously through the thickness of the plate in accordance with a power law distribution as

$$P(z) = P_m + P_c\left(\frac{z}{h} + \frac{1}{2}\right)^n, \quad P_{cm} = P_c - P_m \tag{9}$$

where P represents the effective material property such as Young's modulus E and the thermal expansion coefficient α. Subscripts m and c represent the metallic and ceramic constituents, respectively; and n is the volume fraction exponent. The value of n equal to zero represents a fully ceramic plate, whereas infinite n indicates a fully metallic plate. Since the effects of the variation of Poisson's ratio v on the response of FG plates are very small (Yang *et al*. 2005b, Kitipornchai *et al*. 2006), the Poisson's ratio v is usually assumed to be constant. The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{10}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. T (x, y, z) is the temperature rise through the thickness. Using the material properties defined in Eq. (9), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z)}{1-\nu^2}, \quad Q_{12} = \nu\, Q_{11}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{11}$$

Based on the present trigonometric shear deformation plate theory, the stress resultants are related to the stresses by equations

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{12a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right) g(z)\, dz. \tag{12b}$$

Using Eq. (10) in Eq. (12), the stress resultants of the FG plate can be related to the total strains by

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, \quad S = A^s \gamma \tag{13}$$

where

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, \quad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t, \quad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t, \tag{14a}$$

$$N^T = \left\{N_x^T, N_y^T, 0\right\}^t, \quad M^{bT} = \left\{M_x^{bT}, M_y^{bT}, 0\right\}^t, \quad M^{sT} = \left\{M_x^{sT}, M_y^{sT}, 0\right\}^t, \tag{14b}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{14c}$$

$$A=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix},\quad B=\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},\quad D=\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{14d}$$

$$B^s=\begin{bmatrix} B^s_{11} & B^s_{12} & 0 \\ B^s_{12} & B^s_{22} & 0 \\ 0 & 0 & B^s_{66} \end{bmatrix},\quad D^s=\begin{bmatrix} D^s_{11} & D^s_{12} & 0 \\ D^s_{12} & D^s_{22} & 0 \\ 0 & 0 & D^s_{66} \end{bmatrix},\quad H^s=\begin{bmatrix} H^s_{11} & H^s_{12} & 0 \\ H^s_{12} & H^s_{22} & 0 \\ 0 & 0 & H^s_{66} \end{bmatrix}, \tag{14e}$$

$$S=\left\{S^s_{yz}, S^s_{xz}\right\}^t,\quad \gamma=\left\{\gamma_{yz}, \gamma_{xz}\right\}^t,\quad A^s=\begin{bmatrix} A^s_{44} & 0 \\ 0 & A^s_{55} \end{bmatrix}, \tag{14f}$$

where A_{ij}, B_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B^s_{11} & D^s_{11} & H^s_{11} \\ A_{12} & B_{12} & D_{12} & B^s_{12} & D^s_{12} & H^s_{12} \\ A_{66} & B_{66} & D_{66} & B^s_{66} & D^s_{66} & H^s_{66} \end{Bmatrix} = \int_{-h/2}^{h/2} Q_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right)\begin{Bmatrix} 1 \\ \nu \\ \frac{1-\nu}{2} \end{Bmatrix} dz, \tag{15a}$$

and

$$\left(A_{22}, B_{22}, D_{22}, B^s_{22}, D^s_{22}, H^s_{22}\right)=\left(A_{11}, B_{11}, D_{11}, B^s_{11}, D^s_{11}, H^s_{11}\right),\quad Q_{11}=\frac{E(z)}{1-\nu^2} \tag{15b}$$

$$A^s_{44}=A^s_{55}=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^2 dz, \tag{15c}$$

2.4 Equilibrium and stability equations

The equilibrium equations of the FG plate resting on the Pasternak elastic foundation under thermal loadings may be derived on the basis of the stationary potential energy. The total potential energy of the plate, V, may be written in the form

$$V=U+U_F \tag{16}$$

Here, U is the total strain energy of the plate, and is calculated as

$$U=\frac{1}{2}\int_0^a\int_0^b\int_{-\frac{h}{2}}^{\frac{h}{2}}\left[\sigma_x\left(\varepsilon_x-\alpha\, T\right)+\sigma_y\left(\varepsilon_y-\alpha\, T\right)+\tau_{xy}\gamma_{xy}+\tau_{yz}\gamma_{yz}+\tau_{xz}\gamma_{xz}\right]dzdydx, \tag{17}$$

and U_F is the strain energy due to the Pasternak elastic foundation, which is given by (Benyoucef *et al*. 2006, Ait Atmane *et al*. 2010, Ameur *et al*. 2011)

$$U_F = \frac{1}{2}\int_0^a \int_0^b f_e (w_b + w_s) dydx \tag{18}$$

where f_e is the density of reaction force of foundation. For the Pasternak foundation model

$$f_e = K_W (w_b + w_s) - K_g \nabla^2 (w_b + w_s) \tag{19}$$

where K_W is the Winkler foundation stiffness and K_g is a constant showing the effect of the shear interactions of the vertical elements.

Using Eqs. (7), (8) and (13) and employing the virtual work principle to minimize the functional of total potential energy function result in the expressions for the equilibrium equations of plate resting on two parameters elastic foundation as

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + \overline{N} - f_e = 0 \\ &\frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + \overline{N} - f_e = 0 \end{aligned} \tag{20}$$

with

$$\overline{N} = \left[N_x \frac{\partial^2 (w_b + w_s)}{\partial x^2} + 2N_{xy} \frac{\partial^2 (w_b + w_s)}{\partial x \partial y} + N_y \frac{\partial^2 (w_b + w_s)}{\partial y^2} \right] \tag{21}$$

The stability equations for FG plates may be obtained by means of the adjacent-equilibrium criterion. Let us assume that the state of equilibrium of sandwich plate under thermal loads is defined in terms of the displacement components u_0^0, v_0^0, w_b^0 and w_s^0. The displacement components of a neighbouring state of the stable equilibrium differ by u_0^1, v_0^1, w_b^1, w_s^1 with respect to the equilibrium position. Thus, the total displacements of a neighbouring state are

$$u_0 = u_0^0 + u_0^1, \quad v_0 = v_0^0 + v_0^1, \quad w_b = w_b^0 + w_b^1, \quad w_s = w_s^0 + w_s^1 \tag{22}$$

