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Improving a current method for predicting walking-induced floor vibration

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Abstract. Serviceability rather than strength is the most critical design requirement for vibration-vulnerable floor constructions. Annoying vibrations due to normal walking activity have been observed more frequently on long-span lightweight floor systems in office and commercial retail buildings, raising the need for the development of floor vibration design procedures. This paper highlights some limitations of one of the most commonly used guidelines AISC/CISC DG11, and proposes improvements to this method. Design charts and approximate closed form formulas to estimate the walking response are developed in which various factors relating to the dynamic characteristics of both the floor and the excitation are considered. The accuracy of the proposed formulas and other proposals found in the literature is examined. The proposed modifications would be significant, especially with long-span floors where vibration levels may be underestimated by the current design procedure. The application of the proposed prediction method is illustrated by worked examples that reveal a good agreement with results obtained from finite element analyses and experiments. The presented work would enhance the accuracy and maintain the simplicity and convenience of the design guideline.

Keywords: composite floors; floor vibrations; serviceability; steady state; resonant; walking force; harmonic.

1. Introduction

Modern floor systems are now designed and constructed with longer spans owing to the need for larger column-free spaces in office and commercial retail buildings. The advances in high-strength materials and light weight construction technologies are also altering the dynamic characteristics of the floor systems. Annoying floor vibrations can occur in a system that is in perfect condition from a strength perspective. Serviceability is thus becoming a critical design requirement.

Several procedures for evaluating walking-induced floor vibrations have been developed. They normally provide separate approaches for low frequency floors in which resonance may cause severe vibration amplification and high frequency floors where resonance becomes less important compared with transient response. The threshold natural frequency to distinguish between low and high frequency floors is about 9-10 Hz. In the UK, there is a widely recognised design guide for floor vibration assessments recommended by the Steel Construction Institute (SCI) P073 (Wyatt 1989) and more

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recently P354 (Smith *et al.* 2009). Researchers from Arup developed a methodology based on the well-established principles of modal analysis for predicting footfall-induced vibration in floors and bridges (Willford *et al.* 2005, Willford *et al.* 2007). Their method has been adopted in the CCIP-016 Design Guide published by the UK Concrete Society (Willford and Young 2006). The American Institute of Steel Construction (AISC) and Canadian Institute of Steel Construction (CISC) Design Guide 11 has provided simplified technique for vibration evaluation of floors and footbridges (Murray *et al.* 2003). The AISC/CISC DG11 method is commonly used in North America, Australia and some other countries. For assessment of footfall induced vibration in low frequency floors, the aforementioned documents involve the determination of the peak or root-mean-square acceleration response at resonance, which is then checked against recommended thresholds for human comfort. On the other hand, design procedures developed through European research projects lead to the estimation of floor velocity as a function of floor modal mass, natural frequency and damping ratio from which the associated perception class can be determined (European Commission 2006, Hechler *et al.* 2008).

Given the wide international popularity of the AISC/CISC DG11 and its simplicity for hand calculation, this paper suggests some modifications that improve the prediction of peak acceleration of low frequency floors due to walking.

The following section summarises the procedure outlined in the AISC/CISC DG11 for evaluating walking-induced floor vibration and discusses the need for some modifications.

2. Design for walking excitation using AISC/CISC DG11 method

Using DG11, the floor is idealised as a SDOF system subjected to a simple harmonic force, $F(t)$, as defined by Eq. (1) and its peak acceleration, a_p, can be obtained using Eq. (2)

$$F(t) = \alpha_i P \cos(2\pi i f_p t) \tag{1}$$

$$a_p = \frac{\alpha_i R P}{2\zeta m} \tag{2}$$

where P is the walker weight taken as 0.7 kN, α_i is the dynamic coefficient for the i-th harmonic component of the walking excitation, f_p is the step frequency (pacing rate), ζ is the modal damping ratio and m is the modal mass of the floor. The forcing frequency is selected to match the fundamental frequency of the floor, i.e., $if_p = f_n$ in which f_n is the floor natural frequency. Common forcing frequencies and dynamic coefficients are given in Table 1. As full steady-state resonance may not be achieved for walking, a reduction factor R is introduced with a value of 0.7 for footbridges and 0.5 for floor structures with two-way mode shape configurations, respectively.

Table 1 Forcing frequencies and dynamic coefficients

Harmonic, i	if_p (Hz)	α_i
1	1.6~2.2	0.5
2	3.2~4.4	0.2
3	4.8~6.6	0.1
4	6.4~8.8	0.05

When the dynamic coefficient is approximately related to the frequency using Eq. (3), a simplified design criterion in the form of Eq. (4) is obtained (Murray *et al.* 2003). A constant force, P_0, is taken as 0.29 kN (0.83 × 0.5 × 0.7) for floors and 0.41 kN (0.83 × 0.7 × 0.7) for footbridges.

$$\alpha_i = 0.83\exp(-0.35 i f_p) \tag{3}$$

$$\frac{a_p}{g} = \frac{P_0\exp(-0.35 f_n)}{\zeta W} \le \frac{a_0}{g} \tag{4}$$

Formulas for hand calculation of the natural floor frequency, f_n, and effective weight, W, of simple floor structures are included in the DG11. Peak acceleration as a fraction of the acceleration of gravity, a_p/g, is checked against acceleration limits for human comfort, a_0/g, shown in Fig. 1. Recommended acceleration thresholds relate to various human activities in different environments. For example, peak tolerable acceleration of 0.5% g is typically used for offices with frequency ranging from 4 to 8 Hz while that for outdoor footbridges is 5% g, i.e., ten times higher.

Although the AISC/CISC DG11 method with Eq. (4) is rather simple and convenient for the practising engineer to perform a quick evaluation of floor vibration level, it does not always lead to a conservative design. Annoying vibrations have been found in floor systems that are predicted to be acceptable by this procedure. Marks (2010) has reported the inefficiency of the AISC/CISC DG11 method when comparing predicted footfall induced floor vibrations with measured data obtained from three actual office floors at two commercial buildings in Melbourne (Australia) with spans ranging from 12 to 15 m. Walking tests were performed on the floors from which the measured peak responses were found to be about 1.8 times higher than those estimated using the DG11. The authors also found the same situation observed on another office floor of steel-concrete composite construction as described later in Section 6.2 of this paper.

