Steel and Composite Structures, Vol. 10, No. 4 (2010) 297-315
DOI: http://dx.doi.org/10.12989/scs.2010.10.4.297

Free vibration characteristics of horizontally curved composite plate girder bridges

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(Received October 29, 2009, Accepted June 15, 2010)

Abstract. This paper is concerned with free vibration characteristics and natural frequency of horizontally curved composite plate girder bridges. Three-dimensional finite element models are developed for the girders using the software package LUSAS and analyses carried out on the models. The validity of the finite element models is first established through comparison with the corresponding results published by other researchers. Studies are then carried out to investigate the effects of total number of girders, number of cross-frames and curvature on the free vibration response of horizontally curved composite plate girder bridges. The results confirm the fact that bending modes are always coupled with torsional modes for horizontally curved bridge girder systems. The results show that the first bending mode is influenced by composite action between the concrete deck and steel beam at low subtended angle but, on the girders with larger subtended angle at the centre of curvature such influence is non-existence. The increase in the number of girders results in higher natural frequency but at a decreasing rate. The in-plane modes viz. longitudinal and arching modes are significantly influenced by composite action and number of girders. If no composite action is taken into account the number of girders has no significant effect for the in-plane modes.

Keywords: horizontally curved girders; composite girders; bridges, cross-frames; natural frequency; finite element analysis.

1. Introduction

Horizontally curved girder bridges are commonly employed where the road alignment demands such orientation and, they also provide aesthetically pleasing appearance. Curved bridges can also satisfy the strict demand and tight geometric restrictions placed on highway structures by predetermined roadway configurations. Such arrangements are not popular choice of engineers in view of mathematical complexities associated with the analysis of curved girder systems. The complex behavior can, however, be solved and accounted for in design using today's powerful digital computers and advanced software. With the advent of high speed computers and development of numerical methods, difficulties associated with analysis of curved structures have been largely overcome. The curved girders support reinforced concrete deck slabs which are usually connected to the top flanges of the girders through shear connectors. Designers at the preliminary stage assume sometimes that these girders act independent of the deck slabs resting on them in order to simplify the analysis. The advantage of composite action between the steel

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girders and concrete deck is not considered. The benefits in terms of enhanced ultimate load behaviour and improved shear strength capacity due to the composite action in this type of girders have been studied recently by researchers (Shanmugam *et al*. 2009, Basher *et al*. 2009). The present study is concerned with free vibration characteristics of horizontally curved composite plate girders in bridge construction.

Free vibration analysis is an important phase towards understanding the dynamic characteristics of plate girder bridges. Natural frequencies and mode shapes are the basic vibration properties of a structure and, they influence the response under dynamic loads. In most bridge design codes, provision for dynamic effects is made via the dynamic allowance factors for quasi-static structural analysis based on the fundamental flexural frequency. However, for horizontally curved plate girder bridges, the lateral and coupled modes are more significant, and the situation is more complex as some features could not be revealed by traditional 2D analysis. Most of the codes are not explicit in their recommendations for curved bridges though it is necessary to perform dynamic analysis for this type of bridges to account for curvature effects. Research findings on the behavior of horizontally curved members have been published. These investigations are primarily concerned with the static behavior of curved steel plate girders and, studies on the dynamic responses of horizontally curved composite girders are limited. Published results have shown that the stiffness of a horizontally curved bridge girders decreases due to curvature. For horizontally curved composite plate girder bridges, the parameters which affect the fundamental natural frequency are total number of girders, curvature of the bridge, number of cross-frames, flexural rigidity of the bridge and the mass of the bridge itself. Researchers have investigated recently the dynamic response of horizontally curved bridges. Free vibration analysis of bridges using various methods for straight and skewed composite bridges was carried out by Memory *et al.* (1995) who compared the effects of using the static and dynamic modulus of elasticity for concrete in estimating the natural frequency of vibration. From detailed parametric studies using numerical methods on horizontally curved steel I-Girder bridges connected by cross frames, Davidson *et al*. (1996) concluded that the span length, radius of curvature, flange width and cross-frame spacing have significant influence on the warping-to-bending stress ratio. Free vibration of continuous beam-slab skewed bridges was also examined by Maleki (2002) taking into account the stiffness of elastomeric bearing. Maneetes and Linzell (2003) examined the effects of X and K type cross-frame and lateral bracing with different structural members on the dynamic behavior of horizontally curved steel bridge without composite actions. From the experimental studies on free vibration response it was concluded that lateral displacements would reduce with larger number of cross frames but it has little effect on the vertical displacements.

A three dimensional finite element analysis was used by Barth and Wu (2005) to obtain the natural frequency of composite steel bridges. They proposed equations that represent a significant improvement over the existing prediction methods. These equations, however, are limited to straight composite steel bridges only. Yoon *et al*. (2005) derived a curved beam element to investigate the free vibration characteristics of twin plate girder with varying subtended angles. The numerical formulation was applied to investigate the free vibration characteristics of the bridge by considering the effects of initial curvature, boundary condition, modeling method and degrees of freedom of cross frame. More recently, Lui *et al.* (2006) investigated the dynamic response of horizontally curved steel bridges with composite action. The parameters they considered include curvature, diaphragm spacing and diaphragm stiffness. Each of the diaphragms was assumed to be of a single steel beam and it was modeled using beam element. In the current study, however, K-bracings are used for diaphragms which are modeled using shell elements. The results show that the dynamic response of curved bridges is influenced significantly by the curvature than by the stiffness or diaphragm spacing. Chang and White (2008) presented modeling strategies for curved composite girder bridges ranging from modified line-girder analysis to 3D finite element approach. A full-scale test on curved composite I-girder bridge was carried out to assess the various modeling strategies. It was concluded that a general finite element model with shell elements for the slab, and beam and shell elements for the I-girders provides the most accurate representation of the structural response.

Most of the studies other than the one by Yoon *et al*. (2005) on the dynamic behaviour of horizontally curved bridges do not include the warping stiffness of the girder. Inclusion of the warping stiffness of the girder undoubtedly complicates the problem. However, with the availability of advanced finite element method all these mathematical complexities can be overcome. The finite element computer code LUSAS which includes warping-effects is used in the present study on free vibration characteristics of horizontally curved composite plate girders. There are not many studies which address the issues of composite action between the steel plate girder and reinforced concrete deck. The current studies focus on such composite action and, results are obtained to highlight the effect of composite action. Comparisons are made between the results corresponding to steel plate girders and composite plate girders in order to demonstrate the effect of interaction between steel girder and the concrete slab. The vibration mode shapes of straight and curved bridge girders are also presented and compared to illustrate the effect of curvature on the dynamic behavior.