Accordingly, the stress resultants are divided into two terms representing the stable equilibrium and the neighbouring state. The stress resultants with superscript 1 are linear functions of displacement with superscript 1. Considering all these mentioned above and using Eqs. (20) and (22), the stability equations becomes

$$\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0 \tag{23}$$

$\downarrow$

$$\uparrow$$

$$\begin{aligned}
&\frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0 \\
&\frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} + \overline{N}^1 - f_e^1 = 0 \\
&\frac{\partial^2 M_x^{s1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} + \frac{\partial^2 M_y^{s1}}{\partial y^2} + \frac{\partial S_{xz}^{s1}}{\partial x} + \frac{\partial S_{yz}^{s1}}{\partial y} + \overline{N}^1 - f_e^1 = 0
\end{aligned} \tag{23}$$

with

$$\overline{N}^1 = \left[N_x^0 \frac{\partial^2 \left(w_b^1 + w_s^1 \right)}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 \left(w_b^1 + w_s^1 \right)}{\partial x \partial y} + N_y^0 \frac{\partial^2 \left(w_b^1 + w_s^1 \right)}{\partial y^2} \right] \tag{24a}$$

$$f_e^1 = K_W (w_b^1 + w_s^1) - K_g \nabla^2 (w_b^1 + w_s^1) \tag{24b}$$

The terms N_x^0, N_y^0 and N_{xy}^0 are the pre-buckling force resultants obtained as

$$N_x^0 = N_y^0 = -\int_{-h/2}^{h/2} \frac{\alpha(z)E(z)T}{1-\nu} dz, \quad N_{xy}^0 = 0 \tag{25}$$

2.5 Trigonometric solution to thermal buckling

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eq. (23) for a simply supported FG plate. The following boundary conditions are imposed for the present refined shear deformation theory at the side edges

$$v_0^1 = w_b^1 = w_s^1 = \frac{\partial w_s^1}{\partial y} = N_x^1 = M_x^{b1} = M_x^{s1} = 0 \quad \text{at} \quad x = 0,\ a, \tag{26a}$$

$$u_0^1 = w_b^1 = w_s^1 = \frac{\partial w_s^1}{\partial x} = N_y^1 = M_y^{b1} = M_y^{s1} = 0 \quad \text{at} \quad y = 0,\ b. \tag{26b}$$

The following approximate solution was shown to satisfy both the differential equation and the boundary conditions

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_b^1 \\ w_s^1 \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}^1 \cos(\lambda\, x) \sin(\mu\, y) \\ V_{mn}^1 \sin(\lambda\, x) \cos(\mu\, y) \\ W_{bmn}^1 \sin(\lambda\, x) \sin(\mu\, y) \\ W_{smn}^1 \sin(\lambda\, x) \sin(\mu\, y) \end{Bmatrix} \tag{27}$$

where U_{mn}^1, V_{mn}^1, W_{bmn}^1, and W_{smn}^1 are arbitrary parameters to be determined and $\lambda = m\pi/a$ and $\mu = n\pi/b$. Substituting Eq. (27) into Eq. (23), one obtains

$$[K]\{\Delta\}=0, \tag{28}$$

where $\{\Delta\}$ denotes the column

$$\{\Delta\}=\left\{U_{mn}^{1}, V_{mn}^{1}, W_{bmn}^{1}, W_{smn}^{1}\right\}^{t} \tag{29}$$

and $[K]$ is the symmetric matrix given by

$$[K]=\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} & k_{34} \\ k_{14} & k_{24} & k_{34} & k_{44} \end{bmatrix}, \tag{30}$$

in which

$$\begin{aligned}
k_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
k_{12} &= -\lambda\,\mu\left(A_{12}+A_{66}\right) \\
k_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12}+2B_{66})\,\mu^2] \\
k_{14} &= \lambda\,[B_{11}^{s}\lambda^2 + (B_{12}^{s}+2B_{66}^{s})\,\mu^2] \\
k_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
k_{23} &= \mu\,[(B_{12}+2B_{66})\,\lambda^2 + B_{22}\mu^2] \\
k_{24} &= \mu\,[(B_{12}^{s}+2B_{66}^{s})\,\lambda^2 + B_{22}^{s}\mu^2] \\
k_{33} &= -\left(D_{11}\lambda^4 + 2(D_{12}+2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 + N_x^0\lambda^2 + N_y^0\mu^2 + K_g\left(\lambda^2+\mu^2\right) + K_W\right) \\
k_{34} &= -\left(D_{11}^{s}\lambda^4 + 2(D_{12}^{s}+2D_{66}^{s})\lambda^2\,\mu^2 + D_{22}^{s}\,\mu^4 + N_x^0\lambda^2 + N_y^0\mu^2 + K_g\left(\lambda^2+\mu^2\right) + K_W\right) \\
k_{44} &= -\left(H_{11}^{s}\lambda^4 + 2(H_{12}^{s}+2H_{66}^{s})\lambda^2\mu^2 + H_{22}^{s}\mu^4 + A_{55}^{s}\lambda^2 + A_{44}^{s}\mu^2\right. \\
&\quad \left. + N_x^0\lambda^2 + N_y^0\mu^2 + K_g\left(\lambda^2+\mu^2\right) + K_W\right)
\end{aligned} \tag{31}$$

2.6 Thermal buckling solution

In the following, the solution of the equation $|K| = 0$ for different types of thermal loading conditions is presented. The temperature change is varied only through the thickness.

2.6.1 Buckling of FG plates under uniform temperature rise

The plate initial temperature is assumed to be T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is $\Delta T = T_f - T_i$. Substituting Eq. (31) into the equation $|K| = 0$, the critical buckling temperature change is obtained as

$$\Delta T_{cr} = \frac{1}{\overline{\beta}_1\left(\lambda^2+\mu^2\right)}\frac{\overline{k}_{33}\overline{k}_{44}-\overline{k}_{34}\overline{k}_{43}}{\overline{k}_{33}+\overline{k}_{44}-\overline{k}_{34}-\overline{k}_{43}} \tag{32a}$$

where

$$\overline{\beta}_1 = -\int_{-h/2}^{h/2}\frac{\alpha(z)E(z)}{1-\nu}\,dz. \tag{32b}$$

and

$$\bar{k}_{33} = k_{33} - k_{13}\frac{b_1}{b_0} - k_{23}\frac{b_2}{b_0}, \quad \bar{k}_{34} = k_{34} - k_{14}\frac{b_1}{b_0} - k_{24}\frac{b_2}{b_0}$$
$$\bar{k}_{43} = k_{34} - k_{13}\frac{b_3}{b_0} - k_{23}\frac{b_4}{b_0}, \quad \bar{k}_{44} = k_{44} - k_{14}\frac{b_3}{b_0} - k_{24}\frac{b_4}{b_0} \tag{32c}$$
$$b_0 = k_{11}k_{22} - k_{12}^2, \qquad b_1 = k_{13}k_{22} - k_{12}k_{23}, \quad b_2 = k_{11}k_{23} - k_{12}k_{13},$$
$$b_3 = k_{14}k_{22} - k_{12}k_{24}, \quad b_4 = k_{11}k_{24} - k_{12}k_{14}$$