One of the key reasons leading to an underestimation of floor response may be attributed to the low values for the reduction factor R. The DG11 was calibrated using floors constructed several decades ago. As modern floor systems have longer spans than those in the past, walking paths may be long

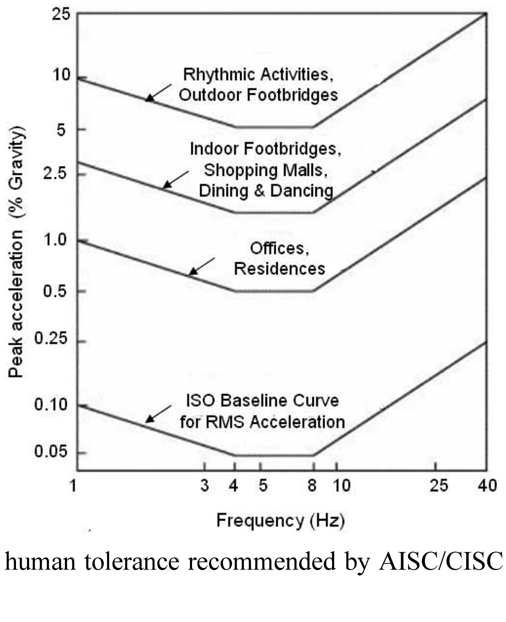


Fig. 1 Peak acceleration for human tolerance recommended by AISC/CISC DG11 (Murray *et al.* 2003)

enough for the vibration energy to build up and hence steady state vibration can in fact occur. The following sections will present an in-depth study of various factors affecting the reduction factor, including floor span, floor natural frequency and step frequency, damping ratio, and contribution from different harmonic components of the walking excitation. Moreover, because the R value may be greater than 1 when several harmonic components are considered together as in Section 3, the term "steady state" factor will now be used instead of "reduction factor".

3. Numerical determination of steady state factor for floor response to walking

3.1 Moving-walk forcing functions

The modelling of stationary walks, i.e., walking on the same spot, is first discussed. When only one harmonic component of the walking excitation that matches the floor frequency is considered, the stationary walk forcing function is given by Eq. (1). A more sophisticated approach involves the inclusion of the first few harmonic components whereby a Fourier series can be used to represent the walking force in the form of Eq. (5)

$$F(t) = P\left[1 + \sum_{i=1}^{4} \alpha_i \cos(2\pi i f_p t + \phi_i)\right] \tag{5}$$

where P, i, a_i, and f_p are defined as for Eq. (1) and ϕ_i is the phase lag (Murray *et al.* 2003). The dynamic coefficient values for different harmonics are given in Table 1. Phase angles can be taken as 0 for the first harmonic and $\pi/2$ for other harmonics (Bachmann and Ammann 1987).

The movement of the walker can now be taken into account by incorporating the mode shape values into the stationary walking force of Eqs. (1) and (5). Floor modal displacement along the walking path (span) with a length L is assumed to follow the mode shape configuration of a simply supported beam in the form of a half-sine function as shown in Fig. 2, which can be included into the stationary forcing functions (Heinemeyer *et al.* 2009). If the walking speed v_p is assumed to be constant along the path, then the coordinate z in Fig. 2 can be replaced by $v_p t$. These manipulations result in forcing functions of Eqs. (6) and (7) for a single-harmonic force and multi-harmonic force, respectively

$$F(t) = \alpha_i P \cos(2\pi i f_p t) \sin\left(\frac{\pi v_p}{L}t\right) \tag{6}$$

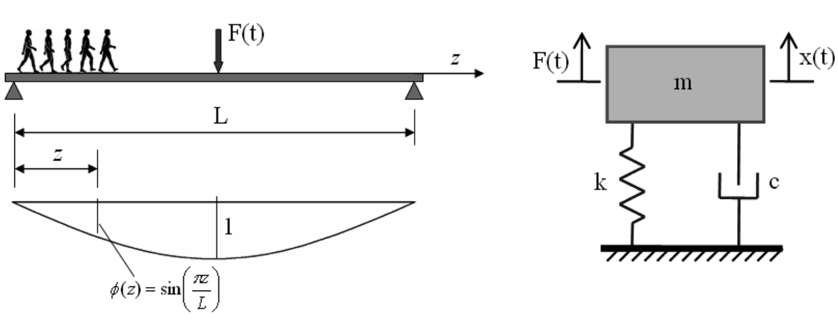


Fig. 2 Simplified mode shape and SDOF model for a beam-like structure subjected to walking excitation

$$F(t) = P\sum_{i=1}^{4}\alpha_i\cos(2\pi i f_p t + \phi_i)\sin\left(\frac{\pi v_p}{L}t\right) \tag{7}$$

It should be noted that the static weight was subtracted from Eq. (7) so that only the dynamic variation in forces is used for analysis. From the relationship between the walking speed, v_p, and step frequency, f_p, given in a tabulated form in Bachmann & Ammann (1987), a curve fitting function for this can be derived as

$$v_p = 1.6667f_p^2 - 4.8333f_p + 4.5 \tag{8}$$

from which v_p in Eqs. (6) and (7) can now be replaced by Eq. (8) in terms of f_p. Typical time traces for forcing functions of Eqs. (6) and (7) are shown in Fig. 3 in which the force is normalised against the walker weight P. This figure represents a walking activity at a normal pacing rate of 2 Hz and walking speed of 1.5 m/s on a walking path of 12 m length.

3.2 Steady state factor

To determine the steady state factor R, we first calculate the peak acceleration a_p due to a person walking from one end of the floor span to the other. The a_p value is then divided by the steady state acceleration a_s to obtain R.

The floor is idealised as a SDOF system with the governing equation of motion as

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{9}$$

where x, m, c, k are the displacement, modal mass, damping coefficient and stiffness of the floor, respectively. Solutions can be found for the cases of single-harmonic force given by Eq. (6) and multi-harmonic force given by Eq. (7). A comparison of the response to these two forcing cases would allow evaluation of the effect of non-resonant harmonic components on the overall floor response. This effect is ignored by the AISC/CISC DG11 design procedure.