2. Finite element analysis

Finite element software LUSAS (1993) was used in the current studies. Three dimensional models were developed by idealizing the flange, web and stiffener plates using shell element in the LUSAS element library. Generally, the shell element can accommodate curved geometry with varying thickness and anisotropic and composite material properties. Concrete slab was idealized by three dimensional hexahedral isoparametric solid continuum elements (HX20) with higher order models capable of modeling curved boundaries. The cross-frames were idealized using the 3D thick beam elements (BMS3). By using the above idealization, the stiffness and the mass matrix of the plate girder including the warping rigidity is taken into account. AASHTO (2005) specification requires full composite action between the reinforced concrete deck slab and the steel girder at serviceability limit state. Thus full composite action was assumed between the girder top flange and deck, which assured nodal compatibility at these locations. Previous findings have shown that the concrete cracking have little effect on the global behavior of the bridge structures and, hence can be neglected by modeling the concrete as un-cracked. Therefore, the concrete slab in the present study was assumed un-cracked and reinforcing steel neglected in the finite element model.

Typical mesh generated with a good aspect ratio for all elements is shown in Fig. 1 for straight and curved composite steel bridge. The mesh was chosen based on the results from convergence studies on this type of bridge girders. The composite bridge girders were modeled as simply supported with Z-axis in the vertical direction. The pin support was simulated by fixing the nodes at the support against movement in the radial and tangential directions. Roller conditions were simulated by fixing the nodes against radial movement with free movement in the tangential direction.

2.1 Accuracy of the finite element modeling

It is important to establish the accuracy of the finite element modeling before undertaking the analysis of the horizontally curved composite plate girders. Neither experimental nor analytical results are

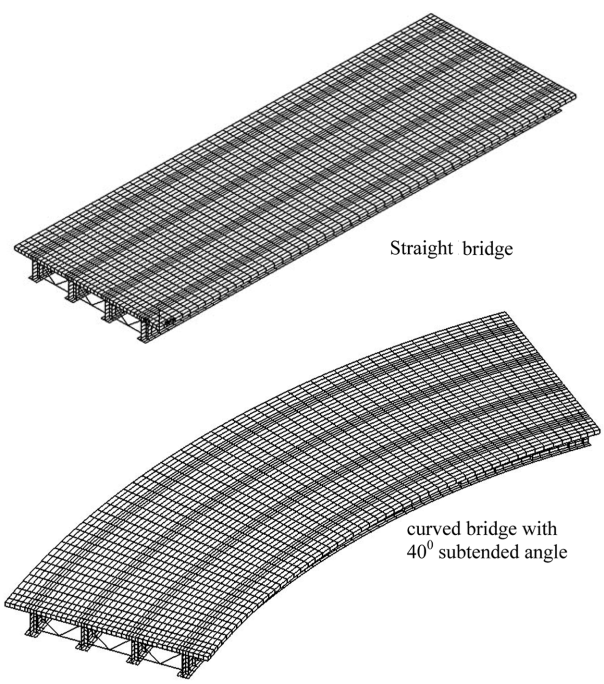
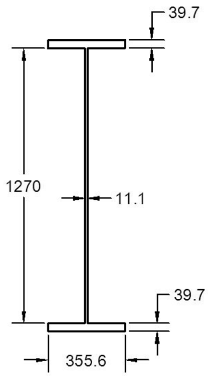


Fig. 1 Typical finite element mesh for composite girder bridges

available in the published literature for horizontally curved composite plate girders. However, analytical results on horizontally curved steel girders have been published by Yoon *et al*. (2005) who has developed a numerical method to predict the vibration of horizontally curved twin steel I-girder bridges. Mello *et al*. (2008) has presented results from dynamic analysis of composite systems made of concrete slabs and steel beams. These results have been used in the present study to assess the accuracy of the finite element modeling. Twin I-girders analyzed by Yoon *et al*. (2005) was modeled by idealizing the flange, web and crossbeams by appropriate elements in the LUSAS element library. Section and geometrical properties, as shown in Fig. 2, and material properties were assumed as per Yoon *et al*. (2005); analyses were carried out to obtain the natural frequencies of the twin girders. A number of twin girders with subtended angle at the centre of curvature varying from 10^o to 90^o were considered in the analyses, and the lowest frequencies presented in Fig. 3 in which the results from the present analyses using LUSAS are given as continuous curves along with those by Yoon *et al*. (2005) as points. Results are given for different modes such as bending, longitudinal, arching and torsion. Natural frequency values corresponding to subtended angle equal to 0^o in Fig. 3 refer to those for straight girder bridge systems. It can be seen from the figure that the two sets of results for all the girders and all the modes are very close, maximum deviation being around 10% in a few cases.

Similarly, a rectangular composite slab supported by four steel columns (Mello *et al*. 2008) was modeled by shell and solid elements in the LUSAS library and, analyzed to obtain the natural frequency for the slab. The results obtained for different span lengths are given for comparison in Table 1 along with the corresponding values by Mello *et al*. (2008). It can be seen from the table that there is close agreement between the two sets of results maximum deviation being 5% thus confirming the ability of LUSAS to predict the vibration behaviour of composite slabs with sufficient accuracy. Based on the comparison for steel beams and composite floor it is concluded that the software package LUSAS



Area (A)	4.278×10^2 cm^2
Moment Inertia about x-axis (I_x)	1.419×10^6 cm^4
Moment Inertia about y-axis (I_y)	2.976×10^4 cm^4
Torsional Constant (K_T)	1.542×10^3 cm^4
Warping Constant (I_ω)	1.275×10^8 cm^6

(all dimensions in mm)

Fig. 2 Cross-section of the girder used by Yoon *et al.*(2005)

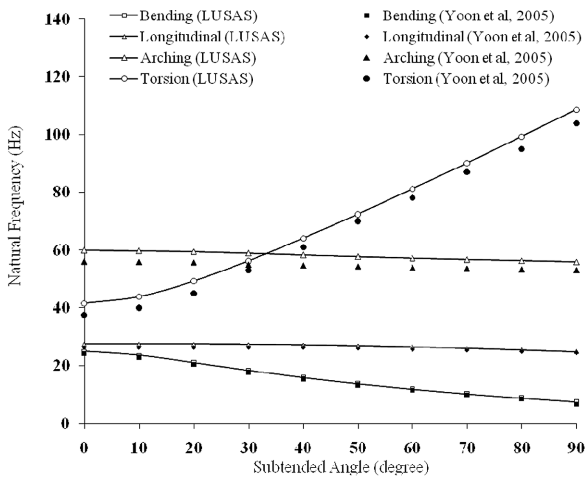


Fig. 3 Results for horizontally curved twin girders

could predict the behavior of composite plate girder bridges accurately. Further analyses for free vibration response of the girders are carried out in the following sections.

3. Analysis of horizontally curved composite plate girder bridges

Having established the accuracy, the finite element package LUSAS was used to carry out further studies on the free vibration behavior of horizontally curved composite plate girder bridges. Parameters such as number of girders, number of cross frames, subtended angle at the centre of curvature and presence of concrete deck that affect the behavior of the system were considered in the analyses. Results obtained from the analyses are presented in the form of plots showing the variation of natural frequency with subtended angle at the centre of curvature, number of girders and number of cross frames. In addition, variations in different types of modes are also included in the results for different configuration of girder systems in order to get an insight into the behavior of these systems.

