Reliability analysis of steel cable-stayed bridges including soil-pile interaction

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Abstract. An efficient and accurate algorithm is proposed to evaluate the reliability of cable-stayed bridges accounting for soil-pile interaction. The proposed algorithm integrates the finite-element method and the response surface method. The finite-element method is used to model the cable-stayed bridge including soil-pile interaction. The reliability index is evaluated based on the response surface method. Uncertainties in the superstructure, the substructure and load parameters are incorporated in the proposed algorithm. A long span steel cable-stayed bridge with a main span length of 1088 m built in China is considered as an illustrative example. The reliability of the bridge is evaluated for the strength and serviceability performance functions. Results of the study show that when strength limit states for both girder and tower are considered, soil-pile interaction has significant effects on the reliability of steel cable-stayed bridges. Further, a detailed sensitivity study shows that the modulus of subgrade reaction is the most important soil-pile interaction-related parameter influencing the reliability of steel cable-stayed bridges.

Keywords: soil-pile interaction; cable-stayed bridges; structural reliability; response surface method; limit state function.

1. Introduction

Steel cable-stayed bridges depend on their strong pile foundations to support their mass and their vertical load demands. When a steel cable-stayed bridge is built on soft or unstable soil, the piles provide support by reaching down to more stable rock in the depths of the ground. Therefore, there is a need to understand the nature of soil-pile and its effects on the bridge structural system.

In the last three decades several researchers have developed numerical and analytical methods to analyze the response of structures accounting for soil-pile interaction. A recent comprehensive review on the subject has been presented by Maekawa *et al.* (2003). However, these methods are deterministic in nature. But in reality there are uncertainties in design variables. These uncertainties include soil properties, pile properties, load magnitude and distribution, etc. Thus, a conventional deterministic approach may have the possibility of leading to unreasonable and unrealistic response prediction of structures accounting for soil-pile interaction due to ignorance of the uncertainties. Therefore, a probabilistic approach is needed to complement the shortcomings of deterministic approaches and to provide more reasonable and realistic response prediction of structures accounting for soil-pile "* Corresponding author, Ph.D., E-mail: chengiin@tsinghua.org.cn

interaction.

To assess random nature associated with structural response accounting for soil-pile interaction, several methods based on probabilistic analysis have been developed (Tandjiria *et al.* 2000, Eloseily *et al.* 2002, Chowdhury *et al.* 1998). However, these methods generally focused on dealing with the response problem of single flexible piles/pile groups and pile-supported dams in the presence of uncertainty. On the other hand, several researchers have investigated this soil variability in their work. Phoon and Kulhawy (1999) categorised the uncertainty of soil properties in their work in order to model soil variability. Similar investigations have been carried out by other researchers like Orr (1993), Frantziskonis and Breysse (2003), Niandou and Breysse (2007), Popescu (1995), Asoaka and Grivas (1982) and many others.

Soil-pile interaction is an important issue that affects the structural performance of cable-stayed bridges. Research works have been conducted to examine the effect of soil-pile interaction on the seismic behavior of cable-stayed bridges (Soneji and Jangid 2008). However, there is no comprehensive investigation on the effect of soil-pile interaction on the reliability of cable-stayed bridges. So, more efforts are still needed; in particular, the effect of uncertainty in soil-pile interaction on the reliability of cable-stayed bridges are to be built throughout the world.

The aims of this study are to develop an efficient and accurate method for the reliability analysis of steel cable-stayed bridges accounting for soil-pile interaction in general, and to investigate the effect of the soil-pile interaction on the reliability of steel cable-stayed bridges in particular.

2. Finite element modeling

The 3D structural models of steel cable-stayed bridges including soil-pile interaction are built and analyzed using the finite element method. The 3D structural models of a typical cable-stayed bridge used in the analyses are shown in Fig. 1. Details about modeling of the superstructures, substructures, and soil–structure interaction effects are presented in the following subsections.



