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# Influence of the cylinder height on the elasto-plastic failure of locally supported cylinders

Arne Jansseune*[1], Wouter De Corte[1], Wesley Vanlaere[2] and Rudy Van Impe[2]

[1]*Department of Civil Engineering, Faculty of Applied Engineering Sciences, University College Ghent, Ghent University Association, Ghent, Belgium*
[2]*Department of Structural Engineering, Faculty of Engineering Sciences, Ghent University, Ghent, Belgium*

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**Abstract.** Frequently, steel silos are supported by discrete supports or columns to permit easy access beneath the barrel. In such cases, large loads are transferred to the limited number of supports, causing locally high axial compressive stress concentrations in the shell wall above the supports. If not dealt with properly, these increased stresses will lead to premature failure of the silo due to local instability in the regions above the supports. Local stiffening near the supports is a way to improve the buckling resistance, as material is added in the region of elevated stresses, levelling these out to values found in uniformly supported silos. The aim of a study on the properties of local stiffening will then be to increase the failure load, governed by an interaction of plastic collapse and elastic instability, to that of a discrete supported silo. However, during the course of such a study it was found that, although the failure remains local, the cylinder height is also a parameter that influences the failure mechanism, a fact that is not properly taken into account in current design practice and codes. This paper describes the mechanism behind the effect of the cylinder height on the failure load, which is related to pre-buckling deformations of the shell structure. All results and conclusions are based on geometrically and materially non-linear finite element analyses.

**Keywords:** cylinder; local supports; elastic buckling; plastic yielding; cylinder height.

## 1. Introduction

A steel silo is typically a cylindrical shell structure supported by local supports (rigid or flexible) or engaged columns (Doerich *et al.* 2009) in order to facilitate emptying operations. Such an elevated silo has the advantage that the bulk solid can be discharged by gravity flow.

However, the discrete supports introduce local forces into the cylindrical shell, resulting in axial compressive stress concentrations in the cylindrical wall directly above the supports, reducing both buckling and yielding resistance of the cylindrical shell (Rotter 2009). This reduction can be addressed, either by an elevated wall thickness in the lower part of the cylinder, by a stepped wall thickness (Knoedel and Ummernhofer 2009), or by adding a stiffener configuration.

In the recent past, the buckling behaviour of locally supported cylinders has been investigated by (Guggenberger *et al.* 2004, Rotter 2004, Vanlaere 2006). A number of stiffening arrangements have been proposed to give the cylindrical shell structure additional strength and stiffness (Doerich 2007,

* Corresponding author, Mr., E-mail: arne.jansseune@hogent.be

Fig. 1 Buckling mode - failure occurs above the upper ring

Vanlaere 2006).

Smaller silo structures are often supported on local brackets, each rigidly connected to a stiff column or floor (Doerich and Rotter 2007). (Vanlaere 2006) considers a configuration comprising two longitudinal, flatbar stiffeners with limited length above each support, combined with circumferential stiffeners arranged at top and bottom of the vertical stiffeners.

This type of steel shell structure is a complex thin-walled structure. The ultimate strength is affected by many parameters (ECCS 2008, EN 1993-1-6 2007): the geometry, the loading conditions, the boundary conditions, the material properties, and the geometric imperfections (Guggenberger 2006). In addition, it is clear that the shell structure has a large number of relevant parameters, and all have a significant influence on the buckling behaviour and the failure load. However, when aiming at future design codes for this type of support conditions, the number of necessary design parameters should be as small as possible, but at the same time the proposed design rule should not be over-conservative. One parameter frequently omitted from design rules is the cylinder height. Indeed, in many contributions (Doerich 2007, Vanlaere 2006) the cylinder height was assumed high enough so that the boundary conditions at the upper edge of the cylinder were sufficiently removed from the location where failure occurs. However, as will be demonstrated in this paper, this assumption is not always conservative. In fact, the effect of the cylinder height on the buckling behaviour is the objective of this paper. The first part describes the considered geometry and the calculation method. The second part discusses results from a geometrically non-linear elastic analyses of a cylinder shell demonstrating the height dependency for both short, medium and high cylinders. Finally, conclusions are drawn.

