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# Three-dimensional free vibration analysis of cylindrical shells with continuous grading reinforcement

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**Abstract.** Three dimensional free vibrations analysis of functionally graded fiber reinforced cylindrical shell is presented, using differential quadrature method (DQM). The cylindrical shell is assumed to have continuous grading of fiber volume fraction in the radial direction. Suitable displacement functions are used to reduce the equilibrium equations to a set of coupled ordinary differential equations with variable coefficients, which can be solved by differential quadrature method to obtain natural frequencies. The main contribution of this work is presenting useful results for continuous grading of fiber reinforcement in the thickness direction of a cylindrical shell and comparison with similar discrete laminate composite ones. Results indicate that significant improvement is found in natural frequency of a functionally graded fiber reinforced cylinder due to the reduction in spatial mismatch of material properties and natural frequency

**Keywords:** functionally graded fiber reinforce; DQM, free vibrations; three-dimensional; orthotropic; cylinder.

## 1. Introduction

Thin and thick shells as structural elements occupy a leadership position in civil, architectural, aeronautical, and marine engineering, since they give rise to optimum conditions for dynamic behavior, strength and stability. In other words, these structures support applied external forces efficiently by virtue of their geometrical shape. The study of the vibration of shells and panels of revolution is an important aspect in the successful applications of these structures. Functionally graded materials (FGM) are a class of composites that have a smooth and continuous variation of material properties from one surface to another and thus can alleviate the stress concentrations found in laminated composites. Typically these materials are made from a mixture of ceramic and metal, or a combination of different materials. Extensive research work has been carried out on this new class of composites since its concept was first introduced and proposed in the late 1980s in Japan.

Despite the evident importance in practical applications, investigations on the dynamic characteristics of FGM shell structures are still limited in number. Among those available, free vibrations of simply supported FGM cylindrical shell was investigated (Loy *et al*. 1999), which was later extended to cylindrical shells under various end supporting conditions (Pradhan *et al*. 2000). Vibration analysis of functionally graded shell was carried out, using a higher-order theory (Patel *et al*. 2005). The free vibrations analysis of functionally graded curved panels was studied by using a higher-order finite

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element formulation (Pradyumna and Bandyopadhyay 2008). Free vibration and dynamic instability of FGM cylindrical panels under combined static and periodic axial forces were studied by using a proposed semi-analytical approach (Yang and Shen 2003). Elastic response analysis of simply supported FGM cylindrical shell under low-velocity impact was presented (Gang *et al.* 1999). Vibrations and wave propagation velocity in a functionally graded hollow cylinder was studied (Shakeri *et al.* 2006). It was assumed the shell to be in plane strain condition and subjected to an axisymmetric dynamic loading. The free vibration of simply supported, fluid-filled orthotropic functionally graded cylindrical shells with arbitrary thickness was investigated (Chen *et al.* 2004). In all the above studies, it was assumed that material properties follow a through-thickness variation according to a power-law distribution in terms of the volume fractions of constituents. Recently three-dimensional steady-state response of a functionally graded fiber reinforced cylindrical panel was studied (Yas and Sobhani 2010). The results indicated the advantages of using functionally graded fiber reinforced shell with graded fiber volume fractions over traditional discretely laminated composite shells.

However, this paper is motivated by the lack of studies in the technical literature concerning to the 3-dimensional vibration analysis of functionally graded fiber reinforced cylinders. The aim is to analyze the influence of continuous grading of fiber reinforcement and comparison of FGM cylinder with discretely laminated composite ones.

The solution is obtained by using numerical technique termed the generalized differential quadrature method (GDQ), which leads to a generalized eigenvalue problem. The mathematical fundamental and recent developments of GDQ method as well as its major applications in engineering are discussed in detail in book (Shu 2000). In recent years, the DQM has been become increasingly popular in the numerical solution for initial and boundary value problems (Bert and Malik 1996). The DQM can yield accurate solutions with relatively fewer grid points. The first application of DQM for composite plates was carried out (Bert *et al.* 1989). It was made use of DQM to determine vibration characteristics of generally laminated beams and cross-ply laminated plates subjected to cylindrical bending respectively (Chen and Bian 2003, Chen 2005). Combination of the state-space method and the technique of DQ based on the three dimensional elasticity theory were used for static and free vibrations analysis of laminated cylindrical panels (Alibegloo and Shakeri 2009, 2007). Three dimensional temperature dependent free vibration analysis of functionally graded material curved panels resting on two parameter elastic foundation was studied through using a hybrid semi-analytic differential quadrature method (Farid *et al.* 2009). 2-D higher- order deformation theory was used for free vibration and stability of functionally graded shallow shells (Matsunaga 2008). Thus another aim of the present paper is to demonstrate on efficient application of the differential quadrature approach, by solving the equations of motion governing the functionally graded orthotropic cylindrical shells.

