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Nonlinear vibration analysis of laminated plates resting on nonlinear two-parameters elastic foundations

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Abstract. In the present manuscript, geometrically nonlinear free vibration analysis of thin laminated plates resting on non-linear elastic foundations is investigated. Winkler-Pasternak type foundation model is used. Governing equations of motions are obtained using the von Karman type nonlinear theory. The method of discrete singular convolution is used to obtain the discretised equations of motion of plates. The effects of plate geometry, boundary conditions, material properties and foundation parameters on nonlinear vibration behavior of plates are presented.

Keywords: Nonlinear dynamics; free vibration; discrete singular convolution; laminated plates; non-linear foundation.

1. Introduction

The increasing use of fibre reinforced composite laminates in some engineering applications such as aerospace applications, space vehicles, aircrafts, civil, mechanical, automobiles, ships, nanocomposites and petro-chemical vessels have necessitated the detailed static and dynamic analysis of structures for their mechanical response (Kant and Mallikarjuna 1989, Kant *et al*. 1992, Liu and Achenbach 1994, Liew and Han 1996, Wang *et al*. 2000, Wang et al. 2001, Liu and Chen 2001, Liew and Huang 2003, Dai *et al*. 2004, Liew *et al*. 2003a, Shukla and Nath 2000). There is much available in the literature concerning nonlinear analysis of laminated plates with or without on an elastic foundation problem (Shen 1999, Teo and Liew 2002, Shen *et al*. 2003, Liew *et al*. 2004, Shukla *et al*. 2005, Zhao and Liew 2009). Kant et al. (1988, 1990) applied finite element method for of laminated plates. Liew *et al*. (2003b) presented layerwise based differential quadrature method for three-dimensional modeling of laminated plates. Liu and Chen (2002) had been applied the element free Galerkin method for analysis of laminated plates. Chebyshev series solution of nonlinear plate problem is presented by Nath and Kumar (1995). Nonlinear transient analysis of thick laminated plates has been investigated by Nath and Shukla (2001).

Most of the literature available concerning linear Winkler foundation model considers linear and nonlinear solutions for laminated plates (Nath *et al.* 1986, Dumir 1988, Dumir and Bhaskar 1988, Shih and Blotter 1993, Shukla and Nath 2000, Nath and Shukla 2001, Shen 2000a, 2000b, Civalek 2004, 2005, 2006, 2007a, Sofiyev 2010). The nonlinear analysis of plates under static and dynamic action

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was investigated by many researchers by this time (Bhaskar and Dumir 1988, Shih and Blotter 1993, Chen and Fung 2004, Chien and Chen 2005, 2006). The resulting field equations of motions being nonlinear, no close form solution could be obtained for most practical problems. So, some numerical and semi-analytical methods have been widely used up to now (Ganapathi *et al.* 1991, Kant and Kommineni 1994, Manoj *et al.* 2000, Shen 2000c, Nath *et al.* 2006). Discrete singular convolution (DSC) method is a relatively new numerical technique in sciences and engineering. The method of discrete singular convolution was proposed to solve linear and nonlinear differential equations by Wei (1999), and later it was introduced to mathematical physics, fluid and solid mechanics (Wei *et al* 1998, 2001, 2002a, 2002b, Wei 2001a, 2001b, Zhao and Wei 2002, 2003, Zhao *et al.* 2002a, 2002b, 2005, Lai and Xiang 2009, Xiang *et al* 2002, 2010a, 2010b, Civalek 2007b, Seçkin and Sarýgül 2009a, 2009b). Frequency values is of fundamental importance in the design of many engineering structures, including beams, plates, shells, panels, turbine blades, circular and conical shells and so on. In this paper, the method of discrete singular convolution (DSC) is applied to nonlinear vibration analysis of laminated plates on nonlinear Winkler-Pasternak foundations. Numerical analyses were carried out to assess the accuracy of the proposed method.

2. Governing equations

A laminated thin rectangular plate of dimensions a, b, and h resting on nonlinear elastic foundations is considered (Fig. 1).

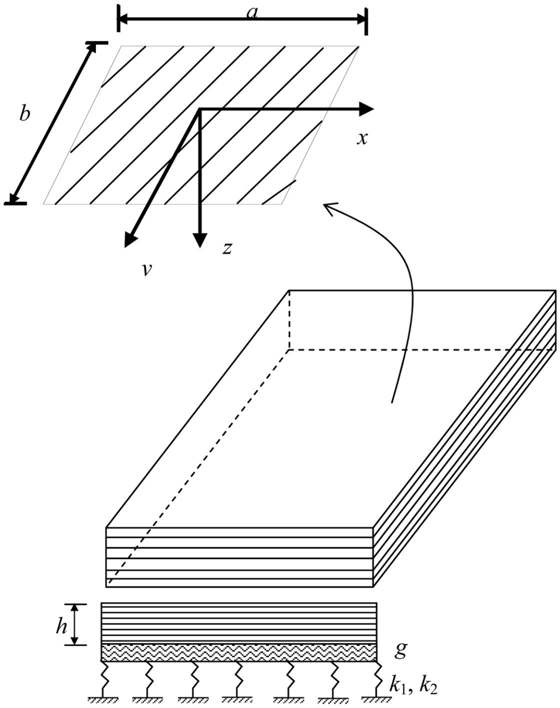

Fig. 1 Geometry and coordinate system of the laminated rectangular plate on nonlinear elastic foundation

