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Determination of reliability index of the retaining wall using artificial intelligence techniques

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Abstract. Reliability analysis of the geo-structures has contributed a lot to the field of Geotechnical Engineering. This area of study gives an overview of the probability of failure of different structures. First-order second-moment method (FOSM) is a method, incorporated in this study, to determine the reliability index of the geo-structures (and other structures as well). In this paper, design of retaining wall is modelled using Functional Network (FN), Genetic Programming (GP) and Group Method of Data Handling (GMDH). These soft computing techniques have removed the cumbersome nature of the problem and have increased the precision of the result. The uncertainties involved in this problem is reduced. As these methodologies are evolved and are heated topics in the artificial intelligence field, they have eliminated the drawbacks of several other soft computing methods involved previously in the reliability problems. These methodologies employ genetic algorithm (GMDH) and make use of domain knowledge along with data knowledge accordingly (FN). These techniques have made problems facile and can produce a precise result. Performance of these methods has been assessed using different performance analysis, criterions and parameters. This paper is a comparative study between FOSM, FN based FOSM, GP based FOSM and GMDH based FOSM.

Keywords: reliability index; FOSM; GP; GMDH; retaining wall

1. Introduction

Geotechnical is a major branch of civil engineering and it has become important to analyze all the aspects and structures of this branch practically, analytically and computationally. Retaining wall is a geotechnical structure which is of sheer importance for the stability of slopes. From geotechnical learnings, it is known that slopes fail in different ways and from technological advancements forecasting the failure using few parameters of the retaining wall (or any other structure) has become easy. Slopes suffer rotational failure, translational failure, compound failure, wedge failure and other failures such as flows and spreads. Remedial measures are taken to avoid the failure of the slope and construction of a retaining wall is among one of the remedies. For the construction of a retaining wall, soil parameters that influence the bearing capacity of the soil along with the earth pressure are measured and evaluated. Primitive parameters that define the failure are cohesion intercept, angle of shearing resistance, unit weight and angle of wall friction.

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Considering these parameters, the factor of safety is calculated based on the sliding criterion. Also, to measure the ability to meet requirements under a specified period, we perform reliability analysis. Reliability analysis has covered a wide range of diversified topics and has let researchers make a comparison between different models with distinct paradigms using a parameter called reliability index which indirectly implies the probability of failure. For reliability analysis, First Order Second Moment Method (FOSM) is widely used but it consumes time (Wu and Kraft 1970, Cornell 1971). It has been remedied by the researchers as they came up with models such as response surface method (Wong 1985, Faravelli 1989), multiple tangent plane surface (Guan and Melchers 1997), multi-plane surfaces method etc. which are used to solve the ambiguities of non-linear limit state surface. But these approaches are limited to nonlinear convex or concave surfaces only, many other models using artificial intelligence technique came into existence to model the system of variables. To model and predict the behavior of unknowns or very complex system depending upon input and output, system identification techniques are applied (Astrom and Eykhoff 1971). Theoretically, knowledge of explicit mathematical input-output relationship is required to model a system of problems. It is known to all that to establish a relation between the inputs and outputs, regression analysis is carried out (Zhang and Goh 2013, Azzouz *et al*. 1976). Assumptions are made and boundaries are levied to make problem facile. Explicit mathematical models are difficult to tackle if the system is not understood properly. Analytical methods of analysis got suppressed by numerical methods which got swiped away by soft computing (Sanchez *et al*. 1997). Artificial intelligence has played a vital role in all the fields and has proved to be the best alternative to solve regression problems. Soft computing methods have gained attention as they compute in an imprecise environment. Neural networks, fuzzy logic and genetic algorithm are the major discoveries or development in the soft computing area and they have great ability to get through with the complex nonlinear system identification and control problems. In this research, we have done the reliability analysis of retaining wall using FOSM and we have modelled our problem using Functional Network (FN), Group Method of Data Handling (GMDH) and Genetic Programming (GP) and then we have compared the reliability indices of all the three models.