In this paper the frequency characteristics of cylindrical shells with continuous grading of fiber volume fraction is obtained. Numerical results are presented and compared to the available results in the literatures for isotropic FGM cylindrical shell to validate the proficiency of DQ approach.

## 2. Problem formulation

Consider a laminated cylindrical shell with thickness "h" placed in the cylindrical coordinate system ($r$, $\theta$, $z$) of Fig. 1. Where $r$, $\theta$, $z$ are in the radial, circumferential and axial directions of the cylinder. It is assumed that the cylinder has been divided into many thin orthotropic layers with equal thickness. Each interface is assumed to be bonded perfectly and no initial defect is considered. For the conventional

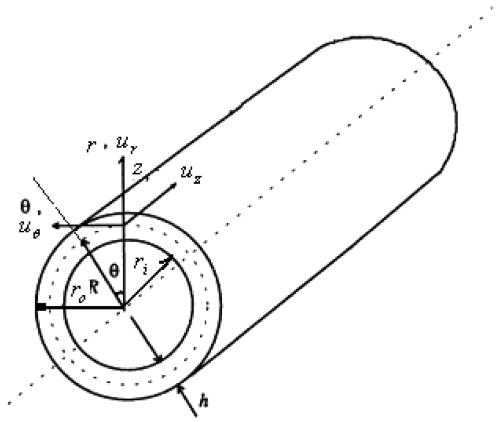


Fig. 1 Geometry of a cylindrical shell

laminated cylinder, each layer has unified fiber volume as well as fiber orientation. However, the fiber volume distribution of functionally graded composite laminate is non-uniform, which is considered to be variable along the thickness direction in this paper. For this purpose, we assume the following linear variation of the reinforcement volume fraction (Pelletier and Vel 2006).

$$V = V_i + (V_o - V_i)\left(\frac{r - r_i}{r_o - r_i}\right)^p \tag{1}$$

Where $V_i$ and $V_o$ which have values that range from 0 to 1, denote the volume fractions on the inner and outer surfaces, respectively. The exponent "$P$" controls the volume fraction profile through the shell's thickness which is set to 1 for numerical results.

The effective mechanical properties of the fiber reinforced cylinder are obtained based on a micromechanical model as follows (Shen 2009, Valery *et al.* 2001)

$$E_{11} = V_f E_{11}^f + V_m E_{11}^m \tag{2}$$

$$\frac{1}{E_{ii}} = \frac{V_f}{E_{ii}^f} + \frac{V_m}{E_{ii}^m} - V_f V_m \frac{v_f^2 E_{ii}^m / E_{ii}^f + v_m^2 E_{ii}^f / E_{ii}^m - 2v^f v^m}{V_f E_{ii}^f + V_m E_{ii}^m} \quad , \qquad i = 2, 3 \tag{3}$$

$$\frac{1}{G_{ij}} = \frac{V_f}{G_{ij}^f} + \frac{V_m}{G_{ij}^m} \quad , \quad i,j = 1, 2, 3 \quad , \quad i \neq j \tag{4}$$

$$v_{ij} = V_f v^f + V_m v^m \quad , \quad i,j = 1, 2, 3 \quad , \quad i \neq j \tag{5}$$

$$\rho = V_f \rho^f + V_m \rho^m \tag{6}$$

Where $E_{ij}$, $G_{ij}$, $v_{ij}$, $\rho$, $V_f$ and $V_m$ are elasticity modulus, shear modulus, Poisson's ratio, density, fiber volume and matrix volume fractions respectively.