Based on the von Karman nonlinear theory, which takes into account moderately large deflections and small strains, the strain and curvatures relations are written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{2}$$

The equations of motion for geometrically nonlinear analysis of laminated plates resting on nonlinear elastic foundation are expressed as (Shih and Blotter 1993)

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{4}$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_x}{\partial x \partial y} + \frac{\partial^2 M}{\partial y^2} + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2} + q(x, y) = ph\frac{\partial^2 w}{\partial t^2} \tag{5}$$

Transverse distributed forces on the plate due foundation can be given (Shih and Blotter 1993)

$$q = k_1 w + k_2 w^3 - k_g \nabla^2 w \tag{6}$$

Where k_1 is the Winkler foundation parameter, k_2 is the nonlinear parameter, k_g is the shear parameter, and ∇^2 is the Laplace operator. For laminated composite plate, the constitutive relationship, the in-plane force and moment resultants can be given by

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \tag{7a}$$

or

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \\ \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{7b}$$

where the laminate stiffness coefficients (A_{ij}, B_{ij}, D_{ij}) are membrane, coupling and flexural stiffness, respectively. These terms are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{z_{k-1}}^{z_k} (1, z, z^2) Q_{ij} dz (i, j=1, 2, 6) \tag{8}$$

$$\begin{Bmatrix} \varepsilon \\ M \end{Bmatrix} = \begin{bmatrix} \bar{A} & \bar{B} \\ -(B)^T & \bar{D} \end{bmatrix} \begin{Bmatrix} N \\ \kappa \end{Bmatrix} \tag{9}$$

Compatibility equation can be written as (Shih and Blotter 1993)

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \varepsilon_{xy}\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{10}$$

Substituting the Eqs. (6-9) into Eq. (5) and Eq. (10) the governing equations of motion can be given in the following form

$$L_{11}(\psi) + L_{12}(w) + L_{13}(w) = 0 \tag{11a}$$

$$L_{21}(w) + L_{22}(\psi) + L_{23}(\psi, w) + L_{24} = 0 \tag{11b}$$

The differential operators given in the governing equations (11a-11b) are listed as

$$L_{11} = A_{22}\frac{\partial^4}{\partial x^4} - 2A_{26}\frac{\partial^4}{\partial x^3 \partial y} + (2A_{12} + A_{66})\frac{\partial^4}{\partial x^2 \partial y^2} - 2A_{16}\frac{\partial^4}{\partial x \partial y^3} + A_{11}\frac{\partial^4}{\partial y^4} \tag{12}$$

$$L_{12} = B_{21}\frac{\partial^4}{\partial x^4} + (2B_{26} - B_{61})\frac{\partial^4}{\partial x^3 \partial y} + (B_{11} + 2B_{22} - 2B_{66})\frac{\partial^4}{\partial x^2 \partial y^2} + (2B_{16} - B_{62})\frac{\partial^4}{\partial x \partial y^3} + B_{12}\frac{\partial^4}{\partial y^4} \tag{13}$$

$$L_{13} = \left(\frac{\partial^2}{\partial x \partial y}\right)^2 + \frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} \tag{14}$$

$$L_{21} = D_{11}\frac{\partial^4}{\partial x^4} + 4D_{16}\frac{\partial^4}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4}{\partial x \partial y^3} + D_{22}\frac{\partial^4}{\partial y^4} \tag{15}$$

$$L_{22} = B_{21}\frac{\partial^4}{\partial x^4} + (2B_{26} - B_{61})\frac{\partial^4}{\partial x^3 \partial y} + (B_{11} + 2B_{22} - 2B_{66})\frac{\partial^4}{\partial x^2 \partial y^2} + (2B_{16} - B_{62})\frac{\partial^4}{\partial x \partial y^3} + B_{12}\frac{\partial^4}{\partial y^4} \tag{16}$$

$$L_{23} = \frac{\partial^2 \psi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 \psi}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} \tag{17}$$

$$L_{24} = k_1 w + k_2 w^3 - k_g \Delta^2 w + \rho h \frac{\partial^2 w}{\partial t^2} \tag{18}$$

3. Discrete singular convolution (DSC)

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics. The method of discrete singular convolution (DSC) was proposed to solve linear and nonlinear differential equations by Wei (1999), Wei *et al* (1998) and later it was introduced to solid and fluid mechanics by many researchers (Wei *et al.* 2001, 2002a, 2002b, Wei 2001a, 2001b, Zhao and Wei 2002, 2003, Zhao *et al.* 2002a, 2002b, 2005, Xiang *et al.* 2002, 2010a, 2010b). In the context of distribution theory, a singular convolution can be defined by (Wei 2001)

$$F(t) = (T^* \eta)(t) = \int_{-\infty}^{\infty} T(t - x)\,\eta(x)dx \tag{19}$$

Where T is a kind of singular kernel such as Hilbert, Abel and delta type, and $\eta(t)$ is an element of the space of the given test functions. In the present approach, only singular kernels of delta type are chosen. This type of kernel is defined by Wei (2001b)

$$T(x) = \delta^{(r)}(x); \qquad (r=0, 1, 2, \dots) \tag{20}$$

where subscript r denotes the rth-order derivative of distribution with respect to parameter x.