Table 1 Comparison of natural frequency for composite floor

Span (L), m	f(Mello *et al.* 2008) Hz	f(LUSAS) Hz	$\frac{f(\text{Mello } et\ al.\ 2008)}{f(LUSAS)}$
5.0	9.35	9.39	0.99
5.5	8.82	8.87	0.99
6.0	8.33	8.40	0.99
6.5	7.86	7.95	0.99
7.0	7.42	7.53	0.99
7.5	7.00	7.13	0.98
8.0	6.60	6.75	0.98
8.5	6.21	6.39	0.97
9.0	5.84	6.05	0.97
9.5	5.49	5.72	0.96
10.0	5.15	5.42	0.95

3.1 Description of the girders

A typical plate girder cross-section as shown in Fig. 4 was chosen for the analyses. The cross-section represents girders used in practice for construction of bridges. The overall girder depth is 1,230 mm. The top and bottom flanges are of 550 mm wide and 30 mm thick with plate slenderness ratio of the outstand equal to 17.9. The web thickness is 13 mm. Transverse stiffeners of 25 mm thick are placed at support and at load points, and intermediate stiffeners of 16 mm thick at other locations with nominal width of outstand equal to 140 mm for all stiffeners. In the case of composite girders, 200 mm thick concrete slab was assumed to act compositely with the steel part of the girder.

The girders are horizontally curved with varying radius of curvature and arc length of 27 m at the centre of the bridge system. Different types of configurations viz. two-girder system, three-girder system and four-girder system have been considered in the analyses. In each case the main girders were connected by cross frames located at 6.75 m interval along the arc length of the curved bridge. Thus there are five cross frames in all for the girder systems. However, the number of cross frames has been varied in the study in which the effect of the number of cross frames is studied. The cross section of the girder systems for different arrangements of the girders are illustrated in Fig. 5(a) along with the plan

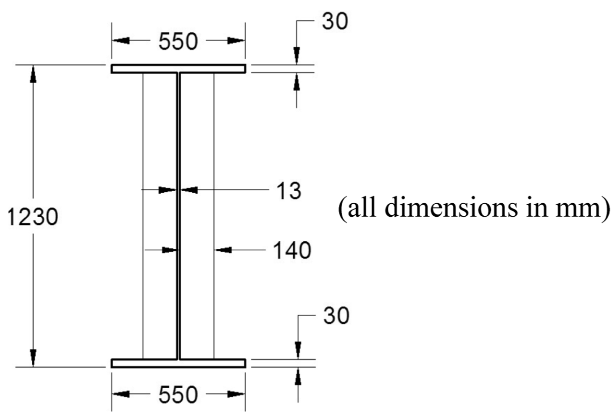
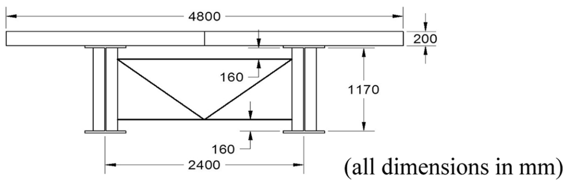
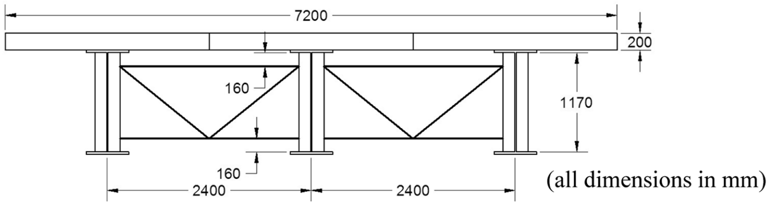


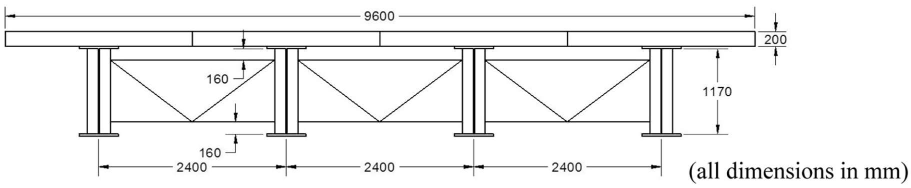
Fig. 4 Cross-section of the girders



Cross-section of the two-girder system



Cross-section of the three-girder system



Cross-section of the four-girder system

Fig. 5(a) Cross-section of different girder configurations

view of the various systems in Fig. 5(b). Front elevation of half of a typical girder is shown in Fig. 6.

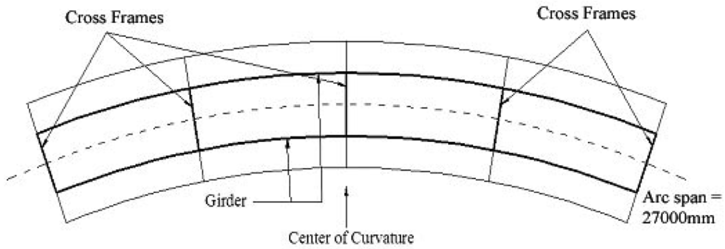
Girder systems ranging from straight to curved in plan with different subtended angle at the centre of curvature were considered in the study. The subtended angle was varied from 10^o to 90^o at the centre of curvature. The deck slab in the composite systems was not provided with super-elevation and, the lines of supports at the ends of the girder systems were orientated in the radial direction. In the analyses, Modulus of elasticity for concrete and steel was assumed as 31,000 MPa and 209,000 MPa, respectively and, the corresponding values for Poisson's ratio were 0.2 and 0.3, respectively. Unit weight for concrete and steel were assumed as 2,400 Kg/m^3 and 7,800 Kg/m^3, respectively. Results obtained from the analyses are presented in the following sections for girder systems with different number of cross frames, systems consisting of steel girders only and those consisting composite girders.

4. Results and discussion

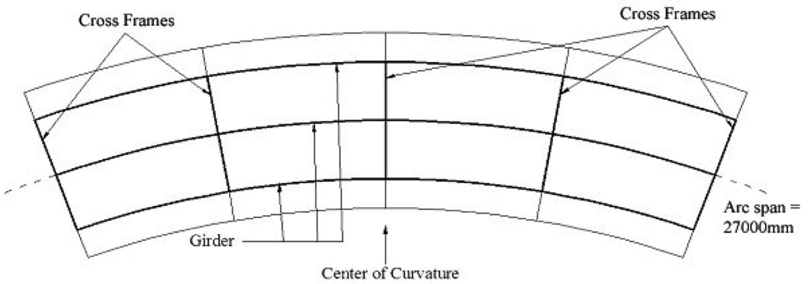
4.1 Number of cross frames

Number of cross frames in the girder systems affects the vibration behavior of the system. As per AASHTO interim guidelines (2005) the maximum distance of the cross-frames are dependent on the parameters such as curvature of the bridge (minimum girder radius within the panel), effective radius of gyration, Young's modulus and yield stress of steel.

