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# Buckling optimization of compressed bars undergoing corrosion

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**Abstract.** This study is devoted to the optimal design of compressed bars under axial compressive forces and exposed to a corrosive environment. The initial volume of the structure is taken as an optimality parameter. Gutman – Zainullin's exponential stress corrosion model is adopted for analysis. Analytical and numerical results are derived for optimal variation of the cross-sectional area of the bar along its axis.

**Keywords:** corrosion; optimization; stability

## 1. Introduction

Elements of many engineering structures are exposed not only to loads and temperatures, but also to various corrosive environments. These factors often appear in highly unfavorable combinations, reducing the load carrying capacity and service life of the structure. Neglecting corrosive environments in analysis may lead to premature and often emergent halting of the system's operation, causing great damage to both the environment and economy. As noted by Morgan (1981), mankind has entered a "century of corrosion." With limited natural resources available these corrosion processes may have a determining influence on the speed of technological advance. On the other hand, development of technology results in the fact that the proportion of accidents involving management errors is projected to be on decline, while the proportion of hardware failures, due to corrosion damage, may be on the rise.

Often the effect of the corrosion on the behavior of the structure is inadequately treated. This occurs when durability due to corrosion is defined as the product of an average corrosion velocity by the period of service.

As noted in Ovchinnikov and Sabitov (1982), apparently the first model describing the corrosion process was the Faraday's 1st Law of Electrolysis linking mass of a substance, current and the process duration. Based on experimental results, we conclude that the corrosive process of structure in an aggressive environment is determined by the temperature, the stress-strain state, the nature of the aggressive environment and time span the structure resides in the corrosive environment. In certain circumstances the governing parameters may also include fluid pressure,

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speed of fluid, or of aggressive gas, characteristic location of elements in the structure and other factors. Different researchers offered various models to describe the same process. In these circumstances, selecting one of many possible corrosion models depends, as aptly noted by Nalimov (1971) on the level of one's "intellectual aestheticism."

The influence of stress on corrosion speed, known as the corrosion rate, was apparently first considered by Dolinskii (1967), who dealt with strength of thin-walled pipes subjected to a continuous corrosion rate as a linear function of stress. Exponential dependence of corrosion in the stress of the structure was proposed by Gutman and Zainullin (1984).

Papers by Potchman and Fridman (1977,1995,1996), Fridman (2002), and Fridman and Zyczkowski (2001) utilized Dolinskii's model for optimization study under corrosion. This study extends the above papers to the exponential stress corrosion model by Gutman and Zainullin to study the stability optimization in the corrosive environment.

## 2. Problem statement

We consider the problem of optimal design employing the criterion of a minimum weight of columns loaded with axial compressive $P$ force, simply supported at both ends (Fig. 1) and prone to corrosion. We adopt the corrosion model introduced by Gutman and Zainullin (1984) which reads

$$\frac{db}{dt} = -2\alpha \exp(\eta t) \exp(\gamma\sigma) \tag{1}$$

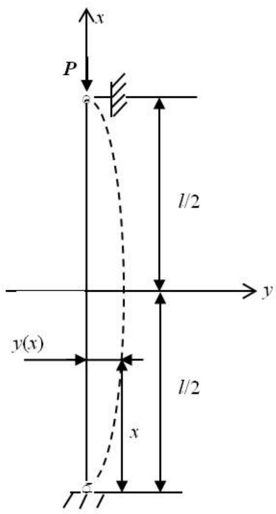


Fig. 1 Bar under axial compressive load $P$

where $\alpha, \eta, \gamma$ - constant coefficients, $\sigma(t) = P/b(t)h$ is the stress, $b_0$ and $b(t)$ is the depth of the rectangular cross section in the initial and current states, respectively, $h$ is the width taken as a constant. We consider the case when the corrosion affects both the top and bottom facets cross-section. This fact explains the factor 2 in Eq. (1)

Separation of variables in Eq. (1) leads, with a notation $a = \gamma P/h$

$$\int \exp(-a/b)db = -2\alpha \int \exp(\eta t)dt \tag{2}$$

Taking into account the familiar series expansion

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{3}$$

we get from Eq. (2)

$$\int\left(1 - a/b + \sum_{2}^{\infty} \frac{(-a)^n}{n!} b^{-n}\right) db = b - a\,ln\,b + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{b^{1-n}}{1-n} = -\frac{2\alpha}{\eta} exp(\eta t) + C \tag{4}$$

