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A higher order shear deformation theory for static and free vibration of FGM beam

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Abstract. In this paper, a higher order shear deformation beam theory is developed for static and free vibration analysis of functionally graded beams. The theory account for higher-order variation of transverse shear strain through the depth of the beam and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. The material properties of the functionally graded beam are assumed to vary according to power law distribution of the volume fraction of the constituents. Based on the present higher-order shear deformation beam theory, the equations of motion are derived from Hamilton's principle. Navier type solution method was used to obtain frequencies. Different higher order shear deformation theories and classical beam theories were used in the analysis. A static and free vibration frequency is given for different material properties. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions.

Keywords: mechanical properties; vibration; strength; deformation; modeling

1. Introduction

In material sciences, a functionally graded material (FGM) is a type of material whose composition is designed to change continuously within the solid. The concept is to make a composite material by varying the microstructure from one material to another material with a specific gradient.

The concept of FGM was first considered in Japan in 1984 during a space plane project. The FGM materials can be designed for specific applications. For example, thermal barrier coatings for turbine blades (electricity production), armor protection for military applications, fusion energy devices, biomedical materials including bone and dental implants, space/aerospace industries, automotive applications, etc.

Static and dynamic analyses of FGM structures have attracted increasing research effort in the last decade because of the wide application areas of FGMs. For instance, Sankar (2001) gave an elasticity solution based on the Euler-Bernoulli beam theory for functionally graded beam subjected to static transverse loads by assuming that Young's modulus of the beam vary exponentially through the thickness. Aydogdu and Taskin (2007) investigated the free vibration

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behavior of a simply supported FG beam by using Euler- Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. Thai and Vo (2012) presented a Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories. Sallai *et al*. (2009) investigated the static responses of a sigmoid FG thick beam by using different beam theories. Ying *et al*. (2008) obtained the exact solutions for bending and free vibration of FG beams resting on a Winkler-Pasternak elastic foundation based on the two- dimensional elasticity theory by assuming that the beam is orthotropic at any point and the material properties vary exponentially along the thickness direction.

Şimşek (2010b) studied the dynamic deflections and the stresses of an FG simply-supported beam subjected to a moving mass by using Euler-Bernoulli, Timoshenko and the higher order shear deformation theories by considering the centripetal, inertia and Coriolis effects of the moving mass.

In the present study, static and free vibration of simply supported FG beams was investigated by using classical beam theory (CBT) (Vel and Batra 2002) and first order shear deformation beam theory (FSDBT) and parabolic shear deformation beam theory (PSDBT) (Benatta *et al*. 2008) and the sinusoidal theory of Touratier (Touratier 1991), and exponential shear deformation beam theory of Karama *et al*. (ESDBT) (2003) and the new refined shear deformation beam theory NRSDBT. The most interesting feature of this theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the beam without using shear correction factors. Then, the present theory together with Hamilton's principle, are employed to extract the motion equations of the functionally graded beams. Analytical solutions for static and free vibration are obtained. Numerical examples are presented to verify the accuracy of the present theory.

2. Problem formulation

Consider a functionally graded beam with length L and rectangular cross section $b \times h$, with b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction.

2.1 Material properties

The properties of FGM vary continuously due to the gradually changing volume fraction of the

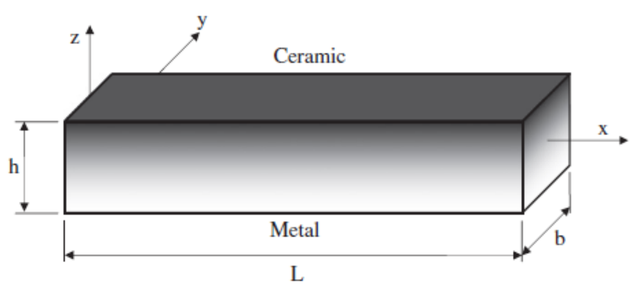

Fig. 1 Geometry and coordinate of a FG beam

constituent materials (ceramic and metal), usually in the thickness direction only. The power-law function is commonly used to describe these variations of materials properties. The expression given below represents the profile for the volume fraction.

$$V_C = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{1a}$$

p is a parameter that dictates material variation profile through the thickness. The value of p equal to zero represents a fully ceramic beam, whereas infinite p indicates a fully metallic beam, and for different values of p one can obtain different volume fractions of metal.

