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Frequency optimization for laminated composite plates using extended layerwise approach

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Abstract. This paper deals with the applicability of extended layerwise optimization method (ELOM) for frequency optimization of laminated composite plates. The design objective is the maximization of the fundamental frequency of the laminated plates. The fibre orientations in the layers are considered as design variables. The first order shear deformation theory (FSDT) is used for the finite element solution of the laminates. Finally, the numerical analysis is carried out to show the applicability of extended layerwise optimization algorithm of laminated plates for different parameters such as plate aspect ratios and boundary conditions.

Keywords: laminated plates; fundamental frequency; finite element solution; extended layerwise optimization method; optimization.

1. Introduction

Laminated composite plates are widely used in the aerospace, automotive, marine and other structural applications because of advantageous features such as high ratio of stiffness and strength to weight and low maintenance cost. Free vibration analysis of laminated composite plates has been an important problem in the design of mechanical, civil and aerospace applications. Vibration can waste energy and create unwanted noise in the motions of engines, motors, or any mechanical devices in operation. When a system operates at the system natural frequency, resonance can happen causing large deformations and even catastrophic failure in improperly constructed structures. Careful designs can minimize those unwanted vibrations.

The optimization of fundamental frequencies for laminated composite plates have been the subject of significant research activities in recent years. For example, Apalak *et al.* (2008) carried out the layer optimisation for maximum fundamental frequency of laminated composite plates under any combination of the three classical edge conditions. Narita (2006) introduced a layerwise optimization (LO) approach to accommodate the finite element analysis for optimizing the free vibration behavior of laminated composite plates with discontinuities along the boundaries. Niu *et al.* (2010) vibro-acoustic optimization of laminated composite plates. Honda and Narita (2011) studied an optimum design method for proposing new types of fiber reinforced composite plates with locally anisotropic structure. Abdalla *et al.* (2007) considered maximisation of the natural frequency of composite panels. Adali and

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Verijenko (2001) presented design of hybrid symmetric laminated plates consisting of high-stiffness surface and low-stiffness core layers. Narita (2003) proposed a new concept of a layerwise optimization approach (LO) to optimize vibration behavior for the maximum natural frequency of laminated composite plates. Narita and Hudgkinson (2005) applied the layerwise optimisation approach to point-supported, symmetrically laminated rectangular plates. Topal and Uzman (2008) maximized fundamental frequencies of simply supported symmetrically laminated composite angle-ply plates with central circular holes with a given material system with respect to fibre orientations. Karakaya and Soykasap (2011) used genetic algorithm and simulated annealing to maximize natural frequency of simply supported hybrid composite plates. The aim of the study was to use two different techniques of optimization on the frequency optimization of composite plates and compare the techniques for their effectiveness. Sadr and Bargh (2011) studied fundamental frequency optimization of symmetrically laminated composite plates using the combination of elitist-genetic algorithm and finite strip method. Umachagi *et al.* (2011) presented the layerwise optimization for the maximization of fundamental frequency of simply supported antisymmetric angle ply laminated composite and sandwich plates. More results can be found in literatures.

On the other hand, layerwise optimization (LO) approach was introduced by Narita who applied this method to the optimization of the laminates in frequency domains. Narita started with a predetermined number of layers in symmetric formation and systematically found the optimal fibre orientations from the outer to the inner layers. His study was restricted by predetermined number of layers. However, this paper deals with extended layerwise optimization method (ELOM) for frequency optimization of laminated plates. Furthermore, this algorithm has no limitations on the number of layers. The design objective is the maximization of the fundamental frequency. The first order shear deformation theory is used for finite element solution of laminates. The design variable is the fiber orientations. Finally, the numerical analysis is carried out to show the applicability of extended layerwise optimization algorithm of laminated plates for different parameters such as plate aspect ratios and boundary conditions.

2. Basic equations

Consider a laminated composite plate of uniform thickness h, having a rectangular plan axb as shown in Fig. 1. The individual layers are assumed to be homogeneous and orthotropic.