Fig. 1 3D Finite Element Model of a typical cable-stayed bridge (i.e., Sutong Bridge)

2.1 Modeling of the superstructures

Regarding modeling of the superstructures, the towers and the piers are modeled as 3D beam elements. The cables are modeled by 3D truss element accounting for geometric nonlinearity due to cable sag. The bridge deck is represented by a single beam and the cross-section properties of the bridge deck are assigned to the beam as equivalent properties.

2.2 Modeling of the substructures

The literature study on the finite-element modeling of piles has revealed that the piles are modeled using 3D beam elements (Mourad and Tabash 1999, Faraji *et al.* 2001). Modeling the abutments using eight-node brick elements requires the integration of stresses to calculate the shears and moments. Accordingly, in this study, the abutments are modeled using Mindlin shell elements (Cook 1995) with six degrees of freedom at each node to accurately simulate shear deformations and bending effects with minimal computational effort and the piles are modeled using 3D beam elements. In addition, to model the rigidity of the deck-abutment joint, the abutment shell elements located within the joint area are assigned a large modulus of elasticity.

2.3 Modeling of soil-pile interaction

For modeling the soil-pile interaction in steel cable-stayed bridges, although the behavior of the foundation soil is nonlinear in nature, a linear elastic behavior is assumed due to the small lateral displacements of the abutments and piles under load effects. In this paper the effects of soil-pile interaction are modeled by attaching continuously distributed springs along the pile (Fig. 2). The elastic stiffness k(z) of the springs along the pile at a depth of z is calculated using the following expression (see Appendix A)



Fig. 2 Schematic of soil-pile interaction model in sandy soil (a) soil-pile interaction model and (b) sandy soil

$$k(z) = 0.91 \cdot E_k \cdot h \cdot z \tag{1}$$

where E_k is modulus of subgrade reaction; *h* is the spacing of the nodes along the pile. It should be mentioned that fundamental assumptions of Eq. (1) is that: (1) the pile-soil interaction force at a given depth only depends on the pile displacement at that depth; (2) the soil–pile interaction for a particular point along the pile is defined by a nonlinear load (*P*)-deformation (*Y*) curve; (3) the nonlinear *P*-*Y* curve is simplified using an elastoplastic curve; and (4) only the elastic portion of this elastoplastic model is used to simulate the force-deformation response of the sandy soil due to small lateral displacement of the piles under live-load effects.

3. Reliability analysis

In this section, some of the important contents of the reliability analysis are briefly presented and they will be used for reliability estimation of steel cable-stayed bridges including soil-pile interaction.

3.1 Limit state function

Structural reliability assessment traditionally considers a limit state to define a failure event. A limit state is a state of the structure including its load at which the structure is just on the point of not satisfying the requirement (Ditlevsen and Madsen 1996). Failure is said to occur when the following condition is satisfied

$$S = G(X) \le 0 \tag{2}$$

where S is the safety margin, X means the vector of the random variables and G(X) is the so-called limit state function which separates the acceptable region from that which is characterized as failure.

The failure modes considered in the present analysis are the serviceability failure and the strength failure of steel cable-stayed bridges. Serviceability failure is defined to occur when a deflection exceeds an allowable deflection limit. The serviceability limit state function is defined as

$$G = \delta_{allow} - y_{max}(X) \tag{3}$$

where δ_{allow} is the allowable maximum deflection specified in codes. In this paper, the allowable maximum deflection of the bridge is considered not to exceed L/400. L is the central span length and $y_{max}(X)$ is the corresponding the maximum deflection estimated from analysis. In the following numerical example, L is equal to 1088 m. Thus, δ_{allow} becomes 2.72 m.