2. Research significance

This study area of amalgamation of civil engineering and artificial intelligence has led to an interdisciplinary approach where different complex design problems are modelled. Different specializations of civil engineering such as geotechnical, structural, concrete, water resource etc. have experienced incorporation of artificial intelligence algorithms such as Neural Networks, the most applied technique for robust calculation and modelling, Genetic Algorithms, Fuzzy logic etc. Other paradigms are also worked upon to solve problems precisely and assure certainty in result. Being a vast branch of implicit and explicit calculations, soft computing has helped in reducing work load and establish better results. Also, it has been seen that soft computing techniques have further led to economic design models when compared to conventional designs. Machine learning beautifully incorporates the conventional approach, establishes mathematical relations and then learns from it. Computer science is evolving like a forest fire and employing their techniques into this discipline has emerged as a need. This paper has elaborated modelling of design of Retaining wall using Functional network, Genetic programming and Group Method of Data Handling. Reliability index is calculated using First Order Second Moment method to measure the safety of the structure in a given time period. To enhance the learning of this probabilistic approach researchers, need to incorporate more data and other optimization techniques. Rapid advancement in the computing techniques must be walked along by the civil engineering scholars as with every modification in the techniques a problem becomes less complex that too unbelievably. This area of research must be held with firm and evolving ideas. More models and optimizing tools must be introduced and applied to the structural design problems.

3. Computing in civil engineering

Civil engineering is a discipline with conventional approaches. From ages when studies got fractioned into different fields, civil engineering was the one with traditionally conventional ideologies. Complexities in the structural problems led to different remedies. Inculcation of computing approach was one in many measures. Incorporating computing techniques into structural problems became a great remedial measure for design problems in structural engineering and other specializations as well (Åström and Eykhoff 1971, Chen 2019). All the networks involved in machine learning are mostly adaptable and versatile. They have the capability of establishing good relations between the input and the output and map them accordingly. They can resolve the complexity of the problem by solving the arduous and nonlinear equations. Several models researched and worked upon in past few decades are Artificial Neural Network (ANN) (Asteris and Mokos 2019, Asteris *et al*. 2019, Apostolopoulou *et al*. 2019) using Back Propagation (BP) algorithm, Adaptive Neuro Fuzzy Inference System (ANFIS), Functional Network (FN), Multivariate Adaptive Regression Spline (MARS), Genetic Algorithm (GA), Genetic Programming (GP), Emotional Neural Network (EmNN), Support Vector Machine (SVM), Relevance vector Machine (RVM), Gaussian Process Regression (GPR) etc. and few optimization techniques inbuilt or used otherwise for tuning the parameters involved in iterative predictions are Particle Swarm Optimization (PSO), Symbiotic Organism Search (SOS) etc. Major areas of civil engineering discipline where these machine learning techniques are brought to use are structural, geotechnical, concrete, water resource etc. (Chandwani *et al*. 2013)

Application of machine learning in structural engineering are

- Economic design of truss by obtaining optimum truss weight.
- Designing RCC single span beam.
- Requirement of resources for conceptual designs.
- Prediction of strength of RCC deep beams.
- Prediction of inelastic moments considering cracks and time effects.
- Determine the moment curvature relationship.
- Prediction of load capacity either normal or eccentric.
- Creating an optimum environment for design of beams.
- Detailed designing of multi-storey building.
- Design of structures based on different criteria.
- For cost optimization problems.

Application of machine learning in geotechnical engineering are

- Settlement of shallow foundations on different soils.
- Prediction of cyclic Resistance Ratio during Liquefaction.
- Soil-structure interaction.
- Influence of different soil parameters on the lateral resistance of pile cap.
- Reliability analysis using reliability index.

• Prediction of failure of different geotechnical structures.
• Optimum and economic design of foundation.
• Factor of safety is predicted at higher confidence levels.

There are other fields too where rigorous work has been done using the soft computing techniques as they do not demand cumbersome calculation and provide accurate results. Also, many civil engineering problems have been rectified using machine learning techniques.

4. Artificial intelligence techniques

4.1 GMDH

ANN has proved to be the best model in the civil engineering field for accuracy and precision in the predicted result (Ochmanski *et al*. 2015, Tarawneh 2013, Uncuoglu 2008). But its main disadvantage that came into knowledge was hidden detected dependencies (Narimen-Zadeh *et. al*. 2003). To overcome this disadvantage, Group Method of Data Handling (GMDH) type Neural Network came into existence. GMDH type NN had been applied to many civil engineering problems including structural, geotechnical problems etc. This model was used by researchers to predict the undrained shear strength of clays, shear wave velocity, pile bearing capacity, lateral displacement induced by liquefaction and liquefaction potential based on geotechnical properties (Kordnaeji *et al*. 2015, Kalantry *et al*. 2009, Ardalan *et al*. 2009, Shooshpasha and Molaabasi 2012, Mola-Abasi *et al*. 2013, Eslami *et al*. 2014). Therefore, this approach has been used in this paper to predict the Factor of Safety (FOS) of the retaining wall.

