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A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates

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Abstract. In this research, a simple but accurate sinusoidal plate theory for the thermomechanical bending analysis of functionally graded sandwich plates is presented. The main advantage of this approach is that, in addition to incorporating the thickness stretching effect, it deals with only 5 unknowns as the first order shear deformation theory (FSDT), instead of 6 as in the well-known conventional sinusoidal plate theory (SPT). The material properties of the sandwich plate faces are assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. The core layer is made of an isotropic ceramic material. Comparison studies are performed to check the validity of the present results from which it can be concluded that the proposed theory is accurate and efficient in predicting the thermomechanical behavior of functionally graded sandwich plates. The effect of side-to-thickness ratio, aspect ratio, the volume fraction exponent, and the loading conditions on the thermomechanical response of functionally graded sandwich plates is also investigated and discussed.

Keywords: sandwich plate; thermomechanical; analytical modelling; functionally graded material; stretching effect

1. Introduction

Sandwich structures are employed in a variety of engineering industries including aircraft, construction and transportation where strong, stiff and light structures are required. The advantages of these structures are that it provides high specific stiffness and strength for a low-weight consideration. Due to the mismatch of stiffness properties between the face sheets and the core, sandwich plates are susceptible to face sheet/core debonding, which is a major problem in sandwich construction, especially under impact loading. To increase the resistance of sandwich

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plates to this type of failure, the concept of a Functionally Graded Material (FGM) is being actively explored in sandwich structure design. FGMs are achieved by gradually changing the composition of the constituent materials along one (or more) direction(s), usually in the thickness direction, to obtain smooth variation of material properties and optimum response to externally applied loading. Increased use of FGMs in various structural applications necessitates the development of accurate theoretical models to predict their response (Yaghoobi and Torabi 2013a, b, c).

In the past, a variety of plate theories have been developed to investigate the functionally graded (FG) plate response. The Classical plate theory (CPT) neglects the transverse shear effects and gives acceptable results for the analysis of thin plates only. However, for moderately thick plates CPT underpredicts deflections and overpredicts buckling loads and natural frequencies. The first-order shear deformation theories (FSDTs) are based on Reissner (1945) and Mindlin (1951) accounts for the transverse shear deformation effect by means of a linear variation of in-plane displacements and stresses through the thickness of the plate, but requires a correction factor to satisfy the free transverse shear stress conditions on the top and bottom surfaces of the plate. Although the FSDT provides a sufficiently accurate description of response for thin to moderately thick plates, it is not convenient to use due to difficulty with determination of the correct value of shear correction factor (Menaa *et al*. 2012). In order to overcome the limitations of FSDT many higher-order shear deformation theories (HSDTs) were developed that involve higher order terms in Taylors expansions of the displacements in the thickness coordinate, notable among them are Reddy and Chin (1998), Matsunaga (2008, 2009), Benachour *et al*. (2011), Xiang and Kang (2013), Bachir Bouiadjra *et al*. (2013), Bakhti *et al*. (2013), Bouderba *et al*. 2013), Zenkour and Sobhy (2013), Sobhy (2013), Klouche Djedid *et al*. (2014), Yaghoobi and Fereidoon (2014) and Zidi *et al*. (2014).

Studies related to FGM sandwich structures by employing HSDTs are few in numbers. Zenkour (2005a, b) studied in detail the bending response, buckling and free vibration of a simply supported FGM sandwich plate using the sinusoidal shear deformation plate theory. Anderson (2003) presented an analytical three dimensional elasticity solution method for a sandwich composite with a functionally graded core subjected to transverse loading by a rigid spherical indentor. An exact thermo elasticity solution for a two-dimensional sandwich structures with functionally graded coating was presented by Shodja *et al*. (2007). In Bhangale and Ganesan (2006), the buckling and vibration of a FGM sandwich beam having viscoelastic layer was studied in thermal environment by using a finite element formulation. Yaghoobi and Yaghoobi (2013) presented analytical solutions for the buckling of symmetric sandwich plates with FGM face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and subjected to mechanical, thermal and also thermo-mechanical loads. In a number of recent articles - see (Bourada *et al*. 2012, Bachir Bouiadjra *et al*. 2012, Tounsi *et al*. 2013, Kettaf *et al*. 2013) - a new refined and robust plate theory for mechanical response and buckling of simply supported FGM sandwich plate with only four unknown functions has been developed. Recently, Ait Amar Meziane *et al*. (2014) extended this new refined plate theory to the vibration and buckling of exponentially graded sandwich plate resting on elastic foundations under various boundary conditions.

Most of these above-mentioned theories neglect the thickness stretching effect (i.e., $\varepsilon_z = 0$) by assuming a constant transverse displacement through the thickness of the plate. This assumption is appropriate for thin or moderately thick FGM plates, but is inadequate for thick FGM plates (Carrera *et al*. 2011, Bessaim *et al*. 2013, Houari *et al*. 2013, Hebali *et al*. 2014, Fekrar *et al*. 2014, Belabed *et al*. 2014, Mantari and Guedes Soares 2014). The effect of the thickness stretching in FG plates was studied by Carrera *et al*. (2011) and Bessaim *et al*. (2013), and it becomes significant in thick plates. Thus, it should be taken into consideration.

This research work aims to present a simple quasi-3D theory with only five unknowns for thermomechanical bending analysis of FGM sandwich plates. The beauty of the present formulation is that, in addition to including the thickness stretching effect ($\varepsilon_z \neq 0$), the displacement field is modeled with only 5 unknowns as the FSDT, instead of 6 as in the well-known conventional sinusoidal plate theory (SPT). The sandwich plate faces are assumed to have isotropic, two-constituent (metal-ceramic) material distribution through the thickness, and the modulus of elasticity, Poisson's ratio, and thermal expansion coefficient of the faces are assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic ceramic material. The plate's governing equations are obtained by using the principle of virtual work. Numerical results for deflections and stresses are investigated. The effects of temperature field on the dimensionless axial and transverse shear stresses of the FGM sandwich plate are studied.