The mechanical constitutive relations, which relates the stresses to the strains are as follows

$$\begin{bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \tau_{t\theta} \\ \tau_{zr} \\ \tau_{z\theta} \end{bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & 0 & 0 \\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0 & 0 & 0 \\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{C}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{C}_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{C}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{zr} \\ \gamma_{z\theta} \end{bmatrix} \tag{7}$$

For plane strain condition (infinite length), and in the absence of body forces, the governing equations are as follows

$$\begin{aligned} \frac{\partial \tau_{z\theta}}{r\partial\theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} &= \rho\frac{\partial^2 u_z}{\partial t^2} \\ \frac{\partial \sigma_\theta}{r\partial\theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} &= \rho\frac{\partial^2 u_\theta}{\partial t^2} \\ \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial\theta} + \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} &= \rho\frac{\partial^2 u_r}{\partial t^2} \end{aligned} \tag{8}$$

Strain-displacement relations are expressed as

$$\begin{aligned} \varepsilon_\theta &= \frac{u_r}{r} + \frac{\partial u_\theta}{r\partial\theta} \quad , \quad \varepsilon_r = \frac{\partial u_r}{\partial r} \\ \gamma_{r\theta} &= \frac{-u_\theta}{r} + \frac{\partial u_\theta}{\partial r} + \frac{\partial u_r}{r\partial\theta} \\ \gamma_{zr} &= \frac{\partial u_z}{\partial r} \quad , \quad \gamma_{z\theta} = \frac{\partial u_z}{r\partial\theta} \end{aligned} \tag{9}$$

Where $u_r$, $u_\theta$ and $u_z$ are radial, circumferential and axial displacement components respectively.

Upon substitution Eq.(9) into (7) and then into (8), the following equations of motion as matrix form are obtained in term of displacement components

$$\begin{bmatrix} K_{1r} & K_{1\theta} & K_{1z} \\ K_{2r} & K_{2\theta} & K_{2z} \\ K_{3r} & K_{3\theta} & K_{3z} \end{bmatrix} \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix} = \begin{Bmatrix} \rho\ddot{u}_r \\ \rho\ddot{u}_\theta \\ \rho\ddot{u}_z \end{Bmatrix} \tag{10}$$

The inner and outer surfaces of the cylinder are traction free

$$\sigma_r = \tau_{rz} = \tau_{r\theta} = 0 \quad \text{at} \quad r = r_i, r_o \tag{11}$$

## 3. Semi-analytical solution

The following functions are assumed for displacement components

$$u_r = \sum_{m=1}^{\infty} \sin(m\theta).U_r(r).e^{i\omega t}, u_\theta = \sum_{m=1}^{\infty} \cos(m\theta).U_\theta(r).e^{i\omega t}, u_z = \sum_{m=1}^{\infty} \cos(m\theta).U_z(r).e^{i\omega t} \tag{12}$$

Where "m" is the circumferential wave numbers and $\omega$ is the natural angular frequency of the vibration.

Upon substituting Eq. (12) into the governing Eq. (10), the coupled partial differential equations reduce to a set of ordinary differential relations as follows:

$$-\bar{C}_{66}\frac{1}{r^2}m^2 U_z(r) + \frac{\partial \bar{C}_{55}}{\partial r}\frac{\partial U_z(r)}{\partial r} + \bar{C}_{55}\left(\frac{\partial^2 U_z(r)}{\partial r^2} + \frac{1}{r}\frac{\partial U_z(r)}{\partial r}\right) = -\omega^2 \rho U_z(r)$$

$$-\bar{C}_{22}\frac{1}{r^2}m^2 U_\theta(r) + \frac{\partial \bar{C}_{44}}{\partial r}\left(-\frac{1}{r}U_\theta(r) + \frac{\partial U_\theta(r)}{\partial r} + \frac{1}{r}mU_r(r)\right) +$$

$$(\bar{C}_{23} + \bar{C}_{44})\frac{1}{r}m\frac{\partial U_r(r)}{\partial r} + \bar{C}_{44}\left(-\frac{1}{r^2}U_\theta(r) + \frac{1}{r}\frac{\partial U_\theta(r)}{\partial r} + \frac{\partial^2 U_\theta(r)}{\partial r^2}\right) +$$