## 2. Stiffener geometry

The stiffener configuration taken from (Vanlaere 2006) comprises two longitudinal stiffeners above each support (Fig. 2). These stiffeners gradually introduce the support load into the shell wall, decreasing the axial compressive stress concentrations above the supports. For the present study, it is assumed that the stiffeners are considered as rigid bodies. As a consequence, buckling will occur in the

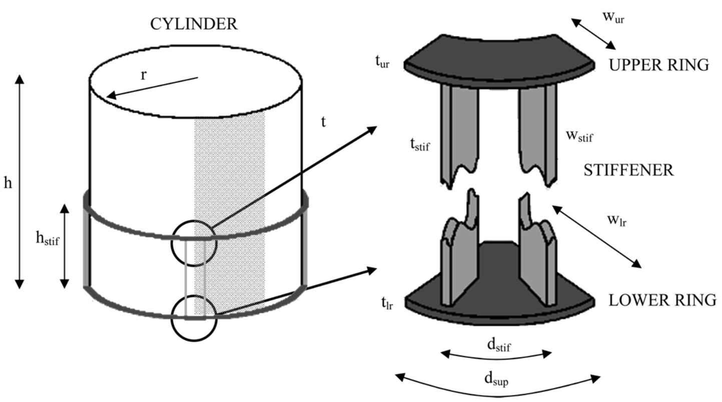


Fig. 2 Geometrical parameters of a locally supported cylinder

cylindrical shell wall but not in the stiffener, and the shape of the rigid stiffeners does not affect the buckling behaviour. Obviously, stiffener buckling and optimum stiffener shape configuration will be the interest of the first author's current and future work, but it will not be the subject of this paper.

Above and below the longitudinal stiffeners, two ring stiffeners are placed. They have been designed in previous work to ensure that the shell keeps its circular form at these locations. In the present study, these ring stiffeners are, in contrast with the longitudinal stiffeners, not assumed to act as rigid bodies.

The shell structure has a limited number of rigid narrow supports equally spaced along the circumference. In this paper, the number of supports is 4. The set of geometrical parameters of a locally supported cylinder as considered in this paper is given in Table 1, using the symbols of Fig. 2. The magnitude of these parameters, scaled to the radius r of the cylinder and valid for flexible vertical stringers, have been determined by experiments on scale models with r = 350 mm in the Laboratory for Research on Structural Models at Ghent University (Vanlaere 2006, Vanlaere *et al.* 2006).

## 3. Numerical model

The structural behaviour of locally supported cylinders is studied by means of numerical simulations with the finite element package ABAQUS (Dassault Systèmes 2009).

In order to reduce the computational time, only a segment of 45 degrees of the cylinder is modelled (shaded area in Fig. 2). Symmetrical boundary conditions are applied on the longitudinal edges. The 45 degree model is validated against a complete model for the analyses indicated in Fig. 4. The top and the bottom of the cylinder should be kept circular by structural connection to the conical hopper and the conical roof, which are not included in the numerical model. A lower circumferential ring is placed at

Table 1 Geometrical parameters

| Cylinder | | Longitudinal stiffeners | | Ring stiffeners | |
|---|---|---|---|---|---|
| r [mm] | 350 | $h_{stif}/r$ [-] | 0.25–1.25 | $t_{ur}/t$ [-] | 1.0 |
| r/t [-] | 500 | $t_{stif}/t$ [-] | 1.0 | $w_{ur}/r$ [-] | 0.06 |
| $h/h_{stif}$ [-] | 1.25–20.0 | $w_{stif}/r$ [-] | 0.06 | $t_{lr}/t$ [-] | 1.0 |
| $d_{sup}/r$ [-] | 0.15 | $d_{stif}/d_{sup}$ [-] | 1.0 | $w_{lr}/r$ [-] | 0.12 |

