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Thermal buckling load optimization of laminated plates with different intermediate line supports

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Abstract. This paper deals with critical thermal buckling load optimization of symmetrically laminated four layered angle-ply plates with one or two different intermediate line supports. The design objective is the maximization of the critical thermal buckling load and a design variable is the fibre orientation in the layers. The first order shear deformation theory and nine-node isoparametric finite element model are used for the finite element solution of the laminates. The modified feasible direction (MFD) method is used for the optimization routine. For this purpose, a program based on FORTRAN is used. Finally, the numerical analysis is carried out to investigate the effects of location of the internal line supports, plate aspect ratios and boundary conditions on the optimal designs and the results are compared.

Keywords: laminated plates; intermediate line supports; thermal buckling load; modified feasible direction method; optimization.

1. Introduction

The laminated composite plate is one of the important structural elements, which is widely used in a variety of high performance engineering systems including aircraft, submarine, automotive, naval and space structures. When the plate is subjected to temperature change, thermally induced compressive stresses are developed in the constraint plate due to thermo-elastic properties and consequently buckling takes place. Thin plate structure becomes unstable at relatively lower temperature and buckles in the elastic region. Hence, thermal buckling represents an important parameter for consideration and plays the significant role in the design of the structures.

A considerable amount of literature exists on thermal buckling of laminated composite plates. For example, Shiau *et al.* (2010) studied thermal buckling behavior of composite laminated plates by making the use of finite element method. Lal *et al.* (2009) examined the effect of random system properties on thermal buckling load of laminated composite plates under uniform temperature rise. Vosoughi *et al.* (2011) investigated thermal postbuckling behavior of laminated composite skew plates. Akhras and Li (2010) extended the finite layer method to the thermal buckling analysis of piezoelectric antisymmetric angle-ply laminates. Rasid *et al.* (2011) improved thermal buckling and thermal post-buckling behaviours of laminated composite plates by embedding shape memory alloy wires within laminated composite plates. Ghomshei and Mahmoudi (2010) implemented differential quadrature

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method for analyzing the thermal buckling behavior of the symmetric cross-ply laminated rectangular thin plates subjected to uniform and/or non-uniform temperature fields. Yapici (2005) presented a thermal buckling analysis of symmetric and antisymmetric angle-ply laminated hybrid composite plates with an inclined crack subjected to a uniform temperature rise. Gilat and Aboudi (2006) studied the analysis of thermal buckling of rectangular active laminated composite plates. Liew *et al.* (2004) investigated thermal buckling and post-buckling analyses for moderately thick laminated rectangular plates that contain functionally graded materials and subjected to a uniform temperature change. Avci *et al.* (2005) presented thermal buckling analysis of symmetric and antisymmetric laminated composite plates containing a hole. Vosoughi *et al.* (2012) studied thermal buckling and postbuckling analysis of laminated composite beams with temperature-dependent material properties.

Research on the subject of thermal buckling load optimization has been reported by many investigators. Topal and Uzman (2008) investigated thermal buckling load optimization of laminated composite plates subjected to uniformly distributed temperature load. The objective function was to maximize the critical temperature capacity of laminated plates and the fibre orientation was considered as design variable. Spallino and Thierauf (2000) presented thermal buckling optimization of laminated composite plates subject to a temperature rise using evolution strategies. Singha *et al.* (2000) maximized buckling temperatures of graphite/epoxy laminated composite plates for a given total thickness considering fibre-directions and relative thicknesses of layers as design variables. Genetic algorithm was employed to optimize as many as ten variables for the five layered plates. Autio (2001) optimized behaviour of a laminated plate with given boundary temperatures and displacement constraints and the optimization problem was expressed in terms of lamination parameters. Mozafari *et al*. (2010) maximized thermal buckling loads of laminated composite plates for a given total thickness. Fibre directions and relative thickness of layers were considered as design variables. The imperialist competitive algorithm was employed to optimize as many as seven variables for the different layered plates. Chen *et al.* (2003) investigated design optimization for structural thermal buckling. The analysis of heat conduction, structural stress and buckling were considered at the same time in the design optimization procedure. The optimization model was constructed and solved by the sequential linear programming or sequential quadratic programming algorithm. Malekzadeh *et al.* (2012) applied the differential quadrature method in conjunction with the genetic algorithms to obtain the optimum buckling temperature of the laminated composite skew plates. Fares *et al.* (2004) presented a multiobjective optimization problem to determine the optimal layer thickness and optimal closed loop control function for a symmetric cross-ply laminate subjected to thermomechanical loadings. The optimization procedure aimed to maximize the critical combination of the applied edges load and temperature levels and to minimize the laminate dynamic response subject to constraints on the thickness and control energy. Lee *et al.* (1999) presented the design of a thick laminated composite plate subjected to a thermal buckling load under a uniform temperature distribution. In design procedures of composite laminated plates for a maximum thermal buckling load, golden section method was used as an optimization routine. Fares *et al.* (2005) presented design and control optimization to minimize the thermal postbuckling dynamic response and to maximize the buckling temperature level of composite laminated plates subjected to thermal distribution varying linearly through the thickness and arbitrarily with respect to the in-plane coordinates.