Invoking initial conditions at $t = 0$ we obtain

$$C = b_0 - a\,ln\,b_0 + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{b_0^{1-n}}{1-n} + \frac{2\alpha}{\eta} \tag{5}$$

When $t = T$, with $T$ denoting the durability, while $b_T$ designating the height of the section at the

$$\frac{\eta}{2\alpha}\left[b_0 - b_T - a \ln \frac{b_0}{b_T} + \sum_{n=2}^{\infty} \frac{(-a)^n}{n!} \frac{\left(b_0^{1-n} - b_T^{1-n}\right)}{1-n}\right] = \exp(\eta T) - 1. \tag{6}$$

## 3. Lagrange function and optimality conditions

The minimum weight of the column is achieved by the optimal distribution of the initial height of the rectangular cross section $b_0$ along column's length. In terms of the volume

$$V = 2\int_0^{1/2} A dx = 2h \int_0^{1/2} b_o(x) dx \tag{7}$$

The conditions of optimality are expressed as equations Euler-Lagrange

$$\delta_y f = f_y - \frac{d}{dx} f_{y'} + \frac{d^2}{dx^2} f_{y''} = 0 \tag{8}$$

$$\delta_{b_o} f = \frac{df}{db_o} = 0 \tag{9}$$

where $f$ is the so-called Lagrange function with additional condition in the form of Eq. (6) as follows

$$f = b_0 h + \lambda(x)\left[-\frac{\eta}{2\alpha}\left(b_0 - b_T - a\,ln\frac{b_0}{b_T} + \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\frac{\left(b_0^{1-n} - b_T^{1-n}\right)}{1-n}\right) + exp(\eta T) - 1\right] \tag{10}$$

The governing equation for buckling of the column reads

$$EIy'' + Py = 0 \tag{11}$$

Note that the buckling takes place at $t > T$ in $h$ direction since $I = bh^3/12$ corresponds to the smaller value of the moment of inertia than its counterpart in perpendicular direction $I = hb^3/12$. The quantity $b_T$ defined as follows

$$b_T = -\frac{12Py}{h^3 Ey''} \tag{12}$$

In these circumstances, Eq. (10) becomes

$$f = b_0 h + \lambda(x)\left\{-\frac{\eta}{2\alpha}\left[b_0 + \frac{12Py}{h^3 Ey''} - a\ln b_0 + a\ln\left(\frac{-12Py}{h^3 Ey''}\right) + \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\frac{\left(b_0^{1-n} - \left(\frac{-12Py}{h^3 Ey''}\right)^{1-n}\right)}{1-n}\right] + \exp(\eta T) - 1\right\}. \tag{13}$$

Evaluation of derivatives yields

$$f_y = \lambda(x)\left\{-\frac{\eta}{2\alpha}\left[\frac{12P}{h^3 Ey''} + a/y - \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3 Ey''}\right)^{1-n} y^{-n}\right]\right\} \tag{14}$$

$$f_{y'} = 0 \tag{15}$$

$$f_{y''} = \lambda(x)\left\{-\frac{\eta}{2\alpha}\left[-\frac{12Py}{h^3E(y'')^2} - a/y'' + \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12Py}{h^3E}\right)^{1-n}(y'')^{n-2}\right]\right\} \tag{16}$$

Substituting these expressions into Eq. (8) results in

$$\begin{aligned}\delta_y f = \lambda(x)\left\{\left[\frac{12P}{h^3Ey''} + a/y - \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3E}\right)^{1-n}\left(\frac{y''}{y}\right)^n\frac{1}{y''}\right]\right\} - \\ -\left(\lambda(x)\left\{\frac{y}{y''}\left[\frac{12P}{h^3Ey''} + a/y - \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3E}\right)^{1-n}\left(\frac{y''}{y}\right)^n\frac{1}{y''}\right]\right\}\right)'' = 0\end{aligned} \tag{17}$$

## 4. Chentsov's method

Hereinafter, we utilize method proposed by Chentsov (1936) (see also Rzhanitsyn (1955) to solve the Eq. (17). We introduce the following notation

$$k = \frac{\lambda(x)y}{y''}\left[\frac{12P}{h^3Ey''} + \frac{a}{y} - \sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3E}\right)^{1-n}\left(\frac{y''}{y}\right)^n\frac{1}{y''}\right] \tag{18}$$

