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Added resistance and parametric roll prediction as a design criteria for energy efficient ships

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Abstract. The increased interest in the design of energy efficient ships post IMO regulation on enforcing EEDI has encouraged researchers to reevaluate the numerical methods in predicting important hull design parameters. The prediction of added resistance and stability of ships in the rough sea environment dictates selection of ship hulls. A 3D panel method based on Green function is developed for vessel motion prediction. The effects of parametric instability are also investigated using the Volterra series approach to model the hydrostatic variation due to ship motions. The added resistance is calculated using the near field pressure integration method.

Keywords: added resistance; 3D panel method; EEDI; parametric roll

1. Introduction

Ships are the most economic and fuel efficient way to transfer cargo in bulk amount across continents. However, the exhaust gases produced by the ships has long term effects on human health and the ecosystem. The International Maritime Organization (IMO) has been working on prevention of air pollution since 1980. The study on CO_2 and other Green House Gas (GHG) emission from ships was presented in October 2000 and potential technical and operational measures for reduction of GHG were identified. In 2005, regulations have been imposed to control emission of NO_x and SO_x gases. In July 2011, IMO made the Energy Efficiency Design Index (EEDI) and Ship Energy Efficiency Management Plan (SEEMP) mandatory for all ships over 400 gross tones (at present excluding ships with steam turbine, diesel-electric and hybrid propulsion) built on or after January 2013. The EEDI is a measure of ship energy efficiency in terms of CO_2 emission per capacity mile (g/t*nm) which must be less than a prescribed value for the specific ship type and size. A reduction factor will be used to implement EEDI in phases to gradually reduce the required limit. To achieve these goals, advancement in the ship hull design, propulsion techniques and alternative energy sources are being investigated.

The propulsive power required for a ship to travel in sea is greatly affected by the added resistance due to waves. Also, the large amplitude roll motion of a vessel increases the fuel

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consumption significantly. Reduction in both added resistance and large amplitude roll requires an optimized hull form design along with additional ship stabilizing techniques such as improved bilge keels and anti-roll tanks. The prediction of a vessels susceptibility towards parametric roll is also of great interest to reduce capsize risk and unsafe vessel operation. The method for estimation of added resistance and parametric roll will be discussed here. The added resistance is calculated assuming the flow to be inviscid and irrotational, the vessel is in deep water and has moderate forward speed ($Fn < 0.3$). At zero forward speed the formulation is exact within linear potential theory where for forward speed case the body is assumed to be slender.

2. Estimation of the added resistance

The added resistance is defined as the increase in resistance in waves compared to the calm water resistance. It is a second order force with respect to the wave amplitude acting in longitudinal direction opposite to the ships forward speed. At zero speed the added resistance is equivalent to the longitudinal drift force (Papanikolaou and Zaraphonitis 1987). There are primarily two ways to calculate the added resistance; either by far-field method introduced by Maruo (1957) or the near-field method introduced by Boese (1970).

The far field methods are based on the diffracted and radiated wave energy and momentum flux at infinity. This method has been later improved by Maruo (1960, 1963) and Joosen (1966). Gerritsma and Beukelman (1972) proposed a similar method based on radiated energy. Salvesen (1978) used this method along with the Salvesen-Tuck-Faltinsen (STF) seakeeping results (Salvesen *et al*. 1970) and found good comparison with the experimental observations. The far-field method has been applied only to the slender strip theory based programs until Iwashita and Ohkusu (1992) used the same in a 3D Green function based panel method and obtained very good results for the added resistance of a fully submerged spheroid. Kashiwagi *et al*. (2010) uses Enhanced Unified Theory (EUT) which is a modified version of Maruo’s method and obtained satisfactory results.

The near-field methods are relatively more intuitive and easier to apply on multi-body problems. The added resistance is found by direct integration of pressure on the submerged hull surface and considering the mean of the second order terms. The method proposed by Boese (1970) was applicable only for head sea and was overly simplified. The method suggested by Faltinsen *et al*. (1980) is so far the most complete in theory, and gives added resistance along with transverse drift forces and yaw moments. They also provide an alternate expression to calculate added resistance and drift forces for short wavelengths. The first 3D panel method implementation of the near-field approach is found in Hsiung and Huang (1995). However, they did not implement the short wavelength case in their calculations.

The work presented here shows evaluation of added resistance using frequency domain 3D panel method based on free-surface Green function. The seakeeping problem including the forward speed effect has been solved using the potential theory presented in Salvesen *et al*. (1970) with the source-sink distribution method as per Hess and Smith(1964) and Garrison(1978). The zero speed results were validated extensively with analytical results for simple shapes and with commercial programs (Guha 2012, Guha and Falzarano 2013). The forward speed results of seakeeping has been validated with results published by ITTC Seakeeping Committee (1978). Then, the method suggested in Faltinsen *et al*. (1980) is used to calculate the added resistance using direct pressure integration over the wetted surface. The short wavelength case is also considered and implemented as per Faltinsen *et al*. (1980).