In order to illustrate the DSC approximation, consider a function $F(x)$. In the method of DSC, numerical approximations of a function and its derivatives can be treated as convolutions with some kernels. According to DSC method, the r*th* derivative of a function $F(x)$ can be approximated as (Wei *et al.* 2002a)

$$F^{(r)}(x) \approx \sum_{k=-M}^{M} \delta^{(r)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (r=0, 1, 2, \ldots,). \tag{21}$$

where Δ is the grid spacing, x_k are the set of discrete grid points which are centered around x, and $2M + 1$ is the effective kernel, or computational bandwidth. It is also known, the regularized Shannon kernel (RSK) delivers very small truncation errors when it use the above convolution algorithm. The regularized Shannon kernel (RSK) is given by (Wei *et al.* 2002b)

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; (\sigma > 0) \tag{22}$$

The researchers is generally used the regularized delta Shannon kernel by this time. The required derivatives of the DSC kernels can be easily obtained using the below formulation (Zhao and Wei 2002, Zhao *et al* 2002a, 2002b)

$$\delta^{(r)}_{\Delta,\sigma}(x - x_j) = \frac{d^r}{dx^r}[\delta_{\Delta,\sigma}(x - x_j)]\big|_{x = x_i} \tag{23}$$

The second-order derivative at $x \neq x_k$, for example can be given as follows (Wei *et al.* 2001)

$$\begin{aligned}\delta^{(2)}_{\sigma,\Delta}(x - x_k) &= -\frac{(\pi/\Delta)\sin(\pi/\Delta)(x - x_k)}{(x - x_k)} \exp[-(x - x_k)^2/2\sigma^2] \\ &\quad -2\frac{\cos(\pi/\Delta)(x - x_k)}{(x - x_k)^2} \exp[-(x - x_k)^2/2\sigma^2] \\ &\quad -2\frac{\cos(\pi/\Delta)(x - x_k)}{\sigma^2} \exp[-(x - x_k)^2/2\sigma^2] + 2\frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)/\Delta} \exp[-(x - x_k)^2/2\sigma^2] \\ &\quad + \frac{\sin(\pi/\Delta)(x - x_k)}{\pi(x - x_k)\sigma^2/\Delta} \exp[-(x - x_k)^2/2\sigma^2] + \frac{\sin(\pi/\Delta)(x - x_k)}{\pi\sigma^4/\Delta}(x - x_k)\exp[-(x - x_k)^2/2\sigma^2]\end{aligned} \tag{24}$$

For $x = x_k$, this derivative is given by

$$\delta^{(2)}_{\sigma,\Delta}(0) = -\frac{3 + (\pi^2/\Delta^2)\sigma^2}{3\sigma^2} = -\frac{1}{\sigma^2} - \frac{\pi^2}{3\Delta^2} \tag{25}$$

For free vibration following periodic form is suitable for the field variables

$$w(x, y, t) = W(X, Y)e^{i\omega t} \tag{26a}$$

$$\psi(x, y, t) = \Psi(X, Y)e^{i\omega t} \tag{26b}$$

By using the Eqs. (26) into Eqs.(11) one obtains

$$\Im_{11}(\Psi) + \Im_{12}(W) + \Im_{13}(W) = 0 \tag{27a}$$

$$\Im_{21}(W) + \Im_{22}(\Psi) + \Im_{23}(\Psi, W) + \Im_{24} = 0 \tag{27b}$$

The differential operators given in the governing equations (27a-27b) are listed as

$$\Im_{11} = A_{22}\frac{\partial^4}{\partial X^4} - 2A_{26}\frac{\partial^4}{\partial X^3 \partial Y} + (2A_{12} + A_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} - 2A_{16}\frac{\partial^4}{\partial X \partial Y^3} + A_{11}\frac{\partial^4}{\partial Y^4} \tag{28}$$

$$\Im_{12} = B_{21}\frac{\partial^4}{\partial X^4} + (2B_{26} - B_{61})\frac{\partial^4}{\partial X^3 \partial X} + (B_{11} + 2B_{22} - 2B_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} + (2B_{16} - B_{62})\frac{\partial^4}{\partial X \partial Y^3} + B_{12}\frac{\partial^4}{\partial Y^4} \tag{29}$$

$$\Im_{13} = \left(\frac{\partial^2}{\partial X \partial Y}\right)^2 + \frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial Y^2} \tag{30}$$

$$\Im_{21} = D_{11}\frac{\partial^4}{\partial X^4} + 4D_{16}\frac{\partial^4}{\partial X^3 \partial Y} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} + 4D_{26}\frac{\partial^4}{\partial X \partial Y^3} + D_{22}\frac{\partial^4}{\partial Y^4} \tag{31}$$