The material properties of FG beams are assumed to vary continuously through the depth of the beam by the rule of mixture (Marur 1999) as

$$P(z) = (P_t - P_b)\, V_C + P_b \tag{1b}$$

where P denotes a generic material property like modulus, P_t and P_b denotes the property of the top and bottom faces of the beam respectively, Here, it is assumed that modules E, G and v vary according to the Eq. (1). However, for simplicity, Poisson's ratio of beam is assumed to be constant in this study for that the effect of Poisson's ratio v on deformation is much less than that of Young's modulus (Delale and Erdogan 1983, Benachour *et al*. 2011).

The variation of Young's modulus in the thickness direction of the P-FGM beam is depicted in Fig. 2, which shows that the Young's modulus changes rapidly near the lowest surface for $p > 1$ and increases quickly near the top surface for $p < 1$.

2.2 Basic assumptions

The assumptions of the present theory are as follows:

(i) The origin of the Cartesian coordinate system is taken at the median surface of the FG beam.

(ii) The displacements are small in comparison with the height of the beam and, therefore, strains involved are infinitesimal.

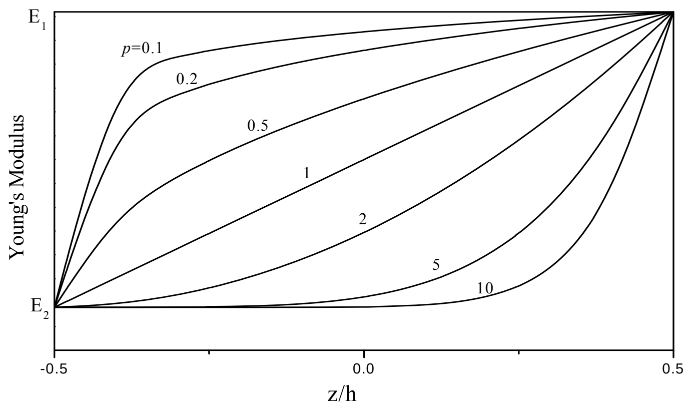

Fig. 2 Variation of Young's modulus in a P-FGM beam

(iii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x, y only.

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{2}$$

(iv) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x.

(v) The axial displacement u in x-direction, consists of extension, bending, and shear components.

$$u = u_0 + u_b + u_s \tag{3}$$

The bending component u_b is assumed to be similar to the displacements given by the classical beam theory. Therefore, the expression for u_b can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \tag{4}$$

The shear component u_s gives rise, in conjunction with w_s, to the parabolic variation of shear strain γ_{xz} and hence to shear stress τ_{xz} through the thickness of the beam in such a way that shear stress τ_{xz} is zero at the top and bottom faces of the beam. Consequently, the expression for u_s can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \tag{5}$$

where

$$f(z) = z - \frac{1}{2}z\left(\frac{1}{4}h^2 - \frac{1}{3}z^2\right) \tag{6}$$

2.3 Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (2)-(6) as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{7a}$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{7b}$$

The strains associated with the displacements in Eq. (10) are

$$\varepsilon_x = \varepsilon_x^0 + z\,k_x^b + f(z)\,k_x^s \tag{8a}$$

$$\gamma_{xz} = g(z)\,\gamma_{xz}^s \tag{8b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\frac{\partial^2 w_s}{\partial x^2},\ \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \tag{8c}$$

$$g(z) = 1 - f'(z) \quad \text{and} \quad f'(z) = \frac{df(z)}{dz} \tag{8d}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x = Q_{11}(z)\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z)\,\gamma_{xz} \tag{9a}$$

where

$$Q_{11}(z) = E(z) \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{9b}$$

2.4 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Thai and Vo 2012)

