Steel and Composite Structures, Vol.14, No. 2 (2013) 121-132 121
DOI: http://dx.doi.org/10.12989/scs.2013.14.2.121

Multiobjective optimum design of laminated composite annular sector plates

Umut Topal*

Karadeniz Technical University, Of Faculty of Technology, Of, Trabzon, Turkey

(Received September 30, 2012, Revised November 23, 2011, Accepted December 01, 2012)

Abstract. This paper deals with multiobjective optimization of symmetrically laminated composite angle-ply annular sector plates subjected to axial uniform pressure load and thermal load. The design objective is the maximization of the weighted sum of the critical buckling load and fundamental frequency. The design variable is the fibre orientations in the layers. The performance index is formulated as the weighted sum of individual objectives in order to obtain the optimum solutions of the design problem. The first-order shear deformation theory is used for the mathematical formulation. Finally, the effects of different weighting factors, annularity, sector angle and boundary conditions on the optimal design are investigated and the results are compared.

Keywords: laminated annular sector plates; multiobjective optimization; buckling; frequency

1. Introduction

Plates, in general, and sector plates in particular have many applications in engineering fields, i.e., aerospace, mechanical and civil engineering as diaphragms, curved bridge decks and end closures of cylindrical vessels. In the limit case that the width of the annular sector plate is small in comparison with its outer radius, it can be treated as a circular curved beam. Composite materials have gained many advantages over their metal counterparts in engineering applications, in particular aerospace engineering. Fiber reinforced laminated composite plates are made of continuous fibers in mainly epoxy resins with different fiber orientations in each layer. The layers can be arranged in a way that the properties of the composite, with respect to the middle plane of the plate, are either symmetrical or nonsymmetrical.

On the other hand, the structures are quite often are subjected to in-plane, external loads and thermal loads which may cause buckling. In addition, the vibration can be problematic when the excitation frequency coincides with the shell’s resonance frequency. Such loadings may occur at different times under in-service conditions, necessitating a design approach which is capable of taking in to account these various loading conditions.

In the literature, there are few researches on the free vibration of laminated annular sector plates. Sharma *et al*. (2005) presented a simple analytical formulation for the eigenvalue problem

*Corresponding author, Professor, E-mail: umut@ktu.edu.tr

of buckling and free vibration analysis of shear deformable laminated sector plates made up of cylindrically orthotropic layers. Xu (2008) established new state space formulations for the free vibration analysis of shear deformable laminated sector plates made up of cylindrically orthotropic layers. Xu (2008) established new state space formulations for the free vibration of circular, annular and sectorial plates by introducing two displacement functions and two stress functions. Houmat (2008) presented a sector *p*-element for the large amplitude free vibration analysis of laminated composite annular sector plates. Malekzadeh (2009) presented an accurate and efficient solution procedure based on the three-dimensional elasticity theory for the free vibration analysis of thick laminated annular sector plates. Malekzadeh *et al.* (2010) studied dynamic response of thick laminated annular sector plates with simply supported radial edges subjected to a radially distributed line load, which moves along the circumferential direction. Ding and Xiu (2000) established the state-space equation of the axisymmetric vibration of laminated annular plates composed of transversely isotropic layers based on the basic equations of three-dimensional theory of elasticity.

Research on the subject of buckling of laminated annular sector has been reported by few investigators. Seifi *et al.* (2012) investigated buckling of composite annular plates under uniform internal and external radial edge loads using energy method. Singhatanadgid and Ungbhakorn (2005) derived the similitude invariants and scaling laws for buckling of polar orthotropic annular plates subjected to radial compressive load and torsional load. Dumir *et al.* (2011) presented axisymmetric postbuckling response of polar orthotropic laminated moderately thick circular and annular plates subjected to uniformly distributed inplane radial compressive load at the outer edge. Pawlus (2011) studied dynamic stability problem of three-layered, annular plate loaded by compressive stress increasing in time.

On the other hand, multiobjective optimization of laminated annular sector has not been investigated by the authors until now. In this study, three different problems are combined as the weighted sum of individual objectives in order to obtain multiobjective optimization solutions to fill this gap. The design objective is the maximization of the weighted sum of the critical buckling load subjected to axial uniform pressure load and thermal load and fundamental frequency. The design variable is the fibre orientations in the layers. The performance index is formulated as the weighted sum of individual objectives in order to obtain the optimum solutions of the design problem. The first-order shear deformation theory is used for the mathematical formulation. Finally, the effects of different weighting factors, annularity, sector angle and boundary conditions on the optimal design are investigated and the results are compared.

