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Investigation of the effect of shell plan-form dimensions on mode-shapes of the laminated composite cylindrical shallow shells using SDSST and FEM

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Abstract. This paper presents the mode-shape analysis of the cross-ply laminated composite cylindrical shallow shells. First, the kinematic relations of strains and deformation are given. Then, using Hamilton's principle, governing differential equations are developed for a general curved shell. Finally, the stress-strain relation for the laminated, cross-ply composite shells are obtained. By using some simplifications and assuming Fourier series as a displacement field, the governed differential equations are solved by the matrix algebra for shallow shells. Employing the computer algebra system called MATHEMATICA; a computer program has been prepared for the solution. The results obtained by this solution are compared with the results obtained by (ANSYS and SAP2000) programs, in order to verify the accuracy and reliability of the solution presented.

Keywords: structural composites; vibration; anisotropy; shell theory; finite element method (FEM)

1. Introduction

A structural composite material consists of two or more constituents combined on a macroscopic scale to form a useful material. Different materials must be put together in a three dimensional body. The goal of this three dimensional composition is to obtain a property which none of the constituents possesses. In other words, the target is to produce a material that possesses higher performance properties than its constituent parts for a particular purpose. Some of these properties are mechanical strength, corrosion resistance, high temperature resistance, heat conductibility, stiffness, lightness and appearance. In accordance with this definition, the following conditions must be satisfied by the composite material. It must be manmade and unnatural. It must comprise of at least two different materials with different chemical components separated by distinct interfaces. It must possess properties, which none of the constituents possesses alone and that must be the aim of its production. The material must behave as a whole, i.e. the fiber and the matrix material (material surrounding the fibers) must be perfectly bonded. Structures composed of composite materials offer lower weight and higher strength and stiffness than those composed of most metallic materials (Qatu 2004).

Shells are common structural elements in many engineering structures, including concrete roofs,

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exteriors of rockets, ship hulls, automobile tires, containers of liquids, oil tanks, pipes, aerospace etc. A shell can be defined as a curved, thin-walled structure. It can be made from a single layer or multilayer of isotropic or anisotropic materials. Shells can be classified according to their curvatures. Shallow shells are defined as shells that have rise of not more than one fifth of the smallest planform dimension of the shell (Qatu 2004). Shells are three-dimensional (3D) bodies bounded by two relatively close, curved surfaces. Since the 3D equations of elasticity are complicated, all shell theories (thin, thick, shallow and deep, etc.) reduce the 3D elasticity problem into a 2D one. This is done usually by Classical Lamination Theory-CLT and Kirchhoff hypothesis. A number of theories exist for layered shells. Many of these theories were developed originally for thin shells and based on the Kirchhoff–Love kinematic hypothesis that straight lines normal to the undeformed mid-surface remain straight and normal to the middle surface after deformation. Among these theories Qatu (2004) uses energy functional to develop equation of motion. Many studies have been performed on characteristics of shallow shells (Qatu 1991, 1992a, 1992b, 1993a, 1993b). Recently, Latifa and Sinha (2005) have used an improved finite element model for the bending and free vibration analysis of doubly curved, laminated composite shells having spherical and ellipsoidal shapes. Large-amplitude vibrations of circular cylindrical shells subjected to radial harmonic excitation in the spectral neighborhood of the lowest resonance are investigated by Amabili (2003). Gautham and Ganesan (1997) deal with the free vibration characteristics of isotropic and laminated orthotropic spherical caps. Liew *et al.* (2002) has presented the elasticity solutions for free vibration analysis of doubly curved shell panels of rectangular planform. Grigorenko and Yaremchenko (2007) have analyzed the stress-strain state of a shallow shell with rectangular planform and varying thickness. Djoudi and Bahai (2003) have presented a cylindrical strain based shallow shell finite element which is developed for linear and geometrically non-linear analysis of cylindrical shells. Reddy (1993) and Librescu (1976) presented studies including the effect of shear deformation for laminated composite shells. Dogan (2010) studied the effects of anisotropy and curvature on free vibration characteristics of laminated composite cylindrical shallow shells. Kumar (2010) presented the analysis of impact response and damage in laminated composite cylindrical shells undergoing large deformations.

The aim of this study is to compare the frequency parameters of the each mode obtained by the theories given in literature and obtained by the finite element formulation for different cases. In this study, formulations of the thick and thin shell theories given by Qatu (2004), have been studied and a computer program coded in Mathematica is developed. The solutions of the problem are also obtained finite element method using commercial programs, named ANSYS and SAP2000. Results obtained by different theories have been compared for different plan-form dimensions, lamination thickness, ratio of radius of curvature equals to 0.1 and elasticity ratio equals to 15 cases. The shell, that has been examined, has quadrangle plan-form varying from square to rectangle. Moreover, lamination thickness has been taken as a variable. For different lamination thicknesses, results of the theories are presented by tables. Cross-ply, 4-layered lamination has been choosen as the material. The elasticity ratio (E_1/E_2) of the material is taken as 15. The results obtained from theories have been compared with literature, ANSYS and SAP2000 by using tables and graphs.

2. Theories

A lamina is made of isotropic homogeneous reinforcing fibers and an isotropic homogeneous material surrounding the fibers, called matrix material (Fig. 1). Therefore, the stiffness of the lamina

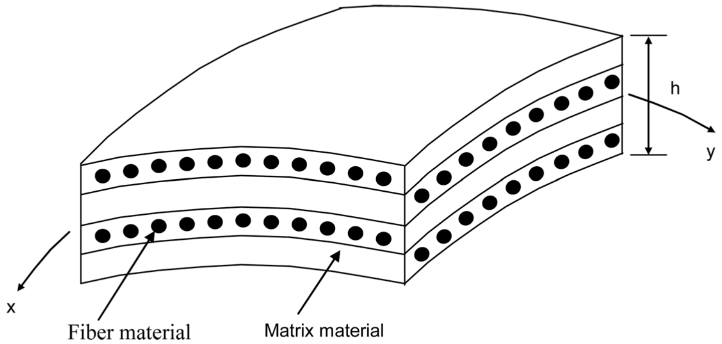


Fig. 1. Fiber and matrix materials in laminated composite shallow shell

varies from point to point depending on whether the point is in the fiber, the matrix or the fiber and matrix interface. Because of these variations, macro-mechanical analysis of a lamina is based on average properties.

There are many theories of shells. Classical shell theory, also known as Kirchhoff-Love kinematic hypothesis, assumes that “The normals to the middle surface remain straight and normal to the mid-surface when the shell undergoes deformation”. However, according to first order shear deformation theory “The transverse normals do not remain perpendicular to the mid-surface after deformation” (Reddy 2003). In addition, classical lamination theory says “laminas are perfectly bonded” (Gurdal *et al.* 1998, Hyer 1997, Reddy and Miravete 1995, Jones 1984). The theory of shallow shells can be obtained by making the following additional assumptions to thin (or classical) and thick (or shear deformation) shell theories. It will be assumed that the deformation of the shells is completely determined by the displacement of its middle surface. The derivation of equations of motion is based on two assumptions. The first assumption is that the shallow shell has small deflections. The second assumption is that the shallow shell thickness is small compared to its radii of curvature. Also, the radii of curvature are very large compared to the in-plane displacement. Curvature changes caused by the tangential displacement component u and v are very small in a shallow shell, in comparison with changes caused by the normal component w.