$$\delta \int_{t_1}^{t_2} (U + V - K)\,dt = 0 \tag{10}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy; and δK is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned} \delta U &= \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta\,\varepsilon_x + \tau_{xz} \delta\,\gamma_{xz})\,dz\,dx \\ &= \int_0^L \left(N \frac{d\delta\,u_0}{dx} - M_b \frac{d^2\delta\,w_b}{dx^2} - M_s \frac{d^2\delta\,w_s}{dx^2} + Q \frac{d\delta\,w_s}{dx} \right) dx \end{aligned} \tag{11}$$

where N, M_b, M_s and Q are the stress resultants defined as

$$(N, M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f)\,\sigma_x dz_{ns} \quad \text{and} \quad Q = \int_{-\frac{h}{2}}^{\frac{h}{2}} g\,\tau_{xz} dz \tag{12}$$

The variation of the potential energy by the applied transverse load q can be written as

$$\delta V = -\int_0^L q(\delta\,w_b + \delta\,w_s)\,dx \tag{13}$$

The variation of the kinetic energy can be expressed as

$$\delta K = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\left[\dot{u}\delta\,\dot{u} + \dot{w}\delta\,\dot{w}\right] dzdx$$

$$= \int_0^L \left\{ I_1\left[\dot{u}_0 \delta \dot{u}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta\dot{w}_b + \delta\dot{w}_s\right)\right] - I_2\left(\dot{u}_0 \frac{d\delta\dot{w}_b}{dx} + \frac{d\dot{w}_b}{dx}\delta\,\dot{u}_0\right) \right.$$
$$+ I_4\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right) - I_3\left(\dot{u}_0 \frac{d\delta\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\delta\,\dot{u}_0\right) + I_6\left(\frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_s}{dx}\right)$$
$$\left. + I_5\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_s}{dx} + \frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right)\right\} dx \tag{14}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_1, I_2, I_3, I_4, I_5, I_6) are the mass inertias defined as

$$\left(I_1, I_2, I_3, I_4, I_5, I_6\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(1, z, f, z^2, zf, f^2\right)\rho(z)dz \tag{15}$$

Substituting the expressions for δU, δV and δT from Eqs. (11), (13) and (14) into Eq. (10) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the functionally graded beam are obtained

$$\delta\,u_0: \quad \frac{dN}{dx} = I_1\ddot{u}_0 - I_2\frac{d\ddot{w}_b}{dx} - I_3\frac{d\ddot{w}_s}{dx} \tag{16a}$$

$$\delta\,w_b: \quad \frac{d^2M_b}{dx^2} + q = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\frac{d\ddot{u}_0}{dx} - I_4\frac{d^2\ddot{w}_b}{dx^2} - I_5\frac{d^2\ddot{w}_s}{dx^2} \tag{16b}$$

$$\delta\,w_s: \quad \frac{d^2M_s}{dx^2} + \frac{dQ}{dx} + q = I_1(\ddot{w}_b + \ddot{w}_s) + I_3\frac{d\ddot{u}_0}{dx} - I_5\frac{d^2\ddot{w}_b}{dx^2} - I_6\frac{d^2\ddot{w}_s}{dx^2} \tag{16c}$$

Eq. (16) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (7), (8), (9) and (12) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_b}{\partial x^3} - B_{11}^s\frac{\partial^3 w_s}{\partial x^3} = I_1\ddot{u}_0 - I_2\frac{d\ddot{w}_b}{dx} - I_3\frac{d\ddot{w}_s}{dx} \tag{17a}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_b}{\partial x^4} - D_{11}^s\frac{\partial^4 w_s}{\partial x^4} + q = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\frac{d\ddot{u}_0}{dx} - I_4\frac{d^2\ddot{w}_b}{dx^2} - I_5\frac{d^2\ddot{w}_s}{dx^2} \tag{17b}$$