Multiplying Eq. (17) by $y$, we represent it in the form

$$ky'' - yk'' = 0 \tag{19}$$

One can check by the direct differentiation that the integral of the latter equation is

$$ky' - yk' = C \tag{20}$$

where $C$ is an arbitrary constant. For further integration one should take into account that $C = 0$ for adopted boundary conditions. Indeed, assuming the buckling mode as being symmetric with respect to the middle cross-section of the column, we establish that $y$, $y''$, $y/y''$ and $k$ are even functions of $x$. Hence, the derivatives of $y$ as well as $k$ would be odd functions of $x$ and thus vanish at $x = 0$. Therefore, letting in Eq. (20) $x = 0$, we get $C = 0$. Thus, Eq. (20) becomes

$$ky' - yk' = 0 \tag{21}$$

Its integral is evaluated by separation of variables

$$k = C_1 y \tag{22}$$

where $C_1$ is a new arbitrary constant. Returning to the original notation in Eq. (18), we find

$$\frac{\lambda(x)y}{y''}\left[\frac{12P}{h^3Ey''}+\frac{a}{y}-\sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3E}\right)^{1-n}\left(\frac{y''}{y}\right)^n\frac{1}{y''}\right]=C_1 y \tag{23}$$

Additionally

$$\delta_{b_o} f = h+\lambda(x)\left[-\frac{\eta}{2\alpha}\left(1-\frac{a}{b_0}+\sum_{n=2}^{\infty}\frac{(-a)^n}{n!}b_0^{-n}\right)\right]=0 \tag{24}$$

from which we determine the Lagrange coefficient

$$\lambda(x)=\frac{2\alpha h}{\eta\left(1-\frac{a}{b_0}+\sum_{n=2}^{\infty}\frac{(-a)^n}{n!}b_0^{-n}\right)} \tag{25}$$

Substituting $\lambda(x)$ into Eq. (23) leads to

$$\frac{2\alpha}{\eta}\left[\frac{12P}{h^2E}+\frac{\gamma P y''}{y}-\sum_{n=2}^{\infty}\frac{(-a)^n}{n!}\left(-\frac{12P}{h^3E}\right)^{1-n}\left(\frac{y''}{y}\right)^n h\right]=\left(1-a/b_0+\sum_{n=2}^{\infty}\frac{(-a)^n}{n!}b_0^{-n}\right)(y'')^2 \tag{26}$$

## 5. Particular cases

First let us consider case (**a**) when in the expansion of exponent in Eq. (3) the first two terms are retained. In this case we get a model of corrosion similar to the model of Dolinskii (1967).

The Eqs. (6) and (26) become, respectively

$$\frac{\eta}{2\alpha}\left(b_0-b_T-\frac{\gamma P}{h}\ln\frac{b_0}{b_T}\right)=\exp(\eta T)-1 \tag{27}$$

$$\frac{2\alpha}{\eta}\left[\frac{12P}{h^2E}+\frac{\gamma P y''}{y}\right]=\left(1-\frac{\gamma P}{hb_0}\right)(y'')^2 \tag{28}$$

We introduce the following non-dimensional quantities

$$A = \exp(\eta T) - 1, \quad B = \frac{\gamma P}{h}\frac{\eta}{2\alpha}, \quad \chi_o = \frac{b_o \eta}{2\alpha},$$
$$\chi_T = \frac{\eta}{2\alpha} b_T = \frac{\eta}{2\alpha}\left(-\frac{12Py}{h^3 E y''}\right) = \frac{-6\eta P}{\alpha h^3 E}\frac{l^2}{4}\frac{V}{V''} = CV/V'' \tag{29}$$

In view of

$$\frac{y}{y''} = \frac{l^2}{4}\frac{V}{V''} \tag{30}$$

we get

$$C = -\frac{1.5\eta P l^2}{\alpha h^3 E}, \quad V = \sqrt{\frac{\eta E}{6\alpha P}}\frac{2h}{l^2} y, \quad V''_{\xi\xi} = \sqrt{\frac{\eta E}{6\alpha P}}\frac{h}{2} y'', \quad \xi = \frac{2x}{l} \tag{31}$$