2.1 Mathematical formulation

The flow around a ship moving with steady forward speed U in a regular wave field with wave propagation angle β can be expressed by a linear velocity potential function $\Phi(x, t)$. The velocity potential can be subdivided into a simple summation of the various components as follows

$$\begin{aligned}\Phi(\vec{x},t) &= \left[-Ux + \phi_S(\vec{x})\right] + \phi_T e^{i\omega_e t} \\ \phi_T &= \left[\phi_I(\vec{x},\beta,\omega_I) + \phi_D(\vec{x},\beta,\omega_I) + \sum_{j=1}^{6}\eta_j\phi_j(\vec{x},\beta,\omega_e)\right]e^{i\omega_e t}\end{aligned} \tag{1}$$

where ω_e denotes the encounter frequency; φ_S represents the perturbation potential due to steady translation; φ_I is the incident wave potential; φ_D is the diffracted wave potential; η_j is the complex motion amplitude and φ_j is the radiation potential due to unit motion in the j^{th} direction. The encounter frequency is expressed in terms of incident wave frequency as

$$\omega_e = \omega_I - \frac{\omega_I^2}{g} U cos\beta \tag{2}$$

The linear incident wave potential is given by

$$\phi_I = \frac{iga}{\omega_I} e^{-ik_I(xcos\beta + ysin\beta)} e^{kz} \tag{3}$$

The radiation and diffraction potentials are obtained by solving a boundary value problem. A panel method is used to distribute sources on the mean submerged hull surface. The 3D zero speed Green function in the frequency domain and sources of unknown strengths are used to define the potential function at zero speed. The Green function satisfies the continuity condition and all other boundary conditions including the free surface and radiation boundary conditions, with the exception of the body boundary condition. The potential at non-zero forward speed are determined from potentials for zero forward speed using the well-known m-terms method given in Salvesen *et al*. (1970). The unknown source strengths are calculated by satisfying the no-penetration body boundary condition which leads to the final evaluation of the first order potentials on the hull surface. This method allows us to analyze slender bodies with moderate forward speed. The accuracy of the result diminishes with bluntness of the body and higher forward speeds. The detailed derivation and benchmark test results are presented in Guha and Falzarano (2014).

The pressure on the body can be found using the Bernoulli's equation. Assuming an inviscid and irrotational flow, the equation for the pressure is

$$P = \frac{1}{2}\rho U^2 - \rho\frac{\partial\Phi}{\partial t} - \frac{1}{2}\rho(\nabla\Phi \times \nabla\Phi) - \rho g z \tag{4}$$

If we neglect the influence of the steady perturbation potential φ_S, we may write

$$P = -\rho gz - \rho \frac{\partial \phi}{\partial t} + \rho U \frac{\partial \phi}{\partial x} - \frac{\rho}{2}\left\{ \left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 \right\} \tag{5}$$

where $\phi = \phi_T e^{i\omega_e t}$ and $\mathrm{z} = \mathrm{z}^{(0)} + \in \mathrm{z}^{(1)} + \in^2 \mathrm{z}^{(2)}$

The hydrostatic and hydrodynamic forces acting on the ship are obtained by integrating the fluid pressure acting over the underwater portion of the hull. The components of the fluid forces acting in each of the six degrees of freedom are thus given by

$$F_{Hj} = -\int_S P n_j ds \qquad j = 1, 2, \ldots, 6 \tag{6}$$

where n_j is the generalized unit normal to the hull surface pointing out of the hull; *P* is the fluid pressure and *S* is the underwater hull surface area. The first order force components are used to formulate the linear equation of motion

$$\sum_{k=1}^{6} [-\omega_e^2 (\mathrm{M}_{\mathrm{jk}} + \mathrm{A}_{\mathrm{jk}}) + i\omega_e B_{jk} + \mathrm{C}_{\mathrm{jk}}]\eta_k = F_j^I + F_j^D \qquad j = 1, 2, \ldots, 6 \tag{7}$$

where M_{jk} is the mass matrix; A_{jk} and B_{jk} are the added mass and damping due to radiated waves; C_{jk} is the hydrostatic stiffness; F_j^I and F_j^D are the wave excitation forces due to incident and diffracted waves respectively. Solving these six coupled equations simultaneously gives the vessel's linear motion response η_j for a given frequency.

2.2 Added resistance

The added resistance is the mean second order wave force on the hull in surge direction. A near field pressure integration method may be used to obtain this force. A perturbation method can be applied to separate various orders of forces from Eq. (6)

2.3 Derivation of forces

Substituting the perturbed entities into Eq. (6), we get the expression for force

$$\mathrm{F} = -(\int_{S_0} ds + \int_{wl} \zeta_r dl)\,(p^{(0)} + \in p^{(1)} + \in^2 p^{(2)})(\mathrm{n}^{(0)} + \in (\theta^{(1)} \times \mathrm{n}^{(0)}) + \in^2 H\mathrm{n}^{(0)}) \tag{8}$$

where ωl is the waterline of the ship, $\theta^{(1)}$ and *H* are first and second order rotation matrices respectively. Separating terms with $\in^0, \in^1$ and $\in^2$ gives the zeroth, first and second order force respectively. The second order force is given by

$$\begin{aligned} \vec{F}^{(2)} = \; & -\int_{S_0} p^{(0)} \left(H\vec{n}^{(0)}\right) ds - \int_{S_0} p^{(1)} (\vec{\theta}^{(1)} \times \vec{n}^{(0)}) ds - \int_{S_0} p^{(2)} \vec{n}^{(0)} ds \\ & -\int_{wl} \zeta_r^{(1)} p^{(0)} (\vec{\theta}^{(1)} \times \vec{n}^{(0)}) dl - \int_{wl} \zeta_r^{(1)} p^{(1)} \vec{n}^{(0)} dl - \int_{wl} \zeta_r^{(2)} p^{(0)} \vec{n}^{(0)} dl \end{aligned} \tag{9}$$