$$B_{11}^s\frac{\partial^3 u_0}{\partial x^3} - D_{11}^s\frac{\partial^4 w_b}{\partial x^4} - H_{11}^s\frac{\partial^4 w_s}{\partial x^4} + A_{55}^s\frac{\partial^2 w_s}{\partial x^2} + q = I_1(\ddot{w}_b + \ddot{w}_s) + I_3\frac{d\ddot{u}_0}{dx} - I_5\frac{d^2\ddot{w}_b}{dx^2} - I_6\frac{d^2\ddot{w}_s}{dx^2} \tag{17c}$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11}, B_{11}, D_{11}, B_{11}^{s}, D_{11}^{s}, H_{11}^{s}\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(1, z, z^{2}, f(z), z\, f(z), f^{2}(z)\right) dz \tag{18a}$$

and

$$A_{55}^{s}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}[g(z)]^{2}\, dz \tag{18b}$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s, can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_m \cos(\lambda\, x)\, e^{i\,\omega\, t} \\ W_{bm} \sin(\lambda\, x)\, e^{i\,\omega\, t} \\ W_{sm} \sin(\lambda\, x)\, e^{i\,\omega\, t} \end{Bmatrix} \tag{19}$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with m th eigenmode, and $\lambda = m\pi/L$. The transverse load q is also expanded in Fourier series as

$$q(x)=\sum_{m=1}^{\infty} Q_m \sin(\lambda\, x) \tag{20}$$

where Q_m is the load amplitude calculated from

$$Q_m=\frac{2}{L}\int_{0}^{L} q(x)\sin(\lambda\, x)dx \tag{21}$$

The coefficients Q_m are given below for some typical loads.

Substituting the expansions of u_0, w_b, w_s from Eq. (19) into the equations of motion Eq. (20), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}-\omega^{2}\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix}=\begin{Bmatrix} 0 \\ Q_m \\ Q_m \end{Bmatrix} \tag{22}$$

where

$$a_{11} = A_{11}\lambda^2\,,\; a_{12} = -B_{11}\lambda^3\,,\; a_{13} = -B_{11}^s\lambda^3\,,\; a_{22} = D_{11}\lambda^4\,,\;\; a_{23} = D_{11}^s\lambda^4\,,\; a_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2 \quad \text{(23a)}$$

$$m_{11} = I_1\,, m_{12} = -I_2\lambda\,,\; m_{13} = -I_3\lambda\,,\; m_{22} = I_1 + I_4\lambda^2\,,\; m_{23} = I_1 + I_5\lambda^2\,,\; m_{33} = I_1 + I_6\lambda^2 \quad \text{(23b)}$$

Table 1 Nondimensional deflections and stresses of FG beams under uniform load

p	Method	$L/h = 5$			$L/h = 20$		
		$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0	CBT*	2.8783	3.7500	–	2.8783	15.0000	–
	ESDBT*	3.1635	3.8083	0.7762	2.8961	15.0145	0.7908
	SSDBT*	3.1649	3.8052	0.7546	2.8962	15.0137	0.7672
	PSDBT*	3.1654	3.8019	0.7330	2.8962	15.0129	0.7437
	Present	3.1654	3.8019	0.7330	2.8962	15.0129	0.7437
0.5	CBT*	4.4401	4.9206	–	4.4401	19.6825	–
	ESDBT*	4.5578	4.6497	0.7738	4.4479	19.6143	0.7908
	SSDBT*	4.8278	4.9969	0.7717	4.4644	19.7014	0.7840
	PSDBT*	4.8285	4.9923	0.7501	4.4644	19.7003	0.7606
	Present	4.8285	4.9923	0.7501	4.4644	19.7002	0.7614
1	CBT*	5.7746	5.7958	–	5.7746	23.1834	–
	ESDBT*	6.2563	5.8944	0.7762	5.8047	23.2078	0.7908
	SSDBT*	6.2586	5.8891	0.7546	5.8049	23.2065	0.7672
	PSDBT*	6.2594	5.8835	0.7330	5.8049	23.2051	0.7437
	Present	6.2594	5.8835	0.7330	5.8049	23.2051	0.7437
2	CBT*	7.4003	6.7676	–	7.4003	27.0704	–
	ESDBT*	8.0666	6.8969	0.7156	7.4420	27.1025	0.7304
	SSDBT*	8.0683	6.8899	0.6931	7.4421	27.1008	0.7058
	PSDBT*	8.0677	6.8824	0.6704	7.4421	27.0989	0.6812
	Present	8.0677	6.8824	0.6704	7.4421	27.0989	0.6812
5	CBT*	8.7508	7.9428	–	8.7508	31.7711	–
	ESDBT*	9.8414	8.1329	0.6403	8.8191	31.8184	0.6554
	SSDBT*	9.8367	8.1219	0.6153	8.8188	31.8156	0.6282
	PSDBT*	9.8281	8.1104	0.5904	8.8182	31.8127	0.6013
	Present	9.8281	8.1104	0.5904	8.8182	31.8127	0.6013
10	CBT*	9.6072	9.5228	–	9.6072	38.0912	–
	ESDBT*	10.9404	9.7343	0.6943	9.6907	38.1438	0.7106
	SSDBT*	10.9419	9.7236	0.6706	9.6908	38.1411	0.6847
	PSDBT*	10.9381	9.7119	0.6465	9.6905	38.1382	0.6586
	Present	10.9381	9.7119	0.6465	9.6905	38.1382	0.6586