2. Basic Equations

Consider a laminated sector plate with total thickness h, outer radius r_o, inner radius r_1, sector angle θ_o and the fibres are arranged in regular form and the sector plate is cut from this ply as depicted in Fig. 1.

The displacement field of the plate based on the first order shear deformation theory is given by the following expressions

$$u(r,\theta,z) = u_o(r,\theta) + z\psi_r(r,\theta)$$
$$v(r,\theta,z) = v_o(r,\theta) + z\psi_\theta(r,\theta) \tag{1}$$
$$w(r,\theta,z) = w(r,\theta)$$

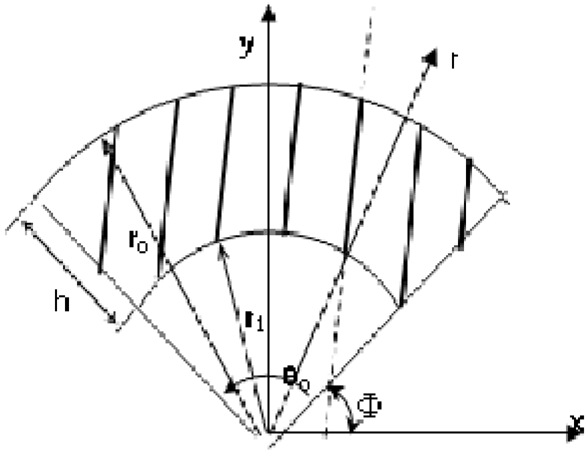


Fig. 1 Geometry of a laminated annular sector plate

where u_o, v_o and w are the displacements of the mid-surface and ψ_r and ψ_θ are the rotations of a normal to the mid-surface about θ and r axis, respectively.

The stress–strain relations for a single lamina in a laminated annular sector plate are given by

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix}_{(k)} \begin{pmatrix} \varepsilon_r - \alpha_r \Delta T \\ \varepsilon_\theta - \alpha_\theta \Delta T \\ \gamma_{r\theta} - \alpha_{r\theta} \Delta T \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} \tau_{\theta z} \\ \tau_{rz} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{pmatrix}_{(k)} \begin{pmatrix} \gamma_{\theta z} \\ \gamma_{rz} \end{pmatrix} \tag{3}$$

where $\overline{Q}_{ij}$ is the transformed reduced stiffnesses, which can be expressed in terms of the orientation angle and the engineering constant of the material. α_r, α_θ, $\alpha_{r\theta}$ are the coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

In-plane strains for the midplane are

$$\varepsilon_r^{(o)} = \frac{1}{r_o(1 - r_1/r_o)}\frac{\partial u}{\partial r}, \varepsilon_r^{(1)} \frac{1}{r_o(1 - r_1/r_o)}\frac{\partial \Psi_r}{\partial r},$$

$$\varepsilon_\theta^{(o)} = \left(\frac{1}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o}\right)\left(u + \frac{1}{\theta_o}\frac{\partial v}{\partial \theta}\right),$$

$$\varepsilon_\theta^{(1)} = \left(\frac{1}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o}\right)\left(\Psi_r + \frac{1}{\theta_o}\frac{\partial \Psi_\theta}{\partial \theta}\right),$$

$$\gamma_{r\theta}^{(o)} = \left(\frac{1}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o}\right)\left(\frac{1}{\theta_o}\frac{\partial u}{\partial \theta}\right) + \frac{1}{r_o(1-(r_1/r_o))}\frac{\partial v}{\partial r}$$

$$- \frac{v}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o} \tag{4}$$

The curvatures are

$$\gamma_{r\theta}^{(1)} = \left(\frac{1}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o}\right)\left(\frac{1}{\theta_o}\frac{\partial \Psi_r}{\partial \theta}\right)$$

$$+ \frac{1}{r_o(1 - r_1/r_o)}\frac{\partial \Psi_r}{\partial r} - \frac{\Psi_\theta}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o},$$

$$\gamma_{rz} = \Psi_r + \frac{1}{r_o(1 - r_1/r_o)}\frac{\partial w}{\partial r},$$

$$\gamma_{\theta z} = \Psi_\theta + \left(\frac{1}{\{r(1-(r_1/r_o)) + (r_1/r_o)\} r_o}\right)\left(\frac{1}{\theta_o}\frac{\partial w}{\partial \theta}\right) \tag{5}$$

where $r = \left(r^* - r_1\right)/\left(r_o - r_1\right)$ and r^* is the radial coordinates.