In this case, the Eqs. (27) and (28) become, respectively

$$\chi_0 - \chi_T - B \ln \frac{\chi_0}{\chi_T} = A \tag{32}$$

$$1 - \frac{B}{\chi_T} = \left(1 - \frac{B}{\chi_0}\right)(V'')^2 \tag{33}$$

The latter equation is derived as follows. From Eq. (28) we get

$$\left(1 - \frac{h^2 E}{-12P}\frac{\gamma P y''}{y}\right) = \left(1 - \frac{\gamma P}{h b_0}\right)\frac{\eta}{2\alpha}\frac{h^2 E}{12P}(y'')^2 \tag{34}$$

In view of the above notations we get

$$\left(1 - \frac{B}{\chi_T}\right) = \left(1 - \frac{B}{\chi_0}\right)\frac{\eta}{24\alpha}\frac{h^2 E}{P}(y'')^2 \tag{35}$$

which results in

$$V''_{\xi\xi} = \sqrt{\frac{\eta E}{6\alpha P}}\frac{h}{2} y'' \tag{36}$$

Taking into account that

$$\frac{y}{y''} = \frac{l^2}{4}\frac{V}{V''}, \quad V = V''\frac{y}{y''}\frac{4}{l^2} = \sqrt{\frac{\eta E}{6\alpha F}}\frac{2h}{l^2} y \tag{37}$$

We find from Eq. (33)

$$\chi_0 = \frac{B\left(V''\right)^2}{V''^2 + B/\chi_T - 1} \tag{38}$$

Substituting this expression into Eq. (32) in view of Eq. (29), we arrive at

$$A = \frac{B(V'')^2}{(V'')^2 + BV''/CV - 1} - \frac{CV}{V''} - B\ln\left[\frac{\dfrac{B(V'')^2}{(V'')^2 + BV''/CV - 1}}{CV/V''}\right] \tag{39}$$

Now we turn to numerical implementation of the procedure. Taking into account symmetry of the buckling mode with respect to *y* axis, we divide the column's half-length into elementary parts of length $\Delta\xi$ as shown in Fig. 2.

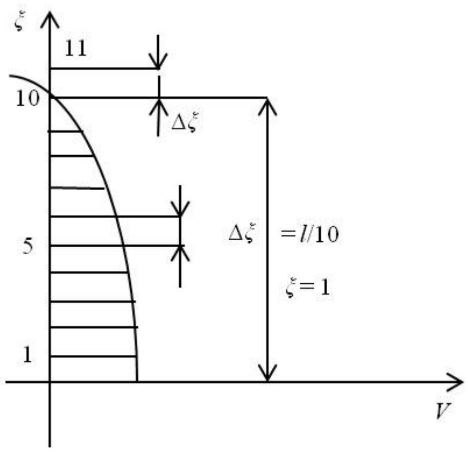


Fig. 2 Discretization of half a column

To determine the shape of the column in this case, in addition to Eq. (39) we use a relationship between the buckling mode and its second derivative in the central finite difference setting

$$V_{i-1} = \Delta\xi^2 V_i'' + 2V_i - V_{i+1} \tag{40}$$

Starting from the arbitrary negative value of $V_{11}$ and $V_{10} = V_{10}'' = 0$ (i.e., $V_9 = -V_{11}$), we determine the value $V_9''$ from Eq. (39) by using random search algorithm Gurvich *et al*. (1979).

This algorithm, which is based on the global random search incorporates the idea of controllable selection of test points and multiple lowering to approach a local extremum. The original method for searching direction choice is proposed in the present contribution. The process of random search is realized as follows; we express

$$\overline{X}_{\varsigma}^{(k+p)} = \overline{X}^{(k)} \pm H_{\Sigma-}$$

$$\overline{H} = \begin{cases} \gamma_2 H & \text{if } F(\overline{X}_{\varsigma}^{(k+p)}) < F(\overline{X}_0^{(k)}) \\ \gamma_1 H & \text{if } F(\overline{X}_{\varsigma}^{(k+L_p)}) \geq F(\overline{X}^{(k)}) \end{cases}$$

where $\sum$ is a single random uniformly distributed vector; $\gamma_1, \gamma_2$ are constants of tension (contraction) of the searched hypercube *H*, where $\gamma_1 \geq 1$; moreover, $\gamma_2 < 1$ and $\gamma_1 \gamma_2 > 1$; $p$ is a number of random realizations of the vector $\overline{X}_{\varsigma}$ at a constant $\overline{H}$, $p = \{1,2,\ldots,L_p\}$; $\overline{X}_0^{(k)}$ are the parameters corresponding to the lowest value obtained at the *k*-th stage of the search $F\left(\overline{X}_0^{(k)}\right)$,while signs $\pm \overline{H}_{\Sigma}$ represent the realization of the double return of the test random point $\overline{X}_{\varsigma}$.