2. Theoretical formulation

Consider a sandwich plate composed of three layers as shown in Fig. 1. Two FG face sheets are made from a mixture of a metal and a ceramic, while a core is made of an isotropic homogeneous material. The material properties of FG face sheets are assumed to vary continuously through the plate thickness by a power law distribution as

$$P^{(n)}(z) = P_2 + \left(P_1 - P_2\right) V^{(n)} \tag{1}$$

where $P^{(n)}$ is the effective material property of FGM of layer n like Young's modulus E, Poisson's ratio v, and thermal expansion coefficient α. P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, (n = 1, 2, 3) defined by

$$\begin{cases} V^{(1)} = \left(\dfrac{z-h_0}{h_1-h_0}\right)^p & \text{for } z \in [h_0, h_1] \\ V^{(2)} = 1 & \text{for } z \in [h_1, h_2] \\ V^{(3)} = \left(\dfrac{z-h_3}{h_2-h_3}\right)^p & \text{for } z \in [h_2, h_3] \end{cases} \tag{2}$$

where p is the power law index ($0 \leq p \leq +\infty$), which dictates the material variation profile through the thickness.

2.1 Kinematics

The displacement field of the present formulation is considered based on the following assumptions: (1) The transverse displacement is superposed into three parts, namely: bending,

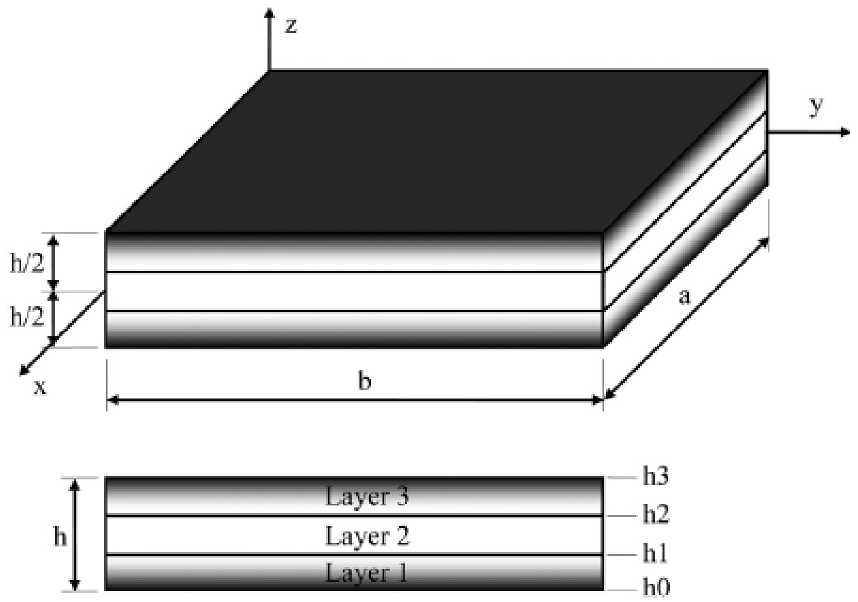

Fig. 1 Geometry and coordinates of rectangular FGM sandwich plate

shear and stretching components; (2) the axial displacement is divided into extension, bending and shear components; (3) the bending parts of the axial displacements are similar to those given by CPT; and (4) the shear parts of the axial displacements give rise to the sinusoidal variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) + g(z)\,\varphi(x,y,t) \end{aligned} \tag{3}$$

where u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively; and the additional displacement φ accounts for the effect of normal stress (stretching effect). The shape functions $f(z)$ and $g(z)$ are given as follows

$$f(z) = z - \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{4}$$

and

$$g(z) = 1 - f'(z) \tag{5}$$

The non-zero strains associated with the displacement field in Eq. (3) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \quad \varepsilon_z = g'(z)\, \varepsilon_z^0, \tag{6}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \varphi \tag{7}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{8}$$

2.2 Constitutive relations

The linear constitutive relations are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\Delta T \\ \varepsilon_y - \alpha\Delta T \\ \varepsilon_z - \alpha\Delta T \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{9}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively.
where $\Delta T = T - T_0$ in which T_0 is the reference temperature.

The applied temperature distribution $T(x, y, z)$ through the thickness are assumed to be

$$T(x,y,z) = T_1(x,y) + \frac{z}{h} T_2(x,y) + \frac{1}{\pi} \sin\left(\frac{\pi z}{h}\right) T_3(x,y) \tag{10}$$

Using the material properties defined in Eq. (1), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = Q_{33} = \frac{E(z)}{1-\nu^2}, \tag{11a}$$

$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu\, E(z)}{1-\nu^2}, \tag{11b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}, \tag{11c}$$

2.4 Governing equations

The governing equations of the present theory are derived using the principle of virtual work; the following expressions can be obtained

$$\int_{-h/2}^{h/2}\int_{\Omega}\left[\sigma_x\delta\,\varepsilon_x + \sigma_y\delta\,\varepsilon_y + \sigma_z\delta\,\varepsilon_z + \tau_{xy}\delta\,\gamma_{xy} + \tau_{yz}\delta\,\gamma_{yz} + \tau_{xz}\delta\,\gamma_{xz}\right]d\Omega\,dz - \int_{\Omega} q\delta\,wd\Omega = 0 \tag{12}$$

where Ω is the top surface and q is the distributed transverse load.

Substituting Eqs. (3), (6) and (9) into Eq. (10) and integrating through the thickness of the plate, Eq. (10) can be rewritten as

$$\begin{aligned}\delta\,U = \int_{\Omega}\Big[& N_x\delta\,\varepsilon_x^0 + N_y\delta\,\varepsilon_y^0 + N_z\delta\,\varepsilon_z^0 + N_{xy}\delta\,\gamma_{xy}^0 + M_x^b\delta\,k_x^b + M_y^b\delta\,k_y^b + M_{xy}^b\delta\,k_{xy}^b \\ & + M_x^s\delta\,k_x^s + M_y^s\delta\,k_y^s + M_{xy}^s\delta\,k_{xy}^s + S_{yz}^s\delta\,\gamma_{yz}^0 + S_{xz}^s\delta\,\gamma_{xz}^0 - q\delta\,w\Big]d\Omega = 0\end{aligned} \tag{13}$$

where the stress resultants (N, M^b, M^s, S^s and N_z) are as follows

$$\begin{aligned}&\left(N_i, M_i^b, S_i^b\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}(1, z, f)\sigma_i dz, && (i = x, y, xy) \\ &S_i^s = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\tau_i g(z)dz, && (i = xz, yz) \\ \text{and}\quad &N_z = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\sigma_z g'(z)dz\end{aligned} \tag{14}$$