Strength failure of steel cable-stayed bridges is defined to failure of several components in strength forming a local or global mechanism. The strength limit state functions for cable, girder and tower members can be defined as

For cable member,
$$G = \sigma_{y1} - \sigma_{ci}$$
 (4)

For girder member,
$$G = \sigma_{y2} - \sigma_{gi}$$
 (5)

For tower member,
$$G = 1.0 - \left(\frac{|F_1|}{AC_1} + \frac{|M_1|}{AM_1} + \frac{|M_2|}{AM_2}\right)$$
 (6)

where σ_{y1} is the yield stress of cables, σ_{ci} is the member stress of cable *i*, σ_{y2} is the yield stress of girder, σ_{gi} is the member stress in the girder at cross-section *i*, F_1 is axial load at the bottom section of the pylon, M_1 and M_2 are bending moments about the element *x* axis and *y* axis at the bottom section of the pylon, respectively, AC_1 are nominal axial compressive capacity at the bottom section of the pylon, AM_1 and AM_2 are nominal bending moment capacity about the element *x* axis and *y* axis at the bottom section of the pylon, and AM_2 are nominal bending moment capacity about the element *x* axis and *y* axis at the bottom section of the pylon, respectively.

3.2 Reliability method

In general, simulation or the first order reliability methods (FORM) can be used for the structural reliability evaluation. The simulation technique may not be desirable for the class of problems considered here, because of computational cost and inherent inefficiency of the approach. In the context of FORM, the limit state function of the structure needs to be defined in an explicit form. However, the limit state function for the class of problems under consideration here is implicit in nature. The response surface method (RSM) can be used to approximately generate the implicit limit state function. Although the RSM method has been applied to solve various structural reliability problems (Tandjiria *et al.* 2000, Bucher and Bourgund 1990, Kim and Na 1997, Das and Zheng 2000, Cheng and Xiao 2005, Wong *et al.* 2005), applications of the RSM to the reliability analysis of steel cable-stayed bridges including soil-pile interaction have not been reported. The present paper thus makes a contribution in this regard.

The main idea of the RSM is to approximate the implicit limit state function by simple and explicit polynomial. A second-order polynomial without cross terms is adopted here. The second-order polynomial can be represented as

$$\hat{g}(X) = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2$$
(7)

where $\hat{g}(X)$ is the approximate limit state function; X_i (i = 1, 2, ..., k) is i th random variable; and b_0 , b_{ii} are unknown coefficients to be determined by solving a set of simultaneous equations. The number of unknown coefficients in (7) is p = 2k + 1. Consider three values of each random variable, namely, a low value, a medium value, and a high value (e.g., $X_i^C - h_i \sigma_{X_i}, X_i^C, X_i^C + h_i \sigma_{X_i}$), where the value of h_i is considered to be 3.0 for the first iteration and 0.99 for the subsequent iterations; X_i and σ_{X_i} is coordinates of the center point and standard deviation of a random variable X_i . The initial center point can be the mean value point. For determining the location of the center point, the iterative linear interpolation scheme suggested by Bucher and Bourgund (1990) is used in this study. A detailed description of the RSM can be obtained from Haldar and Mahadevan (2000).

To evaluate the reliability of steel cable-stayed bridges including soil-pile interaction a deterministic analysis method is very desirable, since the present analysis is based on tracking the uncertainty propagating through the steps of deterministic analysis. Finite element method (FEM) is considered to be the most reliable deterministic analysis method. In this paper, the primary purpose of applying FEM is to compute the values of the above approximate limit state function. For more details concerning the FEM used in this paper, the reader is referred to Cheng and Xiao (2005).

3.3 Sensitivity analysis

An important step in the structural reliability analyses is the sensitivity analysis of reliability indices. This helps identify the important parameters. On the other hand, sensitivity analysis is also useful in reducing the size of problems with a large numbers of random variables. This is because that, in general, only a few variables have a significant effect on the structural reliability (Haldar and Mahadevan 2000). Some literature is available on sensitivity analysis of reliability indices. Liu and Der Kiureghian (1991) presented extensive analyses of reliability sensitivities with respect to parameters defining the random fields. The results indicate relative importance of the basic variables and fields for the static behavior of the plate. Imai and Frangopol (2002) carried out sensitivity analyses of reliability indices with respect to the mean values and standard deviations of the variables of interest. The results indicate that the wind load and resistance are the most influential variables on the reliability of suspension bridges. In this section, sensitivity analysis is used to identify the influences of the random variables on the reliability of steel cable-stayed bridges including soil-pile interaction. Of particular interest in this study is sensitivity measures are formulated as indicated by Hohenbichler and Rackwitz (1986)