2.1. Geometric properties

The vectorial equation of the undeformed surface could be written by the x and y cartesian coordinates as

$$\vec{r} = \vec{r}(x, y) \tag{1}$$

a small increment in $\vec{r}$ vector is given as

$$\overrightarrow{dr} = \vec{r}_{,x}dx + \vec{r}_{,y}dy \tag{2}$$

where $\bar{r}_{,x}$ is the small increment in x direction and $\bar{r}_{,y}$ is the small increment in y direction (Fig. 2). The differential length of the shell surface could be found by dot product of $d\bar{r}$ by itself

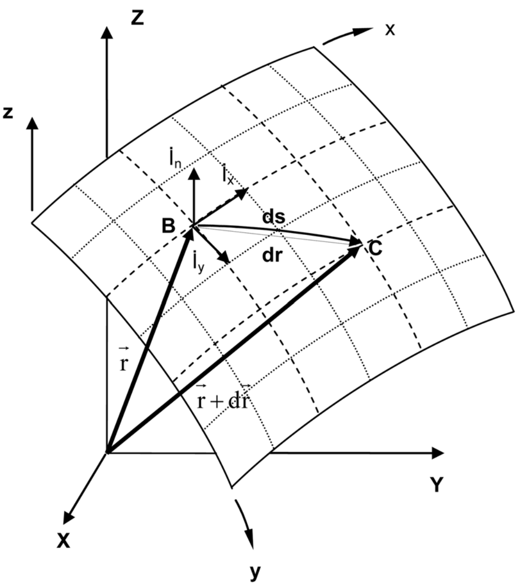


Fig. 2. Coordinates of shell mid-surface

$$ds^2 = \vec{dr} \bullet \vec{dr} = A^2 dx^2 + B^2 dy^2 \tag{3}$$

where *A* and *B* are referred as Lame parameters and defined as

$$A^2 = \vec{r}_{,x} \bullet \vec{r}_{,x} \quad B^2 = \vec{r}_{,y} \bullet \vec{r}_{,y} \tag{4}$$

Eq. (3) is called first fundamental form of the surface. Tangent vector to the surface could be obtained by taking derivative of Eq. (1) with respect to surface length. Then, applying Frenet's formula to the derivative of tangent vector and multiplying both sides by unit normal vector gives second quadratic form.

2.2. Kinematics of displacement

Let the position of a point, on a middle surface, shown by $\vec{r}(x, y)$. If this point undergoes the displacement by the amount of $\vec{U}$ then, final position of that point could be given as

$$\vec{r}'(x, y) = \vec{r}(x, y) + \vec{U} \tag{5}$$

where $\vec{U}$ is the displacement field of the point and defined as

$$\vec{U} = u\vec{i}_x + v\vec{i}_y + w\vec{i}_z \tag{6}$$

where $\vec{i}_x$, $\vec{i}_y$ and $\vec{i}_z$ are the unit vectors in the direction of *x*, *y* and *z*. *u*, *v*, and *w* are the displacements in the direction of *x*, *y* and *z* respectively. Using Eqs. (5) and (6) strains are calculated as

$$\varepsilon_x = \frac{1}{(1+z/R_x)}\left(\frac{1}{A}\frac{\partial u}{\partial x}+\frac{v}{AB}\frac{\partial A}{\partial y}+\frac{w}{R_x}\right)$$

$$\varepsilon_y = \frac{1}{(1+z/R_y)}\left(\frac{1}{B}\frac{\partial v}{\partial y}+\frac{u}{AB}\frac{\partial B}{\partial x}+\frac{w}{R_y}\right)$$

$$\varepsilon_z = \partial w/dz \tag{7}$$

$$\gamma_{xy} = \frac{1}{(1+z/R_x)}\left(\frac{1}{A}\frac{\partial v}{\partial x}-\frac{u}{AB}\frac{\partial A}{\partial y}+\frac{w}{R_{xy}}\right)+\frac{1}{(1+z/R_y)}\left(\frac{1}{B}\frac{\partial u}{\partial y}-\frac{v}{AB}\frac{\partial B}{\partial x}+\frac{w}{R_{xy}}\right)$$

$$\gamma_{xz} = \frac{1}{A(1+z/R_x)}\frac{\partial w}{\partial x}+A(1+z/R_x)\frac{\partial}{\partial z}\left(\frac{u}{A(1+z/R_x)}\right)-\frac{v}{R_{xy}(1+z/R_x)}$$

$$\gamma_{yz} = \frac{1}{B(1+z/R_y)}\frac{\partial w}{\partial y}+B(1+z/R_y)\frac{\partial}{\partial z}\left(\frac{v}{B(1+z/R_y)}\right)-\frac{u}{R_{xy}(1+z/R_y)}$$

where R_x, R_y and R_{xy} are curvatures in x-plane, y-plane and xy-plane respectively.

2.3. Stress strain relation

For an orthotropic media there are 9 stiffness coefficients written in local coordinates.

$$[\sigma] = [Q][\varepsilon] \tag{8}$$

where $[\sigma]$ is the stress matrices, $[Q]$ is the stiffness matrices and $[\varepsilon]$ strain matrices. The stresses in global coordinates are calculated by applying transformation rules. Then, the stresses over the shell thickness are integrated to obtain the force and moment resultants. Due to curvatures of the structure, extra terms must be taken into account during the integration. This difficulty could be overcame by expanding the term $[1/(1+z/R_n)]$ in a geometric series.

2.4. Governing equations

Equation of motion for shell structures could be obtained by Hamilton's principle

$$\delta\int_{t_1}^{t_2}(T+W-U)dt = 0 \tag{9}$$

where T is the kinetic energy of the structure

$$T = \frac{\rho}{2}\int\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right\}dxdydz \tag{10}$$

W is the work of the external forces

$$W = \iint_{xy} (q_x u + q_y v + q_z w + m_x \psi_x + m_y \psi_y) AB dxdy \tag{11}$$

in which q_x, q_y, q_z are the external forces u, v, w are displacements in x, y, z direction respectively. m_x, m_y are the external moments and ψ_x, ψ_y are rotations in x, y directions respectively. U is the strain energy defined as

$$U = \frac{1}{2}\int(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \sigma_{xy}\varepsilon_{xy} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz})dxdydz \tag{12}$$

Solving Eq. (9) gives set of equations called equations of motion for shell structures.