To the best of author's knowledge, critical thermal buckling load optimization of laminated composite plates with intermediate line supports has not been investigated yet. Therefore, in this study thermal buckling load optimization of symmetrically laminated plates with one or two different intermediate line supports is investigated to fill this gap. The design objective is the maximization of

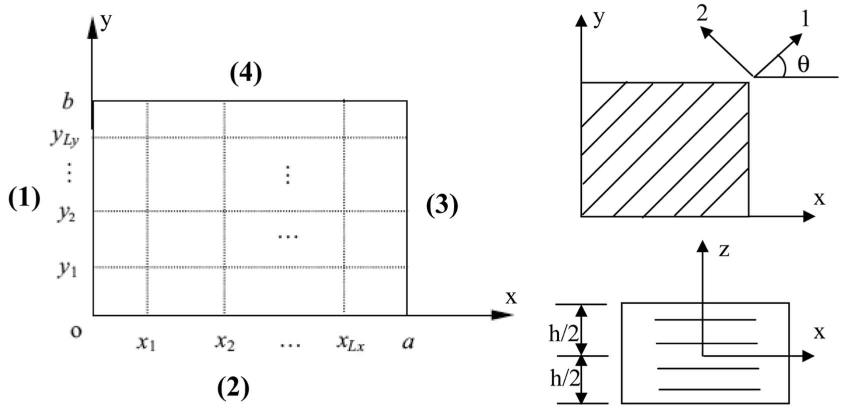

Fig. 1 Geometry of a rectangular laminated plate with internal line supports

the critical thermal buckling load and a design variable is the fibre orientation in the layers. The first order shear deformation theory and nine-node isoparametric finite element model are used for finding the finite element solution of the laminates. The modified feasible direction (MFD) method is used for the optimization routine. For this purpose, a program based on FORTRAN is used. Finally, the numerical analysis is carried out to investigate the effects of location of the internal line supports, plate aspect ratios and boundary conditions on the optimal designs and the results are compared.

2. Basic equations

Consider a laminated composite plate with intermediate line supports of uniform thickness h, having a rectangular plan axb as shown in Fig. 1. The individual layers are assumed to be homogeneous and orthotropic. There are L_x of internal line supports intersecting with the x-axis, while there are L_y of such intersecting with the y-axis. Further, laminated plate has internal line supports which require constraints of zero transverse displacement along the line supports.

Considering the first order shear deformation theory, the displacement fields are expressed as follows

$$\begin{aligned} u(x, y, z, t) &= u_o(x, y, t) + z\psi_x(x, y, t) \\ v(x, y, z, t) &= v_o(x, y, t) + z\psi_y(x, y, t) \\ w(x, y, z, t) &= w_o(x, y, t) \end{aligned} \tag{1}$$

where, u, v, and w are the displacements of a general point (x, y, z) in the plate space in the x, y, and z directions, respectively. The parameters u_o, v_o are the in-plane displacements and w_o is the transverse displacement of a point (x, y) on the laminate middle plane. ψ_x, ψ_y are the rotations of the transverse normal about y and x axes, respectively.

The in-plane strain components (ε_x, ε_y, γ_{xy}) and the transverse shear strains (γ_{xz}, γ_{yz}) can be written as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} + \psi_x \end{Bmatrix} \tag{2}$$

The equations for in-plane stresses (σ_x, σ_y, τ_{xy}) and the transverse shear stresses (τ_{yz}, τ_{xz}) of the *kth* layer for thermal effects can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{(k)} = \begin{Bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{Bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_{(k)} = \begin{Bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{Bmatrix}_{(k)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{4}$$

where $\bar{Q}_{ij}$ are the transformed reduced stiffnesses, α_x, α_y, α_{xy} are the coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

The stress resultants $\{N\}$, stress couples $\{M\}$ and transverse shear stress resultants $\{Q\}$ are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} zdz, \quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = K \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{5}$$

In Eq. (5), K is the shear correction factor. In this study, the shear correction factor is taken 5/6.

3. Finite element formulation

In this study, nine-node Lagrangian rectangular plate element which are five degrees of freedom (u, v, w, ψ_x, ψ_y) is used for the finite element solution of the laminates. The interpolation function of the displacement field is defined as

$$\begin{Bmatrix} u \\ v \\ w \\ \psi_x \\ \psi_y \end{Bmatrix} = \sum_{i=1}^{n} N_i d_i \tag{6}$$

where d_i and N_i are the nodal variables and the interpolation function, respectively. The discrete eigenvalue equation of the static buckling problem of laminates can be derived as

$$([K_b + K_s] - \lambda[K_g])\{u\} = 0 \tag{7}$$

where $[K_b]$, $[K_s]$ and $[K_g]$ are the bending stiffness, shear stiffness and geometric stiffness matrices, respectively. These matrices can be expressed as follows

$$[K_b] = \int_A [B_b]^T[D_b][B_b]dA \tag{8}$$

$$[K_s] = \int_A [B_s]^T[D_s][B_s]dA$$

$$[K_g] = \int_A [B_g]^T[D_g][B_g]dA$$

where

$$[D_b] = \begin{bmatrix} A_{ij} & 0 \\ 0 & D_{ij} \end{bmatrix},\ [D_s] = \begin{bmatrix} k_1^2 A_{44} & A_{45} \\ A_{45} & k_2^2 A_{55} \end{bmatrix},\ [D_g] = \begin{bmatrix} \bar{N}_1 & \bar{N}_{12} \\ \bar{N}_{12} & \bar{N}_2 \end{bmatrix} \tag{9}$$

A_{ij} and D_{ij} $(i, j = 1, 2, 6)$ denote extensional stiffnesses and bending stiffnesses, respectively. A_{ij} and D_{ij} can be calculated as follows

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(l, z^2)dz \qquad (i, j = 1, 2, 6) \tag{10}$$

$$(A_{44}, A_{45}, A_{55}) = \int_{-h/2}^{h/2} (\bar{Q}_{44}, \bar{Q}_{45}, \bar{Q}_{55})dz \tag{11}$$

In Eq. (9), k_1^2 and k_2^2 are the shear correction factors and $k_1^2 = k_2^2 = 5/6$ for the rectangular section. Calculating the critical buckling temperature of buckling due to thermal load is two stage processes. For a specified rise ΔT in temperature the thermal loads are computed and a linear static analysis is carried out to determine the thermal stress resultants. These stress resultants are then used to compute the geometric stiffness matrice, which subsequently used in Eq. (7), to determine the least eigenvalue, λ, and the associated mode shape. The critical buckling temperature, T_{cr}, is calculated as follows

$$T_{cr} = \lambda\Delta T \tag{12}$$

In this study, subspace iteration technique is applied to obtain the numerical solutions of the problem.