The stress resultants $\{N\}$, stress couples $\{M\}$ and transverse shear stress resultants $\{Q\}$ are

$$\begin{Bmatrix} N_r \\ N_\theta \\ N_{r\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{Bmatrix} dz, \begin{Bmatrix} M_r \\ M_\theta \\ M_{r\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{Bmatrix} z dz, \begin{Bmatrix} Q_r \\ Q_\theta \end{Bmatrix} = K \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{rz} \\ \tau_{\theta z} \end{Bmatrix} dz \tag{6}$$

In Eq. (6), K is the shear correction factor. In this study, the shear correction factor is taken 5/6.

2.1 Finite element formulation

In this study, nine-noded Lagrangian rectangular plate elements having five degrees of freedom per node are used for the finite element solution of the laminated annular sector plates. The interpolation of the displacement field is defined as

$$\begin{pmatrix} u \\ v \\ w \\ \psi_r \\ \psi_\theta \end{pmatrix} = \sum_{i=1}^{n} \Phi_i d_i \tag{7}$$

where d_i and Φ_i are the nodal variables and the interpolation function, respectively. Following the standard procedure of the finite element formulation, the stability condition is obtained as

$$\left[K + \lambda K_g\right]\{d\} = 0 \tag{8}$$

in which $[K]$ and $\left[K_g\right]$ are the assembled linear stiffness and geometric stiffness matrixes, respectively. The lowest eigenvalue of the homogeneous system (8) yields the critical buckling load (N_{cr}). On the other hand, the critical temperature buckling load, T_{cr}, can be calculated using Eq. (9)

$$T_{cr} = \lambda \Delta T \tag{9}$$

The free vibration problem of the laminated annular plates becomes as follows

$$\left[K - \omega^2 M\right]\{d\} = \{0\} \tag{10}$$

where $[K]$, $[M]$ and $\lambda = \omega^2$ are the linear stiffness matrix, mass matrix and eigenvector, respectively. Eq. (10) is a set of homogeneous linear equations in the unknown displacements $\{u\}$. For non-trivial solution, the determinant is equal to zero and the eigenvalues correspond to natural frequencies of the laminated plates. The subspace iteration method is used for the frequency analysis.

3. Optimization problem

The optimization problem is formulated in order to find the best orientation angles of fibres in the laminated annular sector plates so that to simultaneously maximize the critical axial pressure buckling load, critical thermal buckling load and fundamental frequency with the laminate configurations. The multiobjective design index, $MODI$, can be describes as follows

$$\mathrm{MODI} = \eta N^* + \xi T^* + \mu \varpi^* \tag{11}$$

where η, ξ and μ are the weighting factors summing the three objective functions with $\eta, \xi, \mu \geq 0, \eta + \xi + \mu = 1$. As the weighting factors are varied, the emphasis of the optimization problem is shifted among various objectives resulting in compromise solutions. The single objective designs can be obtained as special cases by setting $\eta = 1$ or $\eta = 0$. In this study, the optimization problem can be expressed as follows

$$\text{find: } \theta, \quad 0^o \leq \theta_k \leq 90^o,\ \Delta\theta = 5^\circ$$

$$\text{maximize: MODI} \tag{12}$$

In the all computations, the following nondimensionalized quantities are used

$$N^* = \frac{N_{cr}}{N_o}, T^* = \frac{T_{cr}}{T_{cro}}, \varpi^* = \frac{\omega}{\omega_o} \tag{13}$$