The governing equations of equilibrium can be derived from Eq. (11) by integrating the displacement gradients by parts and setting the coefficients δu_0, δv_0, δw_b, δw_s and $\delta\varphi$ to zero separately. Thus one can obtain the equilibrium equations associated with the present simple quasi-3D theory

$$\begin{aligned}&\delta\,u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta\,v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta\,w_b: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \\ &\downarrow\end{aligned} \tag{15}$$

↑

$$\delta w_s: \quad \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + q = 0 \tag{15}$$

$$\delta \varphi: \quad \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = 0$$

By substituting Eq. (6) into Eq. (9) and the subsequent results into Eq. (12), the stress resultants are readily obtained as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_z^0 - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, \quad S = A^s \gamma, \tag{16a}$$

$$N_z = R^a \varphi + L\left(\varepsilon_x^0 + \varepsilon_y^0\right) + L^a\left(k_x^b + k_y^b\right) + R\left(k_x^s + k_y^s\right) - N_z^T, \tag{16b}$$

where

$$N = \left\{N_x, N_y, N_{xy}\right\}, \quad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}, \quad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}, \tag{17a}$$

$$N^T = \left\{N_x^T, N_y^T, 0\right\}, \quad M^{bT} = \left\{M_x^{bT}, M_y^{bT}, 0\right\}, \quad M^{sT} = \left\{M_x^{sT}, M_y^{sT}, 0\right\}, \tag{17b}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}, \tag{17c}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{17d}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{17e}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}, \quad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \quad \begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} Q_{11}^{(n)} \begin{Bmatrix} \nu^{(n)} \\ \nu^{(n)} z \\ \nu^{(n)} f(z) \\ g'(z) \end{Bmatrix} g'(z) dz, \tag{17f}$$

Here the stiffness coefficients are defined as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} Q_{11}^{(n)} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu^{(n)} \\ \frac{1-\nu^{(n)}}{2} \end{Bmatrix} dz, \tag{18a}$$

and

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \quad Q_{11}^{(n)} = \frac{E(z)}{1-\nu^2} \tag{18b}$$

$$A_{44}^s = A_{55}^s = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \frac{E(z)}{2(1+\nu)} [g(z)]^2 \, dz, \tag{18c}$$

The resultant efforts, $N_x^T = N_y^T$, $M_x^{bT} = M_y^{bT}$, $M_x^{sT} = M_y^{sT}$ and N_z^T induced by the thermal effect are expressed by

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_x^{sT} \\ N_z^T \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \frac{E^{(n)}(z)}{1-(\nu^{(n)})^2} \left(1 + 2\nu^{(n)}\right) \alpha^{(n)} T \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z) \end{Bmatrix} dz, \tag{19}$$

2.3 Governing equations in terms of displacements

Introducing Eq. (15) into Eq. (13), the governing equations can be expressed in terms of displacements (δu_0, δv_0, δw_b, δw_s, $\delta\varphi$) and the appropriate equations take the form

$$\begin{aligned} &A_{11} d_{11} u_0 + A_{66} d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 - B_{11} d_{111} w_b \\ &- \left(B_{12} + 2B_{66}\right) d_{122} w_b - \left(B_{12}^s + 2B_{66}^s\right) d_{122} w_s - B_{11}^s d_{111} w_s + L d_1 \varphi = p_1, \end{aligned} \tag{20a}$$

$$\begin{aligned} &A_{22} d_{22} v_0 + A_{66} d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 - B_{22} d_{222} w_b \\ &- \left(B_{12} + 2B_{66}\right) d_{112} w_b - \left(B_{12}^s + 2B_{66}^s\right) d_{112} w_s - B_{22}^s d_{222} w_s + L d_2 \varphi = p_2, \end{aligned} \tag{20b}$$

$$\begin{aligned} &B_{11} d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 \\ &+ B_{22} d_{222} v_0 - D_{11} d_{1111} w_b - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_b - D_{22} d_{2222} w_b \\ &- D_{11}^s d_{1111} w_s - 2\left(D_{12}^s + 2D_{66}^s\right) d_{1122} w_s - D_{22}^s d_{2222} w_s + L^a \left(d_{11}\varphi + d_{22}\varphi\right) = p_3, \end{aligned} \tag{20c}$$

$$\begin{aligned} &B_{11}^s d_{111} u_0 + \left(B_{12}^s + 2B_{66}^s\right) d_{122} u_0 + \left(B_{12}^s + 2B_{66}^s\right) d_{112} v_0 + B_{22}^s d_{222} v_0 - D_{11}^s d_{1111} w_b \\ &- 2\left(D_{12}^s + 2D_{66}^s\right) d_{1122} w_b - D_{22}^s d_{2222} w_b - H_{11}^s d_{1111} w_s \\ &- 2\left(H_{12}^s + 2H_{66}^s\right) d_{1122} w_s - H_{22}^s d_{2222} w_s + A_{44}^s d_{11} w_s + A_{55}^s d_{22} w_s \\ &+ R\left(d_{11}\varphi + d_{22}\varphi\right) + A_{44}^s d_{11}\varphi + A_{55}^s d_{22}\varphi = p_4, \end{aligned} \tag{20d}$$

$$\begin{aligned} &L\left(d_1 u_0 + d_2 v_0\right) - L^a \left(d_{11} w_b + d_{22} w_b\right) + \left(R - A_{44}^s\right) d_{11} w_s + \left(R - A_{55}^s\right) d_{22} w_s \\ &+ R^a \varphi - A_{44}^s d_{11}\varphi - A_{55}^s d_{22}\varphi = p_5, \end{aligned} \tag{20e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \; d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \; d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \; d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1{,}2). \tag{21}$$