2.6.2 Buckling of FG plates subjected to a graded temperature change across the thickness

For FG plates, the temperature change is not uniform. The temperature is assumed to be varied according to the power law variation through-the-thickness as follows

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right)^{\beta} + T_m, \tag{33}$$

where the buckling temperature difference $\Delta T = T_c - T_m$ and T_c and T_m are the temperature of the top surface which is ceramic-rich and the bottom surface which is metal-rich, respectively. β is the temperature exponent ($0 < \beta < \infty$). Note that the value of β equal to unity represents a linear temperature change across the thickness. While the value of β excluding unity represents a non-linear temperature change through-the-thickness.

Similar to the previous loading case, the critical buckling temperature difference ΔT_{cr} can be determined as

$$\Delta T_{cr} = \frac{\bar{k}_{33}\bar{k}_{44} - \bar{k}_{34}\bar{k}_{43} + T_m\bar{\beta}_1\left(\lambda^2 + \mu^2\right)\left(\bar{k}_{33} + \bar{k}_{44} - \bar{k}_{34} - \bar{k}_{43}\right)}{\bar{\beta}_2\left(\lambda^2 + \mu^2\right)\left(\bar{k}_{33} + \bar{k}_{44} - \bar{k}_{34} - \bar{k}_{43}\right)} \tag{34a}$$

where

$$\bar{\beta}_2 = -\int_{-h/2}^{h/2} \frac{\alpha(z)E(z)}{1-\nu}\left(\frac{z}{h} + \frac{1}{2}\right)^{\beta} dz \tag{34b}$$

3. Results and discussion

In this section, various numerical examples are presented and discussed to verify the accuracy of present theory in predicting the critical buckling temperature of simply supported FG plates resting on Pasternak elastic foundation. For numerical results, an Al/Al_2O_3 plate composed of Aluminum (as metal) and Alumina (as ceramic) is considered. The Young's modulus and the thermal expansion coefficient of Aluminum are E_m = 70 GPa and α_m = 23.10^{-6}/°C, respectively, and those of Alumina are E_c = 380 GPa and α_c = 7.410^{-6}/°C, respectively. For verification purpose, the obtained results are compared with those predicted using various plate theories. The description of various displacement models is given in Table 1. In all examples, a shear correction factor of 5/6 is used for FSDT. The Poisson's ratio of the plate is assumed to be constant through the thickness and equal to 0.3. For the linear and non-linear temperature rises through the

Table 1 Displacement models

Model	Theory	Unknown functions
CPT	Classical plate theory	3
FSDT	First-order shear deformation theory (Whitney and Pagano 1970)	5
PSDT	Parabolic shear deformation theory (Reddy 1984)	5
TSDT	Trigonometric shear deformation theory (Zenkour and Sobhy 2011)	5
Present	Present refined plate theory	4

Table 2 Comparison of the critical buckling temperature of a square FG plate under uniform temperature

N	Theory	$b/h=10$	$b/h=20$	$b/h=40$	$b/h=60$	$b/h=80$	$b/h=100$
0	CPT (Javaheri and Eslami 2002)	1709.911	427.477	106.869	47.497	26.717	17.099
	HPT (Javaheri and Eslami 2002)	1617.484	421.516	106.492	47.424	26.693	17.088
	Present	1618.820	421.544	106.495	47.423	26.694	17.089
1	CPT (Javaheri and Eslami 2002)	794.377	198.594	49.648	22.066	12.412	7.943
	HPT (Javaheri and Eslami 2002)	757.891	196.257	49.500	22.037	12.402	7.939
	Present	758.451	196.269	49.502	22.037	12.403	7.9400
5	CPT (Javaheri and Eslami 2002)	726.571	181.643	45.410	20.182	11.352	7.265
	HPT (Javaheri and Eslami 2002)	678.926	178.528	45.213	20.144	11.340	7.260
	Present	678.949	178.510	45.212	20.143	11.340	7.261
10	CPT (Javaheri and Eslami 2002)	746.927	186.732	46.682	20.747	11.670	7.469
	HPT (Javaheri and Eslami 2002)	692.519	183.141	46.455	20.703	11.657	7.462
	Present	692.544	183.133	46.455	20.703	11.656	7.463

thickness, the temperature rises 5 °C in the metal-rich surface of the plate (i.e., T_m 5°C).

The following dimensionless expressions of Winkler's and Pasternak's elastic foundation parameters, as well as the critical buckling temperature difference are used in the present analysis

$$k_1 = \frac{a^4}{D} K_W, \quad k_2 = \frac{a^2}{D} K_g, \quad T_{cr} = 10^{-3} \Delta T_{cr}$$

where $D = E_C h^3 / [12(1-v^2)]$.

Table 3 Comparison of the critical buckling temperature of a square FG plate (a/h = 10) under uniform, linear and nonlinear (β = 3) temperature load

n	Theory	$k_1 = 0, k_2 = 0$			$k_1 = 10, k_2 = 0$			$k_1 = 10, k_2 = 10$		
		Uniform	Linear	Non-linear	Uniform	Linear	Non-linear	Uniform	Linear	Non-linear
0	Present	1.61882	3.22764	6.45528	1.66270	3.31541	6.63082	2.52896	5.04791	10.09582
	TSDT[(a)]	1.61882	3.22764	6.45528	1.66270	3.31541	6.63082	2.52896	5.04791	10.09582
1	Present	0.75845	1.41307	2.82696	0.79935	1.48978	2.98043	1.60674	3.00402	6.00978
	TSDT[(a)]	0.75845	1.41307	2.82696	0.79935	1.48978	2.98043	1.60674	3.00402	6.00978
5	Present	0.67895	1.16006	2.01520	0.73564	1.25765	2.18472	1.85472	3.18391	5.53091
	TSDT[(a)]	0.67895	1.16006	2.01520	0.73564	1.25765	2.18472	1.85472	3.18391	5.53091
10	Present	0.69254	1.21837	2.09718	0.75653	1.33176	2.29235	2.01955	3.56992	6.14487
	TSDT[(a)]	0.69254	1.21837	2.09718	0.75653	1.33176	2.29235	2.01955	3.56992	6.14487