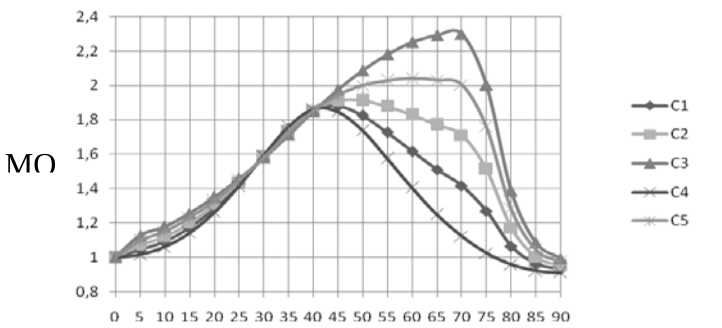
for critical axial pressure buckling load, critical thermal buckling load and fundamental frequency, respectively. The N_o, T_{cro} and ω_o are the external pressure buckling load, thermal buckling load and frequency corresponding to prescribed lamination angles $\left(0^\circ/0^\circ/0^\circ/0^\circ\right)$ for four layered laminated annular sector plates, respectively.

4. Numerical results and discussion

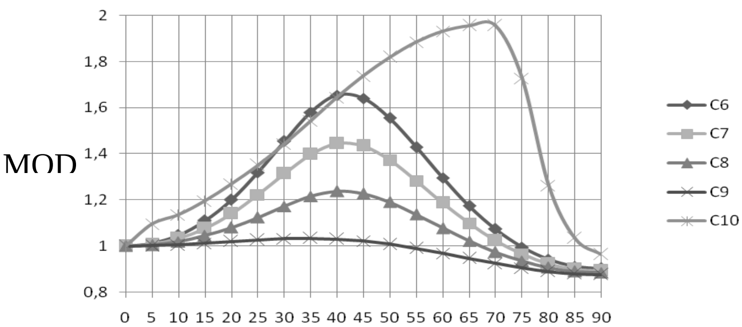
In this study, symmetrically laminated angle-ply $(\theta/-\theta/-\theta/\theta)$ annular sector plates are investigated for optimization problems. Each of the lamina is assumed to be same thickness. Numerical results are given for a typical T300/5208 graphite/epoxy material and the material properties are as below

$E_1 = 181\text{GPa}$, $E_2 = 10.3\text{GPa}$, $G_{12} = G_{13} = 7.17\text{GPa}$, $G_{23} = 2.39\text{GPa}$, $\nu_{12} = 0.28$, $\rho = 1600\,\text{kg/m}^3$, $\alpha_1 = 0.02\text{x}10^{-6}\,{}^\circ\text{C}^{-1}$, $\alpha_2 = 22.5\text{x}10^{-6}\,{}^\circ\text{C}^{-1}$

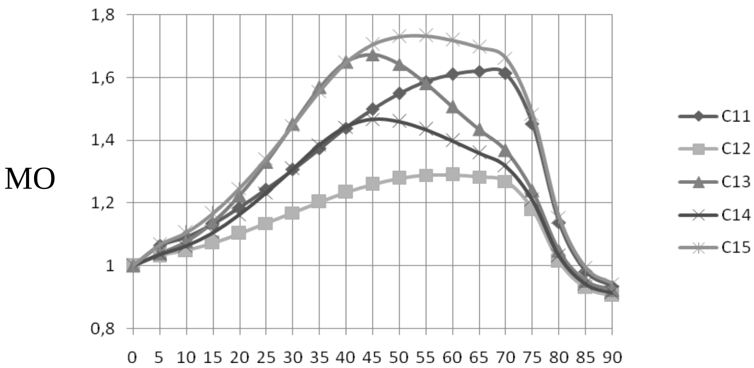
In this study, effect of different weighting factors on the optimal designs is investigated for simply supported four layered $(\theta/-\theta/-\theta/\theta)$ annular sector plates (r_1/r_o =0.1, h/ r_o =0.2, $\theta_o = 60^o$). In Table 1, fifteen different combinations of weighting factors is illustrated. In Figure 2, performance index vs. fibre orientation for different fifteen combinations of weighting factors can be seen. As seen, the weighting factor has a substantial effect on the multiobjective design. The maximum $(\mathrm{MODI})_{max}$ is obtained 2.30 and the optimum fibre orientation is obtained $\theta_{opt} = 70^\circ$ for C3 combination. This means that, the effect of axial unifom pressure buckling load is more dominant than thermal buckling load and fundamental frequency on the multiobjective design. On the other hand, the minimum $(\mathrm{MODI})_{max}$ occurs for C9 combination. That is, fundamental frequency causes minimum multiobjective design index for multiobjective design. In Table 2, the $(\mathrm{MODI})_{max}$ and the optimum fibre orientations are given for different fifteen combinations of weighting factors.