* Results form Ref. (Thai and Vo 2012)

4. Results and discussion

In this section, various numerical examples are presented and discussed to verify the accuracy of present theories in predicting the bending and free vibration responses of simply supported FG beams. The FG beam is taken to be made of aluminum and alumina with the following material properties

Ceramic (P_C: Alumina, Al2O3): E_c = 380 GPa; ν = 0.3; ρ_c = 3960 kg/m3
Metal (P_M: Aluminium, Al): E_m = 70 GPa; ν = 0.3; ρ_m = 2702 kg/m3

And their properties change through the thickness of the beam according to power-law. The bottom surfaces of the FG beams are aluminum rich, whereas the top surfaces of the FG beams are alumina rich.

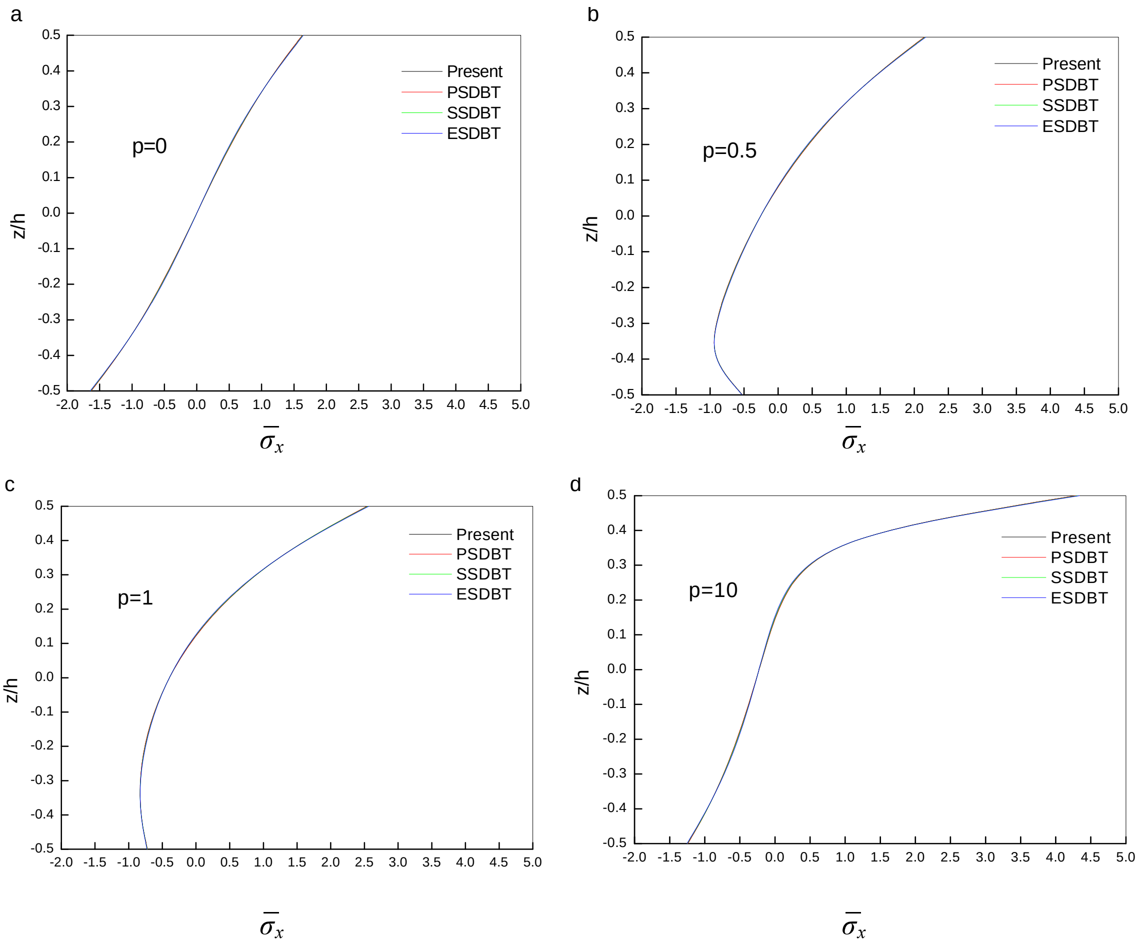

Fig. 3 Variation of nondimensional axial normal stress $\bar{\sigma}_x$ (1/2, z) across the depth of FG beams under uniform load (L = 2h): (a) p = 0; (b) p = 0.5; (c) p = 1 and (d) p = 10