4. Optimization problem

The objective of the design problem is to maximize the critical thermal buckling load of the laminated plates with intermediate line supports. The fibre orientation is taken as design variable. The optimization of the problem is formulated as

Find: θ
Maximize: $(T_{cr})_{max} = \underset{\theta}{max} T_{cr}(\theta)$
Subjected to: $0^\circ \le \theta_k \le 90^\circ$, $\Delta\theta = 1^\circ$ (13)

Critical thermal buckling load, T_{cr}, for a given fibre orientation is determined from the finite element solution of the eigenvalue problems given by Eq. (12). The optimization procedure involves the stages of evaluating critical thermal buckling load and improving the fibre orientation θ to maximize T_{cr}. Thus, the computational solution consists of successive stages of analysis and optimisation until a convergence is obtained and the optimum fibre orientation, θ_{opt}, is determined within a specified accuracy. The modified feasible direction method is used as optimization routine (Topal and Uzman 2008).

5. Numerical Results and Discussion

5.1 Validation and convergence of the present study

In this study, first a convergence study is performed to determine the appropriate finite element mesh to be used in the thermal buckling load analysis of the laminated plate model. Four meshes are developed, with increasing numbers of elements in the x and y directions. In the numerical analysis, simply supported 4-layered cross ply-ply (0°/90°/0°/90°) laminated plates without intermediate line supports are investigated ($b/h = 10$). The material properties are given as below

$E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = G_{13} = 7.17$ GPa, $G_{23} = 2.39$ GPa, $\nu_{12} = 0.28$,
$\alpha_1 = 0.02\times10^{-6}\,{}^\circ C^{-1}$, $\alpha_2 = 22.5\times10^{-6}\,{}^\circ C^{-1}$

As it can be seen from Table 1, there is only a 0.18% difference between the loads calculated for mesh 15×15 and mesh 20×20. This indicates that mesh 20×20 is capable of performing the analysis within a reasonable degree of accuracy.

In this section, the convergence behavior and accuracy of the present study are investigated. A single thin ($b/h = 40$) clamped laminated square plate ($\theta = 45^\circ$) without intermediate line support is considered to compare the present study with the literature results. The material properties are given as below:

$E_1 = 76$ GPa, $E_2 = 5.5$ GPa, $G_{12} = G_{13} = 2.30$ GPa, $G_{23} = 1.5$ GPa, $\nu_{12} = 0.34$,

Table 1 Mesh convergence study of the present study for simply supported 4-layered cross ply-ply (0°/90°/0°/90°) laminated plates without intermediate line supports

Mesh	Critical Temperature T_{cr} (°C)
5×5	5.246×10^4
10×10	4.983×10^4
15×15	4.949×10^4
20×20	4.940×10^4

Table 2. Convergence study of the present study for a clamped square laminated plate without intermediate line support

Critical Temperature	Huang and Tauchert (1992)	Kabir *et al.* (2003)	Present study
T_{cr} (°C)	129.91	131.55	130.04

$\alpha_1 = -4\times10^{-6\circ}C^{-1}$, $\alpha_2 = 79\times10^{-6\circ}C^{-1}$

As it can be seen from Table 2, the results obtained for critical buckling temperature are in very close aggrement with the literature results.

5.2. Optimization problem

In this study, optimization problem is solved for 4-layered angle-ply symmetric ($\theta/-\theta/-\theta/\theta$) laminated plates with intermediate line supports. Each of the lamina is assumed to be same thickness. The optimization results are given for T300/5208 graphite/epoxy material. The material properties are given as below

$E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = G_{13} = 7.17$ GPa, $G_{23} = 2.39$ GPa, $\nu_{12} = 0.28$,
$\alpha_1 = 0.02\times10^{-6\circ}C^{-1}$, $\alpha_2 = 22.5\times10^{-6\circ}C^{-1}$

First, convergence of the present optimization study is compared with the literature results. Therefore, the optimum fibre orientations are obtained for maximum thermal critical temperature with 4-layered simply supported and clamped angle-ply laminated plates without intermediate line supports. It can be seen from Table 3 that, the results compare well with the literature results.