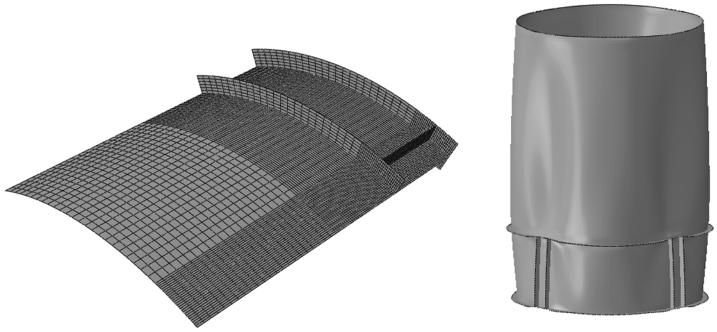


Fig. 3 Numerical model; (a) 45° model, (b) complete model - pre-buckling deformations are shown (deformation scale factor = 50)

the bottom of the cylinder, which coincides with the connection of the conical hopper. In contrast with the connection of the hopper, boundary conditions replace the effect of the roof. These boundary conditions only restrain the displacements in the horizontal plane (in radial and tangential direction). The axial displacements are not restricted (light roof/hopper), except at the rigid supports. The loading condition is an axial load applied uniformly at the upper circumference (Vanlaere 2006).

Shell elements (S8R5) (Dassault Systèmes 2009) are used to model the shell structure. These elements are 8-node doubly curved thin shell elements with reduced integration, using 5 degrees of freedom per node.

A bilinear stress-strain relationship is used to model the behaviour of the material. The elastic, perfectly plastic material is assumed to have a Young's modulus E of 200 GPa, a Poisson's ratio $\nu$ of 0.3, and a yield stress $f_y$ of 235 MPa.

The rigid body behaviour of the stiffeners (See paragraph 2) is not simulated by introducing rigid body elements. Instead, the material behaviour of these elements is modified: a linear stress-strain relationship is used – neglecting effects of plasticity – with a value of Young's modulus that is 200 times larger than the value given above. Although the applicability of this method is verified in (Vanlaere *et al.* 2009) for the purpose of the present study, its overall applicability is the aim of additional research.

In the present study, geometric imperfections are not taken into account, but in future research imperfections will certainly be taken into account (Hübner *et al.* 2006). The aim of the study is to assess the effect of the cylinder height on the failure load of the perfect cylinder rather than to assess the effect of imperfections.

The equilibrium equations are solved in ABAQUS using the modified Riks method (Riks 1972, 1979). This algorithm is especially useful for the study of structural responses that are highly nonlinear (material nonlinearity and geometric nonlinearity). It reduces the computing time significantly while still providing accurate results, regardless of whether the response is stable or unstable.

## 4. Results and discussion

### *4.1 Results from geometrically and materially non-linear elastic analysis (GMNA)*

In previous research, it was assumed that short cylinders had an elevated failure load due to the boundary conditions at the upper edge of the cylinder, which restrict the deformations (Vanlaere 2006). When increasing the cylinder height, this effect becomes less pronounced and the failure load decreases. Once the cylinder height exceeds a critical value, a further increase has no influence on the buckling behaviour and the failure load. As will be demonstrated below by our parametric study, this assumption is not correct.

For this parametric study, the values of the geometrical parameters of the locally supported cylinder are given in Table 1. Only two parameters are varied: the ratio of the stiffener height to the cylinder radius $h_{stif}/r$ between 0.25 and 1.25, and the ratio of the cylinder height to the stiffener height $h/h_{stif}$ between 1.25 and 20.0. Fig. 4 shows the results.

The curves in Fig. 4 for values of $h_{stif}/r$ equal or larger than 0.50 can be divided into three sections: a descending branch (short cylinders), an ascending branch (cylinders with an intermediate cylinder height) and a horizontal line (cylinders with a large cylinder height). Clearly, the ascending branch is not expected and not mentioned in the available literature, and its presence is problematic for simplified design rules. However, it can be explained when studying the studying the deformations and the stresses before buckling (see paragraph 4.2 and 4.3).