The components of the generalized force vector $\{p\}$ are given by

$$\begin{aligned} p_1 &= \frac{\partial N_x^T}{\partial x}, & p_2 &= \frac{\partial N_y^T}{\partial y}, \\ p_3 &= q - \frac{\partial^2 M_x^{bT}}{\partial x^2} - \frac{\partial^2 M_y^{bT}}{\partial y^2}, & p_4 &= q - \frac{\partial^2 M_x^{sT}}{\partial x^2} - \frac{\partial^2 M_y^{sT}}{\partial y^2}, \\ p_5 &= N_z^T, & p_5 &= N_z^T \end{aligned} \tag{22}$$

3. Analytical solutions

Consider a simply supported rectangular plate with length a and width b under transverse load q. To solve this problem, Navier presented the transverse mechanical and temperature loads q, T_1, T_2, and T_3 in the form of a double trigonometric series as

$$\begin{Bmatrix} q \\ T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} q_0 \\ t_1 \\ t_2 \\ t_3 \end{Bmatrix} \sin(\lambda\, x)\sin(\mu\, y) \tag{23}$$

where q_0, t_1, t_2 and t_3 are constants, $\lambda = \pi / a$, $\mu = \pi / b$.

Based on Navier solution method, we assume the following solutions form for displacements (u_0, v_0, w_b, w_s, φ)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \\ \varphi \end{Bmatrix} = \begin{Bmatrix} U\cos(\lambda\, x)\sin(\mu\, y) \\ V\sin(\lambda\, x)\cos(\mu\, y) \\ W_b\sin(\lambda\, x)\sin(\mu\, y) \\ W_s\sin(\lambda\, x)\sin(\mu\, y) \\ \Phi\sin(\lambda\, x)\sin(\mu\, y) \end{Bmatrix} \tag{24}$$

where U, V, W_b, W_s and Φ unknown parameters must be determined. By considered Eqs. (18) and (22), the following equation is obtained

$$[C]\{\Delta\} = \{P\}, \tag{25}$$

where $\{\Delta\} = \{ U, V, W_b, W_s, \Phi\}^t$ and $[C]$ is the symmetric matrix expressed by

$$[C] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{12} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{13} & a_{23} & a_{33} & a_{34} & a_{35} \\ a_{14} & a_{24} & a_{34} & a_{44} & a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & a_{55} \end{bmatrix}, \tag{26}$$

in which

$$
\begin{aligned}
a_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
a_{12} &= -\lambda\,\mu\left(A_{12} + A_{66}\right) \\
a_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2] \\
a_{14} &= \lambda\,[B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\,\mu^2] \\
a_{15} &= L\lambda \\
a_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
a_{23} &= \mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2] \\
a_{24} &= \mu\,[(B_{12}^s + 2B_{66}^s)\,\lambda^2 + B_{22}^s\mu^2] \\
a_{25} &= L\mu \\
a_{33} &= -\left(D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4\right) \\
a_{34} &= -\left(D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\mu^2 + D_{22}^s\,\mu^4\right) \\
a_{35} &= -L^a\left(\lambda^2 + \mu^2\right) \\
a_{44} &= -\left(H_{11}^s\lambda^4 + 2(H_{11}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2\right) \\
a_{45} &= -\left(A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R\left(\lambda^2 + \mu^2\right)\right) \\
a_{55} &= -\left(A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R^a\right)
\end{aligned}
\tag{27}
$$

and the components of the generalized force vector $\{P\} = \{P_1, P_2, P_3, P_4, P_5\}^t$ are expressed by

$$
\begin{aligned}
P_1 &= \lambda\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right), \\
P_2 &= \mu\left(A^T t_1 + B^T t_2 + {}^aB^T t_3\right), \\
P_3 &= -q_0 - h\left(\lambda^2 + \mu^2\right)\left(B^T t_1 + D^T t_2 + {}^aD^T t_3\right), \\
P_4 &= -q_0 - h\left(\lambda^2 + \mu^2\right)\left({}^sB^T t_1 + {}^sD^T t_2 + {}^sF^T t_3\right), \\
P_5 &= -h\left(L^T t_1 + {}^aL^T t_2 + R^T t_3\right).
\end{aligned}
\tag{28}
$$

where

$$\left(A^T,\ B^T,\ D^T\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\frac{E^{(n)}(z)}{1-(\nu^{(n)})^2}\left(1 + 2\nu^{(n)}\right)\alpha^{(n)}\left(1,\ \bar{z},\ \bar{z}^2\right)dz, \tag{29a}$$

$$\left({}^aB^T,\ {}^aD^T\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\frac{E^{(n)}(z)}{1-(\nu^{(n)})^2}\left(1 + 2\nu^{(n)}\right)\alpha^{(n)}\,\overline{\Psi}(z)\left(1,\ \bar{z}\right)dz, \tag{29b}$$

$$\left({}^sB^T,\ {}^sD^T,\ {}^sF^T\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\frac{E^{(n)}(z)}{1-(\nu^{(n)})^2}\left(1 + 2\nu^{(n)}\right)\alpha^{(n)}\,\overline{f}(z)\left(1,\ \bar{z},\ \overline{\Psi}(z)\right)dz, \tag{29c}$$

$$\left(L^T,\ L_a^T,\ R^T\right)=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\frac{E^{(n)}(z)}{1-(\nu^{(n)})^2}\left(1+2\nu^{(n)}\right)\alpha^{(n)}\ \bar{g}'(z)\ \left(1,\ \bar{z},\ \overline{\Psi}(z)\right)dz, \tag{29d}$$

with $\bar{z}=z/h$, $\bar{f}(z)=f(z)/h$ and $\overline{\Psi}(z)=\dfrac{1}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$.

4. Numerical results and discussion

To assess the performance of present theory under mechanical and thermal loads, simply supported functionally graded sandwich plates are considered with following material properties:

- Metal (Titanium, Ti-6Al-4V): P_2 = 66.2 GPa; ν_2 = 1 / 3; α_2 = 10.3 (10^{-6} / K).
- Ceramic (Zirconia, ZrO_2): P_1 = 117.0 GPa; ν_1 = 1 / 3; α_1 = 7.11 (10^{-6} / K).