In this study, 53 different combinations of laminated plates with one or two intermediate line supports are used to investigate the optimization problems as shown in Fig. 2 ($b/h = 10$). The nondimensional thermal buckling load parameter is defined as

$$\bar{T}_{cr} = \alpha_o T_{cr} \times 10^3 \tag{14}$$

where $\alpha_o = 10^{-6}/^\circ C$

Optimization algorithm is applied to laminated plates with any combinations of simply supported (S),

Table 3. Convergence of the optimum fibre orientations for different boundary conditions with literature results

Simply supported			Clamped		
Walker *et al.* (1997)	Singha *et al.* (2000)	Present study	Walker *et al.* (1997)	Singha *et al.* (2000)	Present study
45.1	45.0	45.0	54.3	52.9	54.0

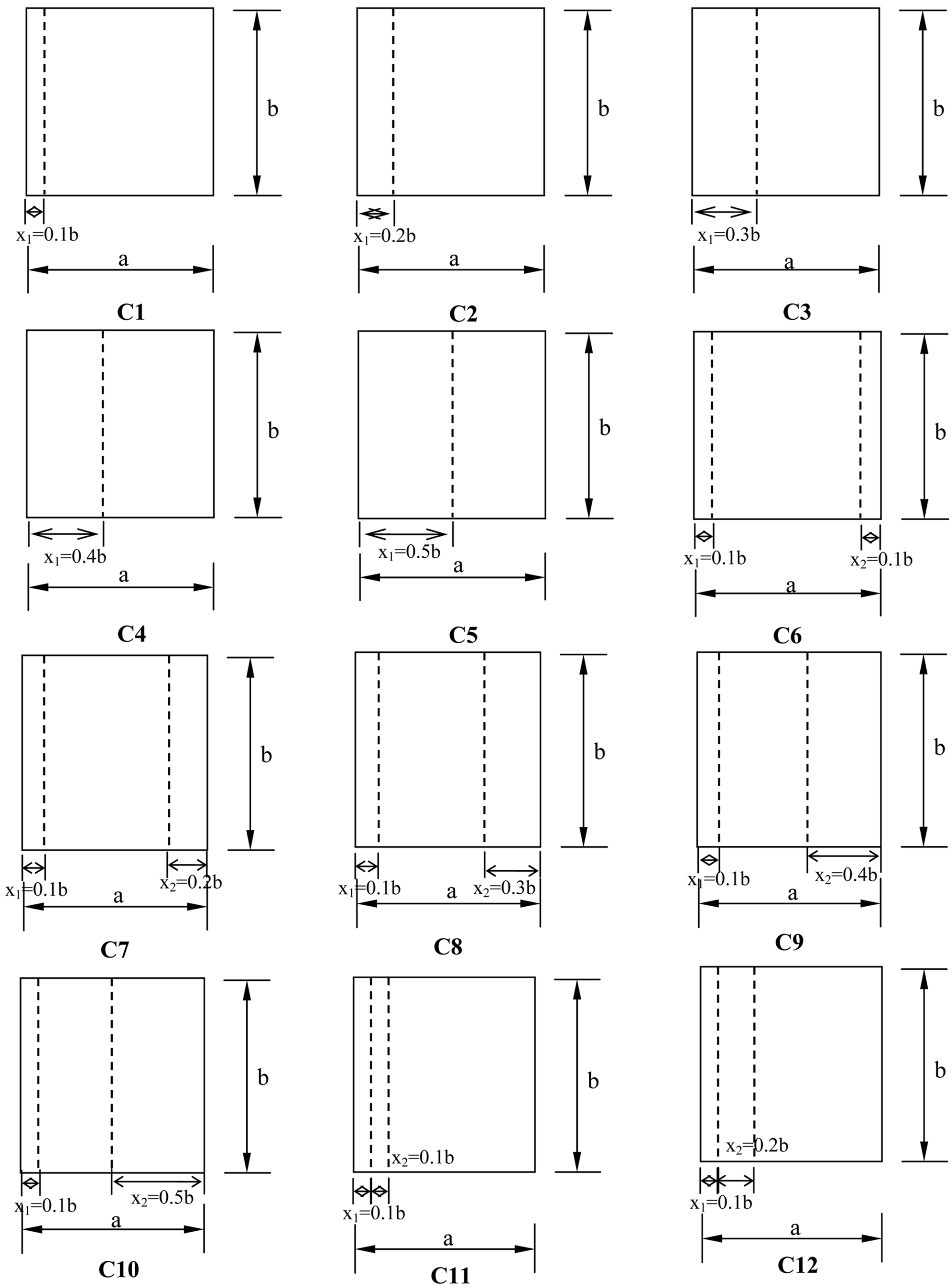

Fig. 2 53 different combinations of laminated plates with one or two intermediate line supports