Substituting all the terms and simplifying gives the expression for the second order force as

$$
\begin{aligned}
\vec{F}^{(2)} \;=\; & -\int_{wl} \frac{1}{2}\rho g \left(\zeta_r^{(1)}\right)^2 \frac{1}{\sqrt{1-n_3^2}} \vec{n}^{(0)} dl \\
& + \int_{S_0} \rho \left(\frac{\partial \phi^{(2)}}{\partial t} - U \frac{\partial \phi^{(2)}}{\partial x} \right) \vec{n}^{(0)} ds \\
& + \int_{S_0} \frac{\rho}{2} \left\{ \left(\frac{\partial \phi^{(1)}}{\partial x} \right)^2 + \left(\frac{\partial \phi^{(1)}}{\partial y} \right)^2 + \left(\frac{\partial \phi^{(1)}}{\partial z} \right)^2 \right\} \vec{n}^{(0)} ds \\
& + \int_{S_0} i\omega_e \rho \left\{ (\eta_1 - \eta_6 y_B + \eta_5 z_B) \frac{\partial \phi^{(1)}}{\partial x} \right. \\
& + (\eta_2 + \eta_6 x_B - \eta_4 z_B) \frac{\partial \phi^{(1)}}{\partial y} \\
& \left. + (\eta_3 - \eta_5 x_B + \eta_4 y_B) \frac{\partial \phi^{(1)}}{\partial z} \right\} \vec{n}^{(0)} ds \\
& - \rho g A^{(0)} \left[\eta_4 \eta_6 x_{B,f} + \eta_5 \eta_6 y_{B,f} + \frac{1}{2}(\eta_4^2 + \eta_5^2) Z_0 \right] \hat{k}
\end{aligned}
$$

$$
\begin{aligned}
& -\omega_e^2 \left\{ -\eta_2 \eta_6 m + \eta_4 \eta_6 m z_g - \eta_6 \eta_6 m x_g \right. \\
& \qquad \left. + \eta_3 \eta_5 m + \eta_4 \eta_5 y_g - \eta_5 \eta_5 m x_g \right\} \hat{i} \\
& -\omega_e^2 \left\{ \eta_1 \eta_6 m + \eta_5 \eta_6 m z_g - \eta_6 \eta_6 m y_g \right. \\
& \qquad \left. - \eta_3 \eta_4 m - \eta_4 \eta_4 m y_g + \eta_4 \eta_5 m x_g \right\} \hat{j} \\
& -\omega_e^2 \left\{ -\eta_1 \eta_5 m - \eta_5 \eta_5 m z_g + \eta_5 \eta_6 m y_g \right. \\
& \qquad \left. + \eta_2 \eta_4 m - \eta_4 \eta_4 m z_g + \eta_4 \eta_6 m x_g \right\} \hat{k}
\end{aligned} \tag{10}
$$

We take a time average of the above force over one time period to obtain the mean second order force on the hull. In regular waves, there is no contribution of the second order potential $\varphi^{(2)}$, hence we can obtain the mean force from the linear analysis.

2.3 Results and discussions

The calculation of added resistance requires solving the linear seakeeping problem including the vessel forward speed. A computer program has been developed at the Marine Dynamics Laboratory at Texas A&M University for the prediction of vessel motion and loads using three dimensional panel method based on Green function. The program has been validated successfully for various structures including floating hemisphere, ship and TLP for zero forward speed with WAMIT (Lee 2013). The forward speed case has been further validated against experimental result published by ITTC Seakeeping Committee (1978) for the container vessel S175. The validation results are published in Guha (2012) and Guha and Falzarano (2013).

The ITTC Container ship S175 (Fig. 1) has been chosen for validation of added resistance calculation for various forward speed. The principal particulars of the vessel are given in Table 1. The motion prediction of the vessel is validated against published results (ITTC Seakeeping Committee 1978) and shown in Figs. 2 and 3. The mean drift force is compared with WAMIT results in Fig. 4 and shown to be in good agreement. The added resistance for head sea condition with F_n=0.15 and 0.25 are shown in Figs. 5 and 6. The results obtained by the in-house program is labeled as “Numerical” in the presented plots.

Table 1 Principal particulars of the container ship S175

Principal Particulars of S175			
Length between perpendiculars	Lpp	m	175
Breadth	B	m	25.4
Side height	D	m	15.4
Design draught	T	m	9.5
Displacement	Δ	t	24168.76
Vertical centre of gravity(from baseline)	z_g	m	9.5
Roll Radius of Gyration	kxx	m	8.33
Pitch Radius of Gyration	kyy	m	42
Yaw Radius of Gyration	kzz	m	35.4

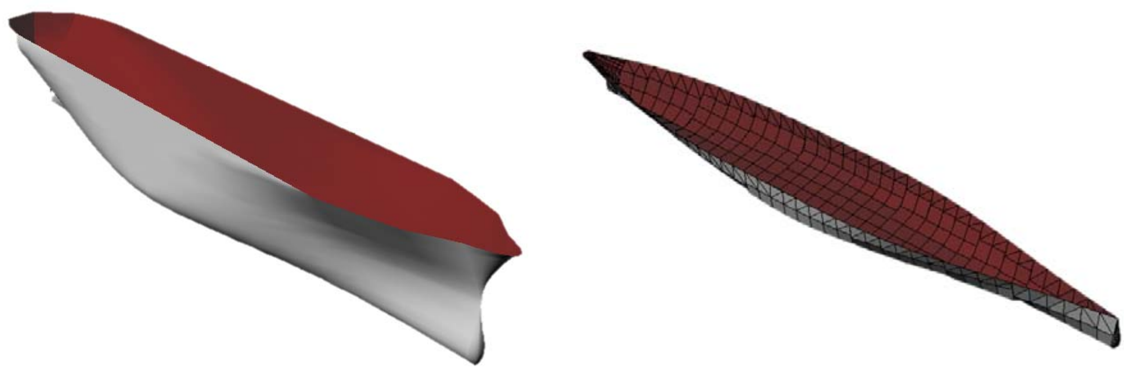

Fig. 1 Hull form (Left) and panel mesh(right) of S175ContainerShip

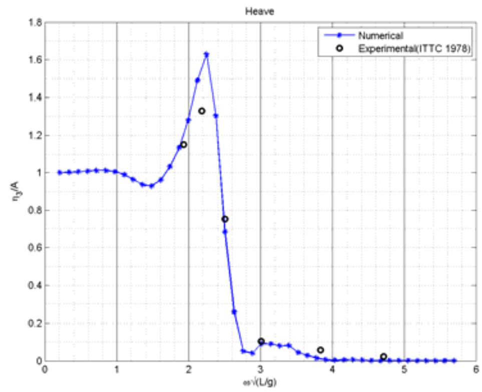

Fig. 2 Comparison of heave response of S175 container ship at F_n=0.275 and β=180 deg