The results are presented in the following normalized forms for displacements and stresses according to Saidi *et al*. (2013) for the purpose of presentation in this article.

- center deflection $\bar{w}=\dfrac{10^3}{q_0a^4/(E_0h^3)+10^3\alpha_0t_2\ a^2/h}w\left(\dfrac{a}{2},\dfrac{b}{2}\right)$,
- axial stress $\bar{\sigma}_x=\dfrac{10}{q_0a^2/h^2+10E_0\alpha_0t_2\ a^2/h^2}\sigma_x\left(\dfrac{a}{2},\dfrac{b}{2},\dfrac{h}{2}\right)$,
- shear stress $\bar{\tau}_{xz}=\dfrac{1}{q_0a/h+E_0\alpha_0t_2a/(10h)}\tau_{xz}\left(0,\dfrac{b}{2},0\right)$.

where the reference values are taken as E_0 = 1 GPa and α_0 = 7.11 10^{-6} / K.

It is assumed, unless otherwise stated, that a / h = 10, a / b = 1, t_1 = 0, and q_0 = t_2 = t_3 = 100. The shear correction factor of FSDT is fixed to be K = 5 /6.

Inspection of Tables 1-4 reveals that the present theory with only five unknowns provides similar results to those predicted by the hyperbolic plate theory (HPT) proposed by Saidi *et al*. (2013) and the sinusoidal plate theory (SPT) developed by Zenkour and Alghamdi (2008) with six unknowns ($\varepsilon_z \neq 0$). This proves that the same accuracy is achievable with the present theory using

Table 1 Comparison of dimensionless center deflections $\bar{w}$ for different FG sandwich square plates (q_0 = t_1 = t_3 = 0, t_2 = 100 and a / h = 10)

p	Theory	$\bar{w}$			
		t_{FGM} / h = 1	t_{FGM} / h = 2 / 3	t_{FGM} / h = 1 / 2	t_{FGM} / h = 4 / 5
0	Present ($\varepsilon_z \neq 0$)	0.461634	0.461634	0.461634	0.461634
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.449863	0.449863	0.449863	0.449863
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.461634	0.461634	0.461634	0.461634
	Ref[(b)] ($\varepsilon_z = 0$)	0.480262	0.480262	0.480262	0.480262

Table 1 Continued

p	Theory	$\bar{w}$			
		$t_{FGM}/h = 1$	$t_{FGM}/h = 2/3$	$t_{FGM}/h = 1/2$	$t_{FGM}/h = 4/5$
1	Present ($\varepsilon_z \neq 0$)	0.614565	0.586124	0.563416	0.599933
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.594840	0.565276	0.542436	0.579538
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.614565	0.586124	0.563416	0.599933
	Ref[(b)] ($\varepsilon_z = 0$)	0.636916	0.606292	0.582342	0.621098
2	Present ($\varepsilon_z \neq 0$)	0.647135	0.618046	0.590491	0.633340
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.627934	0.596416	0.567938	0.612832
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.647135	0.618046	0.590491	0.633340
	Ref[(b)] ($\varepsilon_z = 0$)	0.671503	0.639361	0.609875	0.656142
3	Present ($\varepsilon_z \neq 0$)	0.658153	0.631600	0.602744	0.646475
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.639690	0.610125	0.579769	0.626505
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.658153	0.631600	0.602744	0.646475
	Ref[(b)] ($\varepsilon_z = 0$)	0.683572	0.653671	0.622467	0.670275
4	Present ($\varepsilon_z \neq 0$)	0.662811	0.638705	0.609560	0.652890
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.644833	0.617502	0.586469	0.633395
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.662811	0.638705	0.609560	0.652890
	Ref[(b)] ($\varepsilon_z = 0$)	0.688803	0.661291	0.629533	0.677321
5	Present ($\varepsilon_z \neq 0$)	0.665096	0.642948	0.613842	0.656490
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.647421	0.621990	0.590728	0.637353
	Ref[(b)] ($\varepsilon_z \neq 0$)	0.665096	0.642948	0.613842	0.656490
	Ref[(b)] ($\varepsilon_z = 0$)	0.691420	0.665898	0.634003	0.681343

Table 2 Comparison of dimensionless center deflections $\bar{w}$ for different FG sandwich square plates

p	Theory	$t_{FGM}/h = 0$	$t_{FGM}/h = 0.2$	$t_{FGM}/h = 0.4$	$t_{FGM}/h = 0.6$	$t_{FGM}/h = 0.8$	$t_{FGM}/h = 1$
0	Present ($\varepsilon_z \neq 0$)	0.748424	0.748424	0.748424	0.748424	0.748424	0.748424
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.771340	0.771340	0.771340	0.771340	0.771340
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.796783	0.796783	0.796783	0.796783	0.796783
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.808168	0.808168	0.808168	0.808168	0.808168
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	0.895735	0.895735	0.895735	0.895735	0.895735
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.457873	0.457873	0.457873	0.457873	0.457873
1	Present ($\varepsilon_z \neq 0$)	0.748424	0.825607	0.891560	0.942936	0.979382	1.003408
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.841759	0.906271	0.959745	0.999391	1.026070
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.873745	0.941636	0.996334	1.036213	1.062840
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.886067	0.954808	1.010231	1.050672	1.077690
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	0.979641	1.054630	1.115684	1.160568	1.190728
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.501163	0.539886	0.571450	0.594688	0.610331