$$(\bar{C}_{22} + \bar{C}_{44})\frac{1}{r^2}mU_r(r) = -\omega^2 \rho U_\theta(r)$$

$$-\bar{C}_{44}\frac{1}{r}\left(m\frac{\partial U_\theta(r)}{\partial r} + \frac{1}{r}m^2 U_r(r)\right) + \frac{\partial \bar{C}_{23}}{\partial r}\frac{1}{r}(U_r(r) - mU_\theta(r)) - \bar{C}_{23}\frac{1}{r}m\frac{\partial U_\theta(r)}{\partial r} + \frac{\partial \bar{C}_{33}}{\partial r} + \frac{\partial U_r(r)}{\partial r} +$$

$$(\bar{C}_{22} + \bar{C}_{44})\frac{1}{r^2}mU_\theta(r) + \bar{C}_{33}\left(\frac{\partial^2 U_r(r)}{\partial r^2} + \frac{1}{r}\frac{\partial U_r(r)}{\partial r}\right) - \bar{C}_{22}\frac{1}{r^2}U_r(r) = -\omega^2 \rho U_r(r) \tag{13}$$

A semi – analytical procedure with the aid of DQM technique was recently developed (Chen and Bian 2003). In this method the nth order partial derivative of a continuous function $f(x, z)$ with respect to x at a given point $x_i$ can be approximated as a linear sum of weighted function values at all of the discrete points in the domain of x, *i.e.*,

$$\frac{\partial f^{n(x_i, z)}}{\partial x^n} = \sum_{k=1}^{N} c_{ik}^n f(x_{ik}, z) \quad , \quad (\text{i} = 1,2,...\text{N, n}=1,2,...\text{N -1}) \tag{14}$$

Where N is the number of sampling points, and $c_{ij}^n$ is the $x_i$ dependent weight coefficients (Chen and Bian 2003).

Eq.(14) being applied to Eq.(13), the following equations at an arbitrary sampling point $x_i$ are then obtained.

$$-(\overline{C}_{66})_i\frac{1}{r^2}m^2U_{zi}+\sum_{k=1}^{N}\left(\left(\left(\frac{\partial\overline{C}_{55}}{\partial r}\right)_i+(\overline{C}_{55})_i\frac{1}{r}\right)c_{ik}^{(1)}+(\overline{C}_{55})_ic_{ik}^{(2)}\right)U_{zk}=-\omega^2\rho U_{zi}$$

$$-((\overline{C}_{22})_i\frac{1}{r^2}m^2+\left(\frac{\partial\overline{C}_{44}}{\partial r}\right)_i\frac{1}{r}+(\overline{C}_{44})_i\frac{1}{r^2})U_{\theta i}+\sum_{k=1}^{N}\left(\left(\left(\frac{\partial\overline{C}_{44}}{\partial r}\right)_i+(\overline{C}_{44})_i\frac{1}{r}\right)c_{ik}^{(1)}\right.$$

$$\left.+(\overline{C}_{44})_ic_{ik}^{(2)}\right)U_{\theta k}+\left(\left(\frac{\partial\overline{C}_{44}}{\partial r}\right)_i\frac{1}{r}m+(\overline{C}_{22}+\overline{C}_{44})\frac{1}{r^2}m\right)U_{ri}+(\overline{C}_{23}+\overline{C}_{44})\frac{1}{r}m\sum_{k=1}^{N}c_{ik}^{(1)}U_{rk}$$

$$=-\omega^2\rho U_{\theta t}$$

$$\left(-\left(\frac{\partial\overline{C}_{23}}{\partial r}\right)_i\frac{1}{r}m+((\overline{C}_{22})_i+(\overline{C}_{44}))_i\frac{1}{r^2}m\right)U_{\theta i}+\left(\left(\frac{\partial\overline{C}_{23}}{\partial r}\right)_i\frac{1}{r}-(\overline{C}_{22})_i\frac{1}{r^2}-(\overline{C}_{44})_i\frac{1}{r^2}m^2\right)U_{ri}-$$