In the next step from Eq. (40) we find $V_8$. The process is repeated until the values $V_o$ and $V_o''$ are determined. The solution is validated by evaluating $V_o'$, since the latter must vanish due to the condition $V_1 \approx V_o$. After the values of $V_i$ and $V_i''$ are found, values of $\chi_{Ti}$ and $\chi_{oi}$ are directly evaluated.

Let us consider the particular case (**b**) with $n = 2$. In this case, the Eqs. (6) and (26) lead to the following

$$\frac{\eta}{2\alpha}\left(b_0 - b_T - \frac{\gamma P}{h}\ln\frac{b_0}{b_T} + \frac{\gamma^2 P^2}{2h^2}\left(\frac{1}{b_T} - \frac{1}{b_0}\right)\right) = \exp(\eta T) - 1 \tag{41}$$

$$\frac{2\alpha}{\eta}\left[\frac{12P}{h^2 E} + \frac{\gamma P y''}{y} + \frac{\gamma^2 P^2}{2h}\left(\frac{h^3 E}{12P}\right)\left(\frac{y''}{y}\right)^2\right] = \left(1 - \gamma P / h b_0 + \frac{\gamma^2 P^2}{2h^2 b_0^2}\right)(y'')^2 \tag{42}$$

or the non-dimensional quantities

$$\chi_0 - \chi_T - B\ln\frac{\chi_0}{\chi_T} + \frac{B^2}{2}(1/\chi_T - 1/\chi_0) = A \tag{43}$$

$$1 - \frac{B}{\chi_T} + \frac{B^2}{2\chi_T^2} = \left(1 - \frac{B}{\chi_0} + \frac{B^2}{2\chi_0^2}\right)(V'')^2 \tag{44}$$

From Eq. (44) we find

$$\chi_o = -\frac{B}{2(K-1)}\left(1-\sqrt{2K-1}\right) \tag{45}$$

where

$$K = \frac{1}{(V'')^2}\left(1-\frac{B}{\chi_T}+\frac{B^2}{2\chi_T^2}\right) \tag{46}$$

Substituting the expression into Eq. (43) with Eq. (29), we derive

$$f_1(B,C,V,V'') = A \tag{47}$$

where $f_1(B,C,V,V'')$ depend on $B, C, V$ *and* $V''$.

Let us now consider the special case (**c**) when in Eq. (3) *n* = 3 terms are retained. In this case, the Eqs. (6) and (26) into the following

$$\frac{\eta}{2\alpha}\left(b_0 - b_T - \frac{\gamma P}{h}\ln\frac{b_0}{b_T} + \frac{\gamma^2 P^2}{2h^2}\left(\frac{1}{b_T}-\frac{1}{b_0}\right) + \frac{\gamma^3 P^3}{12h^3}\left(\frac{1}{b_0^2}-\frac{1}{b_T^2}\right)\right) = \exp(\eta T) - 1 \tag{48}$$

$$\frac{2\alpha}{\eta}\left[\frac{12P}{h^2E} + \frac{\gamma P y''}{y} + \frac{\gamma^2 P^2}{2h}\left(\frac{h^3E}{12P}\right)\left(\frac{y''}{y}\right)^2 + \frac{\gamma^3 P^3}{6h^3}\left(\frac{h^3E}{12P}\right)^2\left(\frac{y''}{y}\right)^3 h\right] = \left(1-\gamma P/hb_0 + \frac{\gamma^2 P^2}{2h^2 b_0^2} - \frac{\gamma^3 P^3}{6h^3 b_0^3}\right)(y'')^2 \tag{49}$$

or in a non-dimensional form

$$\chi_0 - \chi_T - B\ln\frac{\chi_0}{\chi_T} + \frac{B^2}{2}\left(\frac{1}{\chi_T}-\frac{1}{\chi_0}\right) + \frac{B^3}{12}\left(\frac{1}{\chi_0^2}-\frac{1}{\chi_T^2}\right) = A \tag{50}$$

$$1-\frac{B}{\chi_T}+\frac{B^2}{2\chi_T^2}-\frac{B^3}{6\chi_T^3} = \left(1-\frac{B}{\chi_0}+\frac{B^2}{2\chi_0^2}-\frac{B^3}{6\chi_0^3}\right)(V'')^2 \tag{51}$$