The displacement field for the first order shear deformation theory can be expressed as

$$u(x, y, z) = u_0(x, y) + z\psi_x(x, y)$$

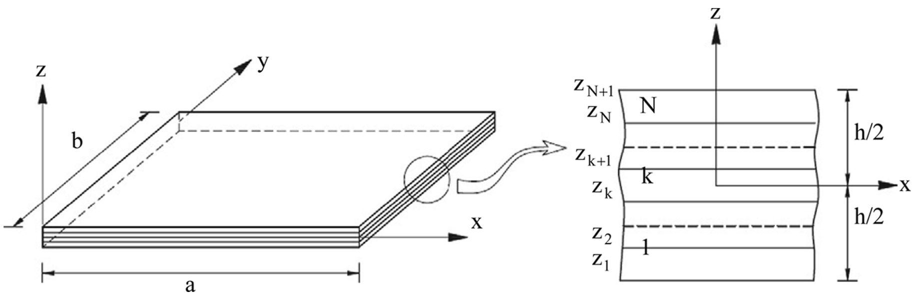

Fig. 1 Geometry and coordinate system of a rectangular laminated composite plate

$$v(x,y,z) = v_0(x,y) + z\psi_y(x,y) \tag{1}$$

$$w(x,y,z) = w_0(x,y)$$

where u, v and w are the displacements of a general point in the x, y and z directions respectively. The parameters u_0, v_0 are the inplane displacements and w_0 is the transverse displacement of a point on the laminate middle plane. The functions ψ_x and ψ_y are the rotations of the normal to the laminate middle plane about x- and y-axes, respectively. The displacement vector at the mid-plane can be defined as

$$\bar{d} = \{u_0, v_0, w_0, \psi_x, \psi_y\}^T \tag{2}$$

Substituting Eq. (1) into the general linear strain-displacement relations, the following relations are obtained.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{vmatrix} - z \begin{vmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{vmatrix}, \quad \begin{vmatrix} \gamma_{yz} \\ \gamma_{xz} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} - \psi_x \end{vmatrix} \tag{3}$$

The stress-strain relations for the *kth* lamina in the element co-ordinates (x, y, z) are written as

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix}_{(k)} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix}_{(k)} \tag{4}$$

$$\begin{pmatrix} \tau_{yz} \\ \tau_{xz} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{pmatrix}_{(k)} \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix}_{(k)} \tag{5}$$

where $\bar{Q}_{ij}$ is the transformed reduced stiffnesses, which can be expressed in terms of the orientation angle and the engineering constant of the material.

For the ideal case in which the system has no damping and no external function, the mathematical statement of Hamilton's principle can be written as

$$\delta \int_{t_1}^{t_2} \left[\frac{1}{2} \int_V \bar{\varepsilon}^T \bar{\sigma} dv - \frac{1}{2} \int_V \dot{u}^T \rho \dot{u} dv \right] dt = 0 \tag{6}$$

where ρ is the mass density of the material, $\dot{u}$ defines the particle velocity vector and $\bar{\sigma} = \{N^T, M^T, Q^T\}^T$. The stress resultants {N}, stress couples {M} and transverse shear stress resultants {Q} are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z dz, \quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = K \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{7}$$

In Eq. (7), K is the shear correction factor. In this study, the shear correction factor is taken 5/6.

3. Finite element formulation

In this study, nine noded Lagrangian rectangular plate elements having five degrees of freedom are used for the finite element solution of the laminated plates. The interpolation function of the displacement field is defined as

$$\begin{pmatrix} u \\ v \\ w \\ \psi_x \\ \psi_y \end{pmatrix} = \left(\sum_{i=1}^{n} N_i d_i \right) \tag{8}$$

where d_i, N_i and n are the nodal variables, the interpolation function and total number of nodals per element, respectively. The generalized mid-surface strains at any point given by Eq. (3) can be expressed in terms of nodal displacements in matrix form as follows

$$\bar{\varepsilon} = \sum_{i=1}^{n} B_i d_i \tag{9}$$

where B_i is a differential operator of shape functions. Substituting for $\bar{\varepsilon}$, $\bar{\sigma}$ and $\dot{u}$ in Eq. (6), we get

$$\int_{t_1}^{t_2} \sum_{e=1}^{NE} \delta d_e^T [K^e d_e + M^e \ddot{d}] dt = 0 \tag{10}$$

where K^e and M^e are the element stiffness and mass matrices, respectively and given by

$$K^e = \int_{v^{(e)}} B^t D B dV$$

$$M^e = \int_{v^{(e)}} N^t \rho N dV \tag{11}$$

where B, D and ρ are the strain-displacement matrix, the rigidity matrix and the mass density, respectively.