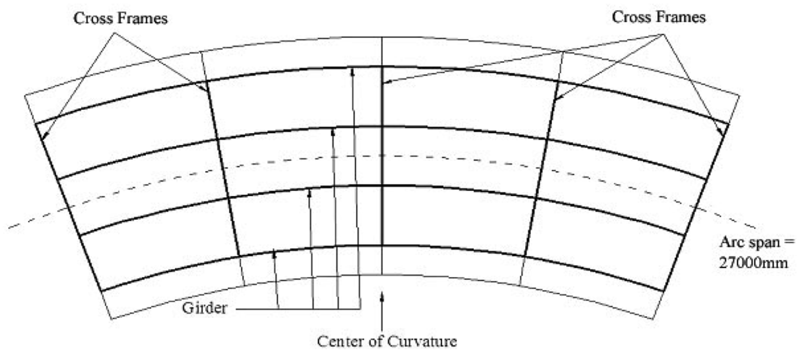
One straight and two horizontally curved simply supported 3- girder systems shown in Fig. 5 were



Plan view of the two-girder system



Plan view of the three-girder system



Plan view of the four-girder system

Fig. 5(b) Plan view of different girder configurations

considered in the study in order to examine the influence of cross frames. Subtended angles equal to 30^o and 60^o at the centre of curvature were considered in these analyses. Number of intermediate cross frames was varied from 0 to 7 for the analyses. Intermediate stiffeners were not explicitly modeled for simplification. Two sets of results were obtained, one set corresponding to steel girders acting alone and the other set for steel girders acting compositely with 200 mm thick concrete deck slab. The results obtained for natural frequency of the first bending mode for the girders are summarized in Fig. 7.

It can be seen from the figure that increase in the number of cross-frames enhances the fundamental natural frequency of the curved bridge. With less number of cross frames or no cross-frames, the fundamental natural frequency is relatively lower compared to the system with adequate number of cross-frames. With the increase in number of cross-frames from 0 to 3 there seems to be a pronounced increase in natural frequency and, it remains constant beyond 3 cross-frames. This is true for all the

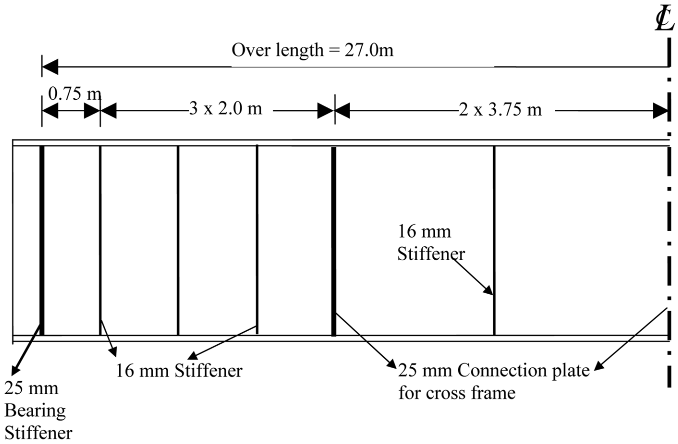


Fig. 6 Front elevation of a typical girder

bridges, irrespective of steel or composite, straight or curved. In the case of straight bridges, however, the number of cross-frames does not seem to influence the natural frequency much. In the case of straight bridges with 3 cross-frames and beyond natural frequency drops significantly (30%) for composite bridge compared to steel bridge. This drop becomes less significant in the case of curved bridges for example around 13% for the curved bridge with subtended angle 30^o and is smaller for the bridge having subtended angle equal to 60^o. As per AASHTO interim recommendations (2005) minimum number of cross-frames required are 3 for straight girder bridges, 5 for bridges with 30^o subtended angle at the centre of curvature, 11 and 17 for bridges with subtended angle 60^o and 90^o, respectively. It can also be seen in the figure that increase in mass due to addition of cross frames results in slight drop in natural frequency in the case of straight girders, steel or composite. The fundamental frequency of the curved girder is naturally lower compared to the straight girder since the curved girders are subjected to both bending and torsional mode. Also, in the case of the curved systems the outer most girders are longer

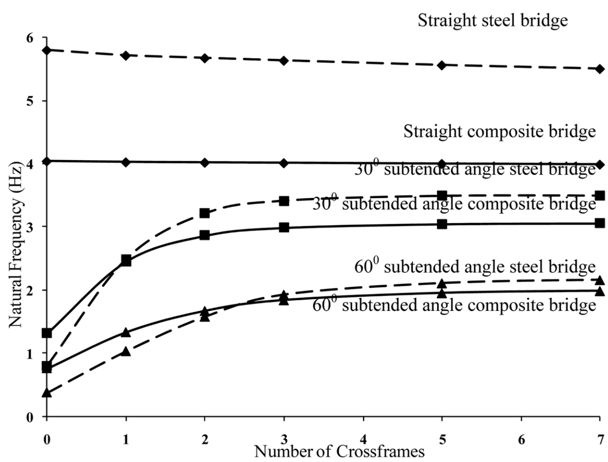


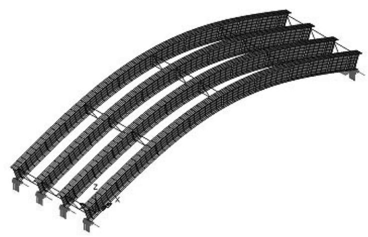
Fig. 7 Variation of natural frequency with number of intermediate cross-frames

and hence influence the torsional behavior significantly. However, with more cross frames in place the unsupported length becomes smaller resulting in better performance in respect of torsional behavior.

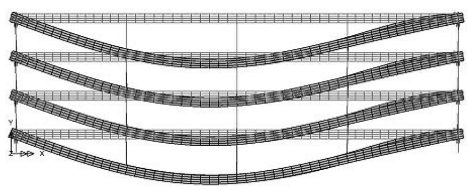
4.2 Mode shapes for steel and composite plate girder bridges

In general, a horizontally curved plate girder bridge has 5 kinds of fundamental vibration modes viz. bending modes, torsional modes, longitudinal modes, arching modes and flapping modes. For convenience, bending, torsional and flapping modes are classified as out-of-plane vibration mode while the longitudinal and arching modes identified as in-plane vibration modes. Mode shapes were obtained for all the girders considered in the analyses. However, these mode shapes are presented only for typical girders viz. straight steel and composite girders and, curved steel and composite girders. These girders contain three intermediate cross frames. Figs.8 and 9 show the mode shapes for steel girder bridges whilst the corresponding mode shapes for composite girder bridges are given in Figs. 10 and 11. In all the figures the five fundamental modes viz. bending mode, longitudinal mode, arching mode, torsion mode and flapping mode are presented.