A numerical integration method was used to solve Eq. (9) for a_p, and a total of 1620 solutions, for either single- or multi-harmonic cases, were found for L varying between 5 and 40 m, ζ between 1% and 5%, and f_n ranging between 3.2 and 8.8 Hz. The step frequency f_p varied between 1.6 and 2.2 Hz, covering pacing rates from slow walk to quite fast walk. Examples of resultant time traces for the

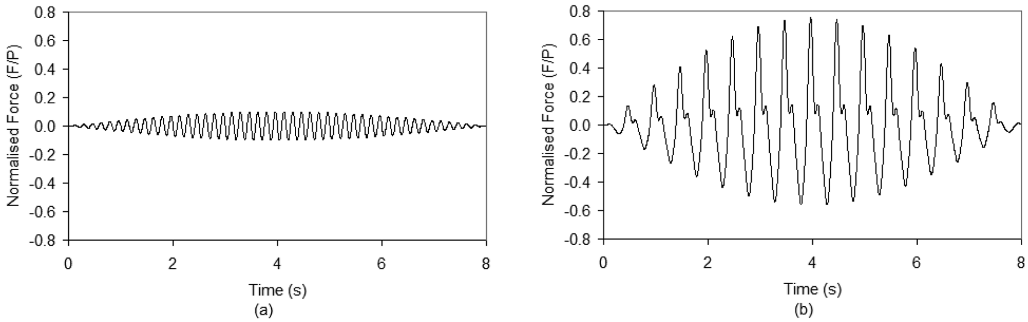


Fig. 3 Typical walking force time history: (a) single harmonic, and (b) multi harmonics

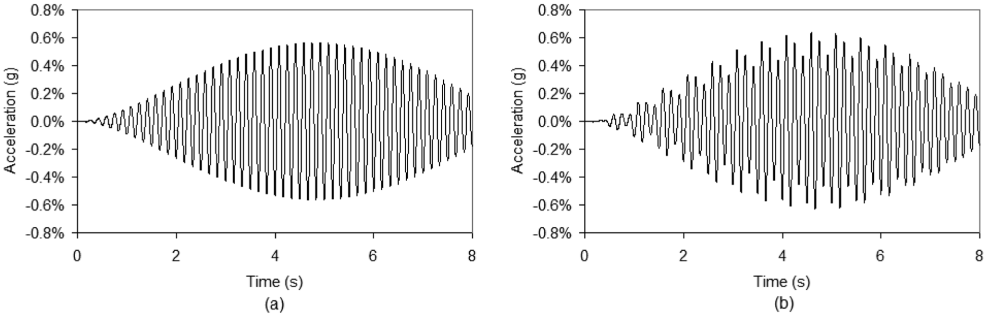


Fig. 4 Typical acceleration time history due to: (a) single harmonic force, and (b) multi-harmonic force

acceleration response obtained from solving Eq. (9) are shown in Fig. 4 for both the single and multi harmonic force cases.

The maximum steady state acceleration a_s of the floor subjected to a simple harmonic force at resonance, given by Eq. (1), can be computed by using the following classical formula (Clough and Penzien 1993)

$$a_s = \frac{\alpha_i P}{2\zeta m} \tag{10}$$

Once a_p and a_s have been calculated, the R value can be determined as the ratio of a_p to a_s. It should be noted that the same a_s value (due to a stationary single harmonic force) is used as a "yardstick" when evaluating the R factor for both the single and multi harmonic force cases. A number of design charts where the R value can be readily determined as a function of the floor span, floor frequency and damping are developed. Examples of design charts for the cases of 6-Hz floors and 7-Hz floors are shown in Fig. 5. In order to further facilitate the design work, closed-form expressions of the steady

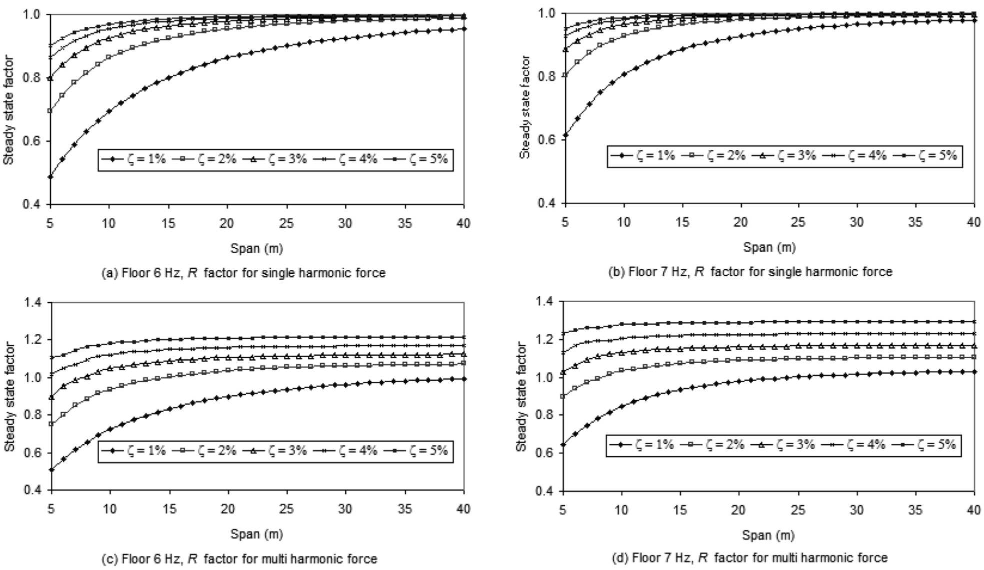


Fig. 5 Steady state factor

state factor R will be developed in Section 4.

3.3 Discussions

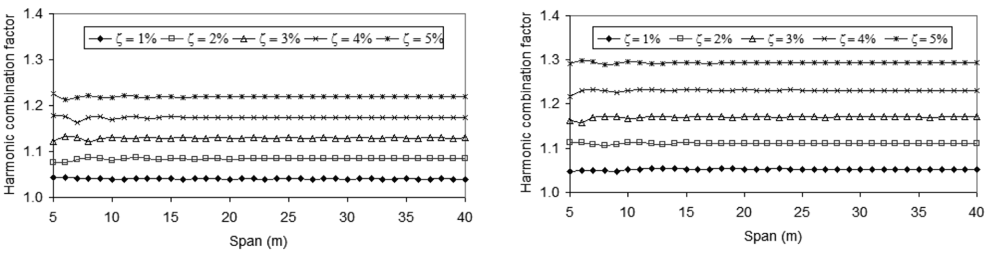
For single-harmonic force, the R factor is, as expected, less than or equal to unity. However, it can be well greater than the conventional values suggested by the AISC/CISC DG11. Several factors affecting the R factor have been identified. The longer the span, the greater the R factor. For instance, R will be 0.69 and 0.89 for spans of 5 m and 12 m, respectively, for a floor with a natural frequency of 6 Hz and damping ratio of 2% excited by a single harmonic force resonant with the floor. Large damping ratios accelerate the occurrence of a steady state motion. For example, to reach an R value of 0.95, a 6-Hz-frequency floor subjected to a single harmonic force needs a span of 38 m, 13 m and 8 m if floor damping ratios are 1%, 3% and 5%, respectively. Different floor natural frequencies result in different R values for the same step frequency as in the case of $f_p = 2$ Hz for $f_n = 4$, 6, and 8 Hz.