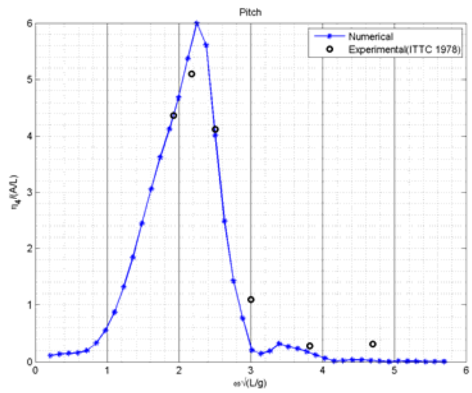

Fig. 3 Comparison of pitch response of S175 container ship at F_n=0.275 and β=180 deg

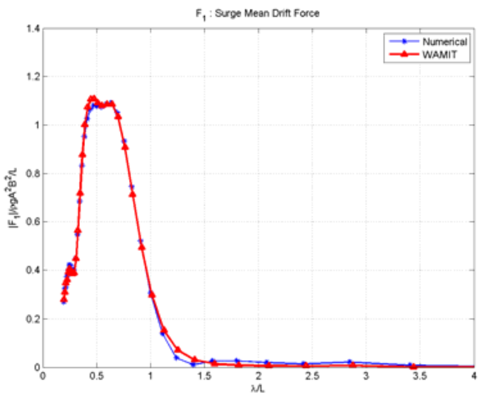

Fig. 4 Surge mean drift force comparison with WAMIT for F_n=0

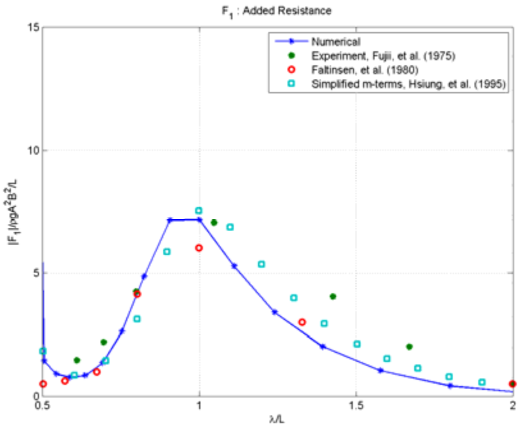

Fig. 5 Added Resistance for S175 at F_n=0.15 for wave heading β=180 deg

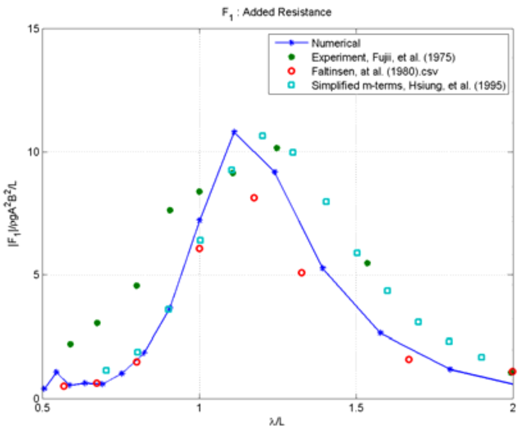

Fig. 6 Added Resistance for S175 at F_n=0.25 for wave heading β=180 deg

3. Parametric roll assessment

The problem of parametric roll of ships is quite old and has been widely investigated by researchers all over the world. For a long time it was believed that parametric roll was a severe phenomenon experienced mostly by fishing vessels in the following sea conditions.

Many researchers tried analyzing this phenomenon. Neves and Rodríguez (2007) used a third order coupled model to analyze the parametric roll of a fishing vessel and compared it with the experimental results. Spyrou (2011) suggested a simple analytical criteria for parametric rolling in the following sea conditions. Other researchers who have studied the parametric roll in following seas include Belenky *et al*. (2008), Gamo *et al*. (2005) and PérezRojas and Abad (2003).

However, later it was realized that the problem was also severe for fine form ships like containerships in head waves. In October1998, the containership APL China encountered extreme weather which resulted in severe loss of cargo. A detailed investigation France *et al*. (2003) of this incident revealed that the primary cause of the catastrophe was the parametric rolling of the ship. After this revelation the research community's interest in parametric rolling of post-Panamax containerships in head seas was aroused.

A lot of researchers have investigated this problem and have come up with different assessments and possible solutions to the problem. Neves and Rodríguez (2007) modeled the problem as a coupled multi degree of freedom problem and has analyzed the effects of coupling between heave and pitch on roll motions. Neves and Rodríguez (2006) also developed a third order coupled model to capture the effect of softening stiffness term and compared against experiments. Paulling (2011) provided a synopsis of the research being performed in the field of parametric roll of ships. He also suggested the use of a few simplified approaches.

Bulian (2005) suggested analytical approximations to evaluate the GM in regular waves and used them to perform simulations in regular waves and compared them against simulations using a GZ look up table to include the quasi-statically calculated stiffness into the roll motion simulations (Bulian 2006). Later Bulian *et al*. (2006) investigated the effect of parametric roll in irregular seas (Bulian *et al*. 2008). Spyrou (2011) (Spyrou *et al*. 2008) has suggested the development of probabilistic framework to come up with analytical criteria for parametric roll.