3.1 Comparison studies

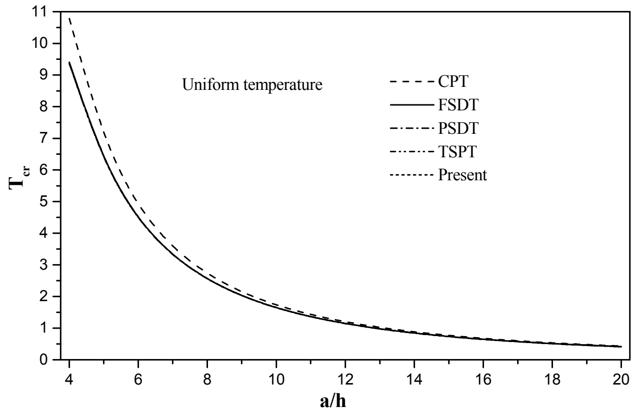
In Table 2, a comparative study is carried out between the results obtained in this study and those reported by Javaheri and Eslami (2002) based on both higher plate theory (HPT) and the classical plate theory (CPT). Results are presented for square FG plate under uniform temperature rise. The results of the present theory show very good agreement with HPT both for thin and thick FG plates and for all values of power law index n.

Another comparative study for the critical buckling temperature difference of FG plate on elastic foundation obtained by the proposed theory and those reported by Zenkour and Sobhy (2011) is illustrated in Table 3. It can be seen that the proposed theory and conventional TSDT (Zenkour and Sobhy 2011) give identical results of the critical buckling temperature for all values of power law index n. It should be noted that the proposed trigonometric theory involves four unknowns as against five in case of conventional TSDT (Zenkour and Sobhy 2011) and HPT (Javaheri and Eslami 2002).

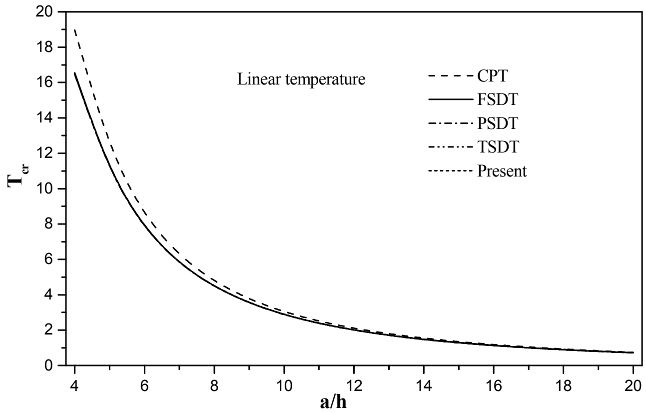
To illustrate the accuracy of present theory for wide range of thickness ratio a/h, the variations of the critical buckling temperature of FG plate on elastic foundation and under a uniform, linear and non-linear temperature load with respect to thickness ratio a/h are illustrated in Fig. 2. The obtained results are compared with those predicted by CPT, FSDT, PSDT and TSDT. It can be seen that the results of present theory and the other shear deformation theories are almost identical, and the CPT overestimates the critical buckling temperature of plate especially for the case of thick plates. Hence, in order to obtain accurate results for thick FG plates, it is necessary to consider the transverse shear deformation effects by using shear deformation theories.

Fig. 3 shows the effects of the aspect ratio a/b on the critical buckling temperature T_{cr} of FG plates (n = 2) under a uniform, linear and non-linear temperature loads. The effect of elastic foundation is considered by employing k_1 = 10 and k_2 = 10. It is observed that, with increasing the plate aspect ratio a/b, the critical buckling temperature difference also increases gradually, regardless of the theory used. Results show the accuracy of proposed theory.

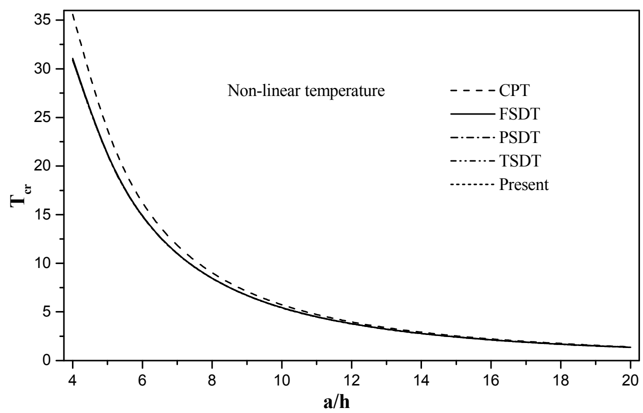
It should be noted that the number of unknown functions in the present theory is four, while that in TSDT (Zenkour and Sobhy 2011), HPT (Javaheri and Eslami 2002), PSDT and FSDT is



(a)

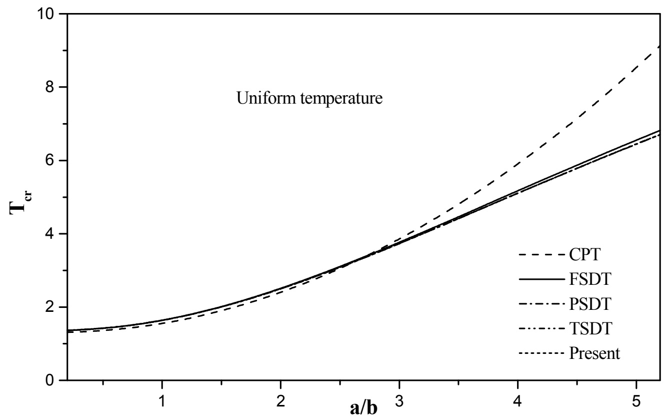


(b)

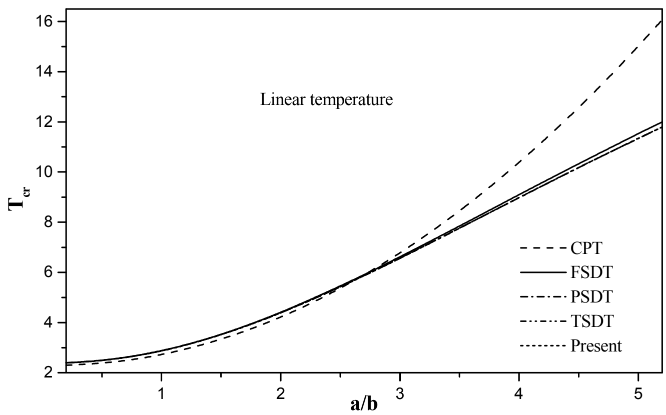


(c)