$$\sum_{k=1}^{N}\left((\overline{C}_{44})_i\frac{1}{r}mc_{ik}^{(1)}+(\overline{C}_{23})_i\frac{1}{r}mc_{ik}^{(1)}\right)U_{\theta k}+\sum_{k=1}^{N}\left(\left(\left(\frac{\partial\overline{C}_{33}}{\partial r}\right)_i+(\overline{C}_{33})_i\frac{1}{r}\right)c_{ik}^{(1)}+(\bar{c}_{33})_ic_{ik}^{(2)}\right)U_{rk}$$

$$U_{zk}=-\omega^2\rho U_{ri} \tag{15}$$

In the above equation $c_{ik}^{(1)}$ and $c_{ik}^{(2)}$ are the weighting coefficients of the first and second order derivatives. In a similar manner the boundary conditions can be discretized. For this purpose, using Eq.(11) and DQ discretization rule for special derivatives, the boundary conditions at $r = r_i$ and $r = r_o$ become

$$(\overline{C}_{23})_1\frac{1}{a}U_{r1}-(\overline{C}_{23})_1\frac{1}{a}mU_{\theta 1}+(\overline{C}_{33})_1\sum_{k=1}^{N}c_{1k}^{(1)}U_{rk}=0$$

$$-(\overline{C}_{44})_1\frac{1}{a}U_{\theta 1}+(\overline{C}_{44})_1\sum_{k=1}^{N}c_{1k}^{(1)}U_{\theta k}+(\overline{C}_{44})_1\frac{1}{a}mU_{r1}=0$$

$$(\overline{C}_{55})_1\sum_{k=1}^{N}c_{1k}^{(1)}U_{zk}=0$$

$$(\overline{C}_{23})_N\frac{1}{b}U_{rN}-(\overline{C}_{23})_N\frac{1}{b}mU_{\theta N}+(\overline{C}_{33})_N\sum_{k=1}^{N}c_{Nk}^{(1)}U_{rk}=0$$

$$-(\overline{C}_{44})_N\frac{1}{b}U_{\theta N}+(\overline{C}_{44})_N\sum_{k=1}^{N}c_{Nk}^{(1)}U_{\theta k}+(\overline{C}_{44})_N\frac{1}{b}m.U_{rN}=0$$

$$(\overline{C}_{55})_N\sum_{k=1}^{N}c_{Nk}^{(1)}U_{zk}=0 \tag{16}$$

In order to carry out the eigenvalue analysis, domain and boundary degrees of freedom are separated and in vector forms they are denoted as (d) and (b), respectively. Based on this definition, the matrix form of the equilibrium equations and the related boundary conditions take the following form

$$\begin{bmatrix} [A_{bb}] & [A_{bd}] \\ [A_{db}] & [A_{dd}] \end{bmatrix} \begin{Bmatrix} \{U_b\} \\ \{U_d\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ -\omega^2[M]\{U_d\} \end{Bmatrix} \tag{17}$$

Where $\{U_d\}$ and $\{U_b\}$ are as follows

$$\{U_d\} = \{\{U_{rd}\}, \{U_{\theta d}\}, \{U_{zd}\}\}^T \tag{18}$$

$$\{U_b\} = \{\{U_{rb}\}, \{U_{\theta b}\}, \{U_{zb}\}\}^T \tag{19}$$

Eliminating the boundary degrees of freedom, this equation become

$$([A] + \omega^2[M])\{U_d\} = \{0\} \tag{20}$$

Where

$$[A] = [A_{dd}] - [A_{db}][A_{bb}]^{-1}[A_{bd}] \tag{21}$$

The above eigenvalue system of equations can be solved to find the natural frequencies of the FGM orthotropic cylindrical shells.

## 4. Numerical results

For numerical computation, sampling points with the following coordinates are used (Shu and Richards 1992).

$$x_i = \frac{1}{2}\left(1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right) \quad , \quad i = 1, 2, \ldots, N \tag{22}$$

To validate the analysis, results for an FGM isotropic cylindrical shell with mechanical properties as: $r_i = 0.25$ m., $r_o = 0.5$ m., on inner surface $E_c = 380$ Gpa, $\rho_c = 3800$ kg/m$^3$, on outer surface $E_m = 70$ Gpa., $\rho_m = 2707$ kg/m$^3$ are compared with similar ones in the literature, see Table 1. The comparison shows that the present results agreed well with those in the literature.