When multiple harmonic forcing components are combined, the resultant R value could exceed unity as can be seen in Figs. 5(c) and (d). Therefore, the peak acceleration of long span floors due to multiple harmonics can be greater than the steady state acceleration due to only one resonant harmonic that matches the floor natural frequency. We can now define a harmonic combination factor, g, as the ratio of the R value due to excitations with multiple harmonics to that due to excitations with the single resonant harmonic only. Plots of g are shown in Fig. 6 for the 6-Hz and 7-Hz floors where resonance would occur at the third harmonic for the former floor and fourth harmonic for the latter one. It is demonstrated interestingly that the g value seems to be generally constant for each frequency and damping ratio when L exceeds about 10 m. Compared with the one-harmonic excitation case, an increase in response of up to 30% can be expected when several harmonics are included in the walking force function. For design convenience a closed form expression for g will be developed in Section 4.

4. Closed form expressions for steady state factor

4.1 Proposal for approximate closed form expressions for R

For the case of single harmonic force, numerical results obtained from Section 3, as typically shown in Figs. 5(a) and (b), and relevant literature (Clough and Penzien 1993, Frýba 1999, Frýba 2009, Ricciardelli and Briatico 2011) suggest that the maximum response to a single harmonic force can approach the steady state response when there are sufficient damping and loading cycles applied to the



(a) γ factor for Floor 6 Hz (b) γ factor for Floor 7 Hz

Fig. 6 Harmonic combination factor

floor. A parameter ε is now defined as Eq. (11), similar to that used in Ricciardelli and Briatico (2011), which is the multiplication of the damping ratio ζ and number of loading cycles n given by Eq. (12)

$$\varepsilon = n\zeta \tag{11}$$

$$n = \frac{2Lf_n}{v_p} \tag{12}$$

The occurrence of a resonance condition is assumed in Eq. (12), i.e., the forcing frequency coincides with the floor frequency f_n so the wavelength of the harmonic force is v_p/f_n.

Attempts have been made to create an approximate closed form expression for the R factor as a function of ε, for the case of singe harmonic force. By curve fitting 1620 ε-R samples obtained from numerical results of Section 3, the authors propose an expression for R as given by Eq. (13)

$$R = -0.0015\ln^5\varepsilon + 0.0119\ln^4\varepsilon - 0.0188\ln^3\varepsilon - 0.0695\ln^2\varepsilon + 0.2604\ln\varepsilon + 0.7570 \tag{13}$$

in which $\ln\varepsilon$ is the natural logarithmic of ε.

To examine the quality of the proposed curve fitting function, the R values from numerical results are plotted against those calculated using Eq. (13), as can be seen in Fig. 7(a). The proposed formula shows very good accuracy with almost all R-R values fitted on a straight line. This formula is further validated by studying the cumulative probability distribution of error, as shown in Fig. 8(a). The error defined here is the ratio of the absolute difference between the R values obtained from numerical results and R values from Eq. (13) to those obtained from numerical results. The 95% fractile error is found to be very minimal at just 0.2%, which indicates an excellent accuracy.

For the case of multi harmonic force, a closed form expression for the harmonic combination factor g is first developed by curve fitting the numerical results obtained from Section 3 as typically shown in Fig. 6. It is found that the γ value not only increases with the damping ratio ζ but also depends on the harmonic number i at which resonance occurs. For instance, i is taken as 3 for 6-Hz floors which would be resonant with the third harmonic of a step frequency of 2 Hz. A closed form solution for γ is proposed as Eq. (14) with a 95% fractile error of 1.1% when compared with numerical results.

$$\gamma = 1.7723i\zeta - 1.0173\zeta + 0.9931 \tag{14}$$

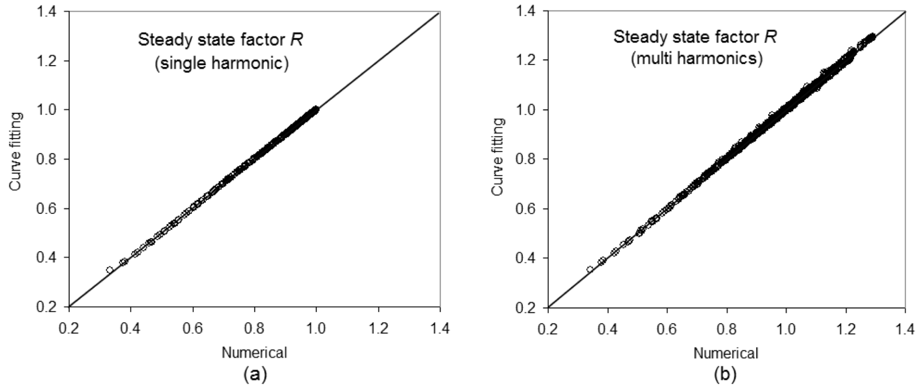


Fig. 7 Numerical vs curve fitting results for R

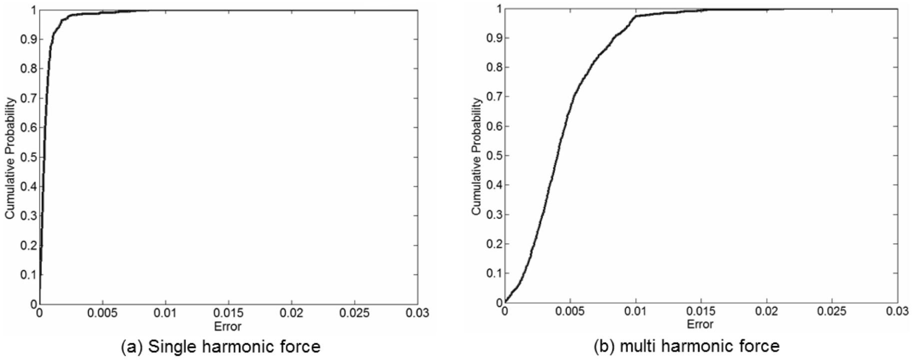


Fig. 8 Cumulative probability distribution of error when fitting R into closed form expressions