$$\begin{aligned}
&\frac{\partial}{\partial x}(BN_x) + \frac{\partial}{\partial y}(AN_{yx}) + \frac{\partial A}{\partial y}N_{xy} - \frac{\partial B}{\partial x}N_y + \frac{AB}{R_x}Q_x + \frac{AB}{R_{xy}}Q_y + ABq_x = AB(\bar{I}_1\ddot{u}^2 + \bar{I}_2\ddot{\psi}_x^2)\\
&\frac{\partial}{\partial y}(AN_y) + \frac{\partial}{\partial x}(BN_{xy}) + \frac{\partial B}{\partial x}N_{yx} - \frac{\partial A}{\partial y}N_x + \frac{AB}{R_y}Q_y + \frac{AB}{R_{xy}}Q_x + ABq_y = AB(\bar{I}_1\ddot{v}^2 + \bar{I}_2\ddot{\psi}_y^2)\\
&-AB\left(\frac{N_x}{R_x} + \frac{N_y}{R_y} + \frac{N_{xy}+N_{yx}}{R_{xy}}\right) + \frac{\partial}{\partial x}(BQ_x) + \frac{\partial}{\partial y}(AQ_y) + ABq_z = AB(\bar{I}_1\ddot{w}^2)\\
&\frac{\partial}{\partial x}(BM_x) + \frac{\partial}{\partial y}(AM_{yx}) + \frac{\partial A}{\partial y}M_{xy} - \frac{\partial B}{\partial x}M_y - ABQ_x + \frac{AB}{R_x}P_x + ABm_x = AB(\bar{I}_2\ddot{u}^2 + \bar{I}_3\ddot{\psi}_x^2)\\
&\frac{\partial}{\partial y}(AM_y) + \frac{\partial}{\partial x}(BM_{xy}) + \frac{\partial B}{\partial x}M_{yx} - \frac{\partial A}{\partial y}M_x - ABQ_y + \frac{AB}{R_y}P_y + ABm_y = AB(\bar{I}_2\ddot{v}^2 + \bar{I}_3\ddot{\psi}_y^2)
\end{aligned} \tag{13}$$

When the shell has small curvature it is referred to as a shallow shell. Shallow shells are defined as shells that have a rise of not more than 1/5th the smallest planform dimension of the shell (Qatu 2004). It has been widely accepted that shallow shell equations should not be used for maximum span to minimum radius ratio of 0.5 or more. For shallow shells, Lame parameters are assumed to equal to one (A=B=1). This gives Eq. (13) in simplified form as

$$\begin{aligned}
&\frac{\partial}{\partial x}N_x + \frac{\partial}{\partial y}N_{yx} + q_x = \bar{I}_1\ddot{u}^2 + \bar{I}_2\ddot{\psi}_x^2\\
&\frac{\partial}{\partial y}N_y + \frac{\partial}{\partial x}N_{xy} + q_y = \bar{I}_1\ddot{v}^2 + \bar{I}_2\ddot{\psi}_y^2\\
&-\left(\frac{N_x}{R_x} + \frac{N_y}{R_y} + \frac{N_{xy}+N_{yx}}{R_{xy}}\right) + \frac{\partial}{\partial x}Q_x + \frac{\partial}{\partial y}Q_y + q_z = \bar{I}_1\ddot{w}^2\\
&\frac{\partial}{\partial x}M_x + \frac{\partial}{\partial x}M_{xy} - Q_y + m_x = \bar{I}_2\ddot{u}^2 + \bar{I}_3\ddot{\psi}_x^2
\end{aligned} \tag{14}$$

$$\frac{\partial}{\partial y}M_y + \frac{\partial}{\partial x}M_{xy} - Q_y + m_y = \bar{I}_2\ddot{v}^2 + \bar{I}_3\ddot{\psi}_y^2$$

Eq. (14) is defined as equation of motion for thick shallow shell. The force and moment resultants are

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{oy} \\ \gamma_{0xy} \\ \kappa_x \\ \kappa_y \\ \tau \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} A_{55} & A_{45} \\ A_{45} & A_{44} \end{bmatrix} \begin{bmatrix} \gamma_{0xz} \\ \gamma_{oyz} \end{bmatrix}$$

The Navier type solution can be applied to thick and thin shallow shells. This type solution assumes that the displacement field of the shallow shells could be represented as sine and cosine trigonometric functions.

Consider a shell with shear diaphragm boundaries on all four edges. That is, boundary conditions for simply supported thick shells

$$N_x = w_0 = v_0 = M_x = \psi_y = 0 \qquad x = 0, a \quad (16)$$

$$N_y = w_0 = u_0 = M_y = \psi_x = 0 \qquad y = 0, b$$

The displacement functions of satisfied the boundary conditions apply

$$u_0(x, y, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} U_{mn}Cos(x_m x)Sin(y_n y)Sin(\omega_{mn}t)$$

$$v_0(x, y, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} V_{mn}Sin(x_m x)Cos(y_n y)Sin(\omega_{mn}t)$$

$$w_0(x, y, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} W_{mn}Sin(x_m x)Cos(y_n y)Sin(\omega_{mn}t) \quad (17)$$

$$\psi_x(x, y, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{xmn}Cos(x_m x)Sin(y_n y)Sin(\omega_{mn}t)$$

$$\psi_y(x, y, t) = \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{ymn}Sin(x_m x)Cos(y_n y)Sin(\omega_{mn}t)$$

Where, $\alpha_m = m\pi/a$, $\beta_n = n\pi/b$, ω_{mn} is national frequency.
Where, U_{mn}, V_{mn}, W_{mn}, $\psi_{\alpha mn}$, $\psi_{\beta mn}$ are arbitrary coefficients.
Substituting the above equations into equation of motion and using a Fourier expansion.

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \psi_{xmn} \\ \psi_{ymn} \end{bmatrix} + \omega_{mn}^2 \begin{bmatrix} -I_1 & 0 & 0 & -I_2 & 0 \\ 0 & -I_1 & 0 & 0 & -I_2 \\ 0 & 0 & -I_1 & 0 & 0 \\ -I_2 & 0 & 0 & -I_3 & 0 \\ 0 & K_{52} & 0 & 0 & -I_3 \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \psi_{xmn} \\ \psi_{ymn} \end{bmatrix} = \begin{bmatrix} -P_x \\ -P_y \\ P_n \\ m_x \\ m_y \end{bmatrix} \quad (18)$$

Following equation can be used directly to find the natural frequencies of free vibrations.