Table 1 Comparison of first natural frequency for various "p"

| "p" | Present (HZ.) | (Shakeri *et al.* 2006) (HZ.) |
|---|---|---|
| 0.001 | 2402.1 | 2441.5 |
| 0.5 | 2985.6 | 3006.2 |
| 5 | 43525.4 | 4394.5 |
| 20 | 4387.3 | 4394.5 |

As an example, it is assumed the FGM cylindrical shell has the following mechanical properties:

$$\frac{E_L}{E_T} = 40, \quad \frac{G_{LT}}{E_T} = 0.5, \quad \frac{G_{TT}}{E_T} = 0.2, \quad v_{LT} = v_{TT} = 0.25, \quad \rho = 1408(Kg/m^3) \tag{23}$$

we characterize the response of orthotropic cylindrical shells with graded fiber volume fractions. The orthotropic cylinder consists of continuous tungsten reinforcement fibers in a copper matrix (W/Cu). These material combinations have found widespread use in high performance application (Miracle 2001). The relevant material properties for the constituent materials are listed in Table 2.

For this material the fibers are oriented at $\phi - 0$, with respect to the axial direction of the cylindrical shell. Here we assume the functionally graded shell has a linear variation starting at $V_w = 0$ (0% Tungsten, 100% Copper) on the inner surface of the shell to $V_w = 0.75$ (75% Tungsten, 25% Copper) on the outer surface. The normalized natural frequency is, $\Omega = \omega r_i \sqrt{\rho^i / E^i}$ ($\rho^i$, $E^i$ are mechanical properties of Copper). A convergence study of the normalized natural frequency is shown in Fig. 2 for various S ratios (the middle surface radius to thickness ratio). As it is noticed, fast rate of convergence of the method is evident and it is found that only seven DQ grid in the thickness direction can yield accurate results. It can also be seen for the considered system the formulation is stable while increasing the number of points and that the use of 50 points guarantees convergence of the procedure.

We now turn our attention to the comparison of the FGM cylindrical shell with discretely laminated 2-layer, 3-layer and 4-layer shell containing [0/0.75], [0/0.375/0.75], [0/0.25/0.500/0.75] volume fractions respectively, as shown in Table 3.

The effect of S ratio on the normalized natural frequency is shown in Fig. 3. As it is observed the normalized natural frequency decreases sharply with increasing the S ratio for thick cylinders and

Table 2 Mechanical properties of the isotropic material

| | Cu | W |
|---|---|---|
| E(Gpa) | 115.0 | 400.0 |
| $v$ | 0.31 | 0.28 |
| $\rho(kg/m^3)$ | 8960 | 19300 |

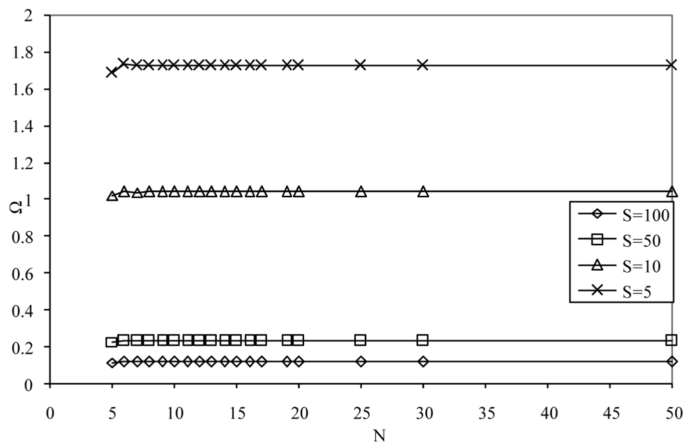


Fig. 2 convergency of the normalized natural frequency of a graded fiber volume fractions cylinder (“m” = 1)