$$\frac{\partial \beta}{\partial \sigma_i} \approx -\beta \alpha_i^2 \tag{8}$$

$$\frac{\partial \beta}{\partial \mu_i} \approx -\alpha_i \tag{9}$$

where β is reliability index, α_i is direction cosine associated with the i-th random variable, μ_i and σ_i are the mean and standard deviation of the i-th random variable, respectively.

4. Illustrative example

This section presents an illustrative example to evaluate the effect of soil-pile interaction on the reliability of steel cable-stayed bridges, and to show the comparative importance of this factor in reliability of steel cable-stayed bridges.

4.1 Description of a bridge

Sutong Bridge, which is a long-span cable-stayed bridge spanning Changjiang River in Jiangsu province, China, is selected herein as an illustrative example. It has a 1088-m central span length, which is the longest central span of any cable-stayed bridge in the world. The bridge span arrangements are (100 + 100 + 300 + 1088 + 300 + 100 + 100) m. Elevation view of the completed bridge is shown in Fig. 3. Pile foundation of Sutong Bridge is shown in Fig. 4. Details concerning other structural parameters are omitted for brevity. Two kinds of loads were considered in the design of the bridge: (1)



Fig. 3 Elevation view of Sutong Bridge (Unit: m)



Fig. 4 Pile foundation of Sutong Bridge (Unit: cm)

dead loads and (2) traffic loads. All dead loads were applied statically in the negative z-direction shown in Fig. 1. For simplicity, the applied traffic loads are assumed to be uniformly distributed on the bridge deck.

4.2 Uncertainties of the example bridge

It is understood that the material properties and geometric parameters of the bridge may fluctuate in the vicinity of the nominal values, caused during the process of structural element manufacturing and erection of the bridge. Therefore, such parameters should be treated as random variables rather than deterministic ones. Besides, soil properties, pile properties and the various loads acting on the bridge are also random variables due to their natures and/or insufficient measured data. Table 1 shows the statistics of these random variables. To study soil-pile interaction effects on the reliability of the bridge, two cases are considered. Case I (considering soil-pile interaction): all random variables shown in Table 1 are considered; Case II (without considering soil-pile interaction): all soil-pile interaction-related variables are not considered. For convenience, the present analysis ignores other parameters for the soil pile interaction even though it is recognized that moment of inertia of pile section, the ultimate soil resistance, etc. do vary randomly (e.g., Tandjiria *et al.* (2000)). These random variables, however, will

not have as significant an influence on structural reliability of steel cable-stayed bridges as modulus of elasticity of piles, pile diameter and modulus of subgrade reaction. Note that modulus of subgrade reaction, E_k is assumed to a random variable in this paper. This assumption is based on Tandjiria *et al.* (2000). Using random variable to model E_k may not be totally justifiable since there is likely to be random variability in spatial distribution as well as in intensity. Nonetheless, the main objective of the