The R factor for multi harmonic force can now be computed as a multiplication of the R value for single harmonic force and its associated harmonic combination factor, as given by Eq. (15)

$$R = (1.7723i\zeta - 1.0173\zeta + 0.9931) \times (-0.0015\ln^5\varepsilon + 0.0119\ln^4\varepsilon - 0.0188\ln^3\varepsilon - 0.0695\ln^2\varepsilon + 0.2604\ln\varepsilon + 0.7570) \tag{15}$$

A plot of approximate values of R to those from simulations for the multi harmonic case is shown in Fig. 7(b), which again demonstrates a good accuracy of the proposed formula. Moreover, the cumulative probability distribution of error in estimating R for multi harmonic force is shown in Fig. 8(b) resulting in a 95% fractile error of just 1.0%.

4.2 Comparison with literature

This paper develops two expressions for the steady factor R for both loading cases: single harmonic and multi harmonics. In the case of multi harmonic force, a unique factor is proposed to take into account the contribution of not only the resonant harmonic but also the non-resonant harmonics to the overall floor response. Different expressions for R found in the relevant literature, however, deal only with the single harmonic case. Comparison of the findings in this paper and the literature is therefore made only for the case of single harmonic force.

In addition to the DG11's proposal that simply assigns a value of 0.5-0.7 to R (Murray *et al.* 2003), some expressions for R have been developed in the literature. Frýba (1999) assumed that the dynamic displacement of a simply supported beam occurs when the moving harmonic force is at midspan of the beam. A formula was developed for the dynamic coefficient which is the ratio of the maximum dynamic displacement to the static displacement (i.e. due to a static concentrated point load) of the beam. Given that the ratio of the steady state displacement to the static displacement is $1/(2\zeta)$ as well documented in the literature (Clough and Penzien 1993), we can change the formula for the dynamic coefficient proposed by Frýba (1999) into an expression for the steady state factor R in the form of Eq. (16)

$$R = \frac{\varepsilon}{1+\varepsilon^2}[\exp(-\pi\varepsilon/2) + \varepsilon] \tag{16}$$

The European guideline EUR 21972 (European Commission 2006) introduced a resonant build-up factor R to allow for the floor response not having reached a steady state condition because of an insufficient number of loading cycles. Using the notation introduced in this paper, the expression for R recommended by the EUR 21972 can be rewritten as

$$R = 1 - \exp(-\pi\varepsilon) \tag{17}$$

Another expression for the resonant build-up factor introduced by the UK design guide CCIP-016 (Willford and Young 2006) is rewritten here in the form of Eq. (18)

$$R = 1 - \exp(-0.55\pi\varepsilon) \tag{18}$$

More recently, Ricciardelli and Briatico (2011), in a comprehensive paper, proposed closed form solutions for the approximate transient response of simply supported beams subjected to a moving harmonic force. An expression for the steady state factor when the force frequency matches the beam fundamental frequency is given in Eq. (19)

$$R = \frac{\varepsilon}{1+\varepsilon^2}\left\{\sqrt{1+\varepsilon^2} + \exp[-\varepsilon(\pi/2 + \arctan(1/\varepsilon))]\right\} \tag{19}$$

Various formulas for estimating the R factor for the single harmonic force case are now compared, including Eqs. (13), (16), (17), (18) and (19) proposed by the authors, Frýba, the EUR 21972, the CCIP-016, and Ricciardelli, respectively. Plots of R versus ε are shown in Fig. 9(a). The R curve derived from the authors' formula almost coincides with and cannot be distinguished from that calculated using Ricciardelli's formula. An excellent agreement between the two formulas, proposed by this paper and Ricciardelli, has therefore been achieved. Fig. 9(b) shows the 95% fractile error when comparing different closed form solutions with numerical results. The two formulas recommended by the authors and Ricciardelli demonstrate an excellent accuracy with a 95% fractile error of 0.2%, i.e. nearly zero. The other formulas reveal inaccuracy with a 95% fractile error of 9%, 23% and 30% in accordance with the CCIP-016, Frýba and EUR 21972 proposals, respectively.

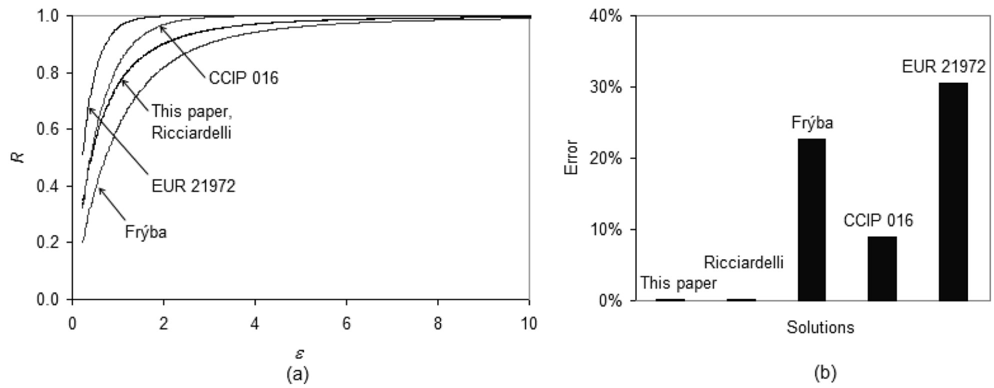


Fig. 9 (a) Various formulas for R factor for single harmonic force, (b) Numerical vs closed form solutions