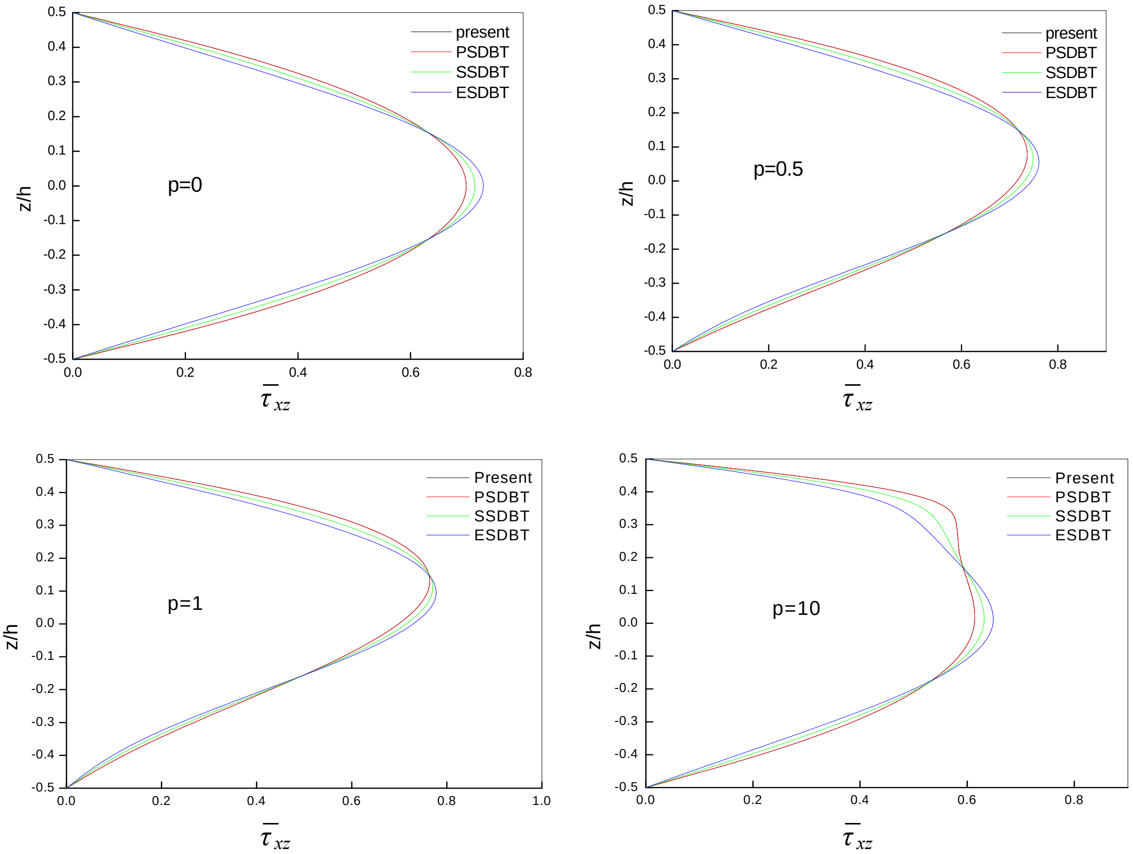

Fig. 4 Variation of nondimensional transverse shear stress $\bar{\tau}_{xz}$ (0, z) across the depth of FG beams under uniform m load ($L = 2h$): (a) $p = 0$; (b) $p = 0.5$; (c) $p = 1$ and (d) $p = 10$

For convenience, the following dimensionless form is used

$$\bar{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right),\ \bar{u} = 100\frac{E_m h^3}{q_0 L^4} u\left(0, -\frac{h}{2}\right),\ \bar{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2}, \frac{h}{2}\right),$$

$$\bar{\tau}_{xz} = \frac{h}{q_0 L}\tau_{xz}(0,0),\ \bar{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$$

4.1 Results for bending analysis

Table 1 contains nondimensional deflection and stresses of FG beams under uniform load q_0 for different values of power law index p and span-to-depth ratio L/h. The obtained results are compared with various shear deformation beam theories (i.e., ESDBT, SSDBT, PSDBT). It can be observed that the values obtained using various shear deformation beam theories (i.e., ESDBT, SSDBT, PSDBT) are in good agreement with the those given by the present theory for all values

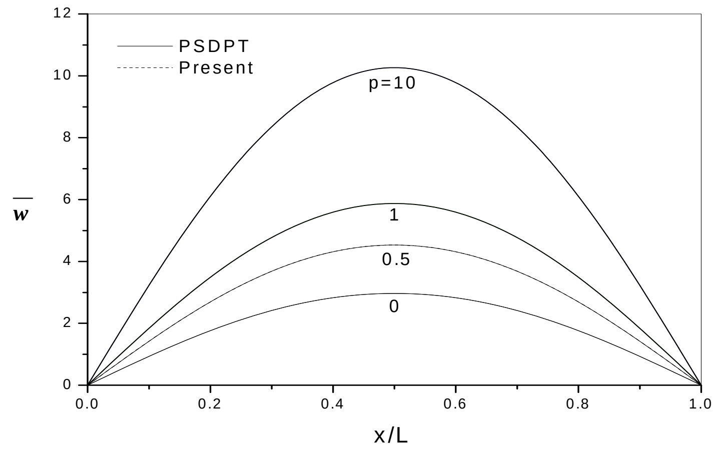

Fig. 5 Variation of the transverse displacement $\bar{w}$ versus non-dimensional length of a FG beam (L = 5h)

Table 2 Variation of fundamental frequency $\bar{\omega}$ with the power-law index for FG beam for L/h = 5

Theory	$p = 0$	$p = 0.2$	$p = 0.5$	$p = 1$	$p = 5$	$p = 10$	Metal
CBT	5.3953	5.0206	4.5931	4.1484	3.5949	3.4921	2.8034
FSDBT	5.1525	4.8066	4.4083	3.9902	3.4312	3.3134	2.6772
ESDBT	5.1542	4.8105	4.4122	3.9914	3.4014	3.2813	2.6781
PSDBT	5.1527	4.8092	4.4111	3.9904	3.4012	3.2816	2.6773
Present	5.1527	4.8081	4.4107	3.9904	3.4012	3.2816	2.6773

Table 3 Variation of fundamental frequency $\bar{\omega}$ with the power-law index for FG beam for L/h = 20