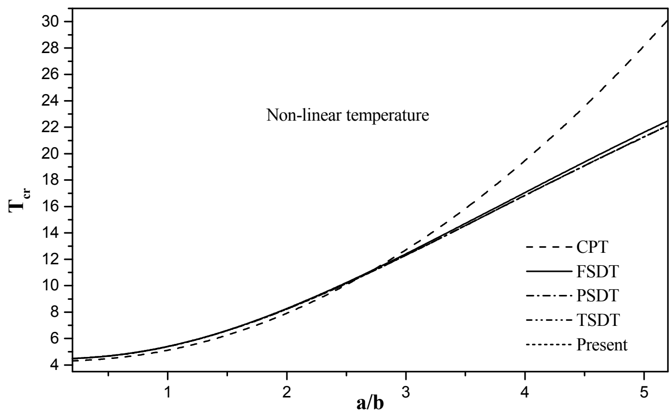
Fig. 2 Comparison of the variation of critical thermal buckling temperature T_{cr} of square FG plate ($n = 2$) on elastic foundation ($k_1 = k_2 = 10$) versus thickness ratio a/h: (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature ($\beta = 3$)



(a)

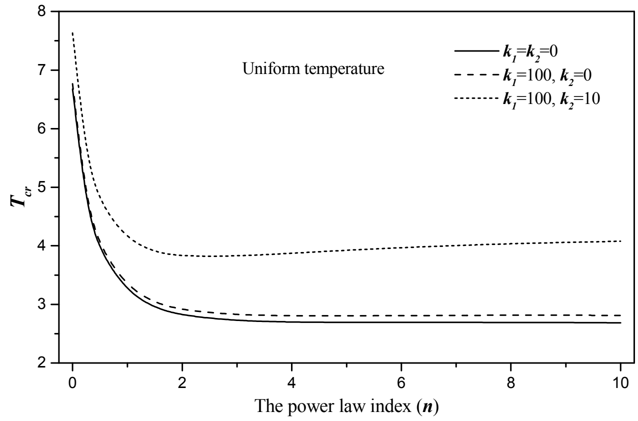


(b)

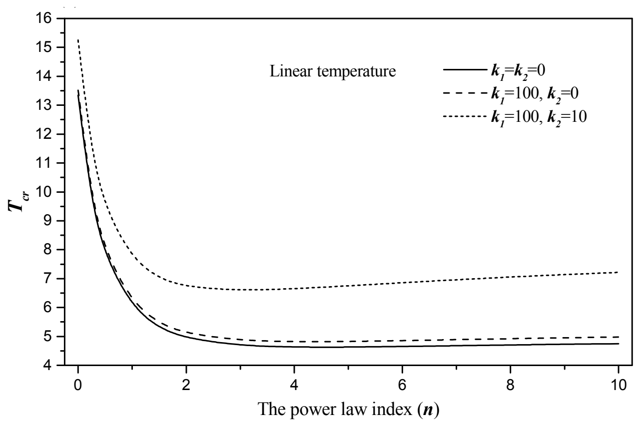


(c)

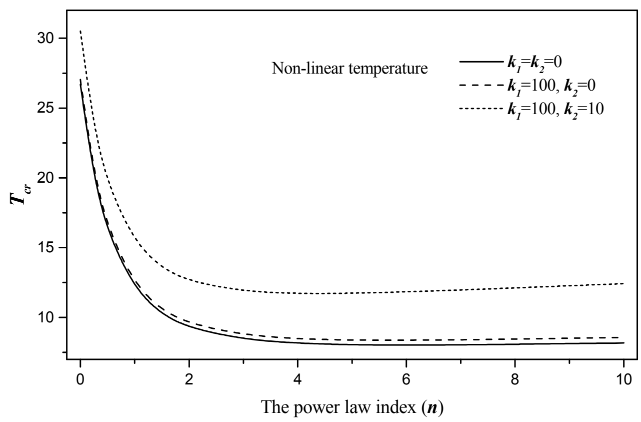
Fig. 3 Comparison of the variation of critical thermal buckling temperature T_{cr} of FG plate (a/h = 10 and n = 2) on elastic foundation ($k_1 = k_2 = 10$) versus a/b: (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature ($\beta = 3$)



(a)



(b)



(c)

Fig. 4 Effect of power law index on the critical buckling temperature difference T_{cr} of FG plates with or without elastic foundations: (a) Uniform temperature; (b) Linear temperature; (c) Non-linear temperature (β = 3)

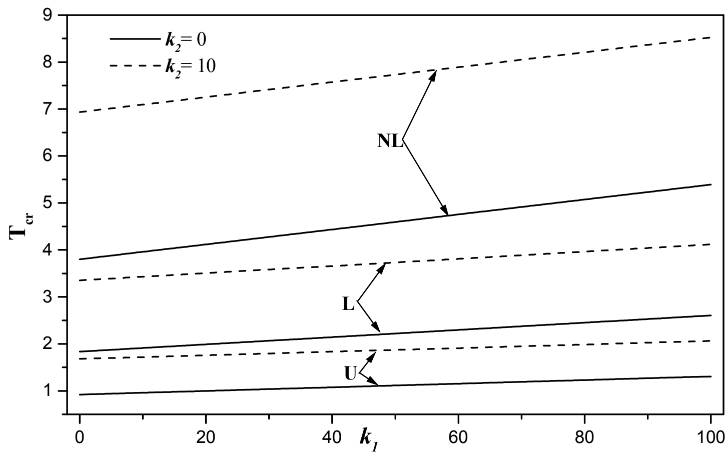


Fig. 5 Effect of elastic modulus of Winkler foundation k_1 on critical buckling temperature of square FG plate (n = 0.5 and a / h = 10) under uniform (U), linear (L) and non-linear (NL) temperature load (β = 3)