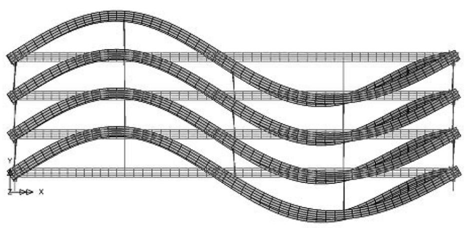
It can be seen from the figures that when a bridge vibrates under bending modes, the entire bridge structure will vibrate in the vertical plane. The bending mode for horizontally curved bridges is always coupled with the torsional behavior. For torsional vibration modes, the whole bridge structure twists



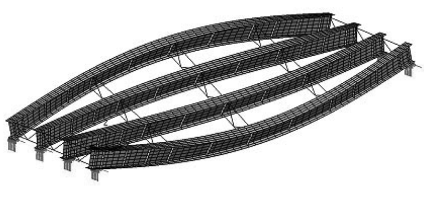
(a) Bending mode



(b) Longitudinal mode



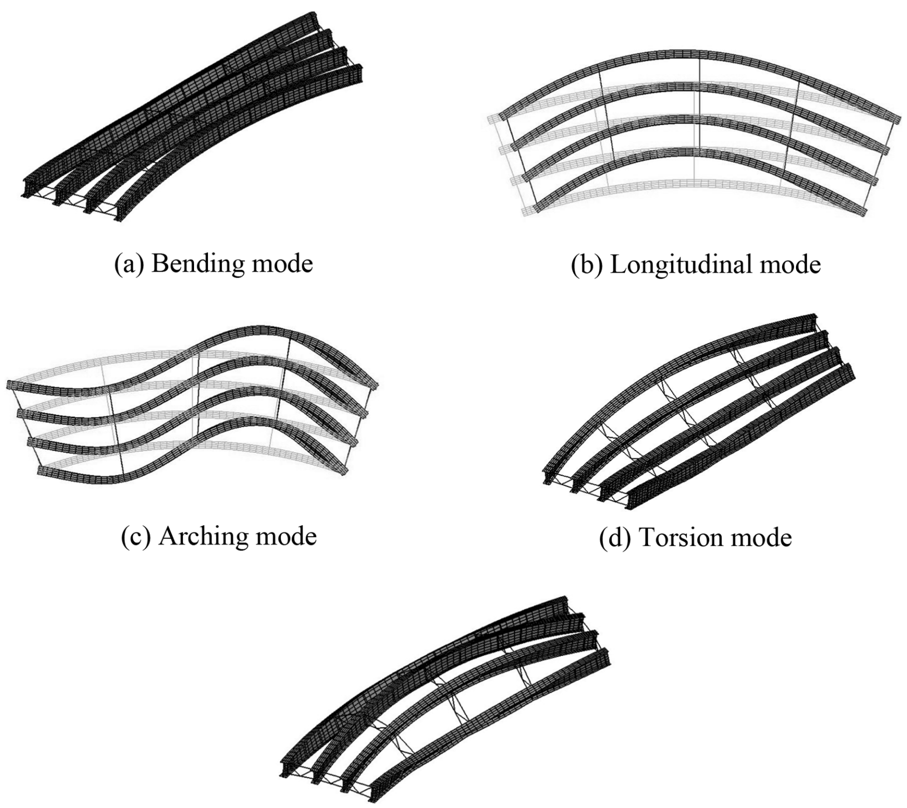
(c) Arching mode



(d) Torsion mode

(e) Flapping mode (3 girders and above only)

Fig. 8 Vibration mode Shapes for straight steel girder bridge systems



(a) Bending mode

(b) Longitudinal mode

(c) Arching mode

(d) Torsion mode

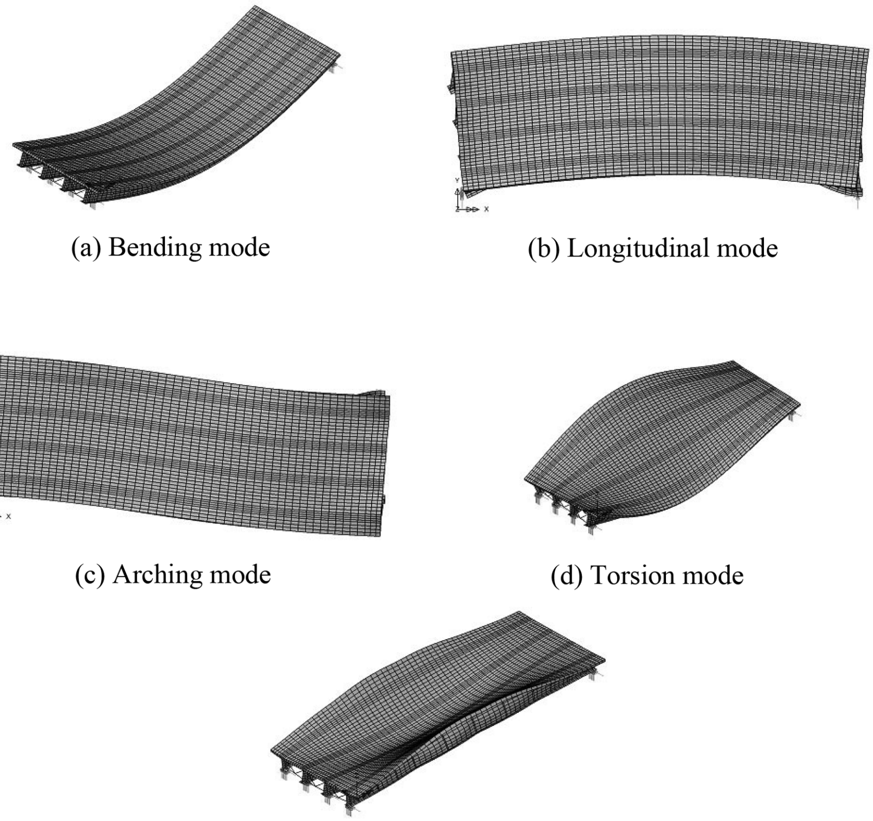
(e) Flapping mode (3 girders and above only)

Fig. 9 Vibration mode Shapes for curved steel girder bridge systems

with the outermost girder vibrating in a direction opposite to the innermost girder. For longitudinal modes, the girder vibrates along the orientation of the bearing at the support showing lateral bending behavior in bridges with low subtended angle. The arching mode shows that the bridge structure vibrates at the second longitudinal mode. For a bridge system with 3 girders and above, there is another distinctive vibration mode called flapping mode. In this mode, the edge girders vibrate vertically in a direction opposite to the intermediate girders. In bending mode, the amplitude of longer girders is larger than that for shorter ones because the stiffness of the longer girders is lower compared to the shorter girders. Effect of lateral-torsional buckling at the un-braced length of the girder becomes more significant with the increase of subtended angle for the torsion and flapping mode. For different vibration modes with very close natural frequency, the vibration modes will inherit some vibration characteristic from each other and form a coupled vibration mode.