$$\Im_{22} = B_{21}\frac{\partial^4}{\partial X^4} + (2B_{26} - B_{61})\frac{\partial^4}{\partial X^3 \partial Y} + (B_{11} + 2B_{22} - 2B_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} + (2B_{16} - B_{62})\frac{\partial^4}{\partial X \partial Y^3} + B_{12}\frac{\partial^4}{\partial Y^4} \tag{32}$$

$$\Im_{23} = \frac{\partial^2 \Psi}{\partial Y^2}\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 \Psi}{\partial X^2}\frac{\partial^2 W}{\partial Y^2} - 2\frac{\partial^2 \Psi}{\partial X \partial Y}\frac{\partial^2 W}{\partial X \partial Y} \tag{33}$$

$$\Im_{24} = K_1 W + K_2 W^3 - G\Delta^2 W + \rho h \omega^2 W \tag{34}$$

Non-dimensional constants for elastic foundations are as follows

$$K_1 = k_1 a^4 / D_{11}, K_2 = k_2 a^4 h^2 / D_{11}, G = k_g a^2 / D_{11}$$

In order to comparison of our results with the literature results two different non-dimensional linear fundamental frequency are defined as below

$$\Omega_{L1} = \omega_{L1} b^2 \sqrt{\frac{\rho_i h_i}{D_{22}}} / \pi^2, \Omega_{L2} = \omega_{L2} b^2 \sqrt{\rho E_{22}} / h$$

4. Method of solution

Using DSC method to discretize the spatial derivatives in Eqs. (27), the derivatives of the displacement components can be given by

$$\Gamma_{11}(\Psi) + \Gamma_{12}(W) + \Gamma_{13}(W) = 0 \tag{35a}$$

$$\Gamma_{21}(W) + \Gamma_{22}(\Psi) + \Gamma_{23}(\Psi, W) + \Gamma_{24} = 0 \tag{35b}$$

The differential operators given in the governing equations (35a-35b) are given as

$$\Gamma_{11} = A_{22}\Re_X^4 - 2A_{26}\Re_X^3\Re_Y + (2A_{12} + A_{66})\Re_X^2\Re_Y^2 - 2A_{16}\Re_Y^4 \tag{36}$$

$$\Gamma_{12} = B_{21}\Re_X^4 + (2B_{26} - B_{61})\Re_X^3\Re_Y + (B_{11} + 2B_{22} - 2B_{66})\Re_X^2\Re_Y^2 + (2B_{16} - B_{62})\Re_X\Re_Y^3 + B_{12}\Re_Y^4 \tag{37}$$

$$\Gamma_{13} = (\Re_X\Re_Y)^2 + \Re_X^2\Re_Y^2 \tag{38}$$

$$\Gamma_{21} = D_{11}\Re_X^4 + 4B_{16}\Re_X^3\Re_Y + 2(D_{12} + 2D_{66})\Re_X^2\Re_Y^2 + 4D_{26}\Re_X\Re_Y^3 + D_{22}\Re_Y^4 \tag{39}$$

$$\Gamma_{22} = B_{21}\Re_Y^4 + (2B_{26} - B_{61})\Re_X^3\Re_Y + (B_{11} + 2B_{22} - 2B_{66})\Re_X^2\Re_Y^2 + (2B_{16} - B_{62})\Re_X\Re_Y^3 + B_{12}\Re_Y^4 \tag{40}$$

$$\Gamma_{23} = \Re_Y^2\Re_X^2 + \Re_X^2\Re_Y^2 - 2\Re_{XY}^2\Re_{XY}^2 \tag{41}$$

$$\Gamma_{24} = K_1(.)_{ij} + K_2(.)_{ij}^3 - G(\Re_X^2 + \Re_Y^2) \tag{42}$$

and the elements of the DSC operators are defined below

$$\Re_X^n(\ ) = \frac{\partial^{(n)}(\ )}{\partial X^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k\Delta X)(\ )_{i+k,j} \tag{43}$$

$$\Re_Y^n(\ ) = \frac{\partial^{(n)}(\ )}{\partial Y^{(n)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(k\Delta Y)(\ )_{i,j+k} \tag{44}$$

$$\Re_X^1\Re_Y^{(n-1)}(\ ) = \frac{\partial^{(n)}(\ )}{\partial X\partial Y^{(n-1)}} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta X)(\ )_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k\Delta Y)(\ )_{i,K+j} \tag{45}$$

$$\Re_X^{(n-1)}\Re_Y^1(\ ) = \frac{\partial^{(n)}(\ )}{\partial X^{(n-1)}\partial Y} = \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n-1)}(k\Delta X)(\ )_{i+k,j} \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(1)}(k\Delta Y)(\ )_{i,K+j} \tag{46}$$

The resulting nonlinear equation has been solved using the Newton-Raphson method (Civalek 2004, 2006, 2007, Baltacıoğlu 2009). The procedure is based on an incremental iterative method. Two type boundary conditions as simply supported and clamped have been considered. Related equations for these boundary conditions are given below.
For simply supported boundary conditions

$$w = 0, M_x = 0 \quad \text{at} \quad x = 0 \text{ and } x = a \tag{47a}$$

$$w = 0, M_y = 0 \quad \text{at} \quad y = 0 \text{ and } y = b \tag{47b}$$

For clamped boundary conditions

$$w = 0, \partial w/\partial x = 0 \quad \text{at} \quad x = 0 \text{ and } x = a \tag{48a}$$

$$w = 0, \partial w/\partial y = 0 \quad \text{at} \quad y = 0 \text{ and } y = b \tag{48b}$$