$$[K]\{\Delta\} + (\omega_{mn})^2[M]\{\Delta\} = 0 \quad (19)$$

$$K_{12} = K_{21} = -(A_{12} + A_{66})x_m y_n$$

$$K_{11} = -A_{11}x_m^2 - A_{66}y_n^2$$

$$K_{13} = K_{31} = \left[\frac{A_{11}}{R_x} + \frac{A_{12}}{R_y}\right]x_m$$

$$K_{14} = K_{41} = -B_{11}x_m^2 - B_{66}y_n^2$$

$$K_{15} = K_{51} = -(B_{12} + B_{66})x_m y_n$$

$$K_{22} = -A_{66}x_m^2 - A_{22}y_n^2$$

$$K_{23} = K_{32} = \left[\frac{A_{12}}{R_x} + \frac{A_{22}}{R_y}\right]y_n$$

$$K_{24} = K_{42} = -(B_{12} + B_{66})x_m y_n$$

$$K_{22} = -B_{66}x_m^2 - B_{22}y_n^2$$

$$K_{33} = -A_{55}x_m^2 - A_{44}y_n^2 - \left[\frac{A_{11}}{R_x^2} + \frac{2A_{12}}{R_x R_y} + \frac{A_{22}}{R_y^2}\right] \quad (20)$$

$$K_{34} = K_{43} = \left[-A_{55} + \frac{B_{11}}{R_x} + \frac{B_{12}}{R_y}\right]x_m$$

$$K_{35} = K_{53} = \left[-A_{44} + \frac{B_{12}}{R_x} + \frac{B_{22}}{R_y}\right]y_n$$

$$K_{44} = -A_{55} - D_{11}x_m^2 - D_{66}y_n^2$$

$$K_{45} = K_{54} = -(D_{12} + D_{66})x_m y_n$$

$$K_{55} = -A_{44} - D_{66}x_m^2 - D_{22}y_n^2$$

$$M_{ij} = M_{ji}$$

$$M_{11} = M_{22} = M_{33} = -I_1 \qquad M_{14} = M_{25} = -I_2 \qquad M_{44} = M_{55} = -I_3$$

$$\text{all other } M_{ij} = 0$$

3. Numerical examples

As an example, a simply supported cylindrical shell which has a ratio of radius of curvature (ratio of shell width/shell radius) equals to 0.1 in one plane and infinite radius of curvature in other plane, has been considered (Fig. 3(a)). The shell, in hand, has a quadrangle planform where the ratio of plan-form dimensions varies from 1 to 4 (a/b = 1, 2, 4). As a material, a laminated composite has been used with a [0º/90º/90º/0º] symmetrical cross-ply stacking sequence (Fig. 3(b)). Ratio of modulus of elasticity (E_1/E_2) which is the ratio of modulus of elasticity in fiber direction to matrix direction, has been taken as 15. Effect of shell thickness ratio that ratio of shell width to shell thickness, a/h = 100, 50, 20, 10 and 5, has been examined.

For each case, the shell has been solved with two theories. First theory used in the solution of composite laminated shallow shell is shear deformation shallow shell theory (SDSST). The second theory is the Finite element model (FEM). Entire structure is meshed by finite elements in this theory. Then assuming a suitable displacement fields for each meshing element, the behavior of the structure

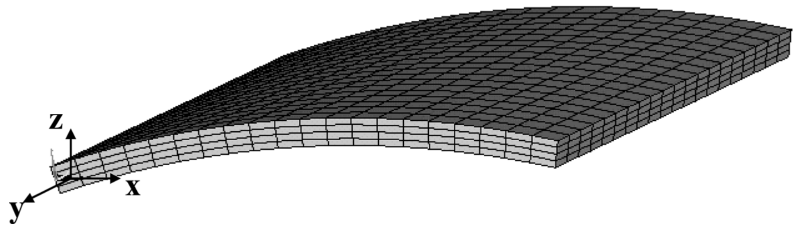


Fig. 3(a). Cylindrical shallow shell

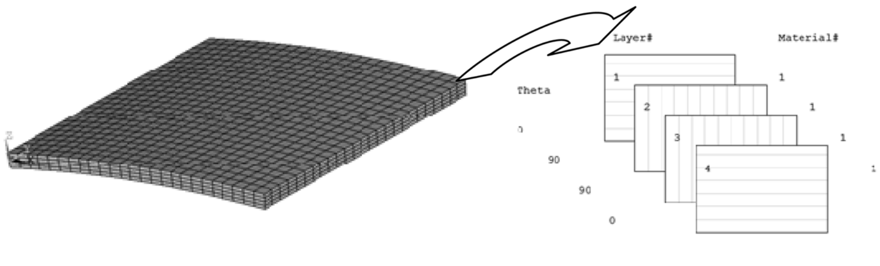


Fig. 3(b). Layered sequence for cylindrical shallow shell

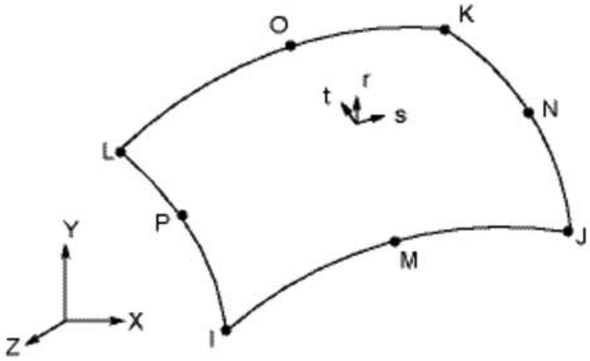


Fig. 4(a). 8-point quadratic elements for ANSYS

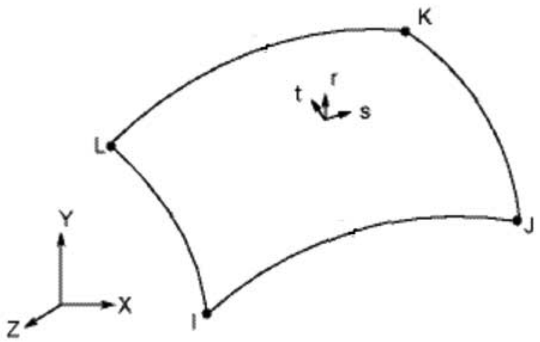


Fig. 4(b). 4-point quadratic elements for SAP2000

Table 1 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes ($a/b = 1$, $a/h = 100$, $E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2 = 0.5$, $\upsilon_{12} = 0.25$ and $K^2 = 5/6$)

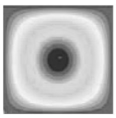
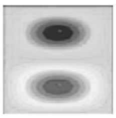
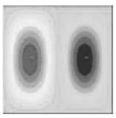
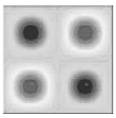
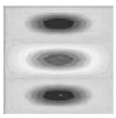
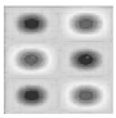
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(2,3)
							
ANSYS	Frequency (ω)	0,06355	0,07793	0,09666	0,11253	0,11819	0,14997
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	28,23443	34,62393	42,94346	49,99755	52,50920	66,62799
Sap2000	Frequency (ω)	0,06335	0,07753	0,09678	0,11208	0,11811	0,14899
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	28,14700	34,44700	42,99911	49,79672	52,47578	66,19629
SDSST	Frequency (ω)	0,03141	0,05973	0,09606	0,11098	0,10982	0,14806
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	13,95614	26,53820	42,67974	49,30593	48,79160	65,78098

Table 2 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 1, a/h = 50, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

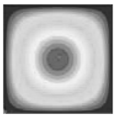
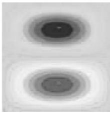
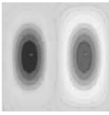
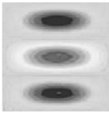
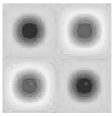
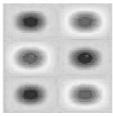
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(1,2)	(2,1)	(1,3)	(2,2)	(2,3)
							