As in the standard finite element procedure, one obtains the global eigenvalue equation is

$$([K] - \omega^2 [M])\{\bar{d}\} = 0 \tag{12}$$

where $[K]$ and $[M]$ are the global stiffness and mass matrices, respectively. Eq. (12) is a set of homogeneous linear equations in the unknown displacements $\{\bar{d}\}$. For non-trivial solution, the determinant is equal to zero and the eigenvalues correspond to free vibration frequencies of the laminated plates. The obtained smallest natural frequency (fundamental frequency) is used as an objective function and will be designed to maximize its value in the present optimization. The subspace iteration method is used for frequency analysis.

4. Extended layerwise optimization algorithm

In this paper, the objective of extended layerwise optimization algorithm is finding the optimum stacking sequence $[\theta_1/\theta_2/\theta_3/..../\theta_N]_{s,opt}$ for the maximum fundamental frequency of laminated plates which can be determined sequentially in the order from the outermost to the innermost layer. The current algorithm basically is illustrated in Fig. 2. The aim of this algorithm is the introduction of new layers in the stack that serve to improve the frequency criterion under consideration. However, there is a major difference in the procedure adopted with that of Narita (2003), in that herein no predetermined number of layers is assumed a priori. Here, the new layers are introduced on the mid-surface of the laminate whose optimal orientation are determined with no limitations as to their number. Another reason for the outward ordering of the successive layers is to place the most effective ones furthest away from the mid-surface. The steps of this algorithm can be expressed as below:

I-1. Assuming constant total laminate thickness equal to h, θ_1 is found so that it would possess the best fundamental frequency criterion. The search for optimal angle is done exhaustively in the -90^o to $+90^o$ domain in increments of $\Delta\theta$ selected.

I-2. Addition of the new layer into the stack which would cause the previously determined layer's thickness to reduce to half and be placed on the top of the stack.

I-N. In this step, by the introduction of the Nth layer based on the same criterion of choice, the thickness of the N-1 layers previously determined would decrease to h/2N. Finally, the new layer must show non-negative improvement of the frequency criterion. The process stops when this improvement becomes less than a predefined value. At the end of stage, a laminate of 2N-layers with the best posture for the frequency criterion is available.

The optimal design problem can be stated mathematically as follows

$$\begin{aligned} &\text{Find: } [\theta_1/\theta_2/\theta_3/..../\theta_N]_s \\ &\text{Maximize: } \omega_1 = \omega_1(\theta) \\ &\text{Subject to: } -90^o \le \theta_k \le 90^o \end{aligned} \tag{13}$$

The fundamental frequency for a given fibre orientation is determined from the finite element solution of the eigenvalue problems given by Eq. (12). The optimization procedure involves the stages of evaluating the fundamental frequency and improving the fiber orientation θ to maximise ω. Thus, the computational solution consists of successive stages of analysis and optimization until a convergence is obtained and the optimal angle θ_{opt} is determined within a specified accuracy.

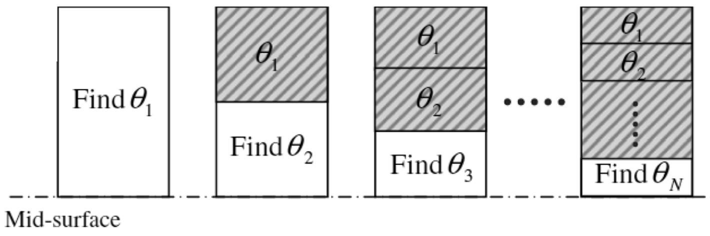

Fig. 2 Stepwise of extended layerwise optimization algorithm

5. Numerical results and discussion

In order to show the applicability of this algorithm, the optimization results of the laminated plates are given for T300/5208 graphite/epoxy material. The material properties are given as below

$E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$, $\rho = 1600$ kg/m^3

In this study, the increments of $\Delta\theta$ is equal to 5°. The results are presented in the nondimensional form as

$$\varpi = \omega a^2 \sqrt{(\rho/E_2)} \tag{14}$$

5.1 Effect of plate aspect ratio on the optimum design

As an example of a multi-angle design case, Table 1 shows the stepwise results of this algorithm for simply supported square laminated plates ($b/h = 25$). The first column indicates the number of steps and the second column indicates the optimum fibre orientations in the layers. The third column and fourth column show the fundamental frequencies and the increases in fundamental frequencies between the steps, respectively. The stopping criterion for $\Delta\varpi$ is taken as 0.005.