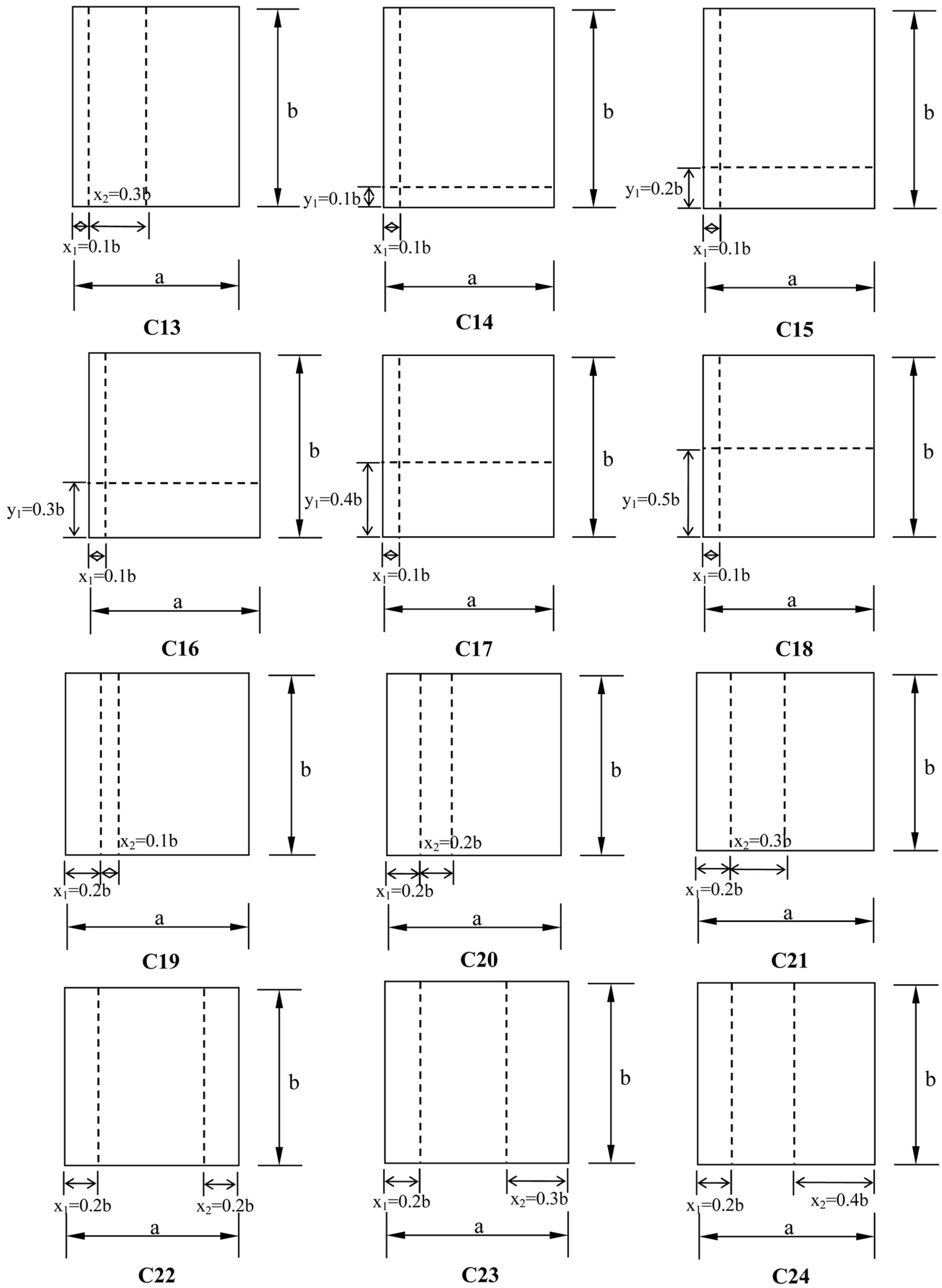

Fig. 2 53 different combinations of laminated plates with one or two intermediate line supports (Continued)