At this point, it is also important to note that if the value of the cylinder height is further increased, the failure phenomenon may change to Euler buckling, which is beyond the scope of this paper. Furthermore, in the case of cylinders with a very small stiffener height, e.g., $h_{stif}/r = 0.25$, failure does not occur in the shell wall, but in the lower ring. This buckling mode leads to a lower failure load, but is not considered further.

### *4.2 Pre-buckling deformations*

The pre-buckling deformations are influenced by the boundary conditions at the top of the cylinder. For short cylinders, these boundary conditions are effectively situated in the vicinity of the location

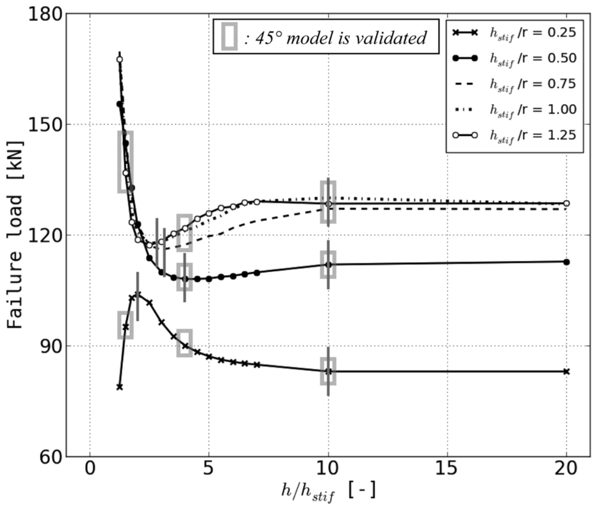


Fig. 4 Failure loads for variations of the cylinder height and the stiffener height.

where failure occurs, i.e., above the upper ring, making the shell wall more difficult to deform. When the cylinder height increases, the boundary conditions are moving further away from the failure location. In such case, the deformations directly above the upper ring are less restrained Fig. 5 demonstrates this effect, showing the pre-buckling radial displacements for various cylinder heights at a load of 100 kN. Outward oriented displacements are positive, inward oriented displacements negative.

For short cylinders (Fig. 5(a)), large pre-buckling deformations are prevented when the upper edge of the cylinder and the associated boundary conditions are near the upper ring. For cylinders with intermediate and large height (Figs. 5(b) and 5(c)), relatively large pre-buckling deformations are visible on both sides of the model. These pre-buckling deformations are situated in two elongated zones above the upper ring, locally creating a modified cylinder curvature (Doerich 2007).

The first zone above the supports ($\theta = 0^o$), which is coloured in blue, corresponds with the inward oriented displacements. These inward displacements become increasingly important with an increasing cylinder height, especially for the short cylinders (transition from Fig. 5(a) to Fig. 5(b)). The second zone on the other side of the model ($\theta = 45^o$), which is coloured in red, corresponds with the outward oriented displacements and becomes more important between Figs. 5(b) and 5(c), thus for the intermediate high cylinders.

The dotted line in Fig. 5 indicates a path in circumferential direction. Along this path, the radial pre-buckling deformations $U_{radial}$ are depicted in Fig. 6, in each case for a load level before failure (100 kN). The circumferential angle $\theta$ varies between $0^o$ (at the supports) and $45^o$ (between the supports). Outward oriented displacements are positive, inward oriented displacements negative. The ratio of the