Table 2 Continued

p	Theory	$t_{FGM}/h = 0$	$t_{FGM}/h = 0.2$	$t_{FGM}/h = 0.4$	$t_{FGM}/h = 0.6$	$t_{FGM}/h = 0.8$	$t_{FGM}/h = 1$
2	Present ($\varepsilon_z \neq 0$)	0.748424	0.845883	0.930539	0.994421	1.035346	1.057609
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.860395	0.943946	1.011993	1.058388	1.084456
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.894003	0.981434	1.050237	1.096095	1.121608
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.906529	0.995042	1.064791	1.111352	1.137297
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	1.001204	1.097973	1.175402	1.227765	1.257304
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.512431	0.562536	0.602673	0.629859	0.645223
3	Present ($\varepsilon_z \neq 0$)	0.748424	0.855272	0.948423	1.016599	1.056867	1.075460
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.869136	0.961579	1.035332	1.082231	1.104836
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.903467	0.999831	1.073875	1.119794	1.141655
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.916083	1.013647	1.088747	1.135420	1.157693
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	1.011279	1.118224	1.202080	1.255041	1.280741
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.517716	0.573152	0.616662	0.644176	0.657539
4	Present ($\varepsilon_z \neq 0$)	0.748424	0.902511	0.958901	1.028393	1.067181	1.082846
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.874209	0.971671	1.048073	1.094108	1.113637
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.908934	1.010269	1.086624	1.131429	1.150192
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.921602	1.024208	1.101684	1.147260	1.166403
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	1.017115	1.129824	1.216678	1.268689	1.290961
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.520783	0.579240	0.624324	0.651345	0.662909
5	Present ($\varepsilon_z \neq 0$)	0.748424	0.864347	0.943749	1.034550	1.072864	1.086419
	Ref[(a)] ($\varepsilon_z \neq 0$)	0.771340	0.877515	0.978164	1.055935	1.100868	1.118027
	SSDT[(a)] ($\varepsilon_z = 0$)	0.796783	0.912488	1.016938	1.094427	1.137993	1.154412
	TSDT[(a)] ($\varepsilon_z = 0$)	0.808168	0.925190	1.030958	1.109609	1.153952	1.170720
	FSDT[(a)] ($\varepsilon_z = 0$)	0.895735	1.020919	1.137289	1.225706	1.276497	1.296101
	CPT[(a)] ($\varepsilon_z = 0$)	0.457873	0.522783	0.583160	0.629064	0.655445	0.665606

a lower number of unknowns than other theories, and clearly highlights how the present theory is simpler and more easily deployed.

Table 1 presents an assessment of the dimensionless center deflection $\bar{w}$ for an FGM sandwich plate subjected to a linear temperature distribution within the thickness. The deflections are analyzed for p = 0, 1, 2, 3, 4, and 5 and different types of sandwich plates. Table 1 shows that the effect of the thickness stretching is to reduce the deflection.

Table 2 compares the deflections of different types of FGM rectangular sandwich plates for p = 0, 1, 2, 3, 4, and 5. It can be concluded that the inclusion of thickness stretching effect serves to make the plate stiffer, and hence, leads to a reduction of deflection. However, the inclusion of shear deformation effect makes the plate more flexible and consequently leads to increase the deflection.

Tables 3 and 4 document, respectively, the values of axial stress $\bar{\sigma}_x$ and transverse shear stress

Table 3 Comparison of dimensionless axial stresses $\overline{\sigma}_x$ for different FG sandwich square plates

p	Theory6	$t_{FGM}/h = 0$	$t_{FGM}/h = 0.2$	$t_{FGM}/h = 0.4$	$t_{FGM}/h = 0.6$	$t_{FGM}/h = 0.8$	$t_{FGM}/h = 1$
0	Present ($\varepsilon_z \neq 0$)	-2.528819	-2.528819	-2.528819	-2.528819	-2.528819	-2.528819
	SSDT ($\varepsilon_z = 0$)	-2.388919	-2.388919	-2.388919	-2.388919	-2.388919	-2.388919
	TSDT ($\varepsilon_z = 0$)	-2.461177	-2.461177	-2.461177	-2.461177	-2.461177	-2.461177
	FSDT ($\varepsilon_z = 0$)	-3.597007	-3.597007	-3.597007	-3.597007	-3.597007	-3.597007
	CPT	-1.706393	-1.706393	-1.706393	-1.706393	-1.706393	-1.706393
1	Present ($\varepsilon_z \neq 0$)	-2.528819	-3.913321	-3.489857	-3.138470	-2.876846	-2.700416
	SSDT ($\varepsilon_z = 0$)	-2.388919	-3.333300	-3.001265	-2.733086	-2.537374	-2.406806
	TSDT ($\varepsilon_z = 0$)	-2.461177	-3.412724	-3.076466	-2.804750	-2.606343	-2.473903
	FSDT ($\varepsilon_z = 0$)	-3.597007	-4.504051	-4.136892	-3.838047	-3.618476	-3.471099
	CPT	-1.706393	-2.193219	-2.003463	-1.848793	-1.734921	-1.658265
2	Present ($\varepsilon_z \neq 0$)	-2.528819	-3.792865	-3.245326	-2.797887	-2.490378	-2.316178
	SSDT ($\varepsilon_z = 0$)	-2.388919	-3.234499	-2.806645	-2.469045	-2.243809	-2.118730
	TSDT ($\varepsilon_z = 0$)	-2.461177	-3.312889	-2.879670	-2.537489	-2.308903	-2.181780
	FSDT ($\varepsilon_z = 0$)	-3.597007	-4.398484	-3.924721	-3.545789	-3.289757	-3.145662
	CPT	-1.706393	-2.137999	-1.892474	-1.695789	-1.562571	-1.487285
3	Present ($\varepsilon_z \neq 0$)	-2.528819	-3.736478	-3.130873	-2.645068	-2.332424	-2.180056
	SSDT ($\varepsilon_z = 0$)	-2.388919	-3.188312	-2.716593	-2.353122	-2.127496	-2.020425
	TSDT ($\varepsilon_z = 0$)	-2.461177	-3.266245	-2.788595	-2.420027	-2.190823	-2.081815
	FSDT ($\varepsilon_z = 0$)	-3.597007	-4.349165	-3.825600	-3.415261	-3.156414	-3.031283
	CPT	-1.706393	-2.112102	-1.840454	-1.627241	-1.492417	-1.426935
4	Present ($\varepsilon_z \neq 0$)	-2.528819	-3.703803	-3.065266	-2.561174	-2.252973	-2.120478
	SSDT ($\varepsilon_z = 0$)	-2.388919	-3.161620	-2.665468	-2.290552	-2.070361	-1.978602
	TSDT ($\varepsilon_z = 0$)	-2.461177	-3.239292	-2.736867	-2.356554	-2.132710	-2.039172
	FSDT ($\varepsilon_z = 0$)	-3.597007	-4.320597	-3.768831	-3.343853	-3.089733	-2.981507
	CPT	-1.706393	-2.097076	-1.810621	-1.589696	-1.457288	-1.400620
5	Present ($\varepsilon_z \neq 0$)	-2.528819	-3.682505	-3.023018	-2.509154	-2.207341	-2.090458
	SSDT ($\varepsilon_z = 0$)	-2.388919	-3.144264	-2.632792	-2.252244	-2.038118	-1.957968
	TSDT ($\varepsilon_z = 0$)	-2.461177	-3.221769	-2.703791	-2.317655	-2.099863	-2.018086
	FSDT ($\varepsilon_z = 0$)	-3.597007	-4.301976	-3.732298	-3.299697	-3.051612	-2.956534
	CPT	-1.706393	-2.087272	-1.791409	-1.566468	-1.437193	-1.387402