In Table 2, effect of plate aspect ratio (a/b) on the optimum results using extended layerwise optimization approach is illustrated. As seen from Table 2, the optimum stacking sequences and the number of layers are the same for $a/b > 1$. On the other hand, the fundamental frequency decreases with increase in the plate aspect ratio.

5.2 Effect of boundary conditions on the optimum design

Extended layerwise optimization algorithm may be applied to laminated plates with any

Table 1 Stepwise results for fundamental frequency for simply supported laminated plates ($a/b = 1$, $b/h = 25$)

Step	Stacking order	ϖ	$\Delta\varpi$
1	[45]	0.5388	-
2	[45/-45]	0.6168	0.078
3	[45/-45/-45]	0.6666	0.050
4	[45/-45/-45/-45]	0.6818	0.015
5	[45/-45/-45/-45/45]	0.6843	0.002

Table 2 Effect of plate aspect ratio (a/b) on the optimum results for simply supported laminated plates ($b/h = 25$)

a/b	Optimum stacking sequence	ϖ
1	[45/-45/-45/-45/45]	0.6843
1.5	[90]	0.4934
2	[90]	0.4831
2.5	[90]	0.4786
3	[90]	0.4763

Table 3 Effect of boundary conditions on the optimum results for square laminated plates (b/h = 25, a/b = 1)

Boundary conditions	Optimum stacking sequence	ϖ
(SSSS)	[45/-45/-45/-45/45]	0.6843
(CSCS)	[0]	1.0372
(CCCC)	[0/90/0/90]	1.0959
(CFCF)	[0]	1.0069
(CFFF)	[0]	0.1690
(SCFF)	[80/-40/60/60/60/65/65]	0.1948
(CCFF)	[65/-30/40/45/45/40/40]	0.2256
(SFCF)	[0]	0.7162
(SSSF)	[0]	0.4814
(SCSF)	[0]	0.4859
(SSCF)	[-5]	0.7260
(SCCF)	[-5]	0.7304
(SSSC)	[90/60/-55/-55/-55/55/60/60]	0.8735
(SSCC)	[0/45/-45/-45/-45/45/45/45/45/45/45/45/-45/-45/-45]	0.9832

combinations of simple support (S), clamped support (C), and free edge (F). Different combinations of the boundary conditions are considered in this study. For example, a clamped-simple-clamped-simple (CSCS) is a specimen with clamped supported on $x = 0$ and $x = a$, and simple supported on $y = 0$ and $y = b$, respectively. In Table 3, effect of different boundary conditions on the optimum results are given using extended layerwise optimization approach ($b/h = 25$, $a/b = 1$). As seen from Table 3, the maximum and minimum fundamental frequencies are obtained for (CCCC) and (CFFF) boundary conditions, respectively. It is obvious from the results that, the optimum stacking sequences and the number of layers can be changeable for different boundary conditions.

6. Conclusions

In this paper, the applicability of the extended layerwise optimization method (ELOM) on frequency optimization of laminated composite plates is investigated. The design objective is the maximization of the fundamental frequency. The fibre orientations in the layers are considered as design variables. The aim of this algorithm is the introduction of new layers in the stack that serve to improve the frequency criterion under consideration. However, there is a major difference in the procedure adopted with that of Narita's layerwise optimization algorithm (LO), in that herein no predetermined number of layers is assumed a priori. Here, the new layers are introduced on the mid-surface of the laminate whose optimal orientation are determined with no limitations as to their number. The limited set of results presented in this paper suggests that the ELOM procedure is an effective technique for determining the optimum laminate lay-ups in laminated plates in spite of increase of the computational effort and time. The optimum stacking sequences and the number of layers are the same for $a/b > 1$. On the other hand, the optimum stacking sequences and the number of layers can be changeable for different boundary conditions. This method may also be applied for other optimization problems and different parameters for laminated structures.

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