Moideen (2010) has looked at the development of 3-D stability charts by Spyrou (2011) (Spyrou *et al*. 2008) suggested the development of. including the non-linear damping as a function of roll amplitude in regular waves (Moideen *et al*. 2012). Umeda *et al*. (2012) and Hashimoto and Umeda (2012) performed time domain simulations with calculating the Froude Krylov and diffraction forces up to the instantaneous waterline at each time step and compared them against experimental time series. Belenky *et al*. (2003) performed time domain simulations and checked the ergodicity of these simulations.

However, a unanimous criteria for parametric roll in head seas has not yet been provided by IMO. In this paper we look at two specific methods of analysis for parametric roll in regular and irregular waves.

3.1 Equation of motion

We begin with the single degree of freedom roll equation of motion as shown in Eq. (11).

$$(I + A(\omega))\ddot{\phi} + B(\omega)\dot{\phi} + B_q\dot{\phi}\,|\,\dot{\phi}\,| + C(t,\phi) = 0 \qquad (11)$$

where

I is the mass moment of inertia of the ship

$A(\omega)$ is the frequency dependent added moment of inertia
$B(\omega)$ is the frequency dependent linear damping
$B_q(\omega)$ is the frequency dependent quadratic damping
$C(t,\varphi)$ is the roll restoring moment of the ship
ϕ is the instantaneous roll angle

3.1.1 Roll restoring moment

The restoring term in Eq. (11) can be expressed in terms of the instantaneous roll restoring arm GZ as in Eq. (12)

$$C(t,\phi) = \rho g \nabla_d GZ(t,\phi) \tag{12}$$

Due to the symmetry of the ship about the centerline, the GZ curve may be assumed to be an odd polynomial function of the instantaneous roll angle. The most simplest case is to consider the linear approximation as shown in Eq. (13)

$$GZ(t,\phi) = GM(t)\phi \tag{13}$$

Where GM(t) is the time varying metacentric height. In the case of following or head seas, the underwater hull form and the water plane area change as the wave passes along the ship. This change will result in the variation of GZ with time. The coefficient GM (t) captures the variation of the under water hull form with time. The resulting roll equation of motion is given by Eq. (14).

$$(I + A(\omega))\ddot{\phi} + B(\omega)\dot{\phi} + B_q\dot{\phi}\,|\,\dot{\phi}\,| + \rho g \nabla_d GM(t)\phi = 0 \tag{14}$$

If the excitation does not consist of a single frequency, then the added mass must be replaced by infinite frequency added mass and the radiation damping by a convolution integral of the roll velocity by an impulse response function. However in this paper, based upon experience and in order to simplify, this effect is neglected and the added mass and radiation damping are assumed to have a constant value equal to their value at the natural frequency of the system.

Non-dimensionalizing the roll equation of motion Eq. (14) by the transformations given in Eq. (15) is given by Eq. (16)

$$\begin{aligned}
&\omega_0 = \sqrt{\frac{\rho g \nabla_d GM_0}{I + A(\omega_0)}} &\qquad& \tau = \omega_0 t \\
&\frac{d}{dt}() = \omega_0 \frac{d}{d\tau}() && \bar{\phi} = \frac{\phi}{\phi_v} \\
&\zeta = \frac{B(\omega_0)}{2(I + A(\omega_0))\omega_0} && \zeta\delta_q = \frac{B_q\phi_v}{I + A(\omega_0)} \\
&GM(t) = GM_0 + \partial GM(t) && \zeta\gamma(t) = \frac{\partial GM(t)}{GM_0}
\end{aligned} \tag{15}$$

where

GM_0 is the calm water metacentric height of the vessel
∂GM(t) is the variation in *GM* from the calm water case due to the presence of waves and body motions
ω_0 is the natural frequency of roll in calm water condition
$\tau=\omega_0 t$ is the rescaled time

ϕ_v is the angle of vanishing stability of the calm water GZ

$\bar{\phi}$ is the roll angle nondimensionalized by ϕ_v

$$\ddot{\bar{\phi}} + 2\zeta\dot{\bar{\phi}} + \zeta\delta_q\dot{\bar{\phi}}\,|\,\dot{\bar{\phi}}\,| + (1+\zeta\gamma(\tau))\bar{\phi} = 0 \tag{16}$$

Dropping the bar, the roll equation of motion can be expressed as Eq. (17)

$$\ddot{\phi} + \phi = \zeta(-2\dot{\phi} - \delta_q\dot{\phi}\,|\,\dot{\phi}\,| - \gamma(\tau)\phi) \tag{17}$$

3.2 Parametric roll in irregular seas

Unlike for the regular wave case, there is no simple analytical solution available for the irregular wave case. Even the analytical methods that do exist have a lot of assumptions which reduce their practical use for analyzing response in severe sea states. Thus the only widespread method of analysis is to perform time domain simulations for Eq. (17).

In order to perform a time domain simulation, it is necessary to correctly estimate the variation of linear stiffness $\gamma(\tau)$ in irregular waves. In this section, the variation in GM with time shall be derived as a Volterra series based on the works of Hua *et al*. (1999) and Moideen (2010) (Moideen *et al*. 2013).