Random variables		Distribution types	Unit	Mean value	Coefficients of variation	Sources
	Modulus of elasticity E_1	Normal	N/m^2	1.95E11	0.1	Cheng and Xiao 2005
	Material density γ_1	Lognormal	kg/m ³	8499.2	0.05	Cheng and Xiao 2005
	Section area A_1			1.2046E-2		
	Section area A_2			1.0891E-2		
	Section area A_3			0.9737E-2	0.05	
Stay	Section area A_4			0.9275E-2		
cable	Section area A_5	Lognormal	m^2	0.812E-2		Cheng and Xiao
	Section area A_6	Dognormai		0.7197E-2		2005
	Section area A_7			0.6273E-2		
	Section area A_8			0.5811E-2		
	Section area A_9			0.5349E-2		
	Section area A_{10}			0.4888E-2		
	Yield stress σ_{yI}	Normal	N/m^2	1770E6	0.20	Wang and Ghosn 2006
	Modulus of elasticity E_2	Normal	N/m^2	3.5E10	0.1	Cheng and Xiao 2005
	Material density γ_2 Lognorm		kg/m ³	2600	0.05	Cheng and Xiao 2005
	Section area A_1			38.6		
	Section area A_2		2	47.4	0.05	
	Section area A_3	Lognormal	m²	39.4	0.05	Cheng and Xiao
	Section area A_4	-		56.3		2003
	Moment inertia I_1		m^4	517.7	0.05	
Pylon	Moment inertia I_2			793.7		
1 91011	Moment inertia I_3	ent inertia I_3		796.5	0.05	Cheng and Xiao
	Moment inertia I_4			1375.3		2005
	nominal axial compressive capacity AC_1	Lognormal	Ν	2.47E9	0.13	
	nominal bending moment capacity about the element x axis AM_1	Normal	Nm	3.50E9	0.074	Wang and Ghosn 2006
	nominal bending moment capacity about the element y axis AM_2	Normal	Nm	2.17E8	0.074	

Table 1 Statistics of the random variables for the Sutong Bridge

	Random variables	Distribution types	Unit	Mean value	Coefficients of variation	Sources
Stiffened girder	Modulus of elasticity E_3	Normal	N/m^2	2.1E11	0.1	Cheng and Xiao 2005
	Section area A_1	Lognormal	m^2	2.4627		Cheng and Xiao 2005
	Section area A_2			2.2587	0.05	
	Section area A_3			2.2010		
	Section area A_4			1.9122		
	Section area A_5			1.7751		
	Section area A_6			1.9839		
	Moment inertia I_1		m^4	7.1382	0.05	Cheng and Xiao 2005
	Moment inertia I_2			6.4080		
	Moment inertia I_3	Lognormal		6.3529		
	Moment inertia I_4	Logiloimai		5.4337		
	Moment inertia I_5			4.7986		
	Moment inertia I_6			5.5661		
	Yield stress σ_{y_2}	Normal	N/m^2	345E6	0.20	Murchison and O'Neill 1984
Pile	Modulus of elasticity E_4	Normal	N/m^2	3.15E10	0.1	Eloseily 1998
	Diameter D_1	Normal	т	2.8	0.016	Eloseily 1998
Modulus of subgrade reaction E_k		Normal	N/m^3	8E6	0.14	Murchison and O'Neill 1984
	Traffic load q	Normal	kN/m	46.2	0.13	Cheng and Xiao 2005

paper is to assess how variability of E_k affects structural reliability. There is clearly more scope for the development of more robust and validated probabilistic model for E_k .

As the objective of this study is to propose an efficient method for evaluating the reliability of steel cable-stayed bridges accounting for soil-pile interaction, all random parameters in the following analysis are based on arbitrary but typical values. On the other hand, since the determination of the correlation of the random parameters is a difficult task without sufficient data, using the independence assumption can greatly simplify the reliability assessment. Therefore, all random parameters considered in the paper are treated as stochastically independent from each other.

4.3 Numerical results and discussions

The reliability of the cable-stayed bridge with and without considering soil-pile interaction (Cases I and II) is evaluated using the proposed RSM algorithm. The results are summarized in Table 2. The probability of failure for the serviceability limit state is compared with Monte Carlo simulation (MCS) results using 2000 simulations. Results from Table 2 show that the probability of failure for the serviceability limit state using the proposed RSM algorithm is very similar to the MCS results, indicating that the algorithm is viable and accurate. For serviceability limit state, the probabilities of failure for the two cases are similar, indicating that the soil-pile interaction may not be an important factor in evaluating the reliability of a long span cable-stayed bridge. However, when strength limit