4.3 Vibration behaviour of steel and composite plate girder bridge systems

Analyses carried out on steel and composite straight or curved girder bridges with 2, 3 or 4 girder systems and varying number of intermediate cross-frames and degree of curvature measured in terms of subtended angle at the centre of curvature have yielded detailed information on vibration behaviour of the bridge systems. However, only salient results are presented herein for discussion. In all these



(a) Bending mode (b) Longitudinal mode

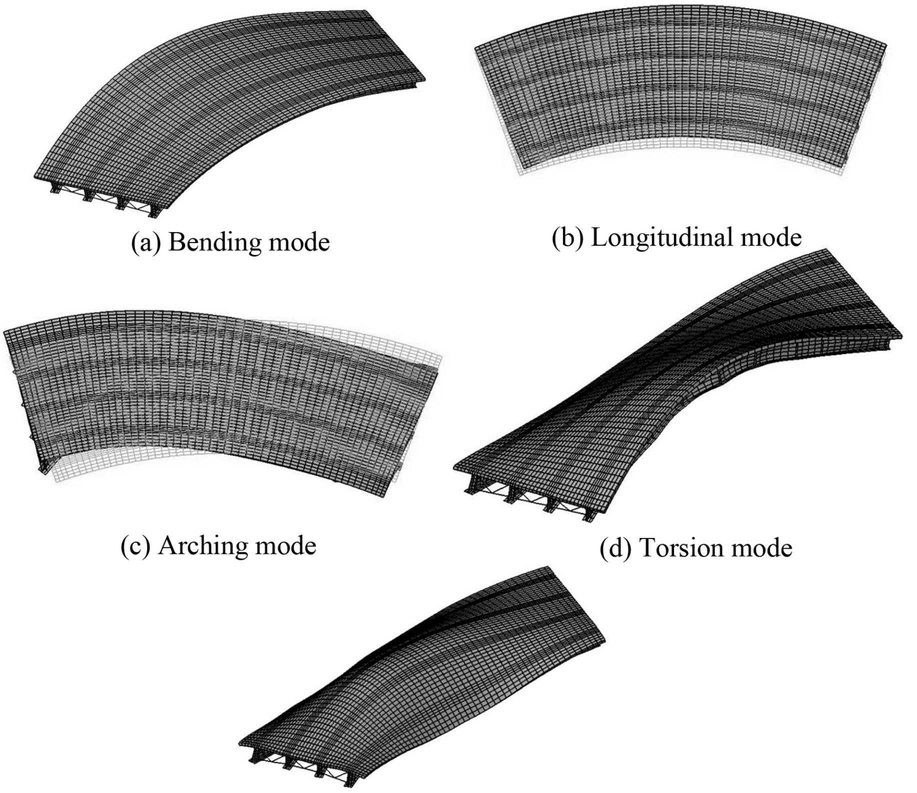
(c) Arching mode (d) Torsion mode

(e) Flapping mode (3 girders and above only)

Fig. 10 Vibration mode shapes for straight composite bridge systems

figures, results corresponding to straight bridge systems are shown as having subtended angle equal to zero. Fig. 12 shows the variation of natural frequency with subtended angle for the first bending mode. Results are plotted for steel and composite girder bridges consisting of 2, 3 or 4 girders connected by three intermediate cross-frames in each case. It is clear from the figure that natural frequency decreases with increase in subtended angle in all cases. Steel girders display higher natural frequency than the corresponding composite bridges up to subtended angle of 30^{o} at the centre of curvature. For subtended angles larger than 30^{0} there is no significant difference between the natural frequencies for steel and composite girders. Bridge systems with 3 or 4 girders show very close natural frequency, larger than that for bridge with two girder systems. This is true irrespective of whether steel or composite girders. More number of girders results in increase of natural frequency but slightly at reduced rate. This is due to the fact that the shorter (inner) girder with the same dimensions as the outer girder tends to stabilize the bridge with the cross-frames and deck slab thus resulting in enhanced natural frequency for the first bending mode of the bridge.

In the case of the bridges with large subtended angles, length of the outermost girder is large and hence reduced stiffness while the mass of the bridge remains constant. The outermost girder tends to vibrate with larger amplitude compared to the inner ones. Therefore, fundamental natural frequency drops significantly as the subtended angle increases. The first bending mode of the steel girder bridges



(a) Bending mode (b) Longitudinal mode

(c) Arching mode (d) Torsion mode

(e) Flapping mode (3 girders and above only)

Fig. 11 Vibration mode shapes for curved composite bridge systems

shows an initial natural frequency of 5.6 Hz compared to the composite plate girder bridges for which the corresponding value is 4Hz. Mass of the steel girder bridges is less than that of the composite plate girder bridges because of additional concrete deck. The increase in mass of the composite plate girder bridges cannot be offset by the increase in vertical stiffness, therefore a lower natural frequency is observed for the composite plate girder bridge system. This phenomenon is only observed at low subtended angle. However, for girders with higher subtended angle the additional mass from the deck slab does not affect much the first bending mode of the composite bridge. It can be observed that the natural frequency of the first bending mode at high subtended angle is almost the same for both non-composite and composite bridges.

Fig. 13 shows the variation of natural frequency with subtended angle for first torsion mode. Results are plotted for steel and composite girder bridge systems consisting of 2, 3 or 4 girders connected by three intermediate cross-frames in each case. It can be seen from the figure that the natural frequency for the first torsion mode increases with subtended angle in the case of steel girders and, the rate of increase, however, drops for the 3-girder and 4-girder systems compared to the 2-girder system. The composite plate girder bridges show lower natural frequency compared to the steel girder bridges. At larger subtended angle the curvature stiffens the girder against torsion deformations. With bigger curvature, however, natural frequency decreases with the increase of the total width of the bridge *i.e.* larger number of girders as shown in the figure for 4-girder system. It can also be observed from the figure

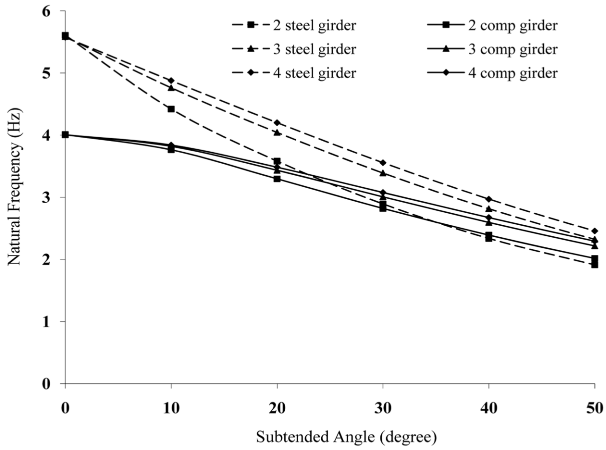


Fig. 12 Variation of natural frequency with subtended angle for first bending mode