(a)



(b)



(c)

Fig. 2 Performance index vs. fibre orientation for different fifteen combinations of weighting factors

Table 1 Fifteen different combinations of weighting factors

Weighting factors	Combinations
η=0.25, ξ=0.75, μ=0	C1
η=0.50, ξ=0.50, μ=0	C2
η=1.00, ξ=0, μ=0	C3
η=0, ξ=1.00, μ=0	C4
η=0.75, ξ=0.25, μ=0	C5
η=0, ξ=0.75, μ=0.25	C6
η=0, ξ=0.50, μ=0.50	C7
η=0, ξ=0.25, μ=0.75	C8
η=0, ξ=0, μ=1.00	C9
η=0.75, ξ=0, μ=0.25	C10
η=0.50, ξ=0, μ=0.50	C11
η=0.25, ξ=0, μ=0.75	C12
η=0.25, ξ=0.50, μ=0.25	C13
η=0.25, ξ=0.25, μ=0.50	C14
η=0.50, ξ=0.25, μ=0.25	C15

Table 2. The $(MODI)_{max}$ and the optimum fibre orientations are given for different fifteen combinations of weighting factors

Combinations	$(MODI)_{max}$	$\theta_{opt}\,(^\circ)$
C1	1.879	45
C2	1.912	50
C3	2.300	70
C4	1.861	40
C5	2.039	60
C6	1.653	40
C7	1.445	40
C8	1.238	40
C9	1.033	35
C10	1.956	65
C11	1.620	65
C12	1.289	60
C13	1.672	45
C14	1.466	45
C15	1.732	55

In this study, effect of annularity (r_1 / r_o) on the optimal designs is investigated for simply supported four layered $(\theta/-\theta/-\theta/\theta)$ laminated annular sector plates (h/ r_o =0.2, $\theta_o = 60^o$, $\eta = 0.25$, $\xi = 0.25$, $\mu = 0.50$). In Figure 3, performance index vs. fibre orientation for different annularity is illustrated. As seen, performance index increases as the annularity increases. The maximum $(MODI)_{max}$ is obtained 5.14 for r_1 / r_o =0.9. On the other hand, the increase in the maximum performance index increases with increase in the annularity. In Table 3, the $(MODI)_{max}$ and the optimum fibre orientations are given for different r_1 / r_o ratios.

In this study, effect of sector angle (θ_o) on the the optimum results is investigated for simply supported four layered $(\theta/-\theta/-\theta/\theta)$ annular sector plates (r_1 / r_o =0.9, h/ r_o =0.2, $\eta = 0.25$, $\xi = 0.25$, $\mu = 0.50$). In Figure 4, performance index vs. fibre orientation for different sector angles can be seen. As seen, as the sector angle increases, the performance index decreases. It is more pronounced for larger fibre orientations. The maximum $(MODI)_{max}$ is obtained 1.912 for

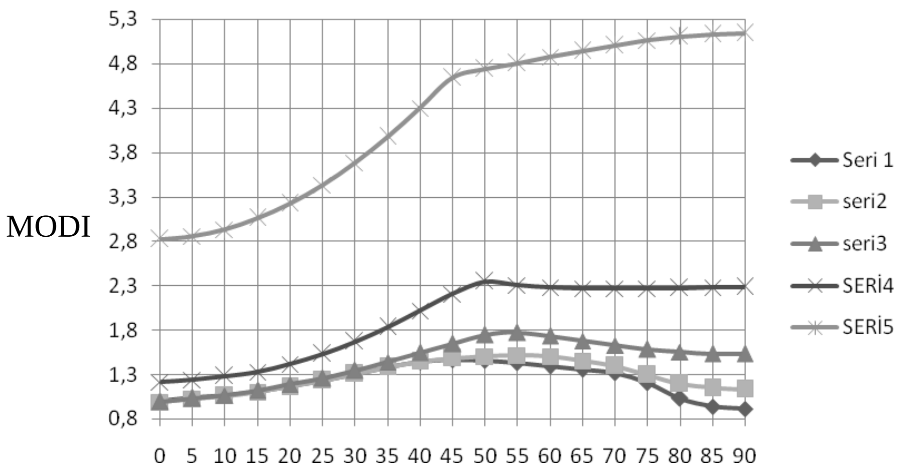


Fig. 3 Performance index vs. fibre orientation for different r_1 / r_o ratios

Table 3. The $(MODI)_{max}$ and the optimum fibre orientations are given for different r_1 / r_o ratios