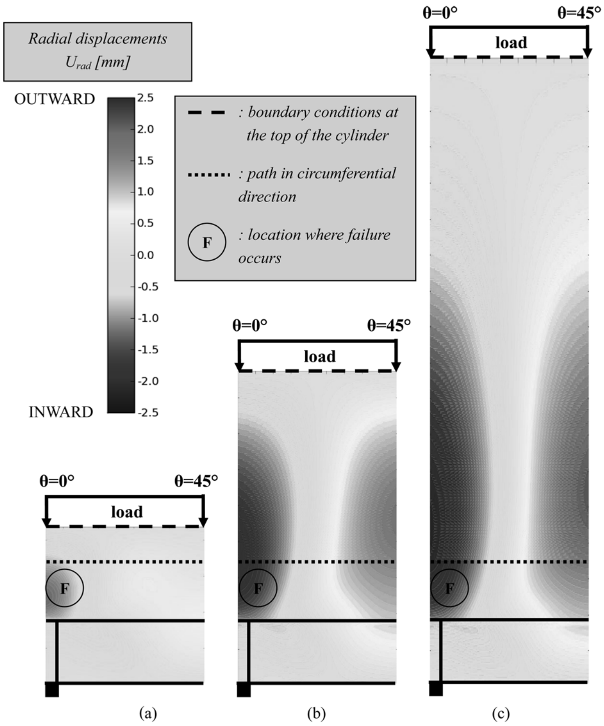


Fig. 5 Pre-buckling deformations patterns; (a) limited cylinder height, (b) intermediate cylinder height, (c) large cylinder height.

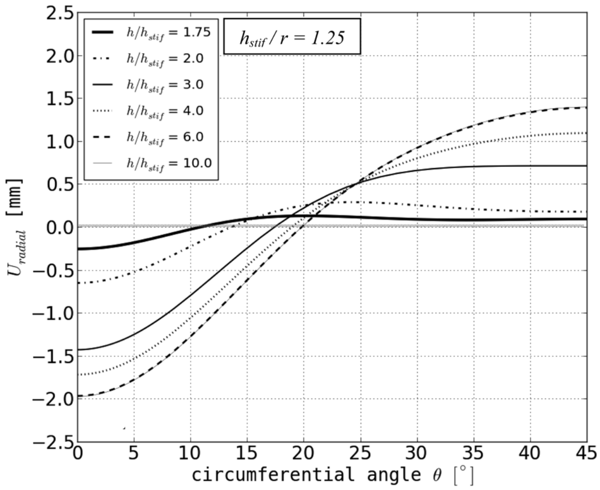


Fig. 6 Pre-buckling deformations in radial direction around the circumference.

stiffener height to the cylinder radius $h_{stif}/r$ is equal to 1.25. Other ratios of $h_{stif}/r$ show similar trends. The ratio of the cylinder height to the stiffener height $h/h_{stif}$ is plotted for the following values: 1.75, 2.0, 3.0, 4.0, 6.0 and 10.0. The radial pre-buckling deformations for high cylinders ($h/h_{stif} > 10.0$) are nearly the same as the pre-buckling deformations of a cylinder with $h/h_{stif} = 10.0$.

Above the supports, the pre-buckling deformations are inward oriented, and on the other side, the deformations are outward oriented. Fig. 6 also illustrates that in case of cylinders with intermediate height, the importance of radial pre-buckling deformations increases with the cylinder height, both for the inward and the outward oriented deformations. For cylinders with a large cylinder height, no further increase of the pre-buckling deformations occurs.

Summarising, it can be concluded from Figs. 5 and 6 that the upper edge of the cylinder and the associated boundary conditions play a major role in the behaviour of the shell wall before buckling. For short cylinders, the boundary conditions prevent large deformations above the upper ring before buckling. The pre-buckling deformations are increasingly important as the height of the cylinder and/or stiffeners increase(s), both for the inward and the outward oriented displacements. When the height exceeds a critical height, the additional height has no influence on the pre-buckling deformations.

### *4.3 Stresses*

In Figs. 7 and 8, the axial stress $\sigma_{axial}$ and the stress in circumferential direction $\sigma_\theta$ are plotted along the circumferential path (dotted line in Fig. 5) for a variable cylinder height, in each case for a situation before failure (a load of 100 kN). Compressive stresses are positive, tensile stresses negative.

A non-uniform stress distribution along the circumference of the cylinders occurs, both for the axial compressive stresses and for the stresses in circumferential direction, strongly affected by the cylinder height and the associated boundary conditions. Furthermore, the stresses in circumferential direction $\sigma_\theta$ are negligible compared to the axial stresses $\sigma_{axial}$.