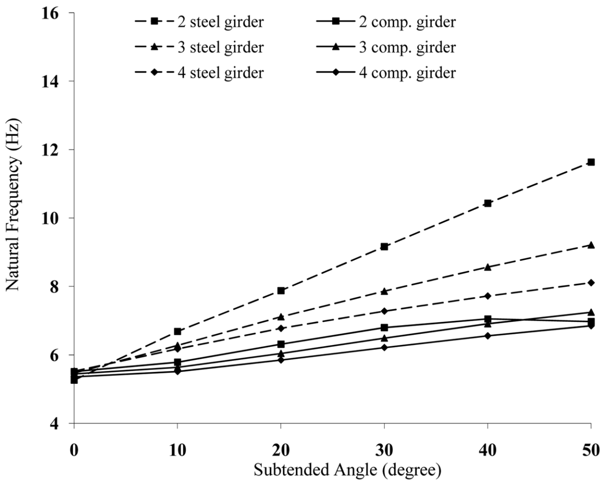


Fig. 13 Variation of natural frequency with subtended angle for first torsion mode

that steel girders display distinct behavior compared to the composite girders. 2-girder steel bridge systems show higher frequency compared to the 3-girder and 4-girder steel systems and, the difference narrows down between 3-girder system and 4-girder system. Natural frequency of the composite girders, however, does not differ much irrespective of the number of girders.

Results for longitudinal mode are shown in Fig. 14 in which natural frequency is plotted against the subtended angle. It can be seen from the figure that the behavior in longitudinal mode of steel girder bridge systems is drastically different from that of the composite plate girder bridge systems. The concrete deck acting compositely with the steel girders increases the moment of inertia and hence improves the longitudinal as well as transverse stiffness of the system. Therefore, natural frequency tends to increase for the composite bridge systems compared to the steel girder bridge systems. The number of girders does not seem to affect the natural frequency of the steel girder bridge systems in

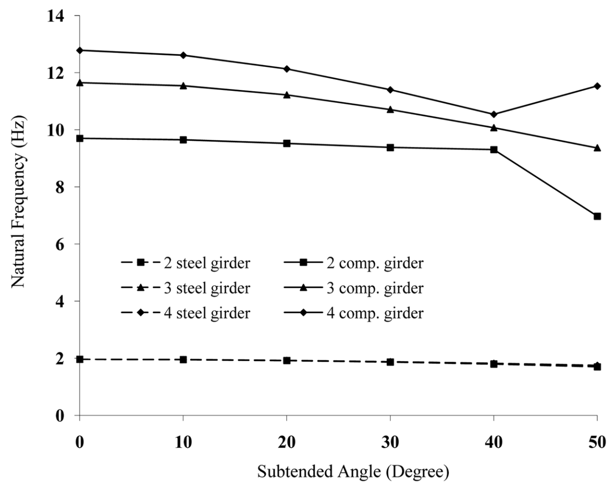


Fig. 14 Variation of natural frequency with subtended angle for longitudinal mode

which the natural frequency varies from 1.6 Hz to 2.0 Hz. Irrespective of the number of girders all three steel girder systems display almost same frequency and, the frequency does not seem to be influenced significantly by the subtended angle. For composite plate girder bridges, however, the natural frequency increases with the increase in total number of girders. For each of the three composite girder systems the natural frequency decreases with subtended angle. At subtended angle between 40^0 and 50^o, the 2-girder and 4-girder systems show some discontinuity in the behavior. In the case of the composite girders, the concrete deck increases the lateral stiffness resulting in larger natural frequency.

Variation of natural frequency with subtended angle is presented in Fig. 15 for arching mode of vibration. It can be seen in the figure that the number of girders does not affect the natural frequency in the case of steel girders. This phenomenon is the same as observed in longitudinal mode. However, a slight drop in natural frequency is observed with the increase of subtended angle. For composite plate girder bridges, natural frequency seems to decrease gradually with the increasing subtended angle.

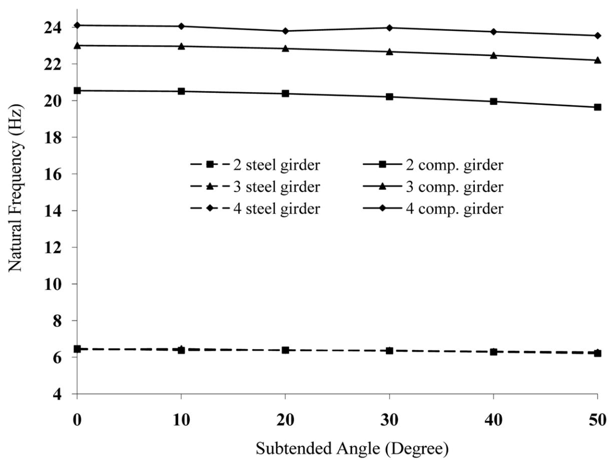


Fig. 15 Variation of natural frequency with subtended angle for arching mode

Also, bridge system with 4 girders display larger frequency compared to the system with less number of girders. For example, in the case of straight bridges the frequency for 4-girder system is around 24.1 Hz whereas the corresponding values for 2-girder and 3-girder systems are 20.6 Hz and 23.0 Hz, respectively. For curved girder with 90^o subtended angles at the centre of curvature the frequency for 2, 3 and 4-girder systems are, respectively 18.2 Hz, 22.0 Hz and 22.1 Hz. Similar to the longitudinal mode, the concrete deck plays an important role on the natural frequency of the arching mode which is dependent upon the lateral stiffness of the bridge consisting of the concrete deck and cross frames. Addition of concrete deck seems to influence substantially the arching mode as shown by the difference in natural frequency corresponding to steel girder systems and composite girder systems. For straight girders, these values are 6.5 Hz and 20.6 Hz for steel girder systems and 2-girder composite systems, respectively, more than 3 times larger for composite girder system.

The results for flapping mode are presented in Fig. 16 in which variation of natural frequency is plotted against subtended angles at the centre of curvature. Flapping mode in which the edge girders vibrate vertically in a direction opposite to the intermediate girders is common in bridge systems with 3 girders and more. Results are presented for 3-girder and 4-girder systems. It can be seen from the figure that the increase in number of girders results in reduced natural frequency. Composite girder, 3-girder or 4-girder systems display smaller frequency compared to steel girder systems. For straight bridges the natural frequency does not differ much between steel and composite girder bridges for 3-girder system. However, in the case of 4-girder system the natural frequency for steel girder is around 24.3 Hz whereas the corresponding value for composite bridge is 22.2 Hz., a drop of 9%. This is not the case for curved girders, particularly those with larger subtended angles.