r_1 / r_o	$(MODI)_{max}$	$\theta_{opt}(^\circ)$
0.1	1.467	45
0.3	1.515	55
0.5	1.775	55
0.7	2.347	50
0.9	5.140	90

$\theta_o = 30^\circ$. In Table 4, the $(MODI)_{max}$ and the optimum fibre orientations are given for different semi-cone angles. One can mention that sector angle has no important effect on the optimum fibre orientations for smaller sector angles.

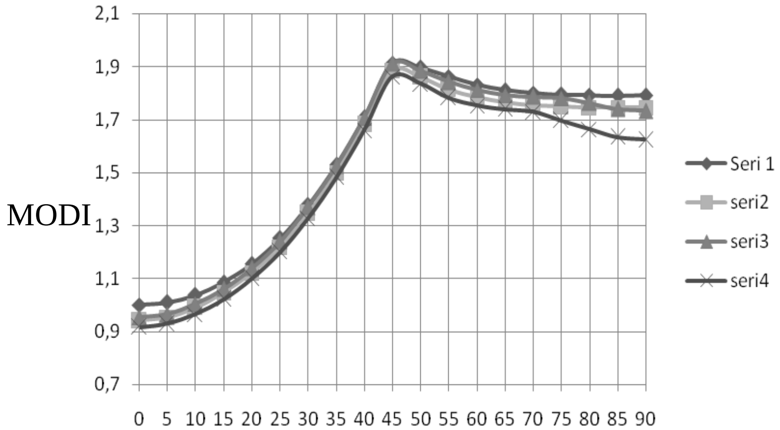


Fig. 4 Performance index vs. fibre orientation for different sector angles

Table 4 The $(MODI)_{max}$ and the optimum fibre orientations are given for different semi-cone angles

θ_o ($^\circ$)	$(MODI)_{max}$	θ_{opt} ($^\circ$)
30	1.912	45
60	1.885	45
90	1.910	45
120	1.782	55

In this study, effect of the boundary conditions on the optimum design is investigated for simply supported four layered $(\theta/-\theta/-\theta/\theta)$ annular sector plates (r_1/r_o =0.1, h/ r_o =0.2, $\theta_o = 60^o$, $\eta = 0.25$, $\xi = 0.25$, $\mu = 0.50$). The different combinations of free (F), simply supported (S) and clamped (C) boundary conditions are considered, viz. (SSSS), (CCCC), (CCSS), (CCFF). Boundary conditions means that the first two letters mean boundary conditions at the straight edges and the other two letters mean those at the curved edges. In Figure 5, performance index vs. fibre orientation for different boundary conditions is given. As seen, the maximum $(MODI)_{max}$ is obtained 1.769 for clamped boundary condition. On the other hand, the minimum $(MODI)_{max}$ is obtained for (CCFF) boundary condition. In Table 5, the $(MODI)_{max}$ and the optimum fibre orientations are given for different boundary conditions.