We find out $\chi_0$ in Eq. (51). We obtain the following cubic equation

$$\chi_0^3 + r_1\chi_0^2 + r_2\chi_0 + r_3 = 0 \tag{52}$$

where

$$r_1 = \frac{B}{K_1 - 1}; \quad r_2 = \frac{B^2}{2(1-K_1)}; \quad r_3 = \frac{B^3}{6(K_1 - 1)}; \quad K_1 = \frac{1 - B/\chi_T + B^2/2\chi_T^2 - B^3/6\chi_T^3}{(V'')^2} \tag{53}$$

The solution is found by applying the Cardan's formula to the equation

$$z^3 + pz + q = 0 \tag{54}$$

where

$$z = \chi_0 + r_1/3, \quad p = \frac{3r_2 - r_1^2}{3}, \quad q = \frac{2r_1^3}{27} - \frac{r_1 r_2}{3} + r_3 \tag{55}$$

If discriminant $D$ is positive, i.e., if

$$D = (p/3)^3 + (q/2)^2 > 0 \tag{56}$$

the solution reads

$$z = u + v, \quad u = \sqrt[3]{-q/2 + \sqrt{D_s}}, \quad v = \sqrt[3]{-q/2 - \sqrt{D}} \tag{57}$$

If discriminant is negative, or if $D < 0$, then

$$z = 2\sqrt[3]{r}\cos(\varphi/3) \tag{58}$$

$$r = \sqrt{-p^3/27}, \quad \cos\varphi = -q/2r \tag{59}$$

Defining $\chi_0$ and substituting into Eq. (50). We have similar to the case where $n = 2$

$$f_2(B, C, V, V'') = A, \tag{60}$$

where $f_2(B, C, V, V'')$ depends on $B, C, V$ and $V''$.

## 6. Numerical results

Optimum shape of the initial form of the height of the rectangular bar $\chi_o(\xi)$ was derived for alloy D16T with the following rates of corrosion models (1): $\sigma = 4.8 \cdot 10^{-4}$ m/year; $\gamma = 0.588 \cdot 10^{-4}$ m$^2$/T; $\eta = 0.091$ year$^{-1}$; $E = 7 \cdot 10^7$ Pa. Parameters have been fixed at $h = 10^{-2}$ m; $P$=10 kH; $T$=10 years; $l$=1 m. In this case the non-dimensional quantities are $A$=1.484; $B$=0.557; $C = -40.625$.

Note this for all three cases, the optimized shapes turn out to be almost identical (Fig. 3).

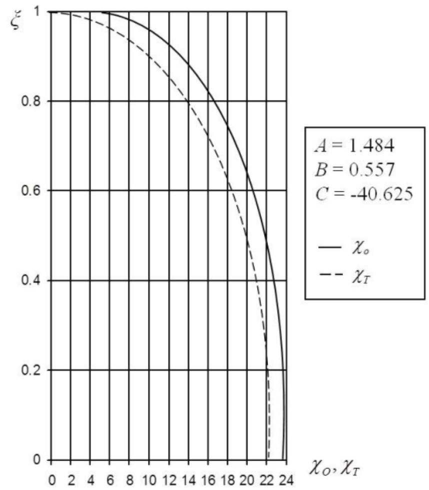


Fig. 3 Optimum initial shape and its final form

Similar results were obtained with the *P*=1.79 T; *T*=7.62 years; *l*=1m. In this case the associated non-dimensional quantities are *A*=1; *B*=1; $C=-73$. Optimum initial shape of the height of the rectangular bar $\chi_o(\xi)$ and its form at $t\approx T$: $\chi_T(\xi)$ are shown in Fig. 4. Here, too, for all three cases, they are almost identical.