Steel and composite girder bridge systems do display significant difference in natural frequency irrespective of whether it is 3-girder or 4-girder systems. In order to determine the extent of difference in natural frequency between different modes the three girder system is chosen, for example, and natural frequency for 1st bending mode, 1st torsion mode, longitudinal mode, arching mode and 1st flapping mode are plotted against subtended angle at the centre of curvature as shown in Fig. 17, for steel girder bridge. The corresponding plots for composite girder bridge systems are presented in Fig. 18.

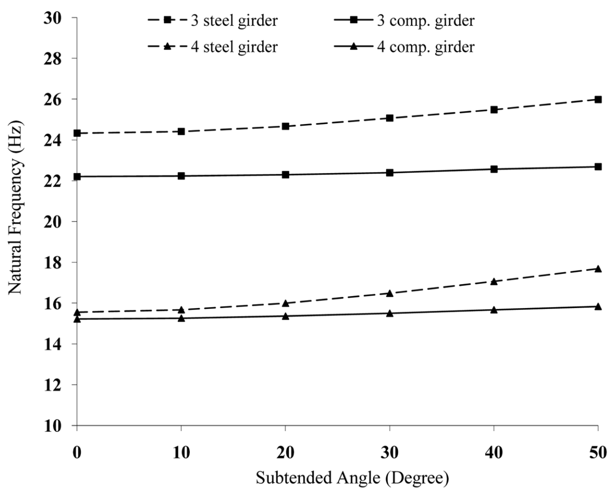


Fig. 16 Variation of natural frequency with subtended angle for flapping mode

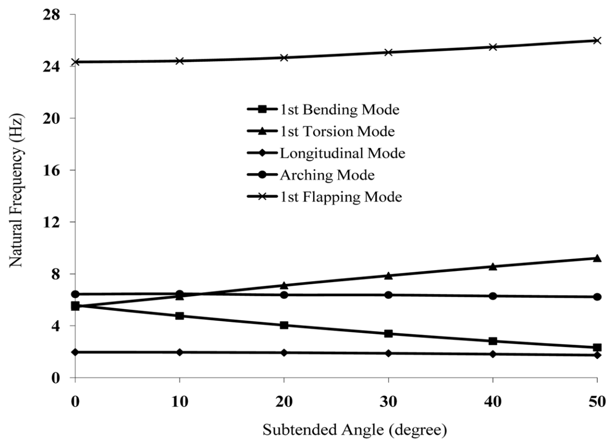


Fig. 17 Variation of natural frequency for 3- steel girder bridge system

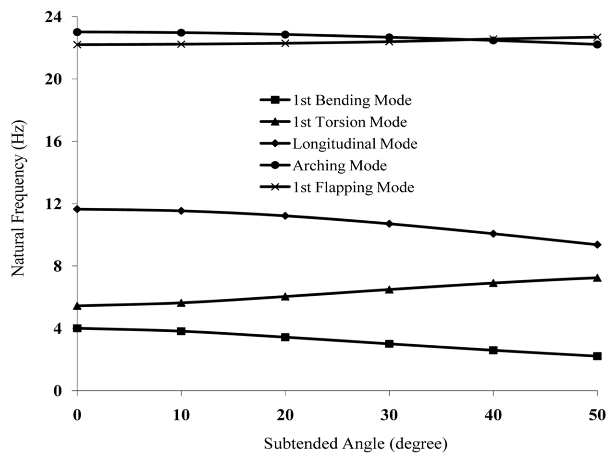


Fig. 18 Variation of natural frequency for 3- composite girder bridge systems

It can be seen from Fig. 17 that natural frequency corresponding to the flapping mode of the steel-girder bridge system is considerably larger than that for all other modes. For straight bridge, flapping mode natural frequency of steel bridge system is around 24.3 Hz whereas the corresponding value for all other modes is less than 7 Hz. In the case of composite straight bridge systems as shown in Fig. 18, both flapping mode frequency and arching mode frequency lie very close to each other, around slightly more than 23 Hz. 1st bending mode frequency tend to decrease with increasing subtended angle in both steel and composite bridge systems. In the case of the steel girder bridge system 1st torsion mode frequency which is smaller than that of arching mode for straight girders tends to be larger than the arching mode for subtended angle more than 10^o. In the case of steel girder bridge systems natural frequency for 1st bending mode and longitudinal mode converge to be almost same for subtended angles beyond 60^0. The fundamental vibration mode for steel girder bridge systems is the longitudinal mode but it is replaced by the bending mode at high subtended angles for all girders considered in this study. For the case of composite bridge systems, the fundamental vibration mode is the bending mode irrespective of the subtended angle and number of girders. The fundamental vibration modes for both steel girder bridge system and composite girder bridge system are different. The longitudinal mode is the fundamental vibration mode for the steel girder bridge system, while the bending mode is the fundamental vibration mode for the composite girder bridge system. This is because the transverse stiffness for the steel girder bridge system is less compared to the flexural stiffness. The concrete deck leads to a significant increase in structure mass and a relatively small increase in structural flexural stiffness and hence the drop in out-of-plane natural frequencies. The in-plane modes for the steel girder bridge system are lower compared to the composite plate girder bridge system. The concrete deck in the case of composite plate girder bridge systems increases the moment of inertia in the transverse direction and hence the corresponding stiffness resulting in larger natural frequency of the in-plane mode.

5. Conclusions

In this paper, a three dimensional finite element model using the computer package LUSAS is developed for free vibration studies of horizontally curved composite plate girder bridge systems. The model is used to carry out analyses to obtain natural frequency and modes of steel and steel-concrete composite bridge girder systems. Parameters such as number of girders, number of cross frames, steel or steel-concrete composite, subtended angle at the centre of curvature have been considered in the study. Through series of numerical modeling, the following characteristics of the horizontally curved girder bridges are observed:

· In the presence of concrete deck, the in-plane vibration modes increase significantly and, a more pronounced increase is observed with the increase of number of girders.
· The number of girders has little or no effect on the lateral vibration modes for steel girder bridges.
· Natural frequency for the bridge systems tends to decrease as the curvature becomes larger except for the torsional and flapping modes which show an increase in natural frequency.
· The presence of concrete deck results in decrease of all the natural frequency corresponding to the out-of-plane modes (bending, torsion and flapping mode) and increases of the natural frequency in respect of in-plane modes (longitudinal and arching mode). The presence of concrete deck and the number of plate girder enhances the natural frequency for the in-plane modes (longitudinal and arching mode).
· The first torsional mode increases with the increase of subtended angle. In the presence of concrete deck the 2-girder and 3-girder systems display a drastic increase in natural frequency.
· The flapping mode exists for 3 or more girders, corresponding natural frequency decreases with the increase of number of girders.

The results presented show that the concrete deck leads to a significant increase in structure mass and a relatively small increase in flexural stiffness relative to the mass increase - hence the decrease in out-of-plane natural frequencies. The information on natural frequencies of various possible modes is useful to the designers while checking for vibration in the bridge systems designed.

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