ANSYS	Frequency (ω)	0,07944	0,11938	0,18897	0,21232	0,21767	0,28999
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	17,64614	26,51915	41,97803	47,16572	48,35279	64,41983
Sap2000	Frequency (ω)	0,07930	0,11923	0,18909	0,21311	0,21687	0,28835
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	17,61559	26,48669	42,00568	47,34158	48,17662	64,05504
SDSST	Frequency (ω)	0,05698	0,10844	0,18873	0,20769	0,21714	0,28929
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	12,65665	24,08922	41,92546	46,13606	48,23742	64,26458

Table 3 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 1, a/h = 20, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

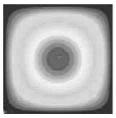
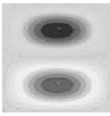
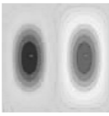
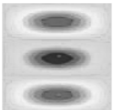
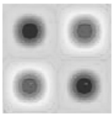
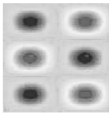
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(1,2)	(2,1)	(1,3)	(2,2)	(2,3)
							
ANSYS	Frequency (ω)	0,14521	0,25968	0,42824	0,48808	0,49454	0,66107
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	12,90291	23,07487	38,05221	43,36976	43,94330	58,74111
Sap2000	Frequency (ω)	0,14547	0,26038	0,42843	0,49041	0,49458	0,65884
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	12,92648	23,13649	38,06929	43,57651	43,94687	58,54254
SDSST	Frequency (ω)	0,13471	0,25517	0,42763	0,48380	0,49407	0,65863
	$\Omega = \omega a^2 \sqrt{\rho/E_2 h^2}$	11,97003	22,67350	37,99775	42,98964	43,90165	58,52445

Table 4 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 1, a/h = 10, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

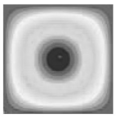
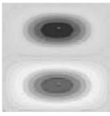
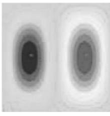
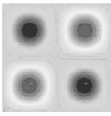
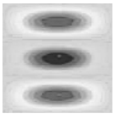
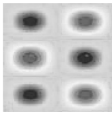
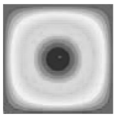
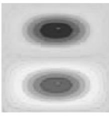
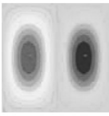
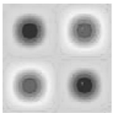
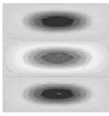
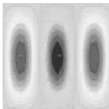
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(2,3)
							
ANSYS	Frequency (ω)	0,25243	0,46610	0,67387	0,79500	0,83811	1,06904
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	11,21531	20,70810	29,93939	35,32112	37,23615	47,49629
Sap2000	Frequency (ω)	0,25383	0,46776	0,67790	0,80235	0,83943	1,06658
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	11,27755	20,78216	30,11844	35,64765	37,29503	47,38708
SDSST	Frequency (ω)	0,24729	0,46168	0,67160	0,79039	0,82572	1,05663
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	10,98675	20,51179	29,83849	35,11589	36,68563	46,94494

Table 5 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 1, a/h = 5, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(3,1)
							
ANSYS	Frequency (ω)	0,39910	0,72639	0,85771	1,05783	1,18046	1,33212
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	8,86581	16,13638	19,05350	23,49916	26,22329	29,59219
Sap2000	Frequency (ω)	0,40231	0,72686	0,86838	1,06704	1,17470	1,33701
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	8,93717	16,14668	19,29044	23,70376	26,09518	29,70096
SDSST	Frequency (ω)	0,39527	0,71586	0,85345	1,04641	1,16160	1,32473
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	8,78066	15,90236	18,95881	23,24535	25,80420	29,42817

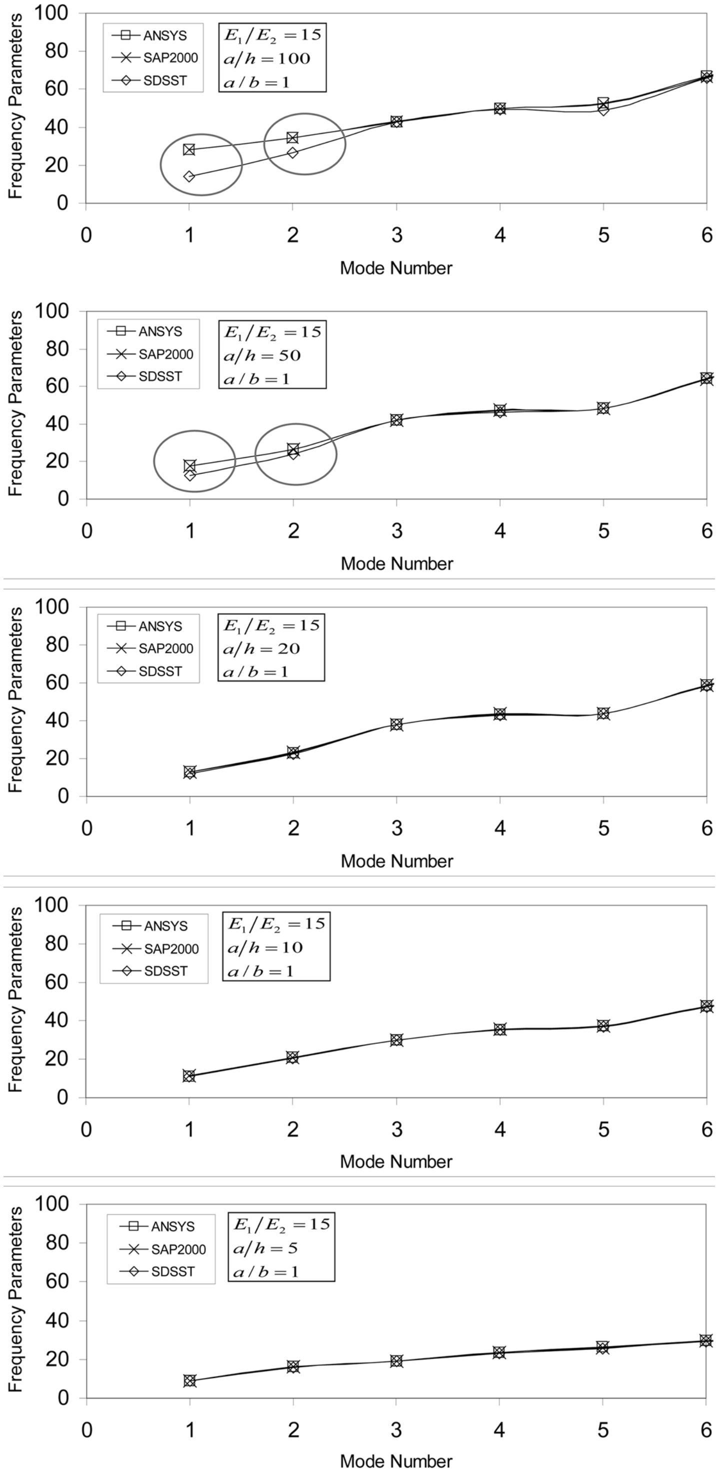


Fig. 5 Comparison of the frequency parameters of the first six modes obtained by SDSST and FEM (For Table 1-5)