For short cylinders ($h/h_{stif} < 3.0$), the axial compressive stresses $\sigma_z$ above the supports ($\theta = 0°$) increase for an increasing cylinder height, while the axial stresses $\sigma_z$ between the supports ($\theta = 45°$) decrease. An opposite trend is observed for the cylinders with an intermediate height ($3.0 \leq h/h_{stif} \leq$

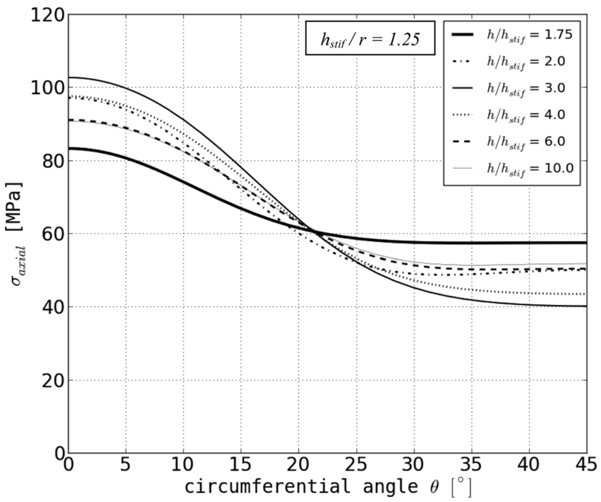


Fig. 7 Pre-buckling axial stress distribution around the circumference.

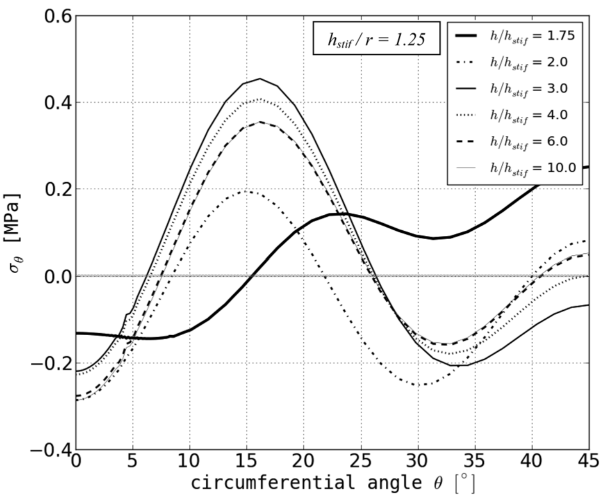


Fig. 8 Pre-buckling circumferential stress distribution around the circumference.

10.0). In case of high cylinders ($h/h_{stif} > 10.0$), the additional height has no further influence on the axial and circumferential stress distribution.

In Table 2, the percentages of decrease/increase of the axial compressive stresses on both sides of the examined circumferential path are presented for a load level of 100 kN. The axial stresses for cylinders of variable cylinder height are compared to the corresponding value of a cylinder with $h/h_{stif} = 3.0$, which is the transition from the short cylinders to the cylinders with an intermediate cylinder height (for $h_{stif}/r = 1.25$). The variation of axial stresses is considerable: above the supports ($\theta = 0^o$), there is a maximum stress reduction of 18.9% for the short cylinders and a maximum stress reduction of 11.6% for the cylinders with a large cylinder height. These results are not negligible. The variable axial stress distribution has a significant influence on shell failure phenomena (yielding and buckling), which are

Table 2 Axial compressive stresses

| $h/h_{stif}$ [-] | $\sigma_z$ [MPa] $\theta = 0°$ | | $\sigma_z$ [MPa] $\theta = 45°$ | |
|---|---|---|---|---|
| 1.75 | 83.2 | -18.9% | 57.4 | +42.8% |
| 2.0 | 97.1 | -5.4% | 50.1 | +24.6% |
| 3.0 | 102.6 | | 40.2 | |
| 4.0 | 97.5 | -5.0% | 43.5 | +8.2% |
| 6.0 | 91.1 | -11.2% | 50.4 | +25.4% |
| 10.0 | 90.7 | -11.6% | 51.8 | +28.9% |

physically caused by these compressive stresses. Since the above mentioned stress reduction occurs at the location of buckling or yielding, the shell wall will be able to withstand a higher failure load.