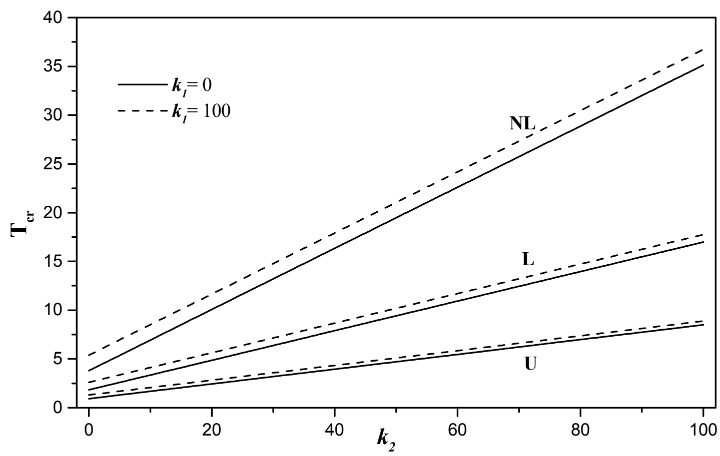


Fig. 6 Effect of Pasternak shear modulus parameter k_2 on critical buckling temperature of square FG plate (n = 0.5 and a / h = 10) under uniform (U), linear (L) and non-linear (NL) temperature load (β = 3)

five. It can be concluded that the present theory is not only accurate but also relatively simple for use in predicting critical buckling temperature of FG plates.

3.2 Parametric studies

Parameter studies are carried out to investigate the effects of thermal loading types and variations of power of FGM, elastic foundation stiffnesses, thickness ratio, and aspect ratio on the critical buckling temperature of FG plates.

In Fig. 4, the effect of power law index n on the critical buckling temperature T_{cr} of FG plate

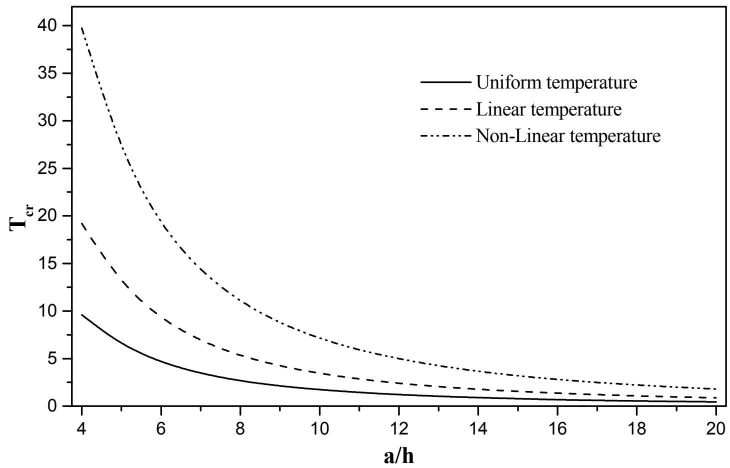


Fig. 7 Effect of the side-to-thickness ratio for different kinds of thermal loads on the critical buckling temperature difference of square FG plate (n = 0.5, β = 3 and k_1 = k_2 = 10)

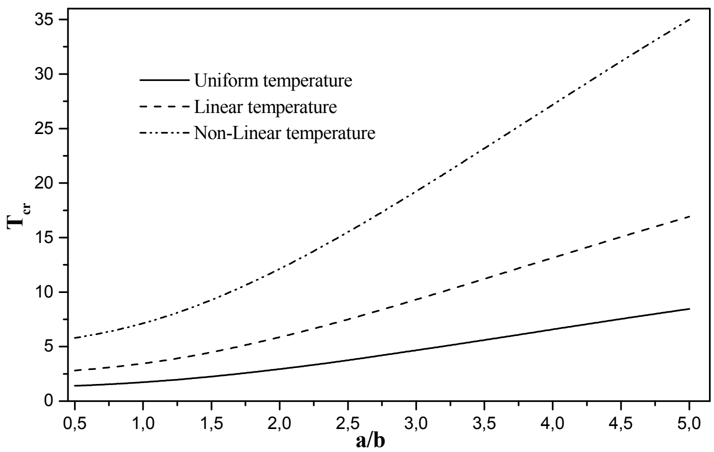
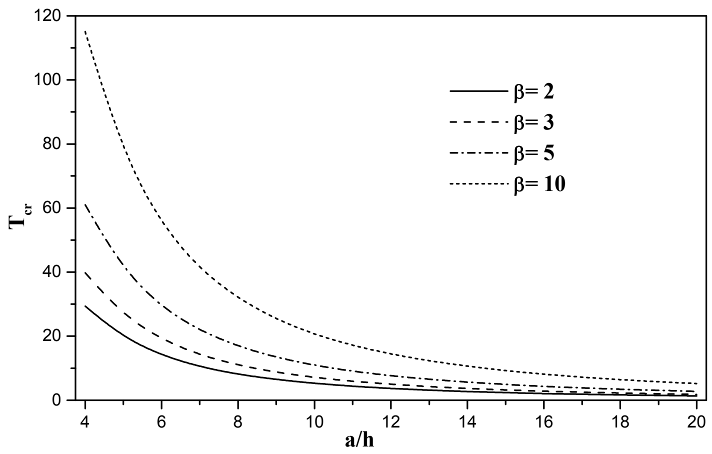


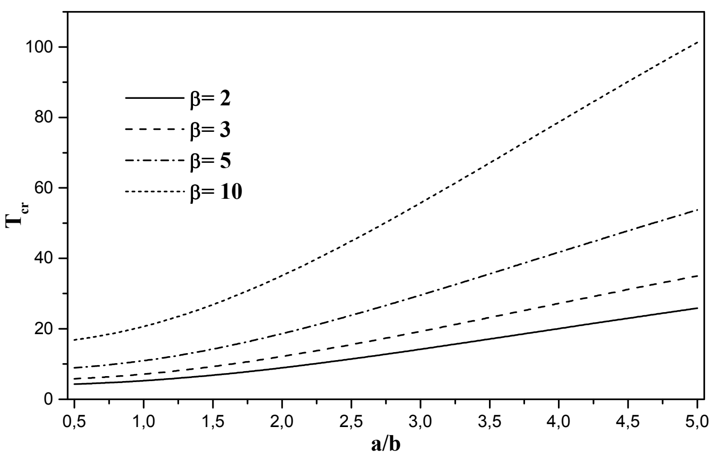
Fig. 8 Effect of the aspect ratio for different kinds of thermal loads on the critical buckling temperature difference of FG plate (n = 0.5, β = 3, a / h = 10 and k_1 = k_2 = 10)

without elastic foundation or resting on Winkler's or Pasternak's elastic foundations is depicted using the present simple trigonometric theory. Results are presented for rectangular FG plate (a / b = 3 and a / h = 10) under uniform, linear and nonlinear (β = 3) temperature change across the thickness. It is noted that T_{cr} decreases rapidly to reach its minimum values and then increases slowly as the power law index n increases. However, for the plate without elastic foundation or resting on one-parameter Winkler's foundation, the variation of the dimensionless critical buckling temperature T_{cr} is almost independent of the power law index n when this latter is higher than 4. It can be also seen that the presence of elastic foundations lead to an increase of the dimensionless critical buckling temperature T_{cr}.