5. Numerical results

In this section, several examples of the linear and nonlinear analyses of anisotropic plates are provided to demonstrate the validity and accuracy of the proposed method. In the following solutions, four sets of orthotropic materials, unless mentioned otherwise, are considered:

Material-I (Glass-Epoxy)
$E_y/E_x = 3, G_{xy}/E_x = 0.5, \nu_y = 0.25$

Material-II (Boron-Epoxy)
$E_y/E_x = 10, G_{xy}/E_x = 0.333, \nu_y = 0.3$

Material-III (Graphite- Epoxy)
$E_y/E_x = 40, G_{xy}/E_x = 0.50, \nu_y = 0.25$

Material-IV (Isotropic)
$E_y/E_x = 1, G_{xy}/E_x = 0.385, \nu_y = 0.3$

In the first example, we consider the nonlinear vibration of cross-ply laminated plate with simply supported edges. A convergence study is carried out with respect to the different number of grid points. From the results shown in Table 1, it is observed that, the convergence is rapid and stable. The convergence results for this case were obtained with a 15 grid point distribution. The results are in good agreement with results given by Ganapathi *et al.* (1991). However, the results given in this table are generally higher those of Kant and Kommineni (1994) stemming from the using higher-order shear deformation theory. Comparison of fundamental frequency of two-layered laminated plates resting on nonlinear elastic foundations is presented in Tables 2 and Table 3. The results from finite element method given by Manoj *et al.* (2000) have also been provided for comparing the accuracy and for verification. From the tables,

Table 1 Comparisons of frequency ratio ($\omega_{NL2} / \omega_{L2}$) of cross-ply ($0^0/90^0/90^0/0^0$) laminated square plate with simply supported edges (Material-III)

	Ganapathi et al.* (1991)	Kant and Kommineni† (1994)	Present DSC results			
			N = 11	N = 13	N = 15	N = 17
0.2	1.04108	1.02808	1.04314	1.04301	1.04301	1.04301
0.4	1.15029	1.13436	1.16022	1.16003	1.16003	1.16003
0.6	1.31653	1.28324	1.33018	1.33005	1.33005	1.33005
0.8	1.51394	1.47890	1.52322	1.52204	1.52204	1.52204
1.0	1.73650	1.68399	1.73838	1.73816	1.73816	1.73816

*First-order shear deformation theory
†Higher-order shear deformation theory

Table 2 Comparisons of fundamental frequency (ω_{L1}) of two-layered laminated plates resting on nonlinear elastic foundations (SSSS plate, Material I, a / b = 1)

Foundation parameters			Manoj et al. (2000)	Present DSC Results
K1	K2	G		
0	0	0	6.8143	6.8229
50	0	0	7.4125	7.4206
50	0	25	11.7873	11.7915
50	25	0	7.4125	7.3938
50	25	50	14.9311	14.926

Table 3 Fundamental frequency (ω_{L1}) of two-layered clamped laminated plates resting on nonlinear elastic foundations (Material I, a/b = 1)

Foundation parameters			Present DSC Results N = 13	Present DSC Results N = 15
K1	K2	G		
0	0	0	13.3608	13.3608
60	0	0	13.7135	13.7135
60	0	30	18.8934	18.8934
60	30	0	13.7146	13.7144
60	30	60	22.0067	22.0066
60	0	60	25.0891	25.0891

it is clear that the results obtained using present method agrees very closely with the literature results (Manoj *et al.* 2000). In the computation N = 15 is used for all type boundary conditions.

Fig. 2 shows the effect of Winkler foundation (K2 = 0, G = 0) parameters on frequency ratio versus amplitude ratio for CCCC isotropic (Material-IV) plate. It can be easily seen that the Winkler parameter has a significant effect on frequency. As expected, frequency ratios decrease as Winkler foundation parameter increases. In Fig. 3 the frequency ratio for a SSSS supported isotropic (Material-IV) plate on Winkler-Pasternak elastic foundation (K1 = 50, K2 = 0) is plotted for different Pasternak parameter. The frequency ratio rapidly decreases with the increasing value of Pasternak parameter (G). An increase in G, however, decreases the frequency ratio more rapidly than an increase in K1.