$\overline{\tau}_{xz}$ for p = 0, 1, 2, 3, 4, and 5 and different types of sandwich plates. In addition, the results of different higher order shear deformation theories (HSDTs) such as sinusoidal shear deformation theory (SSDT), third shear deformation theory (TSDT) and first shear deformation theory (FSDT) are also provided to show the importance of including the thickness-stretching effect. The HSDTs solutions are computed based on a nonlinear variation of in-plane displacements and a constant transverse displacement across the thickness (i.e., thickness-stretching effect is omitted, $\varepsilon_z = 0$).

Fig. 2 shows the effect of the aspect ratio a / b on the dimensionless center deflection $\bar{w}$ for FG sandwich plate. The effect of the mechanical and thermal loads is taken into consideration. It is found that the aspect ratio effect is more pronounced on the thermomechanical bending deflection $\bar{w}$ ($q_0 = t_2 = t_3 = 100$ and) of the FG sandwich plate.

In Figs. 3 and 4, we have plotted the through-the-thickness distributions of the dimensionless axial stress $\bar{\sigma}_x$ and the transverse shear stress $\bar{\tau}_{xz}$ of the FG sandwich plate for $p = 2$ and $t_{FGM} = 0.6h$, respectively. These figures show the great influence played by the different thermal and bending loads on the axial and transverse shear stresses.

Table 4 Comparison of dimensionless transverse shear stresses $\bar{\tau}_{xz}$ for different FG sandwich square plates

p	Theory	$t_{FGM} / h = 0$	$t_{FGM} / h = 0.2$	$t_{FGM} / h = 0.4$	$t_{FGM} / h = 0.6$	$t_{FGM} / h = 0.8$	$t_{FGM} / h = 1$
0	Present ($\varepsilon_z \neq 0$)	0.193097	0.193097	0.193097	0.193097	0.193097	0.193097
	SSDT ($\varepsilon_z = 0$)	0.171604	0.171604	0.171604	0.171604	0.171604	0.171604
	TSDT ($\varepsilon_z = 0$)	0.174481	0.174481	0.174481	0.174481	0.174481	0.174481
	FSDT ($\varepsilon_z = 0$)	0.173624	0.173624	0.173624	0.173624	0.173624	0.173624
1	Present ($\varepsilon_z \neq 0$)	0.193097	0.313620	0.349534	0.341568	0.324311	0.317474
	SSDT ($\varepsilon_z = 0$)	0.171604	0.271618	0.300347	0.293865	0.280890	0.277019
	TSDT ($\varepsilon_z = 0$)	0.174481	0.264677	0.289538	0.284236	0.274133	0.272347
	FSDT ($\varepsilon_z = 0$)	0.173624	0.181504	0.190134	0.199626	0.210115	0.221768
2	Present ($\varepsilon_z \neq 0$)	0.193097	0.338091	0.370811	0.346355	0.315647	0.308622
	SSDT ($\varepsilon_z = 0$)	0.171604	0.292205	0.317892	0.298078	0.275130	0.272583
	TSDT ($\varepsilon_z = 0$)	0.174481	0.282950	0.304910	0.288355	0.270427	0.270952
	FSDT ($\varepsilon_z = 0$)	0.173624	0.184293	0.196359	0.210115	0.225945	0.244354
3	Present ($\varepsilon_z \neq 0$)	0.193097	0.348104	0.376236	0.341227	0.304511	0.3023266
	SSDT ($\varepsilon_z = 0$)	0.171604	0.300600	0.322239	0.294047	0.267073	0.269608
	TSDT ($\varepsilon_z = 0$)	0.174481	0.290349	0.308697	0.285154	0.264327	0.270110
	FSDT ($\varepsilon_z = 0$)	0.173624	0.185719	0.199626	0.215785	0.234789	0.257465
4	Present ($\varepsilon_z \neq 0$)	0.193097	0.353399	0.377769	0.336004	0.296906	0.300779
	SSDT ($\varepsilon_z = 0$)	0.171604	0.305016	0.323396	0.289951	0.261729	0.270017
	TSDT ($\varepsilon_z = 0$)	0.174481	0.294226	0.309711	0.281837	0.260366	0.271755
	FSDT ($\varepsilon_z = 0$)	0.173624	0.186586	0.201639	0.219335	0.240436	0.266029
5	Present ($\varepsilon_z \neq 0$)	0.193097	0.356625	0.378076	0.331797	0.292004	0.301729
	SSDT ($\varepsilon_z = 0$)	0.171604	0.307694	0.323573	0.286687	0.258433	0.272071
	TSDT ($\varepsilon_z = 0$)	0.174481	0.296571	0.309879	0.279200	0.258029	0.274512
	FSDT ($\varepsilon_z = 0$)	0.173624	0.187168	0.203004	0.221768	0.244354	0.272062