Limit state	Serviceability limit state		Strength limit state						
			Stay cable		Stiffened girder		Tower		
Case	Case I	Case II	Case I	Case II	Case I	Case II	Case I	Case II	
Proposed method	$\beta = 1.081$ $P_f = 0.1399$	$\beta = 1.127$ $P_f = 0.1299$	$\beta = 3.309$ $P_f = 4.682\text{E-4}$	$\beta = 3.309$ $P_f = 4.682E-4$	$\beta = 3.038$ $P_f = 1.191E-3$	$\beta = 3.463$ $P_f = 2.671$ E-4	$\beta = 4.115$ $P_f = 1.936$ E-5	$\beta = 4.553$ $P_f = 2.644$ E-6	
Monte Carlo simulation	$\beta = 1.067$ $P_f = 0.1430$	$\beta = 1.100$ $P_f = 0.1357$	-	-	-	-	-	-	

Table 2 Results for Sutong Bridge considering and without considering soil-pile interaction (Cases I and II)

states for both girder and tower are considered, the probabilities of failure for the two cases are found to be different. The results indicate that the consideration of appropriate soil-pile interaction and the uncertainty associated with modeling it significantly affects the reliability estimation of steel cablestayed bridges. Also, the probabilities of failure for Case I is much larger than that for Case II. This implies that without considering soil-pile interaction in the reliability analysis can lead to underestimating the probability of failure. Therefore, soil-pile interaction should be taken into account



Fig. 5 Sensitivity of reliability index of stay cable with respect to (a) mean value; (b) standard deviation



Fig. 6 Sensitivity of reliability index of stiffened girder with respect to (a) mean value; (b) standard deviation



Fig. 7 Sensitivity of reliability index of pylon with respect to (a) mean value; (b) standard deviation

appropriately in reliability analysis of steel cable-stayed bridges, especially when strength limit states for both girder and tower are considered.

As mentioned earlier, soil-pile interaction-related parameters include modulus of elasticity of pile, diameter of pile and modulus of subgrade reaction. To identify the most important parameter, a sensitivity analysis of reliability indices is conducted. The results of the sensitivity analysis for stay cable, girder and tower are presented in Figs. 5-7, respectively. The results show that the modulus of subgrade reaction is the most important parameter influencing the reliability of the bridge. Accurate determination of such a parameter is very important in obtaining reliable probabilistic results. It can also be seen from Figs. 5-7 that the bridge reliability increases as the mean value of the modulus of subgrade reaction. Furthermore, the reliability index of the tower is much more sensitive to the modulus of subgrade reaction than that of the stay cable and girder.

5. Conclusions

In this paper, an efficient and practical method was presented for the reliability analysis of steel cablestayed bridges accounting for soil-pile interaction. Uncertainties in the superstructure, the substructure and load parameters are incorporated in the present method. The proposed approach intelligently integrates the FEM and the RSM. The FEM is used to model the cable-stayed bridge including soil-pile interaction. The reliability index is evaluated based on the RSM. The accuracy of the proposed method is verified with Monte Carlo simulation results.

Using the proposed method, a parametric study was performed for evaluation of the effect of the soilpile interaction on the reliability of a long span cable-stayed bridge. The concerned steel cable-stayed bridge is the Sutong Bridge with a main span length of 1088 m built in China. Two types of limit state functions were considered in the reliability analysis. They are the serviceability and strength limit states. Each limit state was considered separately, since the bridge can fail due to excessive deflection or due to failure of several components in strength forming a local or global mechanism.

This study revealed several very interesting aspects for the reliability of steel cable-stayed bridges accounting for soil-pile interaction. More specifically, when serviceability limit state is considered, the effect of soil-pile interaction is not significant in evaluating the reliability of a long span cable-stayed

bridge. However, when strength limit states for both girder and tower are considered, soil-pile interaction has significant effects on the reliability of steel cable-stayed bridges. Ignoring the effect of soil-pile interaction can lead to the unrealistic probability of failure in the steel cable-stayed bridges. Furthermore, it was found that the modulus of subgrade reaction is the most important soil-pile interaction-related parameter influencing the reliability of steel cable-stayed bridges. Tight control on the parameter is required if a high reliability of a long span cable-stayed bridge is desired.