Table 3 Material volume fractions of 2-layer, 3-layer, 4-layer and FGM

| Type of cylindrical shell | | Material volume fractions |
|---|---|---|
| 2 Layers | 1st lamina | 0%Tungsten, 100%Copper |
| | 2st lamina | 75% Tungsten, 25% Copper |
| 3 Layers | 1st lamina | 0% Tungsten, 100% Copper |
| | 2st lamina | 37.5% Tungsten, 62.5% Copper |
| | 3st lamina | 75% Tungsten, 25% Copper |
| 4 Layers | 1st lamina | 0% Tungsten, 100% Copper |
| | 2st lamina | 25% Tungsten, 75% Copper |
| | 3st lamina | 50% Tungsten, 50% Copper |
| | 4st lamina | 75% Tungsten, 25% Copper |
| FGM | inner surface | 0% Tungsten, 100% Copper |
| | outer surface | 75% Tungsten, 25% Copper |

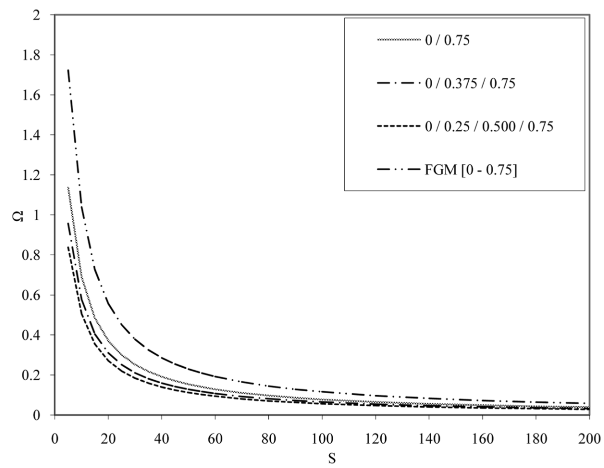


Fig. 3 Effect of S ratio on the normalized natural frequency for graded fiber volume fractions, FGM, 2-layer, 3-layer, 4-layer ("m" = 1)

remains unaltered for thin ones. It is also noticed the normalized natural frequency of a graded fiber volume fractions is higher than a similar discrete laminated and closer to 2-layer laminated one. Similar results can be obtained from Fig. 4. In this figure, comparison is made between graded fiber volume fractions and discrete laminated cylinders for various values of circumferential wave numbers.

According to Fig. 5, for thick functionally graded fiber reinforced cylindrical shells, the normalized natural frequency increases with the increase of the circumferential wave numbers at a higher rate and for thin ones this increase is very slow.

In this section we compare the normalized natural frequency of the considered functionally graded fiber reinforced(0% Tungsten, 100% Copper on the inner surface and 75% Tungsten, 25% Copper on the outer surface) with a composite cylindrical shell made of 75% Tungsten fiber, 25% Copper and also an isotropic cylinder made of Copper. Table 4 shows this comparison for different m. As observed the normalized natural frequency of a functionally graded fiber reinforced is lower than the composite

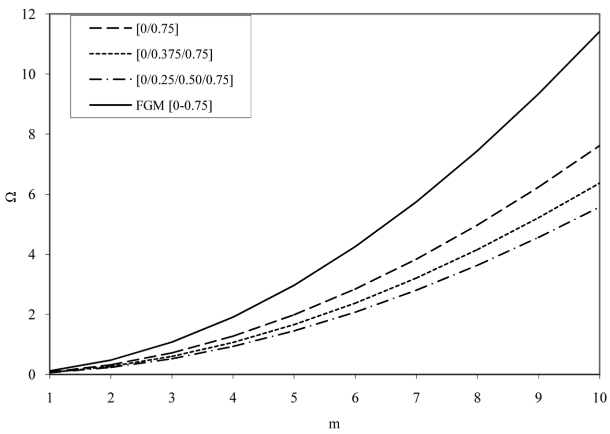


Fig. 4 Variation of the normalized natural frequency against “m”(S = 100)

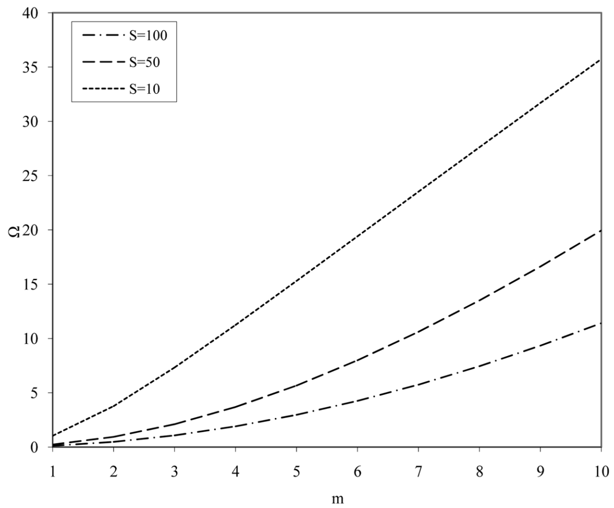


Fig. 5 Variation of the normalized natural frequency versus “m” for different S ratios