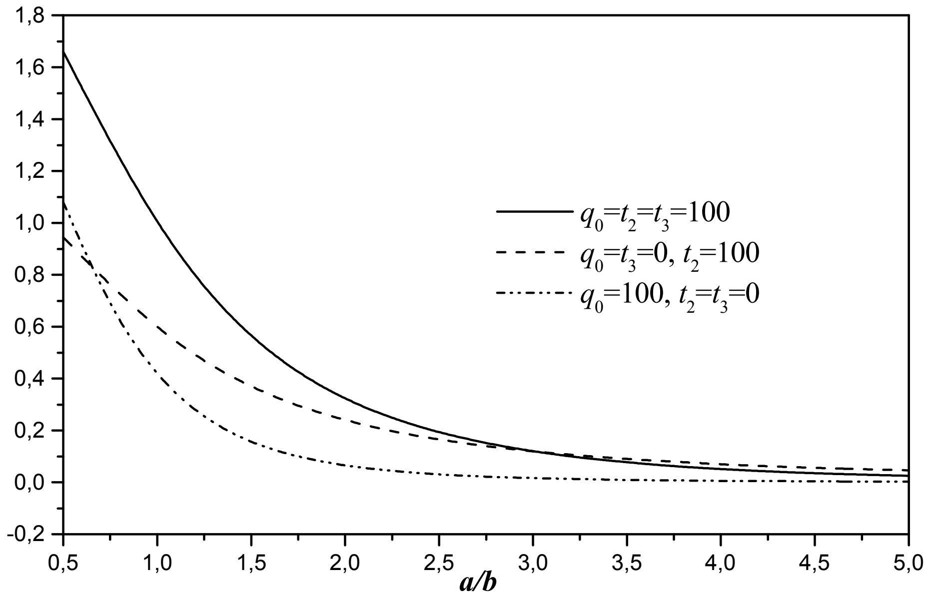

Fig. 2 Effect of mechanical and temperature loads on the dimensionless center deflection of FG sandwich plate versus a / b ($t_{FGM} = 0.6h$, $p = 2$)

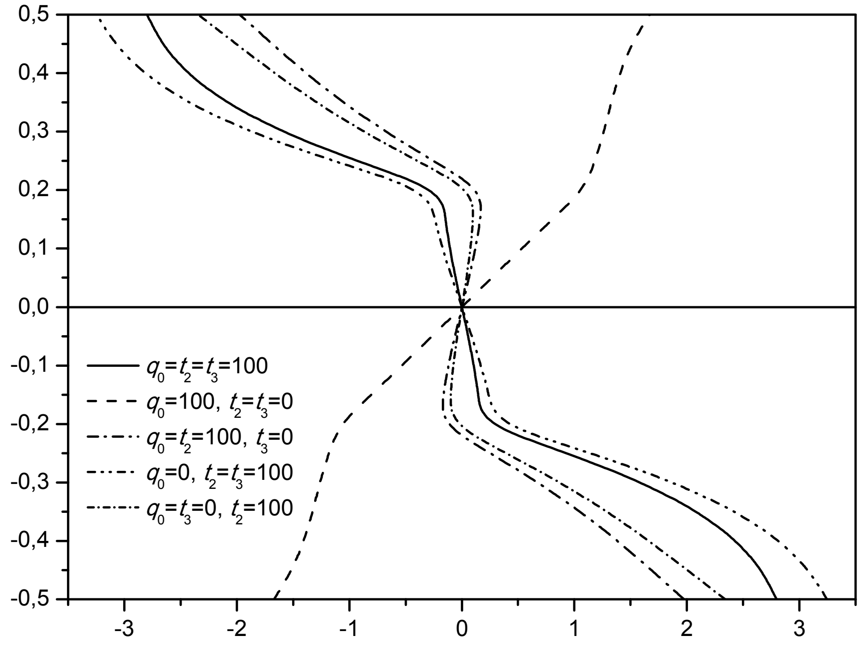

Fig. 3 Effect of mechanical and temperature loads on the dimensionless axial stress of FG sandwich plate ($t_{FGM} = 0.6h$, $p = 2$)

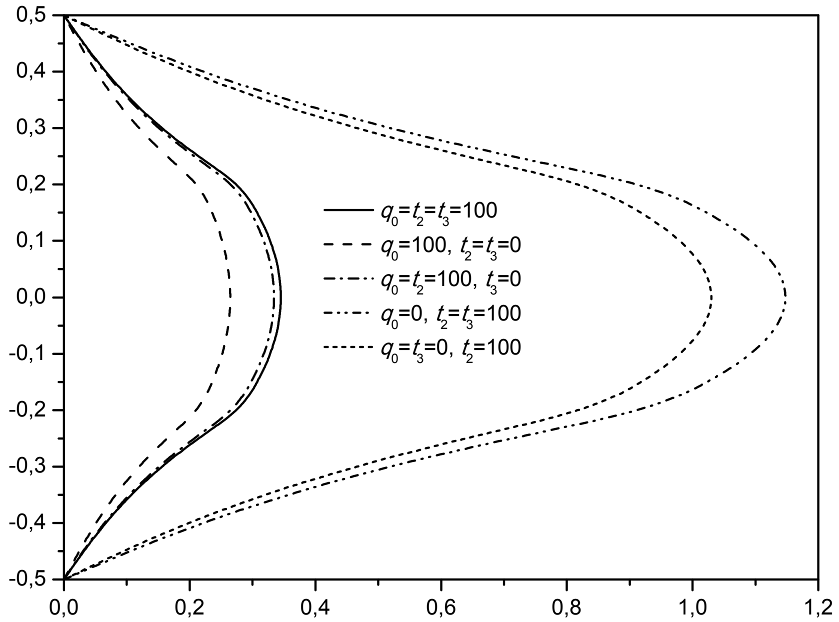

Fig. 4 Effect of mechanical and temperature loads on the dimensionless transverse shear stress of FG andwich plate ($t_{FGM} = 0.6h$, $p = 2$)

5. Conclusions

A new 5-unknowns quasi-3D sinusoidal plate theory with stretching effect for the thermomechanical bending of FG sandwich plates is presented in this work. The main assumption of this formulation is the decomposition of the transverse displacement into bending and shears components. This theory is free of the zero in-plane resultant forces assumption used in developing the other four variables shear deformation theories and hence have the potential to be used for modeling of the nonlinear FG plate problems. Results demonstrate that the proposed theory is able to provide very accurate results compared with the CPT, FSDT and other HSDTs with higher number of unknowns and so deserve special attention and offer potential for future research. The main point that can be outlined from the present study is that the thickness stretching effect is more pronounced for thick plates and it needs to be taken in consideration in the modeling.

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