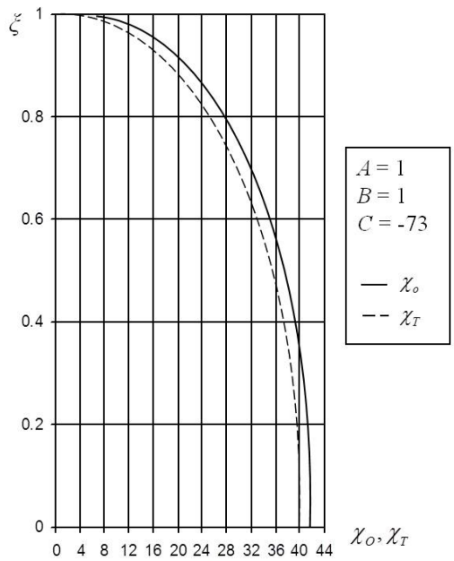


Fig. 4 Optimum initial and final shapes for *B*=1

For Fig. 5 the numerical results were obtained for the following data: *P*=1.79 T; *T*=7.62 years; *l*=1 m; *A*=1; *C*=-73; *B*=5. The new value of *B* with remaining parameters kept is associated with a dramatic increase on corrosion rate, i.e., of the parameter $\gamma = 0.588 \times 5 = 2.94 \cdot 10^{-4}$ m$^2$/T. Optimum initial form is characterized by the height of the rectangular bar $\chi_o(\xi)$ and its form when *t*≈*T*, namely $\chi_T$ as shown in Fig. 5.

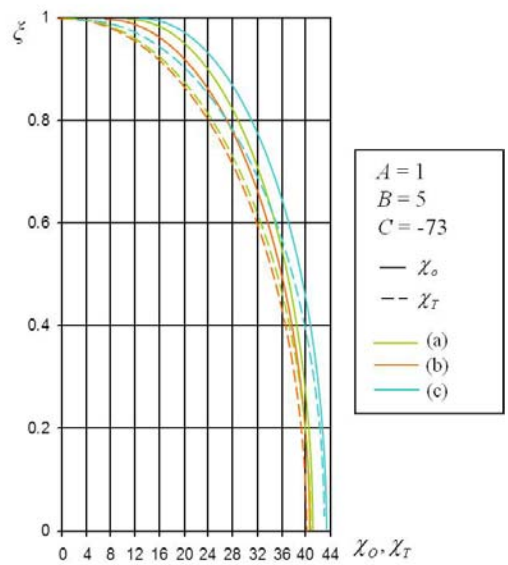


Fig. 5 Optimum initial and final shapes for *B*=5

## 7. Conclusions

In this paper the general and particular optimization solutions are obtained for the Gutman-Zainullin corrosion model.

The results of numerical evaluation shown in Figs. 3 and 4 show that the increase in the number of terms in the expansion in Eq. (2) does not produce significant changes for the chosen sets of parameters. In all three cases, the optimal initial shape of the height of the rectangular bar $\chi_o(\xi)$ and of its counterpart for *t*≈*T*, namely $\chi_T(\xi)$ are almost identical. This closeness in results may have the following explanation. By substituting values obtained for $\chi_0, \chi_T$ in Eq. (50), one observes that the terms in front of *B* and even more in front of *B* tend to zero. Increasing the value of factor *B* (with other values fixed at $\alpha = 4.8 \cdot 10^{-4}$ m/year; $\gamma = 0.588 \cdot 10^{-4}$ m$^2$/T; $\eta = 0.091$ year$^{-1}$; *E*=$7 \cdot 10^6$ T/m$^2$) should lead also to increase of the force *P,* this in turn leading to a sharp increase in $\chi_0, \chi_T$. Since these terms appears in the denominators in (50), then respectively, the contribution of terms containing $B^2$ and $B^3$ are decreased.

An increased slight difference (between 1.5-10%) in all three cases occurs when the corrosion rate depends strongly on *B*, as shown in Fig. 5.

Summing up the above, the use of corrosion model by Dolinskii allows one to get a sufficiently accurate of the optimization problem for bars in axial compression in corrosive environment, when corrosion rate does not depend strongly on *B*. However, when corrosion rate strongly depends on *B*, one is recommended to utilize the model by Gutman and Zainullin. The results of his study include, as a particular case, the analysis associated with the Dolinskii's corrosion model. As such it represents a generalization of previous study by Fridman and Zyczkowski (2001). It appears that the future studies on buckling optimization ought to include linear and/or exponential corrosion models depending on the corrosive environment the structure is residing in.

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