cylinder and larger than the isotropic one. That is because the amount of strengthening Tungsten fibers decreases from inner surface to the outer surface in continuous fiber reinforced cylinder, however the composite one has unified Tungsten fiber volume fraction.

Further comparison is noticed in Fig. 6 for various values of S ratios. As observed the normalized natural frequency of the continuous fiber reinforced of the cylinder falls between those for composite and isotropic cylinders.

## 5. Conclusion

The Differential Quadrature Method has been used to study 3-Dimensional free vibration analysis of continuous grading fiber reinforced cylinders. The effective mechanical properties of the fiber reinforced cylinder are obtained based on a micromechanical model. The dynamic equilibrium

Table 4 Comparison of the normalized natural frequency against m (S = 100)

| m | Composite cylinder | Continuous fiber reinforced cylinder | Isotropic cylinder |
|---|---|---|---|
| 1 | 0.12808 | 0.11564 | 0.10423 |
| 2 | 0.5277 | 0.4765 | 0.42948 |
| 3 | 1.1905 | 1.07523 | 0.9687 |
| 4 | 2.1117 | 1.90760 | 1.7181 |
| 5 | 3.2846 | 2.96801 | 2.6720 |
| 6 | 4.70134 | 4.24955 | 3.8239 |
| 7 | 6.3524 | 5.74422 | 5.16595 |
| 8 | 8.2275 | 7.44311 | 6.6895 |
| 9 | 10.315 | 9.33667 | 8.3856 |
| 10 | 12.605 | 11.4148 | 10.244 |

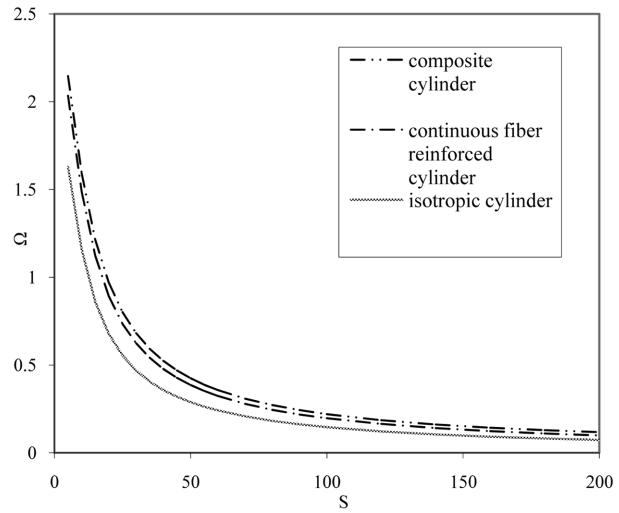


Fig. 6 Comparison of the normalized natural frequency against S (m = 1)

equations are discretized with the present method giving a standard linear eigenvalue problem. The effectiveness of this method in predicting free vibration behavior of a functionally graded fiber volume fractions cylinders was checked by comparing its results for isotropic FGM cylinder with corresponding numerical in the literature. From this study, some conclusions can be made:

· It was shown that only seven DQ grid in the thickness direction can yield accurate results.

· The formulation is stable while increasing the number of points and that the use of 50 points guarantees convergence of the procedure.

· Results indicate that improvement is found in natural frequency of a functionally graded fiber volume fraction cylinder with respect to a similar discrete laminated composite cylinder,

· Results indicate natural frequency of a continuous fiber reinforced cylinder falls between those for composite and isotropic ones.

· It is shown, upon increasing the middle surface radius to thickness ratio, S, in a thick functionally graded volume fractions cylinder the normalized natural frequency decreases rapidly and finally reaches a constant value in the thin cylinder and this behavior is similar to discretely laminated composite cylinders.

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