4.3 Discussion

For single harmonic excitations, the formula suggested by the EUR 21972 overestimates the steady state factor and provides greatest values of R when compared with other formulas. The authors found that Eq. (17) is actually based on the response to a stationary force rather than a moving force, which can explain this situation. Indeed, the displacement response of a SDOF system with low damping subjected to a stationary harmonic force at resonance can be approximately expressed by Eq. (20) (Clough and Penzien 1993)

$$x(t) = \frac{1}{2\zeta}\frac{F_0}{k}[\exp(-\zeta\omega_n t) - 1]\cos\omega_n t \tag{20}$$

in which F_0 is the forcing magnitude and $\omega_n = 2\pi f_n$ is the natural circular frequency. Fig. 10 shows that the peak response values build up with time to reach the steady state. It can be seen from the equation above that the ratio of the maximum displacement at an instance t to the steady state displacement is $1 - \exp(-2\pi f_n \zeta t)$. Let t be the crossing time L/v_p and using the notations defined by Eqs. (11) and (12), the expression for R in the form of Eq. (17) can be obtained. The CCIP-016 formula has the same form as that of the EUR 21972 except for the number 0.55 that tends to reduce the R values. The formula proposed by Frýba lacks accuracy when providing R values lower than they would be. This may be because the maximum response would occur at the instance the force is past the beam's midspan (see Fig. 4(a)) rather than at exactly the midspan as assumed.

In the case of multi-harmonic excitations the R factor can exceed unity. This characteristic of R has not been mentioned in literature. The comprehensive work of Ricciardelli and Briatico (2011) provided separate solutions for the response to a single harmonic force at resonance and the response to a single harmonic force away from resonance, however a unique factor that can include the effects of all forcing components with different frequencies and dynamic coefficients (and hence magnitudes) on the overall response was not developed. This problem has been addressed in this paper by the introduction of Eq. (15), which can further facilitate the design work.

5. Proposal for a floor vibration prediction method

It is proposed that the peak acceleration a_p is predicted by modifying the steady state response a_s due

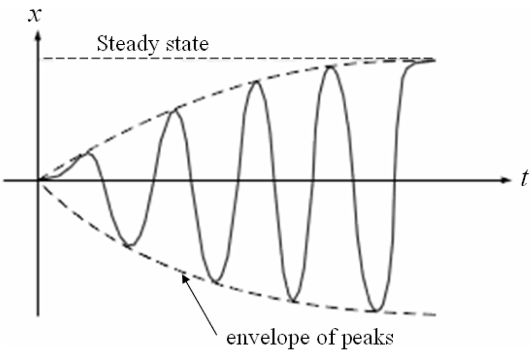


Fig. 10 Response to stationary harmonic force at resonance

to a simple harmonic excitation. A simplified formula for a_p is given by Eq. (21)

$$a_p = \frac{\alpha_i RP}{2\zeta m} \tag{21}$$

where α_i is obtained from Table 1 Forcing frequencies and dynamic coefficients by selecting the lowest harmonic for which the forcing frequency can coincide with the floor natural frequency. The steady state factor R can be calculated using Eq. (15). A higher value for the walker's weight P (rather than 700 N) may be used as real statistical data obtained from recent census in Australia (ABS 2008), the US (Ogden *et al.* 2004), Canada (Shields *et al.* 2008) and England (NatCen and UCL 2009) revealed that the average weight of an adult normally exceeds 800 N. The natural frequency f_n and modal mass m for simple floor structures can be determined analytically using established simplified formulas (Murray *et al.* 2003; Hechler *et al.* 2008; Smith *et al.* 2009). For complicated floors, finite element (FE) modal analysis is a better approach to obtain f_n and m. Recommended values for the damping ratio ζ can be obtained from the AISC/CISC DG11, depending primarily on the non-structural components and furnishings.

The dynamic coefficient α_i should be taken from Table 1 rather than computed using Murray's approximate expression of Eq. (3). This is because Eq. (3) expresses a decrease in a_i with an increase in f_p within each harmonic, as can be seen in Fig. 11(a), which may not be reasonable. Results from measurements within each harmonic have shown that dynamic load increases as pacing rate increases. For example, Fig. 11(b) shows the measured dynamic coefficient for the first four harmonics obtained with 3 males walking across an instrumented floor strip (Rainer and Pernica 1986). In addition, extensive experimental data obtained from about 1000 individual walking traces performed by 40 people at University College in London, England (Kerr and Bishop 2001) revealed an increase in the first dynamic coefficient with the step frequency.

For a conservative design and allowing for a flexible fit-out in modern offices, it can be assumed that the walker will cross over the location of maximum modal displacement at which the response is evaluated. When a more accurate calculation is needed, then actual modal displacement with value less than 1 at the position of the annoyed person could be considered from which a_p may be reduced to some extent. However, these simplifications might be compensated by the fact that two-way floor systems may have several closely-spaced modes tending to increase the response due to a single mode. The application of the proposed prediction method is illustrated in the following worked examples along with comparisons with FE results and experimental findings.

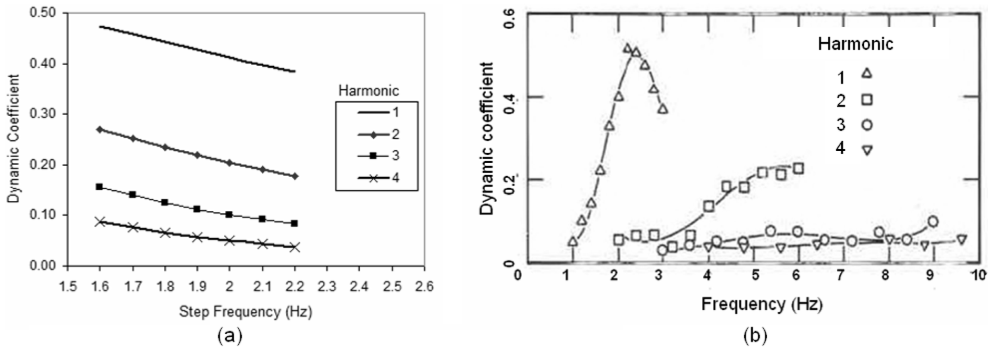


Fig. 11 Dynamic coefficients: (a) Murray's formula, and (b) measured by Rainer and Pernica (1986)