Table 6 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes ($a/b = 2$, $a/h = 100$, $E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2 = 0.5$, $\upsilon_{12} = 0.25$ and $K^2 = 5/6$)

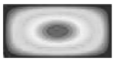
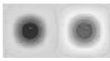
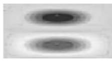
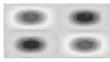
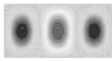
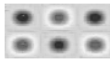
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(1,2)	(2,2)	(3,1)	(3,2)
							
ANSYS	Frequency (ω)	0,07792	0,11249	0,18540	0,21282	0,22168	0,29480
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	34,61891	49,97761	82,36936	94,55278	98,49128	130,97482
Sap2000	Frequency (ω)	0,07778	0,11259	0,18557	0,21215	0,22282	0,29343
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	34,55585	50,02286	82,44702	94,25532	98,99809	130,36618
SDSST	Frequency (ω)	0,05973	0,11098	0,18157	0,21131	0,22081	0,29434
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	26,53820	49,30593	80,66823	93,88395	98,10270	130,77349

Table 7 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes ($a/b = 2$, $a/h = 50$, $E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2 = 0.5$, $\upsilon_{12} = 0.25$ and $K^2 = 5/6$)

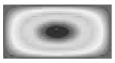
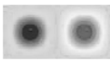
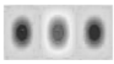
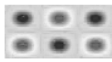
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(1,2)	(2,2)	(3,1)	(3,2)
							
ANSYS	Frequency (ω)	0,11926	0,21740	0,35193	0,41267	0,42690	0,56922
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	26,49197	48,29461	78,17942	91,67195	94,83408	126,44829
Sap2000	Frequency (ω)	0,11932	0,21788	0,35265	0,41186	0,42890	0,56706
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	26,50691	48,40143	78,33847	91,49295	95,27785	125,96995
SDSST	Frequency (ω)	0,10844	0,21714	0,34949	0,41221	0,42653	0,56931
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	24,08922	48,23742	77,63627	91,57016	94,75090	126,46831

Table 8 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes ($a/b = 2$, $a/h = 20$, $E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2 = 0.5$, $\upsilon_{12} = 0.25$ and $K^2 = 5/6$)

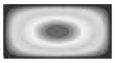
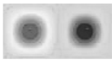
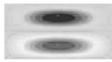
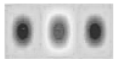
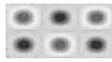
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(1,2)	(3,1)	(2,2)	(3,2)
							
ANSYS	Frequency (ω)	0,25892	0,49330	0,79960	0,88795	0,92777	1,20684
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	23,00722	43,83322	71,05101	78,90111	82,43907	107,23740
Sap2000	Frequency (ω)	0,26010	0,49652	0,80115	0,89395	0,92696	1,20654
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	23,11143	44,11987	71,18831	79,43422	82,36723	107,21014
SDSST	Frequency (ω)	0,25517	0,49407	0,79181	0,88538	0,92231	1,19924
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	22,67350	43,90165	70,35833	78,67289	81,95432	106,56195

Table 9 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes ($a/b = 2$, $a/h = 10$, $E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2 = 0.5$, $\upsilon_{12} = 0.25$ and $K^2 = 5/6$)

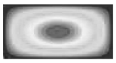
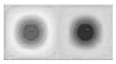
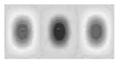
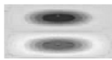
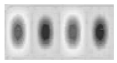
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(1,2)	(2,2)	(4,1)
							
ANSYS	Frequency (ω)	0,46414	0,79268	1,24490	1,28418	1,45120	1,71915
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	20,62120	35,21768	55,30951	57,05468	64,47528	76,37966
Sap2000	Frequency (ω)	0,46710	0,80322	1,26224	1,28411	1,44645	1,73019
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	20,75257	35,68595	56,07962	57,05168	64,26426	76,87014
SDSST	Frequency (ω)	0,46168	0,79039	1,23691	1,26276	1,43127	1,70701
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	20,51179	35,11589	54,95428	56,10285	63,58975	75,84034

Table 10 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 2, a/h = 5, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

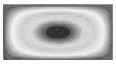
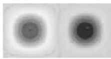
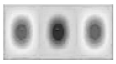
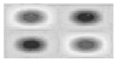
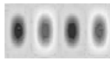
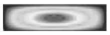
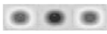
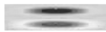
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(1,2)	(2,2)	(4,1)
							
ANSYS	Frequency (ω)	0,72392	1,05567	1,47224	1,65675	1,82708	1,91077
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	16,08155	23,45110	32,70494	36,80375	40,58761	42,44668
Sap2000	Frequency (ω)	0,72689	1,06790	1,48665	1,65160	1,81231	1,90836
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	16,14737	23,72266	33,02497	36,68933	40,25941	42,39308
SDSST	Frequency (ω)	0,71586	1,04641	1,45848	1,63584	1,80720	1,89415
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	15,90236	23,24535	32,39929	36,33916	40,14588	42,07741

Table 11 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 4, a/h = 100, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(1,2)
							
ANSYS	Frequency (ω)	0,18532	0,21253	0,29433	0,43302	0,62177	0,67799
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	82,33340	94,42237	130,76809	192,38685	276,24350	301,22453
Sap2000	Frequency (ω)	0,18521	0,21197	0,29404	0,43532	0,63037	0,67924
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	82,28619	94,17535	130,63942	193,40802	280,06512	301,77794
SDSST	Frequency (ω)	0,18157	0,21131	0,29434	0,43355	0,62192	0,67621
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	80,66823	93,88395	130,77349	192,61986	276,31394	300,43386

has been obtained. In this paper, two different finite element package programs, ANSYS and SAP2000 have been used. The structure is meshed by 25 × 25 elements in ANSYS model. A 8-noded quadratic element is considered as a meshing element named as SHELL99 [14]. The element has 100 layers to model the composite materials used in the structure. For each layer geometric and material properties is entered to program. Furthermore, thicknesses of each layer, fiber orientations and stacking sequence must be entered carefully. During solution process, subspace and block Lanczos mode extracting methods used separately to calculate first 30 frequencies. SAP2000 structural analysis packet program has been used also to verify ANSYS results. The structure is meshed by 25 × 25 elements in SAP2000 model. Four-node quadrilateral shell elements have been used by SAP2000 finite element program.