### *4.4 Elastic buckling*

The failure load for pure elastic failure is the axial load whereby the critical buckling stress is reached. The critical buckling stress $\sigma_{ax,cr}$ of a perfect cylindrical shell subjected to a uniform axial compression and with classical boundary conditions is given by (Timoshenko 1910)

$$\sigma_{ax,cr} = \frac{E}{\sqrt{3 \cdot (1-\nu^2)}} \cdot \frac{t}{r} = 0.605 \cdot E \cdot \frac{t}{r} \tag{1}$$

In this formula, E is the Young's modulus, $\nu$ the Poisson's ratio, r the radius of the cylinder and t the thickness of the shell wall. The corresponding failure load can be calculated by multiplying the cross-sectional area of the shell wall by the critical buckling stress.

$$F_{ax,cr} = A_{shell} \cdot \sigma_{ax,cr} = (2 \cdot \pi \cdot r \cdot t) \cdot \sigma_{ax,cr} \tag{2}$$

The above mentioned formulas indicate that the failure load is a function of the local radius/ curvature. In the models that are examined in the present paper, the pre-buckling deformations are significantly influenced by the height of the cylinder, because the boundary conditions at the upper edge of the cylinder restrict nearby deformations. The most important pre-buckling deformations are elongated zones with a modified curvature. Fig. 9 represents a horizontal cut of the cylindrical shell at the height where failure occurs. In this figure, the radius $r_0$ is the radius of the perfect cylindrical shell.

For short cylinders ($h/h_{stif} < 3.0$), the inward oriented pre-buckling deformations above the supports become increasingly important with an increasing cylinder height (see Figs. 5 and 6). The flattened shell wall above the supports causes a smaller curvature or a larger effective radius of curvature $r_{in}$ at the region where buckling occurs. The reduced critical buckling stress $\sigma_{ax,cr}$ is caused by this larger radius of curvature $r_{in}$ (Eq. (1)), which leads to a reduced buckling load $F_{ax,cr}$ (Eq. (2)). In Eq. (2), the amount of material of the shell wall $A_{shell}$ remains constant. The decreasing failure load is also observed in Fig. 4.

For cylinders with an intermediate cylinder height ($3.0 \leq h/h_{stif} \leq 10.0$), an ascending branch of the failure load is observed in Fig. 4. This can be explained by the more important outward oriented pre-buckling deformations between the supports (see Figs. 5 and 6), causing a smaller effective radius $r_{out}$. The latter leads to a larger critical buckling stress $\sigma_{ax,cr}$ (Eq. (1)) and a larger buckling load $F_{ax,cr}$ (Eq. (2)).

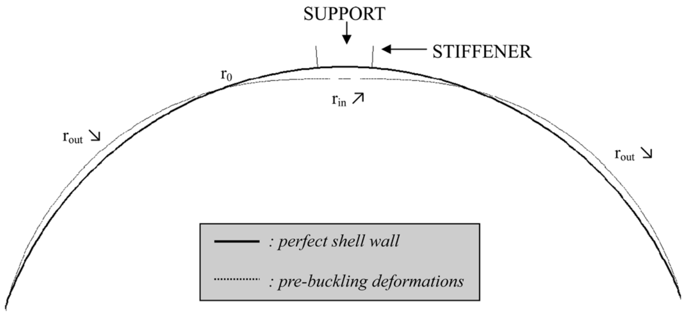


Fig. 9 Partial horizontal cut of the shell with the pre-buckling deformations.

Based on the previous considerations, it may be understood that the increasing failure load of cylinders with increasing intermediate height (the ascending branch in Fig. 4) can be explained by the outward oriented pre-buckling deformations above the upper ring. For increasing height, and corresponding pre-buckling deformations, both the critical buckling stress and the axial stress distribution in circumferential direction are positively influenced. The latter means that possible effects of plasticity will occur at higher load levels. Since elastic buckling is positively affected, the ascending branch in Fig. 4 is inevitable.