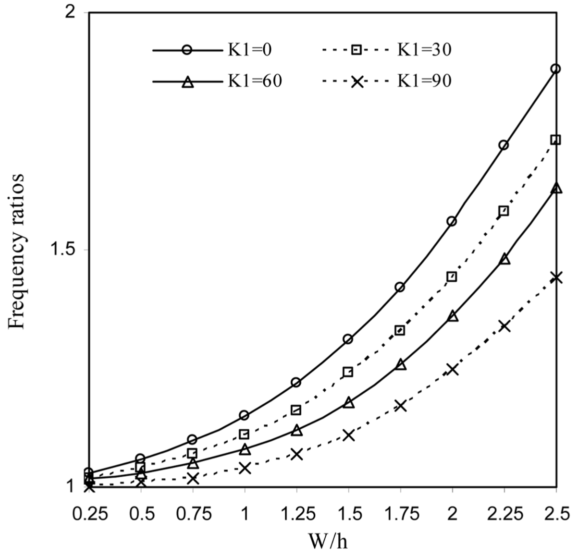

Fig. 2 Variation of frequency ratios for CCCC plates (K2 = 0, G = 0, Material-IV) for different Winkler parameters

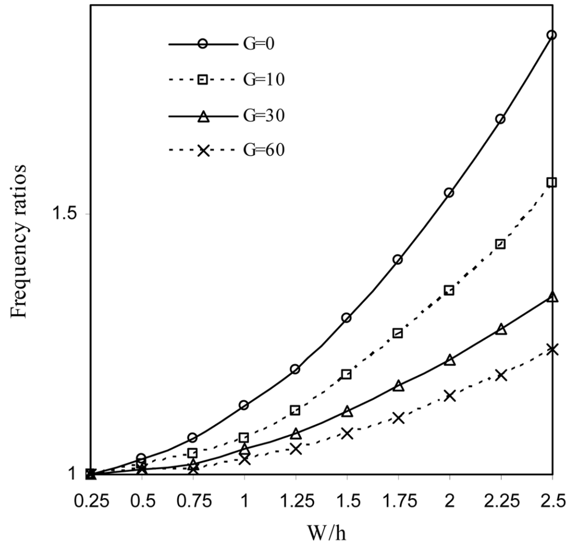

Fig. 3 Variation of frequency ratios for SSSS plates (K2 = 0, K1 = 50, Material-IV) for different Pasternak parameters

Fig. 4 shows plots of frequency ratio versus W/h for CCCC laminated (0/90) plates (K1 = 90, G = 10, K2 = 0) of various materials. It is concluded from this figure that the material properties have a significant effect on frequencies. The effects of lamination scheme and number of layers on the frequency ratio of laminated plates on nonlinear foundation (K1 = 40, G = 40, K2 = 30) with CCCC boundary conditions have been studied, and the results are shown in Fig. 5. The frequency ratio is lower for 0/90/0/90 laminated plate, and it is higher for 0/90 plate. It can be shown from these figures (Figs. 4 and 5) that the

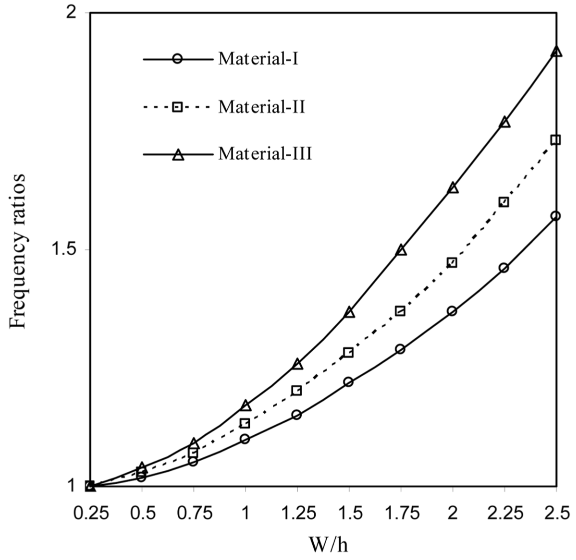

Fig. 4 Variation of frequency ratios for CCCC laminated (0/90) plates (K1=90, G=10, K2=0) for different material parameters

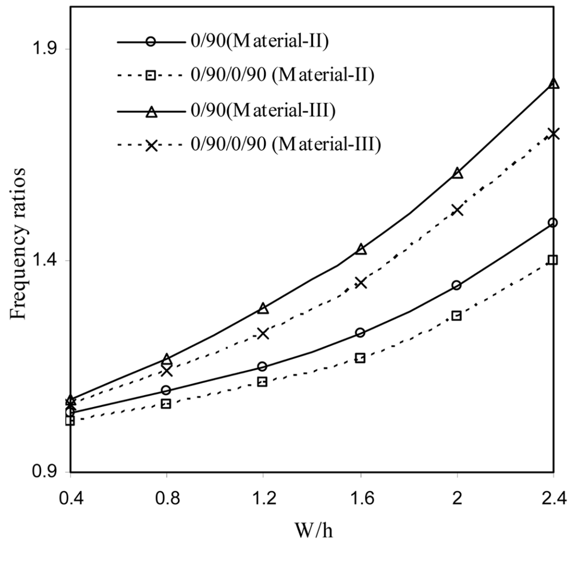

Fig. 5 Variation of frequency ratios for CCCC laminated plates (K1=40, K2=30, G=40) for different material parameters

effect of number of layers and material properties is significant.