Although previous studies have evaluated the reliability of steel cable-stayed bridges, the effect of soil-pile interaction is not taken into account in the reliability analysis of steel cable-stayed bridges. This work provided an improved understanding of the reliability of steel cable-stayed bridges accounting for soil-pile interaction.

It should be noted that each uncertain quantity is simply represented by a single random variable and the spatial variability is not taken into account in this study. However, in reality many structural parameters such as material properties, geometric parameters, loads and modulus of subgrade reaction show random variability in spatial distribution as well as in intensity, which could have considerable effect on the actual reliability of steel cable-stayed bridges. The question of spatial variability will be the subject of a separate study.

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Appendix A. Derivation of Eq.1

Generally, the soil-pile interaction for a particular point along the pile is defined by a nonlinear load P-Y curve, where P is lateral soil resistance per unit length of the pile; and Y is lateral deflection. A typical P-Y curve for soil subjected to lateral movement of a pile is shown with a solid line in Fig.8. This highly nonlinear behavior is simplified using an elastoplastic curve displayed on the same figure with a dashed line. The elastic portion is defined with a slope equal to the secant soil modulus E_s and the plastic portion is defined as the ultimate soil resistance per unit length of pile P_u . In this study, only the elastic portion E_s of this elastoplastic model is used to simulate the force-deformation response of the sandy soil due to small lateral displacement of the piles under live-load effects. Presumably P_u varies with depth so then does E_s . The sandy soil modulus E_s can be calculated using the following expression

$$E_s = \frac{P_u}{2 \cdot \Delta_{50}} \tag{10}$$

where Δ_{50} is the pile deflection at 50% of the ultimate soil resistance as described below. The *p*-*y* curve for sandy soil is expressed as (Murchison and O'Neill 1984)



Fig. 8 A typical *p*-*y* curve and its elastoplastic idealization for sandy soil

$$p = \overline{A} \cdot p_u \tanh\left[\frac{E_k z}{\overline{A} p_u}y\right]$$
(11)

As shown in Fig. 8, the ultimate sandy soil resistance is $\overline{A} \cdot p_u$, and the pile deflection corresponding to $0.5 \cdot \overline{A} \cdot p_u$ is Δ_{50} . Thus

$$p = \overline{A} \cdot p_u \tanh\left[\frac{E_k z}{\overline{A} p_u} \Delta_{50}\right] = 0.5 \cdot \overline{A} \cdot p_u$$
(12)

$$\frac{E_k z}{A p_u} \Delta_{50} = a \tanh(0.5) = 0.5493$$
(13)

$$\Delta_{50} = 0.5493 \frac{\overline{Ap_u}}{E_k z} \tag{14}$$

Substituting (14) into (10), E_s can be rewritten as

$$E_s = \frac{0.5\overline{A} \cdot p_u}{\Delta_{50}} = \frac{0.5\overline{A} \cdot p_u}{0.5493 \frac{\overline{A}p_u}{\overline{E_k z}}} = 0.91E_k z$$

The elastic stiffness k(z) of the springs along the pile at a depth of z is calculated by multiplying the E_s by the spacing of the nodes along the pile h. Thus

$$k(z) = 0.91 \cdot E_k \cdot h \cdot z$$

References

Asoaka, A. and Grivas, D.A. (1982). Spatial variability of the undrained strength of clays. ASCE, J. Eng. Mech., **108**(5), 743-756.