Although Eq. (1) is not immediately applicable for locally supported cylinders, resemblance is found in the fact that failure will occur in the zone of minimal curvature (maximum radius) when taking into account the pre-buckling deformations.

### *4.5 Plastic yielding*

The cylinders considered in this study have a cylinder radius to the thickness ratio equal to 500. In other words, they will fail by elastic buckling. For these analyses, no conclusions can be drawn for failure due to plastic yielding.

### *4.6 Comparison to the results of Guggenberger for locally supported unstiffened cylinders*

The results of this study have been compared to the results of Guggenberger for locally supported unstiffened cylinders (Guggenberger 2006). Those results are limited to a ratio of $h/r = 2.0$, and consequently do not show a height dependency. A comparison for the parameter set (4 rigid supports with $d_{sup}/r = 0.15$, $r/t = 500$, $h/r = 1.875–2.1875$, $h_{stif}/r = 1.25$), corresponding to an equal total cylinder height, indicates a similar value of the non-dimensional critical support forces $\sigma_{GNA}/\sigma_0 \cdot d/r$. However, for the parameter set (4 rigid supports with $d_{sup}/r = 0.15$, $r/t = 500$, $h/r = 3.00–3.50$, $h_{stif}/r = 1.25$), corresponding to an equal unstiffened cylinder height, a smaller value is found in this study. These values indicate that the Guggenberger results for locally supported cylinders without vertical stiffeners match the results from this paper for an equal total cylinder height to radius ratio very closely, but not for a unstiffened cylinder height to radius ratio.

### *4.7 Overview*

Based on previous findings, we will describe the relation between the cylinder height and the failure load (Fig. 4).

#### *4.7.1 Short cylinders*

For short cylinders, buckling of the cylinder above the terminations of the longitudinal stiffeners is obstructed by the proximity of the boundary conditions (replacing the effect of the conical roof which is not included in the numerical model) at the upper rim of the cylinder. The displacements in radial and tangential direction are restrained in such way that the upper edge of the cylinder is kept circular. This results in an elevated failure load, which lowers quickly as the cylinder height increases.

#### *4.7.2 Cylinders with an intermediate cylinder height*

From a certain height, the failure load will not further decrease, but instead starts increasing with the cylinder height. Such "intermediate" cylinders fail at the location indicated with the letter 'F' in Fig. 5. At this location, the pre-buckling deformations in radial direction are negative, i.e. in inward direction. Elastic buckling will occur at an increasingly higher failure load, because for the same total load on the cylinder, the compressive axial stress at the location of the local failure phenomenon decreases with an increasing height of the cylinder and/or stiffeners.

For any cylinder height in this branch, a larger stiffener height enhances this effect, because the axial compressive stresses are even more smeared out in circumferential direction, thereby increasing the slope of the rising branch (Fig. 4). Due to this, the difference between the minimum of the curve and the asymptote value increases.

#### *4.7.3 Cylinders with a large height*

Once the cylinder height surpasses a critical height, a further increase has no influence on the pre-buckling deformations, the stresses before buckling and the failure load. Clearly, the distance between the top of the cylinder and the area where the deformations occur has become too large for the boundary conditions at the upper rim to play a role.

Of course, if the height of the cylinder is further increased, the failure phenomenon may change to flexural Euler buckling and at that point the cylinder height will again become an important parameter.

## 5. Conclusions

When analyzing the collapse of cylindrical shell structures on rigid supports, the number of parameters to investigate is large. In order to reduce this number, the cylinder height is frequently eliminated assuming that, when choosing this value large enough, it has no further influence on the failure load. The results from this paper have proven that this assumption is not correct. Indeed, both the boundary conditions at the upper edge, as well as pre-buckling deformations, and the distribution of compressive axial stresses in circumferential direction, significantly influence the failure load. Eventually, and from a certain value only, the cylinder height indeed ceases to influence the bearing capacity. However, at that point, the value of the failure load exceeds the ones found at intermediate heights. Neglecting that observation would lead to a non-conservative design rule. Clearly, future design rules should either be based on the minimum values at intermediate heights, or should contain the cylinder height as a parameter.

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