The local breadth B, sectional moment about the keel M and the sectional area A at a section x and draft $T(x)$ are expressed as Taylor series expansion of the sectional draft change z as shown in Eq. (18)

$$\begin{aligned} B(x,T(x)+z) &= B(x,T(x)) + c_1(x)z + \frac{1}{2!}c_2(x)z^2 + \ldots \\ M(x,T(x)+z) &= M(x,T(x)) + d_1(x)z + \frac{1}{2!}d_2(x)z^2 + \ldots \\ A(x,T(x)+z) &= A(x,T(x)) + e_1(x)z + \frac{1}{2!}e_2(x)z^2 + \ldots \end{aligned} \tag{18}$$

where

$$\begin{aligned} c_1(x) &= \frac{\partial B}{\partial z}(x,T(x)) & c_2(x) &= \frac{\partial^2 B}{\partial z^2}(x,T(x)) \\ d_1(x) &= \frac{\partial M}{\partial z}(x,T(x)) & d_2(x) &= \frac{\partial^2 M}{\partial z^2}(x,T(x)) \\ e_1(x) &= \frac{\partial A}{\partial z}(x,T(x)) & e_2(x) &= \frac{\partial^2 A}{\partial z^2}(x,T(x)) \\ & & &\text{and so on} \end{aligned} \tag{19}$$

The metacentric height GM for any ship is given as in Eq. (20).

$$GM = BM + KB - KG \tag{20}$$

The metacentric radius BM and vertical center of buoyancy KB at any instantaneous position are given by Eqs. (21) and (22) respectively.

$$BM = \frac{1}{12\nabla_d}\int_L B^3(x, T(x) + r(x))dx \tag{21}$$

$$KB = \frac{1}{\nabla_d}\int_L M(x, T(x) + r(x))dx \tag{22}$$

where

T(x) is the local draft at station *x* of the ship in calm water

r(x) is the relative wave elevation (including the effects of heave and pitch) at station *x* of the ship.

The vertical center of gravity KG at any instantaneous position including the effect of pitch motion is given by Eq. (23). The second term in Eq. (23) takes into account the lowering of the keel point due to the pitching of the vessel.

$$KG = \frac{1}{\nabla_d}\int_L A(x, T(x) + r(x))(KG_0 + x(\eta_5 - \alpha_{trim}))dx \tag{23}$$

where

η_5 is the instantaneous pitch angle

α_{trim} is the static trim of the vessel in calm water

KG_0 is the vertical center of gravity in the calm water condition

Substituting Eqs. (21)-(23) into Eq. (20) leads to Eq. (24)

$$\begin{aligned}\partial GM = \;& \frac{1}{\nabla_d}\int_L \Big(\frac{B^3(x, T(x) + r(x))}{12} + M(x, T(x) + r(x)) \\ & - A(x, T(x) + r(x))(KG_0 + x(\eta_5 - \alpha_{trim}))\Big)dx \\ & - GM_0\end{aligned} \tag{24}$$

Substituting Eq. (18) into Eq. (24) leads to Eq. (25)

$$\partial GM = \sum_i \partial GM_i \tag{25}$$

where

$$\partial GM_1 = \frac{1}{\nabla_d} \int_L [\frac{B^2(x,T(x))c_1(x)}{4} + d_1(x) - KG_0 e_1(x)] r(x) dx$$
$$\partial GM_2 = \frac{1}{\nabla_d} \int_L [\frac{3B^2(x,T(x))c_2(x) + 3B(x,T(x))c_2^2(x)}{12} + d_2(x) \tag{26}$$
$$- KG_0 e_2(x)] r^2(x) dx - \frac{1}{\nabla_d} \int_L x e_1(x) \eta_5 r(x) dx$$

and so on

In other words, the various orders of GM variations can be expressed as Eq. (27).

$$\partial GM(t) = \sum_i [\int_L G_i(x) r^i(x,t) dx + \int_L R_{i-1}(x) r^{i-1}(x,t) dx] \tag{27}$$

where

$$G_1(x) = \frac{1}{\nabla_d}[\frac{B^2(x,T(x))c_1(x)}{4} + d_1(x) - KG_0 e_1(x)]$$
$$G_2(x) = \frac{1}{\nabla_d}[\frac{3B^2(x,T(x))c_2(x) + 3B(x,T(x))c_2^2(x)}{12} + d_2(x)$$
$$- KG_0 e_2(x)] \tag{28}$$
$$G_3(x) = \frac{1}{\nabla_d}[\frac{3B^2(x,T(x))c_3(x) + 6B(x,T(x))c_1(x)c_2(x) + c_1^3(x)}{12}$$
$$+ d_3(x) - KG_0 e_3(x)]$$

$$R_1(\omega, x) = -\frac{x e_1(x) \eta_5(\omega)}{\nabla_d}$$
$$R_2(\omega, x) = -\frac{x e_2(x) \eta_5(\omega)}{\nabla_d} \tag{29}$$

The surface wave elevation can be considered as a Fourier Series shown in Eq. (30)

$$\eta(x,t) = \frac{1}{2} \sum_{n=1}^{N} a_n [e^{i(k_n x - \omega_n t + \beta_n)} + e^{-i(k_n x - \omega_n t + \beta_n)}] \tag{30}$$

Assuming the effect of heave and pitch of the vessel, the relative wave elevation can be expressed as in Eq. (31)

$$r(x,t) = \frac{1}{2}\sum_{n=1}^{N} a_n[e^{i(k_n x)} - \eta_3(\omega_n) + x\eta_5(\omega_n)]e^{i(-\omega_n t+\beta_n)}$$
$$+\frac{1}{2}\sum_{n=1}^{N} a_n[e^{-i(k_n x)} - \overline{\eta_3}(\omega_n) + x\overline{\eta_5}(\omega_n)]e^{-i(-\omega_n t+\beta_n)} \tag{31}$$

where

$\eta_3(\omega)$ is the heave complex RAO
$\eta_5(\omega)$ is the pitch complex RAO
The relative wave elevation can further be expressed as shown in Eq. (32)

$$r(x,t) = \frac{1}{2}\sum_{n=1}^{N} a_n[v(\omega_n,x)e^{-i(\omega_n t-\beta_n)} + \overline{v}(\omega_n,x)e^{i(\omega_n t-\beta_n)}] \tag{32}$$

where

$$v(\omega_n,x) = [e^{i(k_n x)} - \eta_3(\omega_n) + x\eta_5(\omega_n)]$$
$$\overline{v}(\omega_n,x) = [e^{-i(k_n x)} - \overline{\eta_3}(\omega_n) + x\overline{\eta_5}(\omega_n)] \tag{33}$$