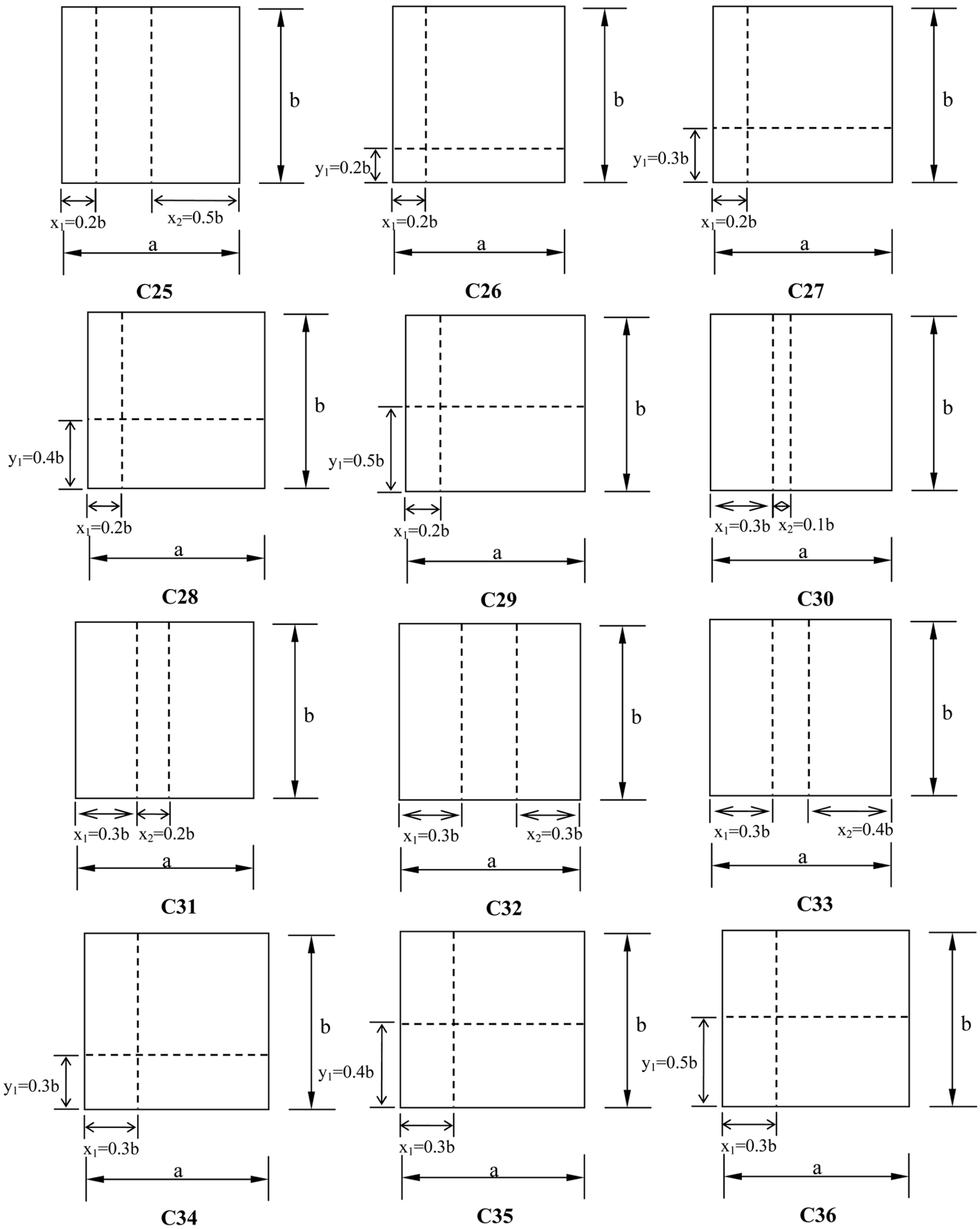

Fig. 2 53 different combinations of laminated plates with one or two intermediate line supports (Continued)

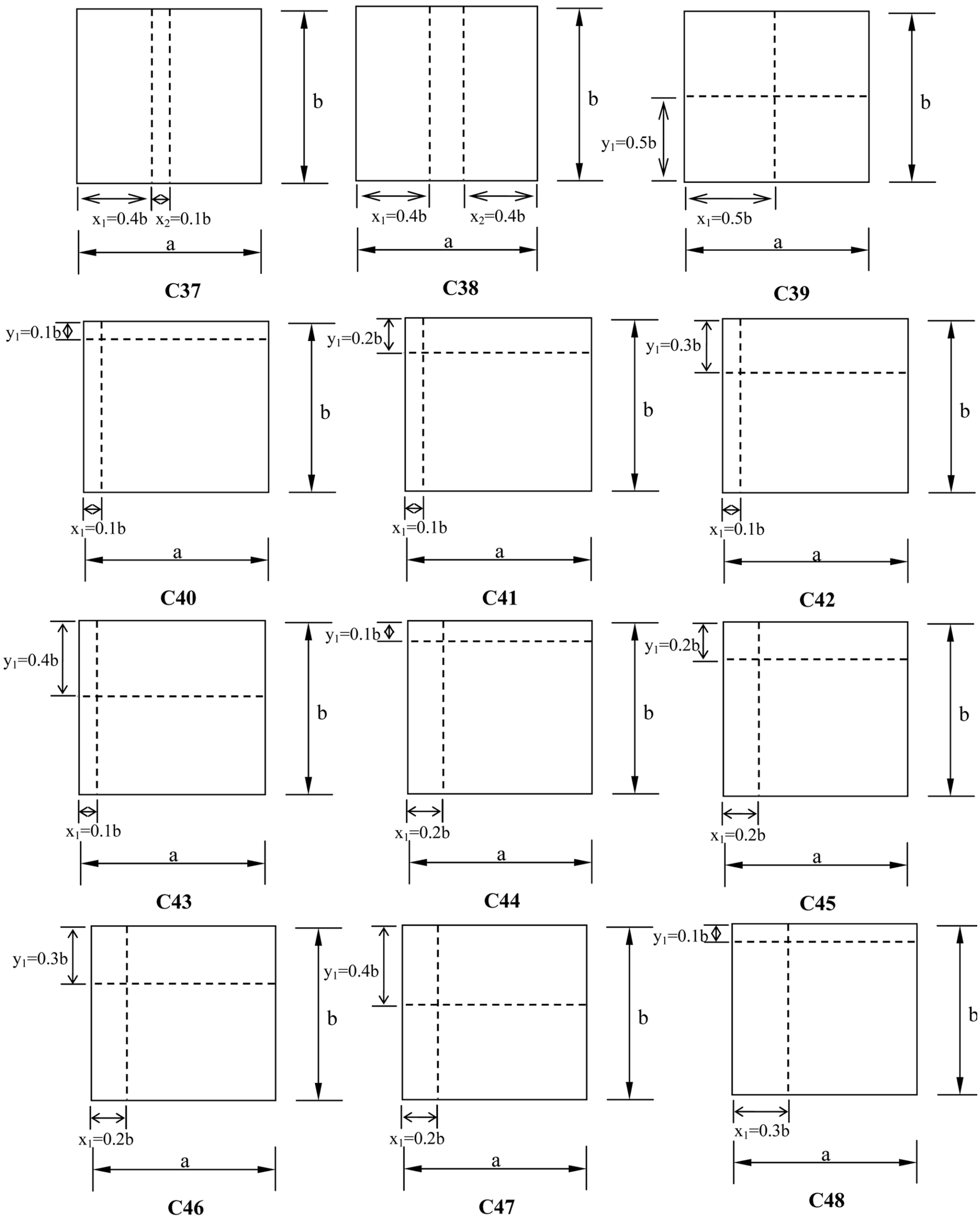

Fig. 2 53 different combinations of laminated plates with one or two intermediate line supports (Continued)