Fig. 5 shows the effect of Winkler modulus parameter on the critical buckling temperature T_{cr}



(a)



(b)

Fig. 9 Effect of the non-linearity parameter β on the critical buckling temperature difference of FG plate (n = 0.5 and $k_1 = k_2 = 10$) versus: (a) the side-to-thickness ratio a / h ($a / b = 1$); (b) the aspect ratio a / b ($a / h = 10$)

of square FG plate with or without the shear foundation layer. Irrespective of the type of the temperature load, T_{cr} increases linearly as the Winkler modulus parameter k_1 increases with the presence or the absence of the shear foundation layer.

Fig. 6 shows the effect of Pasternak shear modulus parameter on the critical buckling temperature T_{cr} of square FG plate with the presence or the absence of the Winkler foundation layer. Whatever the type of temperature load is, T_{cr} increases linearly as the shear stiffness of elastic foundation k_2 increases with the presence or the absence of Winkler foundation layer. From the results presented in Figs. 5 and 6, it can be observed that the results are more sensitive to the variation of k_2 than that of k_1 especially in the case of non-linear temperature change across the thickness.

The effects of the side-to-thickness ratio a/h and the aspect ratio a/b for different kinds of thermal loads on the critical buckling temperature difference are presented in Figs. 7 and 8, respectively. It is noticed that a decrement occurs for T_{cr} as a/h increases, whereas the opposite is the case a/b increases. Also, the critical buckling temperature difference of the FG plate under linear temperature distribution across the thickness is greater than the one under uniform temperature raise and less than the one under nonlinear temperature distribution across the thickness.

The effect of the temperature exponent β on the critical buckling temperature T_{cr} of the FG plate is depicted in Fig. 9. It is seen that T_{cr} is very sensitive to the variation of β. Indeed, it is noticed from Fig. 9 that the T_{cr} increases with the increase of the non-linearity parameter β.

4. Conclusions

Thermal buckling behavior of simply supported FG plates under different types of thermal loading (uniform, linear and higher order temperature distribution through the thickness) and resting on a Winkler-Pasternak elastic foundation has been analyzed by using an efficient and simple trigonometric shear deformation theory. Unlike the conventional trigonometric shear deformation theory, the proposed trigonometric shear deformation theory contains only four unknowns. All comparison studies show that the critical buckling temperature obtained by the proposed theory with four unknowns is almost identical with those predicted by other shear deformation theories containing five unknowns. It can be concluded that the proposed theory is accurate and efficient in predicting the thermal buckling responses of FG plates resting on elastic foundation. The formulation lends itself particularly well in analysing FG plates under various boundary conditions (Sobhy 2013, 2014, Ait Amar Meziane *et al.* 2014, Heireche *et al.* 2008) which will be considered in the near future.

References

Abdelaziz, H.H., Ait Atmane, H., Mechab, I., Boumia, L., Tounsi, A. and Adda Bedia, E.A. (2011), "Static analysis of functionally graded sandwich plates using an efficient and simple refined theory", *Chinese J. Aeronaut*., **24**(4), 434-448.

Ait Atmane, H., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2010), "Free vibration analysis of functionally graded plates resting on Winkler-Pasternak elastic foundations using a new shear deformation theory", *Int. J. Mech. Mater. Des*., **6**(2), 113-121.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater*., **16**(3), 293-318.

Ameur, M., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2011), "A new trigonometric shear deformation theory for bending analysis of functionally graded plates resting on elastic foundations", *KSCE J. Civil Eng*., **15**(8), 1405-1414.

Avramidis, I.E. and Morfidis, K. (2006), "Bending of beams on three-parameter elastic foundation", *Int. J. Solids Struct*., **43**(2), 357-375.

Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. (2012), "Thermal buckling of functionally graded plates according to a four-variable refined plate theory", *J. Therm. Stresses*, **35**(8), 677-694.

Bachir Bouiadjra, R., Adda Bedia, E.A., Tounsi, A. (2013), "Nonlinear thermal buckling behavior of functionally graded plates using an efficient sinusoidal shear deformation theory", *Struct. Eng. Mech., Int.*

J., **48**(4), 547-567.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos*.: *Part B*, **60**, 274-283.

Benachour, A., Daouadji, H.T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos*. *Part B*, **42**(6), 1386-1394.

Benyoucef, S., Mechab, I., Tounsi, A., Fekrar, A., Ait Atmane, H. and Adda Bedia, E.A. (2010), "Bending of thick functionally graded plates resting on Winkler-Pasternak elastic foundations", *Mech*. *Compos*. *Mater*., **46**(4), 425-434.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw*. *Struct*. *Mater*., **15**(6), 671-703.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013) "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos*. *Struct*., *Int*. *J*., **14**(1), 85-104.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw*. *Struct*. *Mater*., **14**(1), 5-33.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int*. *J*. *Comput*. *Methods*, **11**(6), 1350082.

Cheng, Z.Q. and Batra, B.C. (2000), "Exact correspondence between eigenvalues of membranes and functionally graded simply supported polygonal plate", *J. Sound Vib*., **229**(4), 879-895.

Cheng, Z.Q. and Kitipornchai, S. (1999), "Membrane analogy of buckling and vibration of inhomogeneous plates", *ASCE J. Eng*. *Mech*., **125**(11), 1293-1267.

Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos*. *Struct*., *Int*. *J*., **17**(1), 69-81.

El Meiche, N., Tounsi, A., Ziane, N., Mechab, I. and Adda Bedia, E.A. (2011), "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate", *Int*. *J*. *Mech*. *Sci*., **53**(4), 237-247.

Fekrar, A., El Meiche, N., Bessaim, A., Tounsi, A. and Adda Bedia, E.A. (2012), "Buckling analysis of functionally graded hybrid composite plates using a new four variable refined plate theory", *Steel Compos*. *Struct*., *Int*. *J*., **13**(1), 91-107.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**(4), 795-810.