Similarly the pitch time series can also be expressed as a Fourier Series as in Eq. (34)

$$\eta_5(t) = \frac{1}{2}\sum_{n=1}^{N} a_n[\eta_5(\omega_n)e^{-i(\omega_n t-\beta_n)} + \overline{\eta_5}(\omega_n)e^{i(\omega_n t-\beta_n)}] \tag{34}$$

Substituting Eqs. (28), (29), (32) and (34) into Eq. (27) gives the first and second order GM transfer functions. The first order transfer function and the corresponding 1st order GM variation is given by Eqs. (35) and (36) respectively.

$$f_1(\omega) = \int_L G_1(x)v(\omega,x)dx$$
$$\overline{f_1}(\omega) = \int_L G_1(x)\overline{v}(\omega,x)dx \tag{35}$$

$$\partial GM_1(t) = \sum_{n=1}^{N} a_n[f_1(\omega_n)e^{-i(\omega_n t-\beta_n)} + \overline{f_1}(\omega_n)e^{i(\omega_n t-\beta_n)}] \tag{36}$$

Similarly, the second order transfer function and second order GM variation are given by Eqs. (37) and (38) respectively.

$$
\begin{aligned}
u_2(\omega_m,\omega_n) &= \int_L [G_2(x)v(\omega_m,x)v(\omega_n,x) + R_2(\omega_m,x)v(\omega_n,x)]dx \\
\bar{u}_2(\omega_m,\omega_n) &= \int_L [G_2(x)v(\omega_m,x)\bar{v}(\omega_n,x) + R_2(\omega_m,x)\bar{v}(\omega_n,x)]dx \\
v_2(\omega_m,\omega_n) &= \int_L [G_2(x)\bar{v}(\omega_m,x)v(\omega_n,x) + \bar{R}_2(\omega_m,x)v(\omega_n,x)]dx \\
\bar{v}_2(\omega_m,\omega_n) &= \int_L [G_2(x)\bar{v}(\omega_m,x)\bar{v}(\omega_n,x) + \bar{R}_2(\omega_m,x)\bar{v}(\omega_n,x)]dx
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\partial GM_2(t) = \sum_{m=1}^{M}\sum_{n=1}^{N} & a_m a_n \{u_2(\omega_m,\omega_n)e^{-i[(\omega_m+\omega_n)t-\beta_m-\beta_n]} \\
& \bar{u}_2(\omega_m,\omega_n)e^{-i[(\omega_m-\omega_n)t-\beta_m+\beta_n]}\} + \\
& \sum_{m=1}^{M}\sum_{n=1}^{N} a_m a_n \{v_2(\omega_m,\omega_n)e^{i[(\omega_m-\omega_n)t-\beta_m+\beta_n]} \\
& \bar{v}_2(\omega_m,\omega_n)e^{i[(\omega_m+\omega_n)t-\beta_m-\beta_n]}\}
\end{aligned} \tag{38}
$$

Thus $\gamma(t)$ in Eq. (37) can be expressed as Eq. (39)

$$
\gamma(t) = \frac{\partial GM}{GM_0} = \frac{\partial GM_1(t) + \partial GM_2(t)}{GM_0} \tag{39}
$$

Where $\partial \mathrm{GM}_1(t)$ and $\partial \mathrm{GM}_2(t)$ are obtained from Eqs. (36) and (38) respectively. Transforming the time scale in $\gamma(t)$ and substituting into Eq. (17), multiple time domain simulations for various sea states can be performed to evaluate the susceptibility of the hull form to parametric roll in irregular long crested seas.

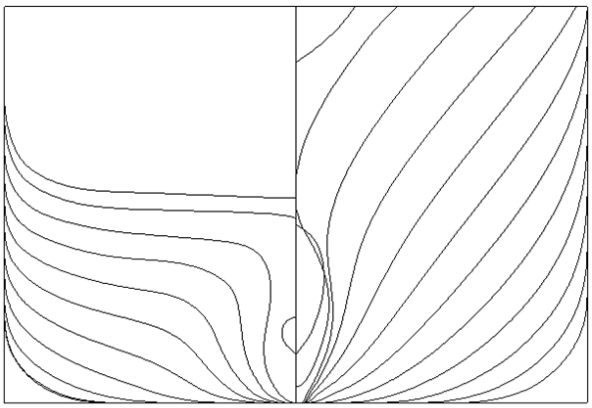

Fig. 7 Body Plan of C11 hull form

3.3 Example: APL China hull form

In this section, above method of Volterra series is applied to a modified hull form of APL China. The details of the ship are given in Table2. The body plan of the hull form is shown in Fig. 7.

A simulation of the ship in a Bretschneider Spectrum of significant wave height of H_S =5m and modal period of T_m =13.16 *sec* has been performed. The absolute value of the 1st order transfer function is shown in Fig. 8. The second order transfer functions are shown in Fig. 9. The second order GM variation is observed to be much smaller than the first order GM variation. However, it was found that the first order GM variational ways has a zero mean and is not enough to model the asymmetric GM variation always the calm water GM_0. The second order GM variation has a non-zero mean component which captures the asymmetric behavior more accurately.

The resulting wave elevation, GM variation and roll motion time series are shown in Figs. 10 -12. Fig. 12 shows the roll motion time series to have approximately twice the period of the excitation which demonstrates the Mathieu type behavior in the irregular seas. The effect of second order transfer function should not be confused with a 2nd order roll phenomenon as the second order transfer function only relates to the modeling of an asymmetric variation of GM about the calm water GM.