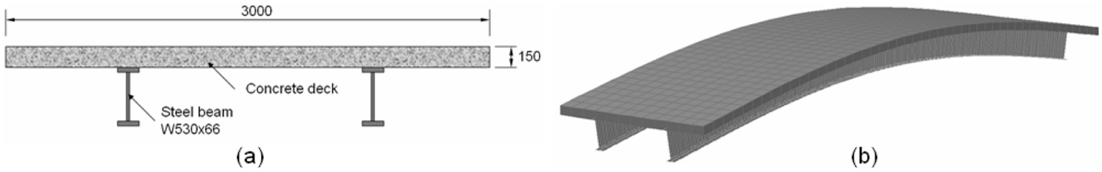


Fig. 12 A footbridge model: (a) cross section, and (b) fundamental mode shape

6. Worked examples

6.1 A typical generic footbridge

This example is identical to the one used in the AISC/CISC DG11 (Murray *et al.* 2003). A simply supported outdoor footbridge has a span of 12 m with a cross section shown in Fig. 12(a). The total weight, W, of the bridge including the weight of concrete slab and steel beams is 145.2 kN. An FE model for the bridge structure was created in SAP2000 computer software (CSI 2009). The first natural mode shape was obtained as shown in Fig. 12(b) with the corresponding fundamental frequency of 6.70 Hz, which is in excellent agreement with that computed analytically by Murray *et al.* (2003). The walking force was modelled as a combination of four harmonic components using Eq. (7) in which a step frequency of 1.67 Hz was assumed to match its fourth harmonic frequency with the bridge frequency. To be consistent with the DG11 example, a walker weight of 700 N, damping ratio of 1% and dynamic coefficients calculated from Eq. (3) were used in the FE model. The maximum acceleration of the bridge obtained from an FE time history analysis is 3.36% g.

Similar result for the bridge response can be obtained using the simple procedure outlined in section 5. Indeed, the substitution of $\alpha = 0.079$ computed from Eq. (3), $R = 0.88$ calculated using Eq. (15), $\zeta = 1\%$, $P = 700$ N and $m = 1/2\ W/g = 7400$ kg into Eq. (21) yields $a_p = 3.35\%$ g. An excellent agreement between the response predicted from FE time history analysis and that from the proposed procedure is achieved.

In comparison if a value of 0.7 was used for R as recommended in the DG11 then the peak response would be underestimated at 2.70% g. This example demonstrates that although the investigated structure has a low damping (1%), its span is long enough (12 m) for the R factor to be greater than the conventional value suggested in the DG11, resulting in a higher thus more conservative predicted acceleration response.

6.2 A real composite floor case study

Disturbing floor vibrations were reported from the tenants occupying an office floor of steel-concrete composite construction at a multi-story building in Melbourne, Australia. The most annoying area was located at the north-west corner of the building with floor beam spans of up to about 12.7 m as shown in Fig. 13. This floor bay has two perpendicular long corridors crossing the bay centre. The distance from the intersection of these corridors to the closest work station is just about 1 m, which is too small to avoid the vibration effects.

A number of physical heel-drop tests were conducted on the problematic bay, resulting in an estimated resonant frequency of approximately 6.2 Hz and modal damping ratio of about 3.0%. Time history of the floor bay response to an actual heel-drop excitation, along with its Fourier transformation

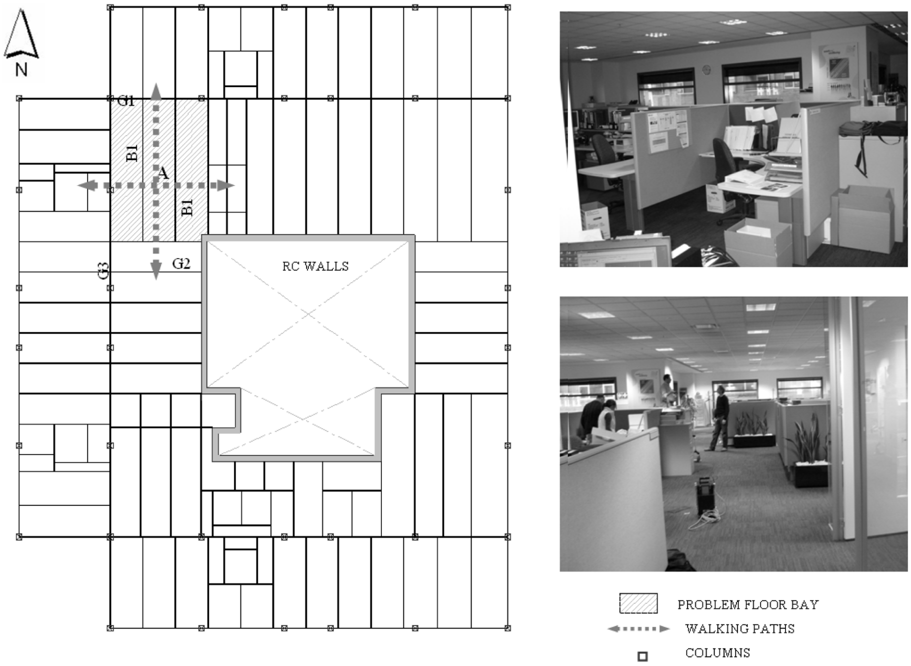


Fig. 13 Framing details in plan view and fit-out of a real office floor

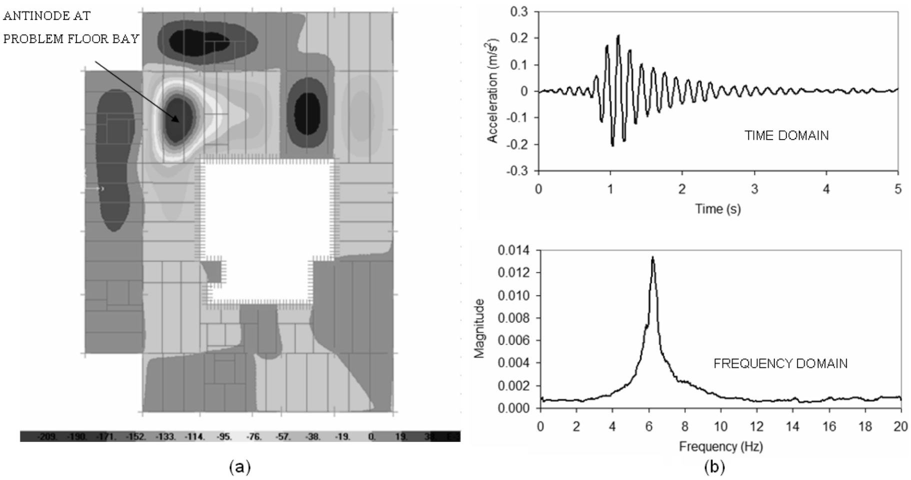


Fig. 14 (a) Resonant mode shape, and (b) floor response to a heel drop excitation

to frequency domain, is presented in Fig. 14(b). A detailed FE model was created and calibrated using SAP2000 software (CSI 2009) from which the resonant mode shape for the problematic bay was obtained as shown in Fig. 14(a) with its associated natural frequency being estimated at 6.22 Hz.