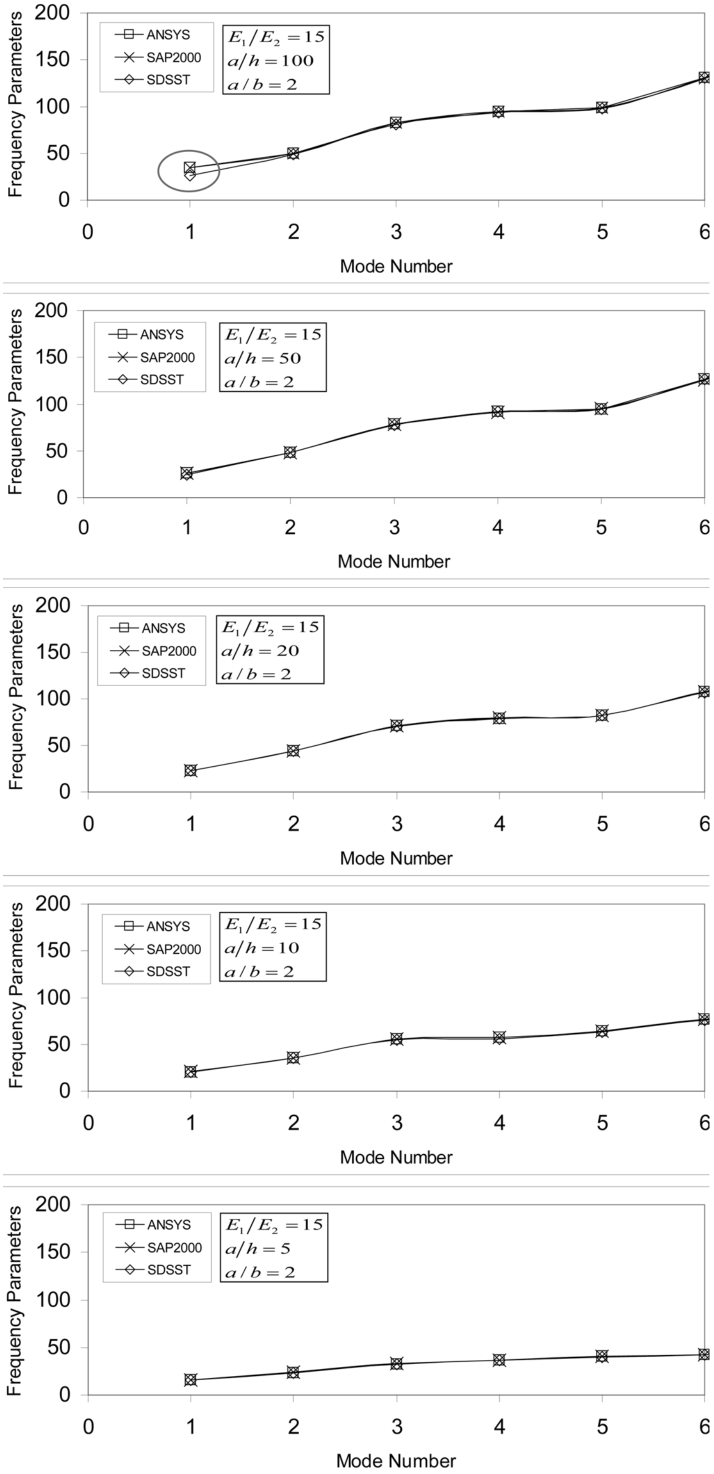


Fig. 6 Comparison of the frequency parameters of the first six modes obtained by SDSST and FEM (For Table 6-10)

Table 12 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 4, a/h = 50, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

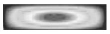
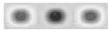
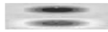
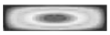
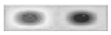
Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(1,2)
							
ANSYS	Frequency (ω)	0,35159	0,41156	0,56754	0,81907	1,14476	1,28793
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	78,10386	91,42575	126,07630	181,95182	254,30190	286,10602
Sap2000	Frequency (ω)	0,35169	0,41131	0,56812	0,82321	1,15502	1,28962
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	78,12654	91,37078	126,20453	182,87084	256,58138	286,48176
SDSST	Frequency (ω)	0,34949	0,41221	0,56931	0,82016	1,14361	1,27804
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	77,63627	91,57016	126,46831	182,19351	254,04536	283,90945

Table 13 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 4, a/h = 20, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
							
ANSYS	Frequency (ω)	0,79798	0,92344	1,20148	1,58814	2,02911	2,49371
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	70,90644	82,05435	106,76106	141,11823	180,30178	221,58517
Sap2000	Frequency (ω)	0,79879	0,92568	1,20752	1,59750	2,03432	2,47995
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	70,97852	82,25376	107,29696	141,94975	180,76527	220,36228
SDSST	Frequency (ω)	0,79181	0,92231	1,19924	1,58069	2,01536	2,47385
	$\Omega = \omega a^2 \sqrt{\rho / E_2 h^2}$	70,35833	81,95432	106,56195	140,45676	179,08006	219,82083

Orthotropic and lamination properties of the problem could be modeled by using this element. Regardless of the point used, both programs provided same geometric shape as seen in Fig. 4(a)-(b).

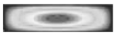
Mesh density is extremely important. In this study, the structure is meshed by 25 × 25 elements in SAP2000 and ANSYS models. In this study, the structure was also modeled by the mesh element 50X50. However, analysis results demonstrated that increasing the number of mesh did not significantly alter the analysis results. Therefore, in this study, the structure is meshed 25 × 25 for the ease of solving problem as well as saving computer analysis time.

The governing Eq. (14) (using SDSST theory) derived in the theory section are solved by using Mathematica program separately. Furthermore, ANSYS packet program has been used in solution. The

Table 14 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 4, a/h = 10, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(,)
							
ANSYS	Frequency (ω)	1,28088	1,44541	1,74356	2,11521	2,52312	2,83047
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	56,90792	64,21800	77,46421	93,97609	112,09918	125,75464
Sap2000	Frequency (ω)	1,28100	1,44683	1,74748	2,11563	2,50501	2,82076
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	56,91329	64,28101	77,63831	93,99505	111,29466	125,32320
SDSST	Frequency (ω)	1,26276	1,43127	1,72622	2,09284	2,49546	
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	56,10285	63,58975	76,69392	92,98233	110,87050	

Table 15 Comparison of the frequencies (ω) and nondimensional frequency parameters ($\Omega = \omega a^2\sqrt{\rho/E_2h^2}$) of the shallow shell obtained by Shear Deformation Shell Theory (SDSST) and Finite Element Method (ANSYS & SAP2000) for six modes (a/b = 4, a/h = 5, E_1/E_2 = 15, $G_{12}/E_2 = G_{13}/E_2 = G_{13}/E_2$ = 0.5, υ_{12} = 0.25 and K^2 = 5/6)

Method	Frequency(ω) and Nondimensional Frequency(Ω) Parameters	Mode shapes					
		(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(,)
							
ANSYS	Frequency (ω)	1,65357	1,82388	2,09176	2,41996	2,78649	2,83082
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	36,73306	40,51643	46,46720	53,75802	61,90020	62,88503
Sap2000	Frequency (ω)	1,65086	1,81651	2,07583	2,38615	2,71988	2,82115
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	36,67282	40,35273	46,11337	53,00681	60,42048	62,67017
SDSST	Frequency (ω)	1,63584	1,80720	2,07347	2,39988	2,76426	
	$\Omega = \omega a^2\sqrt{\rho/E_2h^2}$	36,33916	40,14588	46,06099	53,31189	61,40650	

geometry of the shell structures has been created using arc-length method in ANSYS. Then, area element has been defined between the arc lines. Finally using SHELL99 finite element, the area has been meshed. Similar procedures have been applied in SAP2000 program. Due to software differences, modeling steps differs from ANSYS to SAP2000. Furthermore, finite elements also differs in both program hence, FEM results have little difference.