Table 2 Particulars of the C11 Pram Hull form

Particulars (Units)	Value
Length between Pependiculars (*m*)	255.302
Breadth (*m*)	40
Depth (*m*)	29.25
Draft Aft (*m*)	11.5
Draft Fwd (*m*)	11.5
Displacement from Maxsurf (m^3)	6.74×10^4
Displacement (*tonnes*)	6.9085×10^4
Displacement (*kgs*)	6.9085×10^7
KG (*m*)	18.374
GM (*m*)	1.965
Roll Gyradius (*m*)	16.73
Roll Inertia (kgm^2)	1.93365×10^{10}
Roll Added Inertia (kgm^2)	4.49590×10^9
Natural Period of Roll (*sec*)	26.58
Natural Frequency of Roll (*rad/sec*)	0.2364

4. Conclusions

Two different approaches to improve the EEDI of a ship have been proposed. The first is to predict added resistance of a ship in forward speed efficiently. A near field pressure integration method is used using a 3D panel method program to predict the added resistance. The hull form can be optimized for better EEDI based on the added resistance prediction.

The second approach is to reduce the ship motion inirregular seas so as to enable the ship to operate for most of its voyage at its design speed. Two methods have been proposed for prediction of parametric roll of ship in both regular and irregular seas.

The regular wave method utilizes Floquet theory, and the irregular wave method is based on a Volterra series approach. These two methods will help determine the the magnitude of roll motion in regular and irregular seas. Based on these, the hull form can be effectively optimized to reduce its susceptibility to parametric roll and hence improve the EEDI of the vessel.

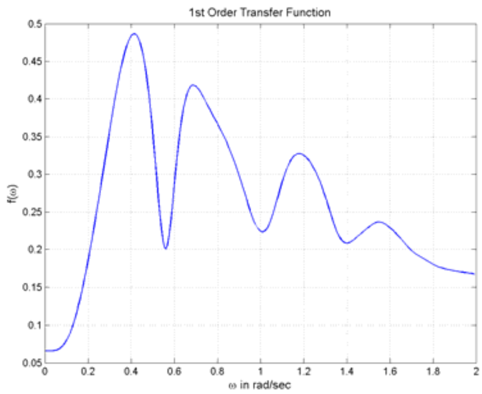

Fig. 8 The1stordertransferfunction for GM variation ∂GM_1

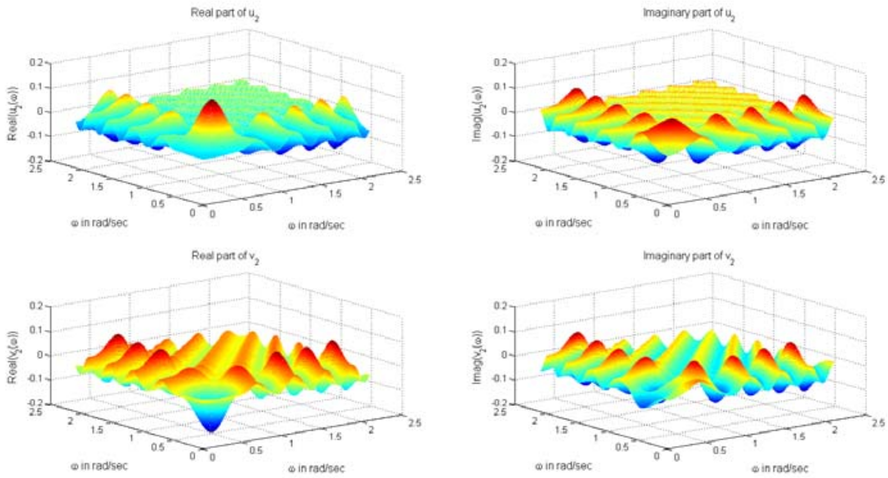

Fig. 9 The 2nd order transfer function for GM variation ∂GM_2

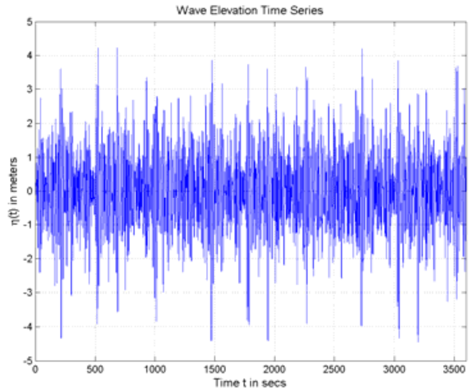

Fig. 10 Wave elevation time series

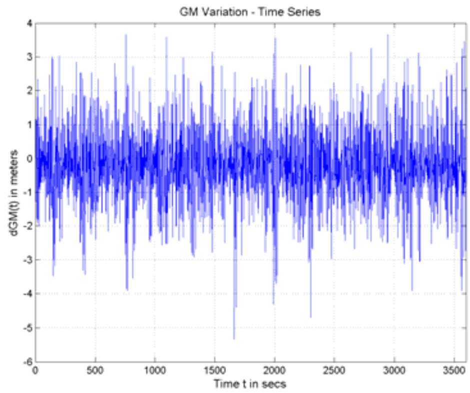

Fig. 11 Variation of GM from calm water condition in waves

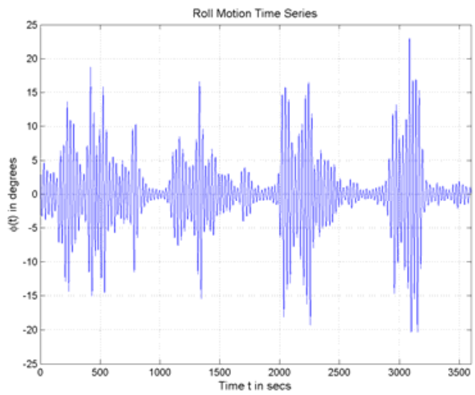

Fig. 12 Roll motion time series

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