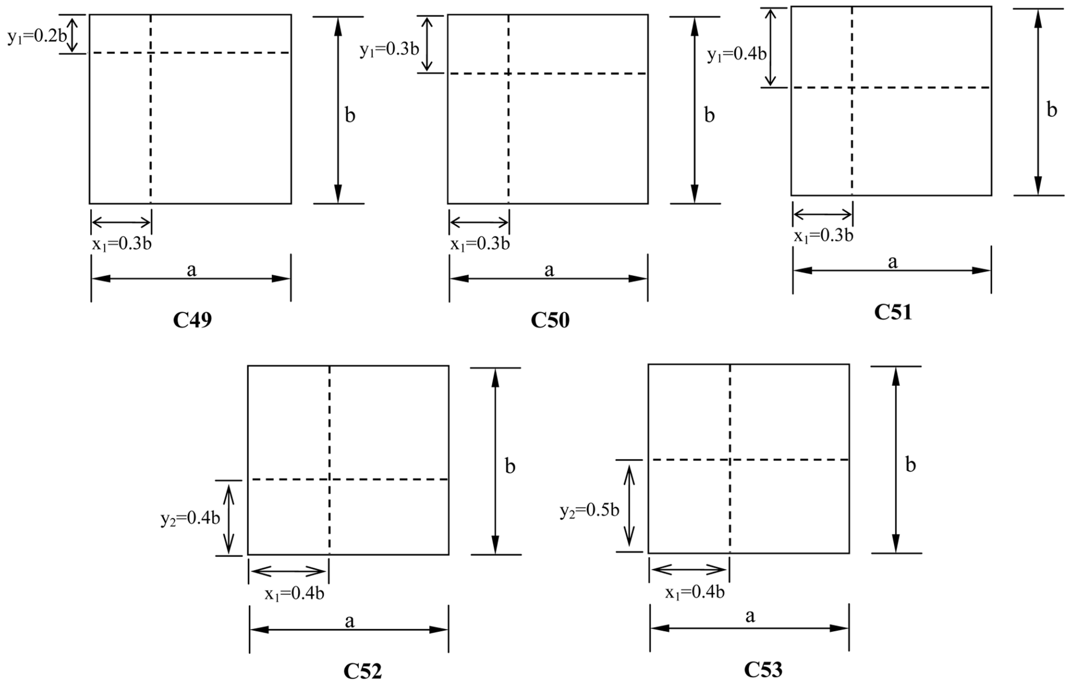

Fig. 2 53 different combinations of laminated plates with one or two intermediate line supports

clamped support (C), and free edge (F). Different combinations of the boundary conditions are considered in this study. For example, a clamped-simply supported-clamped-simply supported (CSCS) is a specimen with clamped supported on $x = 0$ and $x = a$, and simply supported on $y = 0$ and $y = b$, respectively. In Fig. 3, effect of the position of one intermediate line support on the critical thermal buckling load for square laminated plates for different boundary conditions is investigated. The location of one intermediate support is shifted from left ($x_1 = 0.1b$) to the center ($x_1 = 0.5b$) for laminated square plates

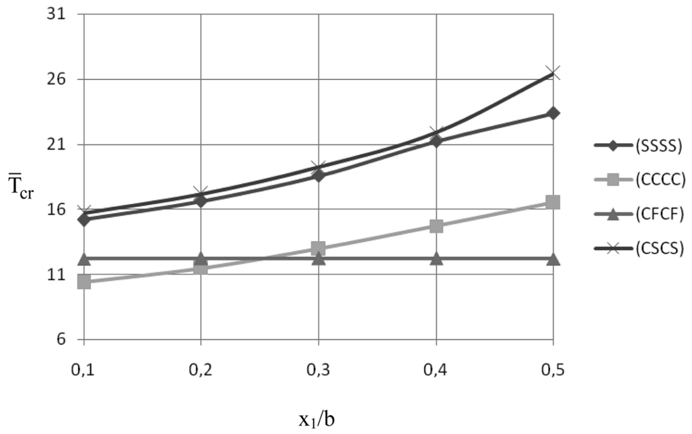

Fig. 3 Effect of the position of one intermediate line supports on the thermal buckling load for square laminated plates for different boundary conditions

Table 4 Optimum fibre orientations for different one intermediate line supports for different boundary conditions

Boundary conditions	θ_{opt} (°)				
	C1	C2	C3	C4	C5
(SSSS)	45	45	45	45	45
(CCCC)	40	38	38	39	38
(CFCF)	0	0	0	0	0
(CSCS)	39	39	39	39	38

Table 5 Effects of one and two intermediate line supports on the critical thermal buckling load for simply supported laminated square plates ($b/h = 10$)

Combinations	$\bar{T}_{cr}$	Combinations	$\bar{T}_{cr}$
C1	15.180	C27	20.766
C2	16.635	C28	23.177
C3	18.585	C29	24.951
C4	11.661	C30	21.977
C5	23.375	C31	26.255
C6	17.043	C32	26.255
C7	18.773	C33	26.206
C8	21.021	C34	26.725
C9	23.871	C35	31.626
C10	24.757	C36	22.492
C11	16.692	C37	24.895
C12	18.711	C38	26.347
C13	21.500	C39	30.298
C14	16.473	C40	16.308
C15	17.772	C41	17.524
C16	19.555	C42	19.279
C17	22.001	C43	21.756
C18	23.982	C44	17.524
C19	18.828	C45	18.570
C20	21.752	C46	20.159
C21	25.575	C47	22.516
C22	20.757	C48	19.279
C23	23.310	C49	20.159
C24	26.350	C50	21.550
C25	25.575	C51	23.712
C26	19.021	C52	27.292
		C53	28.199

(C1, C2, C3, C4 and C5). As it can be seen from Fig. 3, critical thermal buckling load increases when the internal line support moves from the plate edge to the plate center for (SSSS), (CCCC) and (CSCS) boundary conditions. On the other hand, critical thermal buckling load is almost constant for (CFCF) boundary condition. It can be said that, locating the internal line support at the plate center generally maximizes critical thermal buckling load, a considerable significance in engineering practice for laminated plate with one intermediate line support. The maximum critical thermal buckling load occurs for $x_1 = 0.5b$ for (CSCS) boundary condition, whereas the minimum critical thermal buckling load occurs for $x1 = 0.1b$ for (CCCC) boundary condition. In Table 4, the optimum fibre orientations are given for different intermediate line supports for different boundary conditions.