The problem defined at the beginning has been solved by FEM and Mathematica program (Fig. 3). The results obtained by FEM and MATHEMATICA, have been compared in tables and graphs.

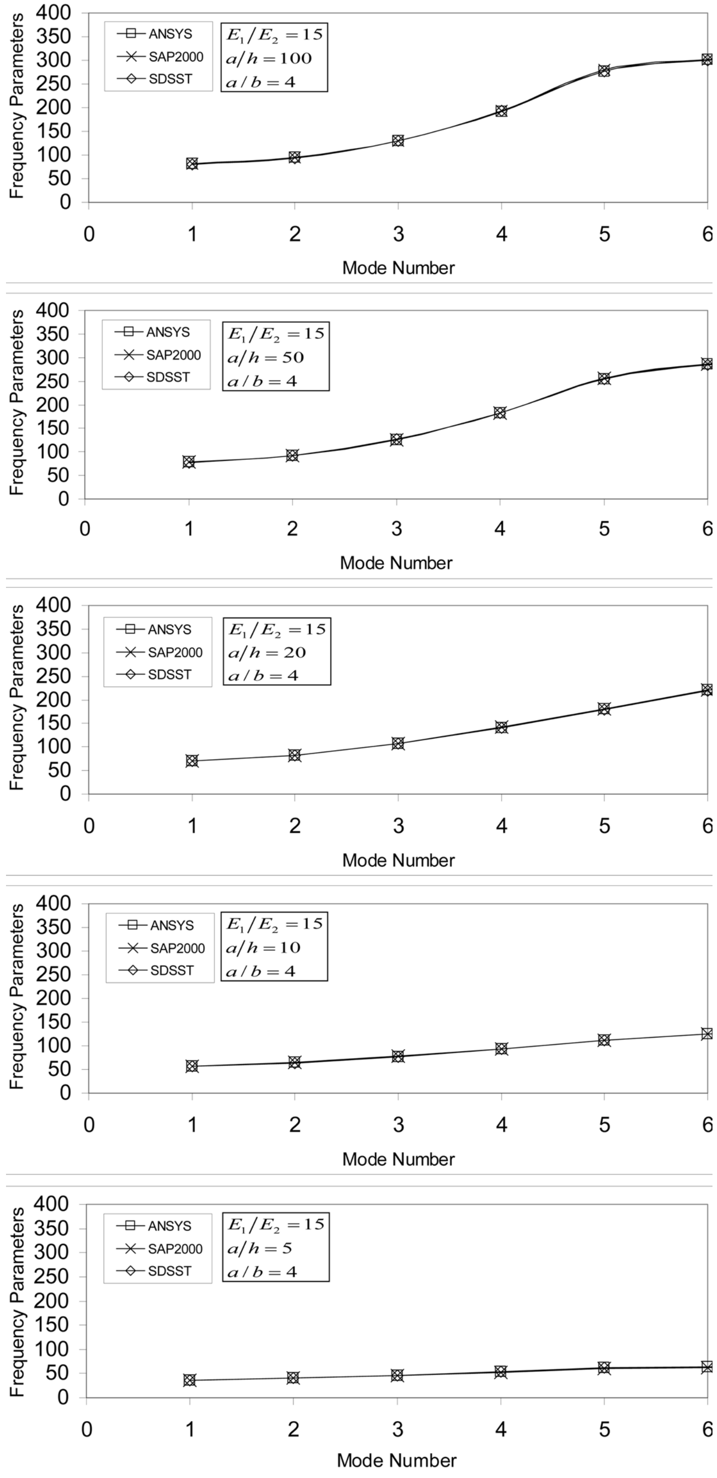


Fig. 7 Comparison of the frequency parameters of the first six modes obtained by SDSST and FEM (For Table 11-15)

4. Conclusions

The results obtained from the SDSST and FEM analysis results are compared. In literature, there are not enough FEM and SDSST results considering shell plan-form dimensions and giving higher modes. Generally, results obtained from first mode or first three modes have been found in literature considering a typical shell plan-form dimension and/or a typical shell thickness. It should be noticed that in order to get better knowledge about the agreement between FEM and SDSST higher modes must be calculated. Therefore, in this study, the higher modes up to sixth mode have been calculated.

In this study, the first six non dimensional frequency parameters, obtained from SDSST and FEM, have been given. A considerable differences have been observed in the first mode results for both theories (Figs. 5 and 6). These differences get smaller and almost reaches zero in higher modes. Hence, considering the first mode and using one method could result in misunderstanding the real behavior of the structure. Therefore, all modes must be calculated and results should be compared to see the overall picture.

In the case of thin and square plan-form dimensions of shallow shell, SDSST's and FEM's first mode results differ from each other (Fig. 5) This difference gets smaller for second mode and becomes almost zero for the higher modes. However, it should be seen that as the shell thickness increases, the results almost coincide for all modes.

Furthermore, the effect of shell plan-form dimensions on free vibration frequency parameters has been studied for three cases (a/b = 1,2,4). As the shell plan-form changes from square (a/b = 1) to rectangle (a/b = 4), the results of SDSST and FEM get closer. The difference between first mode results of SDSST and FEM when the plan-form goes from square to rectangle has been disappeared in thin shallow shell.

Finally, free vibration frequency analysis results showed that when the layer thickness increases, the free vibration frequencies obtained by using FEM and SDSST seems to be compatible with each other. For obtaining the equations of shear deformation shallow shell theory (SDSST), some omissions and assumptions made in literature are also accepted in this study. Thus, the solution of the problem can be simplified. For instance $\varepsilon_z = 0$, $\sigma_z = 0$ and higher order terms are neglected when equations are obtained. The governing Eq. (7) includes the term $(1 + z/R_n)$. This term can be expanded in a geometric series form. For thin shells, this term $(1 + z/R_n)$ is close to 1. Therefore, numerical investigations revealed that such expansion did not give better results for thin shallow shells (Fig. 5). These assumptions means that the effects of these parameters are almost zero for the thick shell elements, however, as results suggest the above mentioned assumptions are more effective for the thin shells elements. Moreover the effects of $(1 + z/R_n)$ term is expanded in a geometric series form, $\varepsilon_z = 0$, $\sigma_z = 0$ can be neglected for the thick shell elements, however for the thin shells those parameters showed significant effect on the free vibration frequencies.

As it is well known, SAP 2000 and ANSYS are finite element method (FEM) based softwares. Also both programs use the same equations in the calculations. Therefore, obtaining same results from the both programs is expected and usual. In this study, rectangular mesh with a regular geometry element was used. The 4-point quadratic elements are used in SAP2000. For the ANSYS analysis program, on the other hand, the 8-point quadratic elements are used. As clearly seen in the analysis results, choosing 4 or 8 points quadratic element type did not significantly alter the results for free vibration frequency analysis. Because, regardless of the point used, both programs provided same geometric shape. The reason behind this behavior can be attributed to the geometry of the studied element.

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