Theory	$p = 0$	$p = 0.2$	$p = 0.5$	$p = 1$	$p = 5$	$p = 10$	Metal
CBT	5.4777	5.0967	4.6641	4.2163	3.6628	3.5546	2.8462
FSDBT	5.4603	5.0827	4.6514	4.2051	3.6509	3.5415	2.8371
ESDBT	5.4604	5.0829	4.6516	4.2051	3.6483	3.5389	2.8372
PSDBT	5.4603	5.0829	4.6516	4.2050	3.6485	3.5389	2.8372
Present	5.4606	5.0817	4.6511	4.2050	3.6486	3.5389	2.8371

of power law index p and span-to-depth ratio L/h. Due to ignoring the shear deformation effect, CBT underestimates deflection of moderately deep beams (L/h = 5).

Figs. 3-4 show the variations of axial stress $\bar{\sigma}_x$, and transverse shear stress $\bar{\tau}_{xz}$, respectively, through the depth of a very deep beam (L = 2h) under uniform load. In general, the present theory and all shear deformation beam models give almost identical results, except for the case of transverse shear stress $\bar{\tau}_{xz}$.

Fig. 5 illustrates the variation of the non-dimensional transversal displacement $\bar{w}$ versus non-dimensional length for different power law index p. It can be seen also that the present beam theory gives almost identical results to PSDBT. In addition, the results show that the increase of

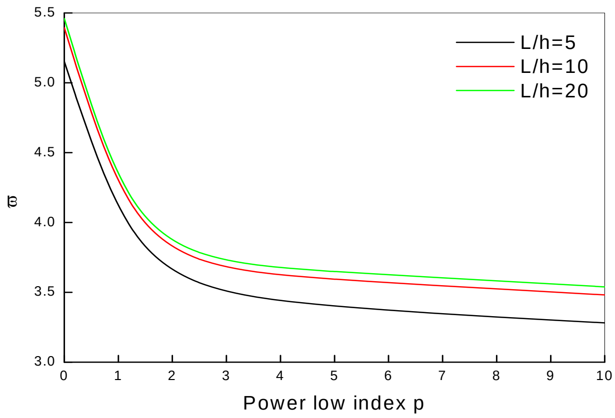

Fig. 6 Variation of the fundamental frequency $\bar{\omega}$ of FG beam with power-law index p

the power law index p leads to an increase of transversal displacement $\bar{w}$.

4.2 Results for free vibration analysis

Tables 2 and 3 shows the nondimensional fundamental frequencies $\bar{\omega}$ of FG beams for different values of power law index p and span-to-depth ratio L/h. The calculated frequencies are compared with those given by Şimşek (2010a) using various beam theories. It should be noted that the results reported by Şimşek (2010a) based on various shear deformation beam models in which the shear strains are approximated in terms of shear rotations instead of shear components of bending rotation as in this study. An excellent agreement between the present solutions and results of Şimşek (2010a) is found.

Fig. 6 shows the non-dimensional fundamental natural frequency $\bar{\omega}$ versus the power law index p for different values of span-to-depth ratio L/h using the present theory. It is observed that an increase in the value of the power law index leads to a reduction of frequency. The highest frequency values are obtained for full ceramic beams ($p = 0$) while the lowest frequency values are obtained for full metal beams ($p \to \infty$). This is due to the fact that an increase in the value of the power law index results in a decrease in the value of elasticity modulus. In other words, the beam becomes flexible as the power law index increases, thus decreasing the frequency values. It can be also seen that the span-to-depth ratio L/h has a considerable effect on the non-dimensional fundamental natural frequency $\bar{\omega}$ where this latter is reduced with decreasing L/h. This dependence is related to the effect of shear deformation.

5. Conclusions

A New shear deformation beam theory is proposed for bending and frees vibration analysis of functionally graded beams. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. It is based on the assumption that the transverse displacements consist of bending and shear components. Based on the present beam theory, the equations of motion are derived from Hamilton's principle. Numerical examples show that the proposed theory gives solutions which are almost identical with those obtained using other shear deformation theories.

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