Hadji, L., Ait Atmane, H., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2011), "Free vibration of functionally graded sandwich plates using four-variable refined plate theory", *Appl*. *Math*. *Mech*., **32**(7), 7925-7942.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng*. *Mech*., **140**(2), 374-383.

Heireche, H., Tounsi, A., Benzair, A., Maachou, M. and Adda Bedia, E.A. (2008), "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Physica E*., **40**(8), 2791-2799.

Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int*. *J*. *Mech*. *Sci*., **76**, 467-479.

Javaheri, R. and Eslami, M.R. (2002), "Thermal buckling of functionally graded plates based on higher order theory," *J. Therm*. *Stress*., **25**(1), 603-625.

Kaci, A., Tounsi, A., Bakhti, K. and Adda Bedia, E.A. (2012), "Nonlinear cylindrical bending of functionally graded carbon nanotube-reinforced composite plates", *Steel Compos*. *Struct*., *Int*. *J*., **12**(6), 491-504.

Karama, M., Afaq, K.S. and Mistou, S. (2003), "Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity", *Int. J. Solid. Struct.*, **40**(6), 1525-1546.
Kettaf, F.Z., Houari, M.S.A., Benguediab, M. and Tounsi, A. (2013), "Thermal buckling of functionally graded sandwich plates using a new hyperbolic shear displacement model", *Steel Compos. Struct, Int. J.*, **15**(4), 399-423.
Khalfi, Y., Houari, M.S.A. and Tounsi, A. (2014), "A refined and simple shear deformation theory for thermal buckling of solar functionally graded plates on elastic foundation", *Int. J. Comput. Methods*, **11**(5), 1350077.
Kitipornchai, S., Yang, J. and Liew, K.M. (2006), "Random vibration of the functionally graded laminates in thermal environments", *Comput. Method. Appl. Mech. Eng.*, **195**(9-12), 1075-1095.
Klouche Djedid, I., Benachour, A., Houari, M.S.A., Tounsi, A. and Ameur, M. (2014), "A *n*-order four variable refined theory for bending and free vibration of functionally graded plates", *Steel Compos. Struct., Int. J.*, **17**(1), 21-46.
Koizumi, M. (1993), "The concept of FGM", *Ceram Trans*, *Funct Gradient Mater*, **34**, 3-10.
Nedri, K., El Meiche, N. and Tounsi, A. (2014), "Free vibration analysis of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory", *Mech. Compos. Mater.*, **49**(6), 641-650.
Noda, N., Hetnarski, R.B. and Tanigawa, Y. (2003), *Thermal Stresses*, (Second Edition), Taylor & Francis, New York, NY, USA.
Pasternak, P.L. (1954), "On a new method of analysis of an elastic foundation by means of two foundation constants", Cosudarstrennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture, Moscow, USSR, 1-56. [in Russian]
Reddy, J.N. (1984), "A simple higher-order theory for laminated composite plates", *ASME J. Appl. Mech.*, **51**(4), 745-752.
Reddy, J.N. and Cheng, Z.Q. (2002), "Frequency correspondence between membranes and functionally graded spherical shallow shells of polygonal plan form", *Int. J. Mech. Sci.*, **44**(5), 967-985.
Sadoune, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2014), "A novel first-order shear deformation theory for laminated composite plates", *Steel Compos. Struct., Int. J.*, **17**(3), 321-338.
Sobhy, M. (2013), "Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions", *Compos. Struct.*, **99**, 76-87.
Sobhy, M. (2014), "Generalized two-variable plate theory for multi-layered graphene sheets with arbitrary boundary conditions", *Acta Mech.*, **225**(9), 2521-2538.
Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.
Whitney, J.M. and Pagano, N.J. (1970), "Shear deformation in heterogeneous anisotropic plates", *J. Appl. Mech.*, **37**(), 1031-1036.
Woo, J., Meguid, S.A., Stranart, J.C. and Liew, K.M. (2005), "Thermomechanical postbuckling analysis of moderately thick functionally graded plates and shallow shells", *Int. J. Mech. Sci.*, **47**(8), 1147-1171.
Yaghoobi, H. and Fereidoon, A. (2014), "Mechanical and thermal buckling analysis of functionally graded plates resting on elastic foundations: An assessment of a simple refined nth-order shear deformation theory", *Compos.: Part B*, **62**, 54-64.
Yaghoobi, H. and Torabi, M. (2013a), "Exact solution for thermal buckling of functionally graded plates resting on elastic foundations with various boundary conditions", *J. Therm. Stresses*, **36**(9), 869-894.
Yaghoobi, H. and Torabi, M. (2013b), "Post-buckling and nonlinear free vibration analysis of geometrically imperfect functionally graded beams resting on nonlinear elastic foundation", *Appl. Math. Model.*, **37**(18-19), 8324-8340.
Yaghoobi, H. and Yaghoobi, P. (2013), "Buckling analysis of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions: an analytical approach", *Meccanica*, **48**(8), 2019-2035.

Yamanouchi, M., Koizumi, M., Hirai, T. and Shiota, I. (1990), "On the design of functionally gradient materials", *Proceedings of First International Symposium on Functionally Gradient Materials*, Sendai, Japan, October.

Yang, J., Liew, K.M. and Kitipornchai, S. (2005a), "Second-order statistics of the elastic buckling of functionally graded rectangular plates", *Compos. Sci. Technol.*, **65**(7-8), 1165-1175.

Yang, J, Liew, K.M. and Kitipornchai, S. (2005b), "Stochastic analysis of compositionally graded plates with system randomness under static loading", *Int. J. Mech. Sci.*, **47**(10), 1519-1541.

Zenkour, A.M. (2009), "The refined sinusoidal theory for FGM plates on elastic foundations", *Int. J. Mech. Sci.*, **51**(11-12), 869-880.

Zenkour, A.M. and Sobhy, M. (2010), "Thermal buckling of various types of FGM sandwich plates", *Compos. Struct.*, **93**(1), 93-102.

Zenkour, A.M. and Sobhy, M. (2011), "Thermal buckling of functionally graded plates resting on elastic foundations using the trigonometric theory", *J. Therm. Stresses*, **34**(11), 1119-1138.

Zenkour, A. and Sobhy, M. (2012), "Elastic foundation analysis of uniformly loaded functionally graded viscoelastic sandwich plates", *J. Mech.*, **28**(3), 439-452.

Zenkour, A. and Sobhy, M. (2013), "Dynamic bending response of thermoelastic functionally graded plates resting on elastic foundations", *Aerosp. Sci. Technol.*, **29**(1), 7-17.

Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

CC