In Table 5, effects of one and two intermediate line supports on the critical thermal buckling load are investigated for simply supported laminated square plates. As it can be seen from Table 5, the maximum critical thermal buckling load occurs for laminated plates with two intermediate supports for C35. In Table 6, effects of one and two intermediate line supports on the optimum fibre orientations are given. As it can be seen, the effect of intermediate line supports on the optimum fibre orientations can be

Table 6 Effects of one and two intermediate line supports on the optimum fibre orientations for simply supported laminated square plates

Combinations	θ_{opt} (°)	Combinations	θ_{opt} (°)
C1	45	C27	45
C2	45	C28	45
C3	45	C29	45
C4	45	C30	45
C5	45	C31	45
C6	44	C32	45
C7	44	C33	44
C8	44	C34	45
C9	44	C35	45
C10	45	C36	45
C11	45	C37	45
C12	45	C38	45
C13	45	C39	45
C14	45	C40	45
C15	45	C41	45
C16	45	C42	45
C17	45	C43	45
C18	45	C44	45
C19	45	C45	45
C20	45	C46	45
C21	45	C47	45
C22	44	C48	45
C23	44	C49	45
C24	44	C50	45
C25	45	C51	45
C26	45	C52	45
		C53	45

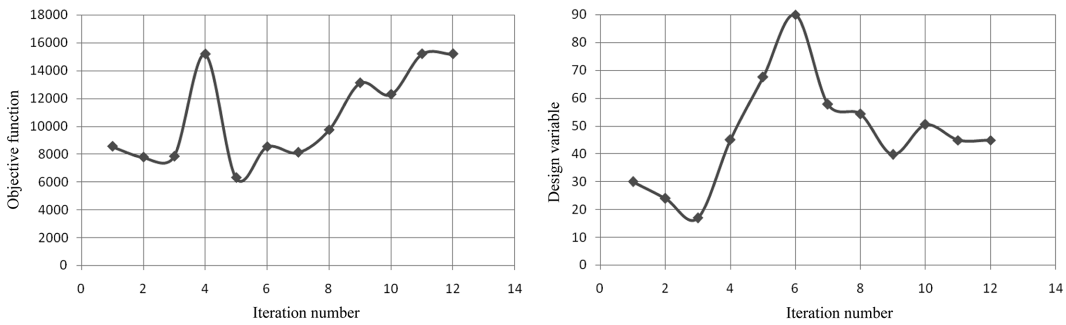

Fig. 8 Iteration histories of simply supported laminated plates for C1

negligible.

In Fig. 8, iteration histories are given at the end of the optimization process of simply supported laminated plates for C1.

In this study, effect of plate aspect ratio (a/b) on the optimum results is investigated for simply supported laminated plates for C35 (b/h = 10). As it can be observed from Fig. 9, the maximum critical thermal buckling load occurs for square laminated plates. On the other hand, the critical thermal buckling load reaches at a constant value for larger plate aspect ratios. In Table 7, effect of plate aspect ratio on the optimum fibre orientations is illustrated. As it can be seen, the optimum fibre orientations $\theta_{opt} = 0^o$ for $a/b > 1$.

6. Conclusions

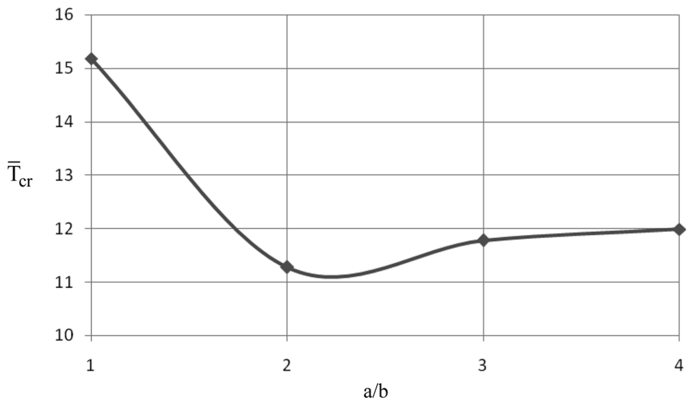

Fig. 9 Effect of plate aspect ratio on the critical thermal buckling load

Table 7 Effect of plate aspect ratio on the optimum fibre orientations

a/b	θ_{opt} (°)
1	45
2	0
3	0
4	0

This paper deals with critical thermal buckling load optimization of symmetrically laminated four layered angle-ply plates with one or two different intermediate line supports. The design objective is the maximization of the critical thermal buckling load and a design variable is the fibre orientation in the layers. 53 different combinations of laminated plates with one or two intermediate line supports are used to investigate optimization problems. The location parameter can be optimized to improve the critical thermal buckling load. The optimal location of the internal line support in increasing the critical thermal buckling load varies from plate to plate. Thus for laminated plates, locating the internal line support at the plate center generally maximizes the critical thermal buckling load, a considerable significance in engineering practice for laminated plate with one intermediate line support. As plate aspect ratio increases, critical thermal buckling load decreases because of diminishing of the rigidity of the plate. Plate aspect ratio has no effect on the optimum fibre orientations for $a/b > 1$. This problem can be investigated for more internal supports and different parameters and the results can be compared.

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