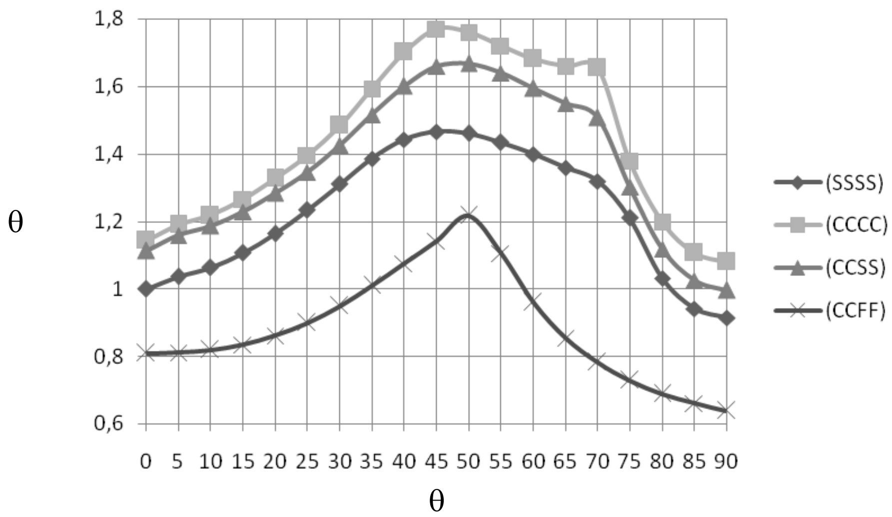


Fig. 5 Performance index vs. fibre orientation for different boundary conditions

Table 5 The $(\mathrm{MODI})_{max}$ and the optimum fibre orientations are given for different boundary conditions

Boundary conditions	$(\mathrm{MODI})_{max}$	$\theta_{opt}\,(^\circ)$
(SSSS)	1.467	45
(CCCC)	1.769	45
(CCSS)	1.668	50
(CCFF)	1.216	50

5. Conclusions

In this study, a multiobjective optimization is carried out for symmetrically laminated composite annular sector plates subjected to axial uniform pressure load and thermal load. The design objective is the maximization of the weighted sum of the critical buckling load and fundamental frequency. The design variable is the fibre orientations in the layers. Results are presented for different weighting factors, annularity, sector angle and boundary conditions. As seen from the results that, the weighting factor has a substantial effect on the multiobjective design. The effect of axial unifom pressure buckling load is more dominant than thermal buckling load and fundamental frequency on the multiobjective design. On the other hand, fundamental frequency causes minimum multiobjective design index for multiobjective design. Performance index increases as the annularity increases. The increase in the maximum performance index increases with increase in the annularity. As the sector angle increases, the performance index decreases. It is more pronounced for larger fibre orientations. The sector angle has no important effect on the optimum fibre orientations for smaller sector angles. The maximum $(\mathrm{MODI})_{max}$ is obtained for clamped boundary condition. Finally, it can be said from the results that, the multiobjective optimization can change the behavior of the laminates substantially. Therefore, all effects must be considered at the optimization stage of the laminates.

References

Ding, H.J. and Xu, R.Q., (2010), “Free axisymmetric vibration of laminated transversely isotropıc annular plates”, *J. Sound Vib*., **230**, 1031–1044.

Dumi, P.C., Dube, G.P. and Mallick, A., (2011), Axisymmetric postbuckling of moderately thick laminated annular plate using FSDT”, *Compos. Struct*., **51**(3), 311-318.

Houmat, A., 2008, “Large amplitude free vibration of shear deformable laminated composite annular sector plates by a sector p-element”, *Int. J. Non-Linear Mech*., **43**(9), 834–843.

Malekzadeh, P., 2009, “Three-dimensional free vibration analysis of thick laminated annular sector plates using a hybrid method”, *Compos. Struct*., **90**(4), 428–437.

Malekzadeh, P., Haghighi, M.R.G. and Gholami, M., (2010), “Dynamic response of thick laminated annular sector plates subjected to moving load”, *Compos. Struct*., **92**(1), 155–163.

Pawlus, D., (2011), “Solution to the problem of axisymmetric and asymmetric dynamic instability of three-layered annular plates”, *Thin-Walled Struct*., **49**(5), 660-668.

Seifi, R., Khoda-yari, N.and Hosseini, H., (2012), “Study of critical buckling loads and modes of cross-ply laminated annular plates”, *Compos. Part B: Eng*., **43**(2), 422-430.

Sharma, A., Sharda, H.B. and Nath, Y. (2005), “Stability and vibration of thick laminated composite sector plates”, *J. Sound Vib*., **287**(1-2), 1–23.

Singhatanadgid, P. and Ungbhakorn, V., (2005), “Scaling laws for buckling of polar orthotropic annular plates subjected to compressive and torsional loading”, *Thin-Walled Struct*., **43**(7), 1115-1129.

Xu, R.Q., (2008), “Three-dimensional exact solutions for the free vibration of laminated transversely isotropic circular, annular and sectorial plates with unusual boundary conditions”, *Arch. App. Mech*., **78**, 543–558.

CC