The maximum floor acceleration response was predicted by FE simulation with two load cases: one for walking at the same spot and the other for walking along the corridor with the associated forcing functions as described by Eqs. (5) and (7), respectively. The step frequency was assumed to be 2.07 Hz to produce a resonant condition at the third harmonic and hence the worst floor response. The weight of the walker was taken as 800 N. Both load cases can be modelled as equivalent concentrated time-dependent forces applied at the floor bay centre which is the antinode of the mode shape. The response of the problematic floor bay was also determined at this location. The time trace for floor response obtained from FE analysis is shown in Fig. 15, revealing a peak acceleration of 0.78% g and 0.74% g for stationary walk and moving walk, respectively.

Several walking tests were carried out on the real floor in which a walker attempted to maintain a pacing rate of around 2 Hz. These tests include walking at a spot close to the bay centre and walking along the corridor. Typical time traces for the measured floor response are shown in Fig. 16. The measured peak response to one-spot walk fluctuated between different tests, ranging from 0.75% to 0.85% g while the observed response to moving walk could reach a maximum of 0.65% g. Differences in floor response level between FE modelling and measurements are acceptable and inevitable, especially when it was quite difficult for the walker to maintain the same walking speed and stride length for the whole walking path. Moreover, a perfect resonant condition could not occur as it was unlikely that an exact resonant pacing rate of 2.07 Hz was continuously performed.

The simplified procedure outlined in Section 5 can be used to predict the floor bay response to walking. The floor bay can be approximated as a SDOF system with a modal mass m of 20600 kg obtained from the FE model. The substitution of $\alpha = 0.1$ for the 3rd harmonic, $R = 1.05$ calculated using Eq. (15), $\zeta = 3.0\%$, $P = 800$ N, and $m = 20600$ kg into Eq. (21) yields $a_p = 0.69\%$ g. It can be seen that a good agreement among the proposed simplified prediction method, FE time history analysis and field measurement is achieved.

Results from the proposed method, FE simulation and experimental studies all reveal that floor peak

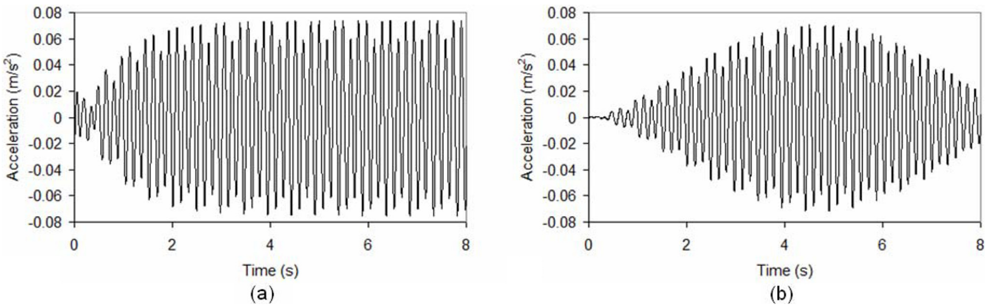


Fig. 15 FE-predicted floor response to (a) walking at the same spot, and (b) walking along the corridor

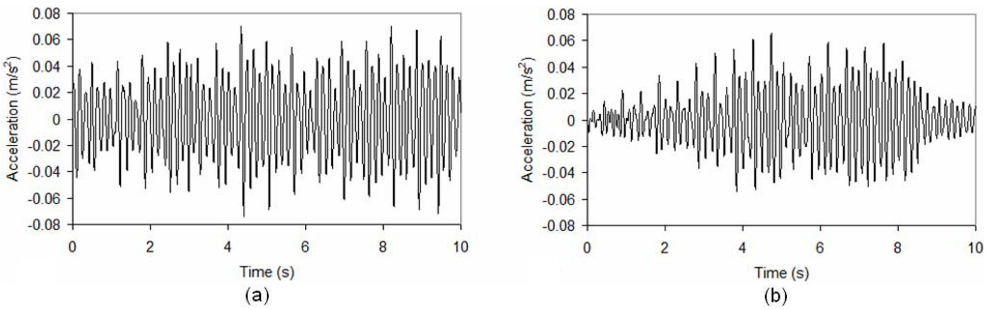


Fig. 16 Measured floor response to (a) walking at the same spot, and (b) walking along the corridor

response could exceed the threshold of 0.5% g for human comfort in an office environment and remedial measures should hence be targeted to rectify the problematic floor. However, if a value of 0.5 was used for R as recommended in the AISC/CISC DG11 then the predicted response would be 0.33% g, which is well within the comfort threshold. This low value of R artificially truncates half of the response level and leads to an unrealistic prediction.

7. Conclusions

An in-depth investigation of various factors affecting the steady state factor R has clearly shown that its conventional values of 0.5~0.7 as suggested by the popular guideline AISC/CISC DG11 might result in non-conservative design. The conventional range of R is found to be suitable for floors with spans of up to about 7 m and damping ratio of 1-2% subjected to only one harmonic component of the walking force. Increasing floor spans would enable the number of footstep crossing the floor and the build-up of vibration energy to be large enough for a steady state motion to occur and hence an increase in R. When several harmonic components of the walking force are considered together in long-span floors, the R value could even exceed 1.0, meaning that peak acceleration due to multiple harmonic components can be greater than the steady state response due to only one harmonic that coincides with the floor frequency.

Practical design graphs and closed form solutions have been developed in this paper with the most significant finding relating to the introduction of a unique factor that can include the effects of both the resonant harmonic and non-resonant harmonics with different forcing frequencies and magnitudes on the overall response. Comparison with the literature was also performed for the case of single-harmonic excitation. Worked examples have highlighted an acceptable agreement in floor response estimated by the proposed method with those obtained from FE time history analysis and experiments. The presented work would thus enhance the accuracy on one hand and maintain the simplicity and convenience of the current AISC/CISC DG11 on the other hand.

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