The effects of nonlinear foundation parameter on frequency ratio of orthotropic plate are studied and results are depicted in Figs. 6 and 7. As similar the results presented in Figs. 2 and 3, the effect of the increase in Winkler and Pasternak foundation parameters is to decrease the frequency ratio. However, it can be seen from these figures that frequency ratio increase with increasing value of nonlinear foundation

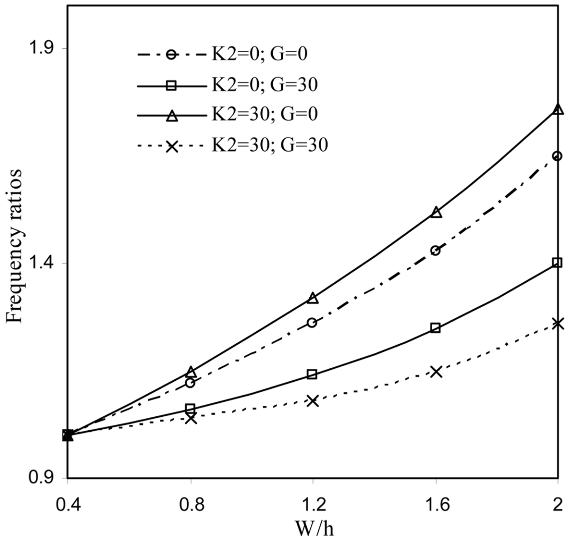

Fig. 6 Variation of frequency ratios for SSSC plates (K1 = 40, Material-I) for different foundation parameters

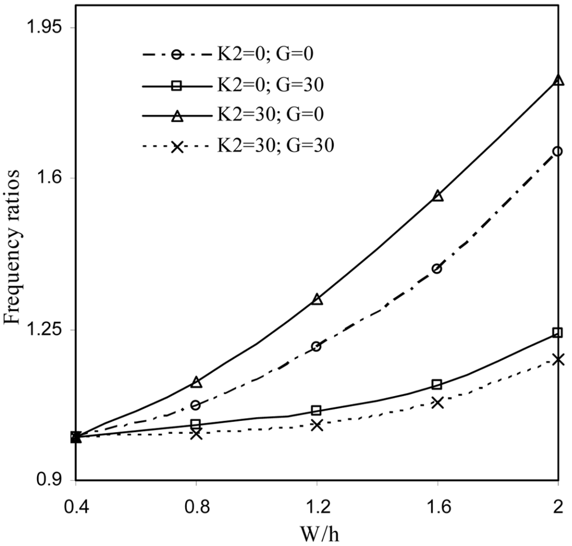

Fig. 7 Variation of frequency ratios for SSSS laminated plates (K1 = 40, Material-II) for different foundation parameters

parameter (K2) for similar Winkler (K1) and Pasternak parameter (G). It is concluded from these figures that nonlinear foundation parameter (K2) of foundation exhibit a significant effect on the nonlinear frequency response of plate than the linear Winkler parameter.

The effect of boundary conditions on frequency response has also been investigated and results presented in Fig. 8. The lowest frequency ratios are obtained for CCCC plates, as expected. The effect of foundation and material parameters, layer numbers, and aspect ratios are investigated, and results are

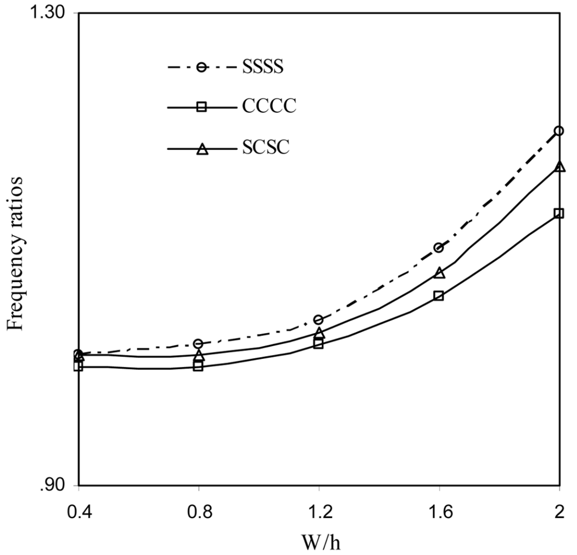

Fig. 8 Variation of frequency ratios for CCCC laminated (0/90) plates (K1 = 40, K2 = 30, G = 30, Material-II) for different boundary conditions

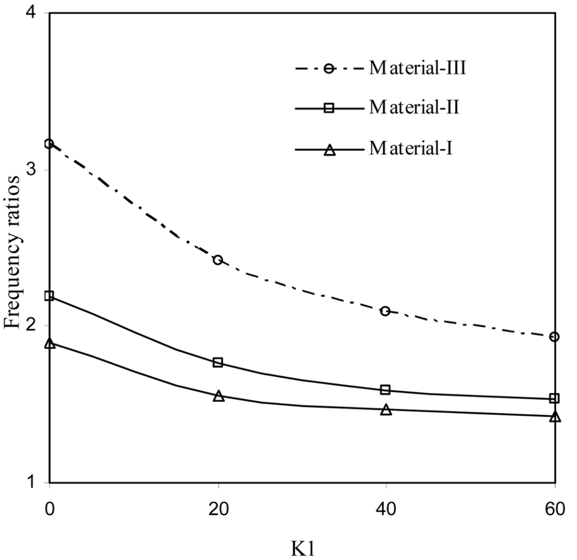

Fig. 9 Variation of frequency ratios for SSSS laminated (0/90) plates (K2 = 0, G = 0) for different material parameters

depicted in Figs. 9-12. The frequency ratio increase with increasing value of E_1/E_2. The aspect ratio is significant on nonlinear frequency response for laminated plate on elastic foundation. It is also shown that an increase in the aspect ratio results in an increase in the frequency for a given material and foundation parameters.