- Bucher, C.G. and Bourgund, U. (1990), "A fast and efficient response surface approach for structural reliability problems", *Structural Safety*, 7(1), 57-66.
- Cheng, J. and Xiao, R.-C. (2005), "Serviceability reliability analysis of cable-stayed bridges", *Struct. Eng. Mech.*, **20**(6), 609-630.
- Chowdhury, M.R., Wang, D., and Haldar, A. (1998), "Reliability assessment of pile-supported structural systems", *J. Struct. Eng.*, ASCE, **124**(1), 80-88.
- Cook, R.D. (1995), Finite-element modeling for stress analysis, Wiley, New York.
- Das, P.K. and Zheng, Y. (2000), "Cumulative formation of response surface and its use in reliability analysis", *Probab. Eng. Mech.*, 15(4), 309-315.
- Ditlevsen, O. and Madsen, H.O. (1996), Structural reliability methods, John Wiley & Sons Ltd, Chichester, England.
- Eloseily, K.H. (1998), Reliability assessment of pile groups in sand. PhD dissertation of University of Maryland.
- Eloseily, K.H., Ayyub, B.M. and Patev, R. (2002), "Reliability assessment of pile groups in sands", J. Struct. Eng., ASCE, 128(10), 1346-1353.
- Faraji, S., Ting, J.M., Crovo, D.S. and Ernst, H. (2001), "Nonlinear analysis of integral bridges: Finite-element model", J. Geotech. Geoenviron. Eng., 127(5), 454-461.
- Frantziskonis, G. and Breysse, D. (2003), Influence of soil variability on differential settlements of structures, *Comput. Geotech.*, **30**(3), 217-230
- Haldar, A. and Mahadevan S. (2000), *Reliability assessment using stochastic finite element analysis*, John Wiley & Sons, New York.
- Hohenbichler, M. and Rackwitz, R. (1986), "Sensitivity and importance measures in structural reliability", *Civil Engineering Systems*, **3**(4), 203-210.
- Imai, K. and Frangopol, D.M. (2002), "System reliability of suspension bridges", Structural safety, 24, 219-259
- Kim, S. and Na, S. (1997), "Response surface method using vector projected sampling points", *Structural Safety*, **19**(1), 3-19.
- Liu, P.-L. and Der Kiureghian, A. (1991), "Finite element reliability of geometrically nonlinear uncertain structures", J. Eng. Mech., ASCE, 117(8), 1806-1825.
- Maekawa, K., Pimanmas, A. and Okamoura, H. (2003), "Nonlinear soil structure interaction", London: Spon Press.
- Mourad, S. and Tabash, W.S. (1999), "Deck slab stresses in integral abutment bridges", *Journal of Bridge Engineering*, **4**(2), 125-130.
- Murchison, J.M. and O'Neill, N.M. (1984), "Evaluation of P-Y relationships in cohesionless soils", *Proceedings* of ASCE Symposium on analysis and design of pile foundations. San Francisco (CA): 174-191
- Niandou, H. and Breysse, D. (2007), "Reliability analysis of a piled raft accounting for soil horizontal variability", *Comput. Geotech.*, **34**(2), 71-80.
- Orr, T.L.L. (1993), Probabilistic characterization of Irish till properties. In B.O. Skipp, editor, *Risk and reliability in ground engineering*, pages 126-133, London, November. Institution of Civil Engineers, Thomas Telford.
- Phoon, K.-K. and Kulhawy, F.H. (1999), "Evaluation of geotechnical property variability", *Canadian Geotechnical Journal*, **36**, 625-639.
- Popescu, R. (1995), Stochastic variability of soil properties: data analysis, digital simulation, effects on system behaviour. PhD thesis, Princeton University.
- Soneji, B.B. and Jangid, R.S. (2008), "Influence of soil-structure interaction on the response of seismically isolated cable-stayed bridge", *Soil Dyn. Earthq. Eng.*, **28**(4), 245-257.
- Tandjiria, V., The, C.I. and Low, B.K. (2000), "Reliability analysis of laterally loaded piles using response surface methods", *Structural Safety*, 22(4), 335-355.
- Wang, J. and Ghosn, M. (2006), "Hybrid data mining/genetic shredding algorithm for reliability assessment of structural systems", J. Struct. Eng., ASCE, 9, 1451-1460.
- Wong, S.M., Hobbs, R.E. and Onof, C. (2005), "An adaptive response surface method for reliability analysis", *Structural Safety*, 27(4), 287-308.