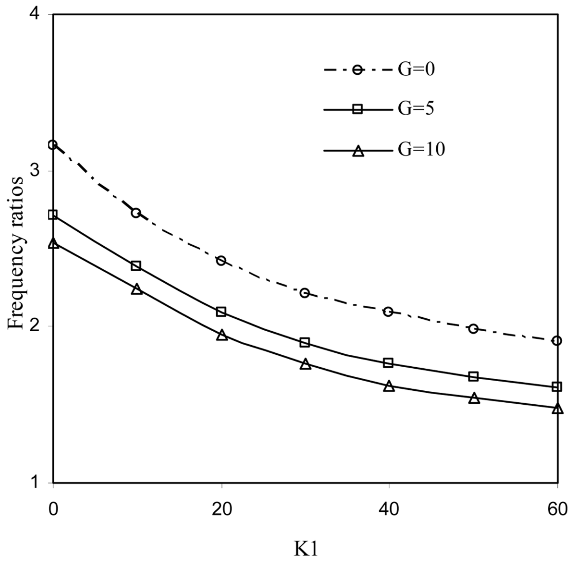

Fig. 10 Variation of frequency ratios for SSSS laminated (0/90) plates (K2 = 0, Material-III) for different Pasternak parameters

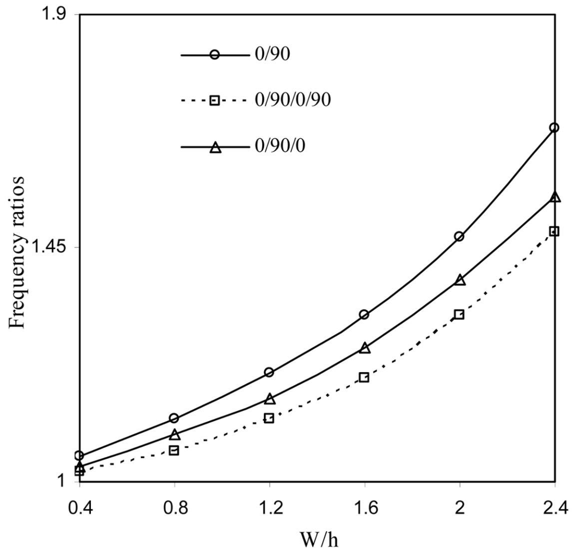

Fig. 11 Variation of frequency ratios for CCCC laminated plates (K1 = 40, G = 40, K2 = 40, Material-II) for different lamination

6. Conclusions

In the present study, a regularized Shannon delta kernel based discrete singular convolution method is adopted for the linear and nonlinear vibration analysis of isotropic, orthotropic and laminated plate resting on nonlinear elastic foundations. The applicability of the DSC method is demonstrated from the

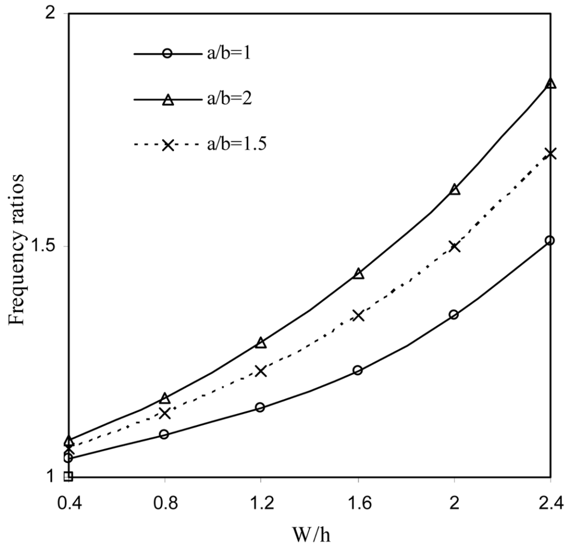

Fig. 12 Variation of frequency ratios for CCCC laminated (0/90) plates (K1=40, G=40, K2=30 Material-II) for different aspect ratios

linear and nonlinear solution of plates on elastic foundations. We cannot obtain reliable results for standard grid distributions in free boundary treatment. The effects of some geometric and material properties of plates and parameters of foundations on free vibration of laminated plates are investigated. It is concluded from the results that the material type, number of layers and aspect ratio of plate are significant on frequency ratio. The effect of nonlinear foundation parameter on frequency ratios is higher than the linear Winkler parameter and Pasternak parameter. The influence of Ey/Ex on frequency ratio has a significant effect than the number of layers. Even though the analysis presented is for the nonlinear vibration case, the postbuckling analysis of laminated plates on nonlinear elastic foundation also can be analyzed using the numerical method is presently under study.

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