For the first time, the GMDH was used for the multivariate analysis method for modelling and identifying complex systems and was developed by Ivankhnenko (Ivankhenko 1971). GMDH algorithm, in which coefficients of the model are estimated by means of least squares method, has been classified into complete induction or incomplete induction and these represent the Combinatorial (COMBI) and Multilayered Iterative Algorithms (MIA) respectively (Farlow 1984).

Aim of the present study is to model and predict the Factor of Safety of the retaining wall using c, φ, γ and δ as input parameters. To perform such structural operations, genetic algorithms are established with a new approach. This GMDH type NN architecture consists of neurons in hidden layers and their connectivity configuration combined with regression methods to solve for an optimal set of coefficients which are appropriate for the quadratic expression which will further be used for modelling and prediction of the factor of safety (FOS).

4.1.1 Review of GMDH type NN

GMDH constructs a function in the feed-forward network based on the second-degree transfer function. This algorithm automatically determines the hidden or other layers along with neurons in those layers, input variables and optimal model structure. GMDH model connects pairs of neurons in each layer using quadratic polynomial producing new neurons in the next layer. This type of representation is useful for mapping input and output. The normal definition for this type of identification problem is to work out a function $\hat{f}$ that can be used approximately instead of the actual one, f or the prediction of output $\hat{y}$ for a given input vector U= (u_1, u_2, u_3,……….,u_n) as near as possible to actual output y.

Given M observation of multi-input-single-output data pairs

$$y_i = f(u_{i1}, u_{i2}, u_{i3}, \dots, u_{in}) \quad (\text{i}= 1, 2, 3, \dots, \text{M}) \tag{1}$$

Now, it is easy to train GMDH type neural network to predict the output values $\hat{y}_i$ for given input vector U= (u_{i1}, u_{i2}, u_{i3}, …, u_{in}), that is

$$\hat{y}_i = \hat{f}(u_{i1}, u_{i2}, u_{i3}, \dots, u_{in}) \quad (\text{i}= 1, 2, 3, \dots, \text{M}) \tag{2}$$

Further, our concerned problem is to determine a GMDH type NN network's algorithm that minimizes the square of difference between observed (actual) and predicted values

$$\sum_{i=1}^{M} [\hat{f}(u_{i1}, u_{i2}, u_{i3}, \dots, u_{in}) - y_i]^2 \rightarrow \quad min \tag{3}$$

Connection between input variables and output values can be established and expressed using the renowned complicated discrete series called Volterra functional series also known as Kolmogorov-Gabor polynomial i.e.,

$$y = a_0 + \sum_{i=1}^{n} a_i u_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} u_i u_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} u_i u_j u_k + \tag{4}$$

Representation of the above mathematical description can be done as a system of partial quadratic polynomial consisting of two variables only (or in terms of network, neuron).

$$\hat{y} = G(x_i, x_j) = a_o + a_1 u_i + a_2 u_j + a_3 u_i u_j + a_4 u_i^2 + a_5 u_j^2 \tag{5}$$

Eq. (5) is repeatedly used in a network to create a mathematical relation amidst input and output shown in equation 4. Coefficients of equation 5 are evaluated using regression technique (Jamali *et. al*. 2009), also it is ensured that difference between the actual measured output y and predicted output (or here the calculated one using equations) $\hat{y}$. Also, using the quadratic form, a tree of polynomial is created whose coefficients are obtained using least square criterion. Coefficient of each quadratic function, defined in equation 5 i.e., G_i, is worked upon so as to fit the output in the whole set of input output data pairs which were obtained by manual calculations.

$$E = \frac{\sum_{i=1}^{M} \left(y_i - G_i()\right)^2}{M} \rightarrow \quad min \tag{6}$$

In GMDH algorithm, combination of all the possible pairs of independent variables (input) out of n number of variables are considered and worked out using regression analysis. Regression polynomial is formed which best fits the dependent observations $(y_i, i = 1,2,3, \dots, M)$ in a lest square sense. As a result of the analysis, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will occur in the first hidden layer of the network from the set of observed values i.e. $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, 3, \dots, M)\}$ where p,q Є {1,2, …, n). it is now possible to construct M data triples using the observation data in the form of (23)

$$\begin{matrix} u_{1p} & u_{1q} & \vdots & y_1 \\ u_{2p} & u_{2q} & \vdots & y_2 \\ \dots & \dots & \vdots & \dots \\ u_{Mp} & u_{Mq} & \vdots & y_M \end{matrix} \tag{7}$$

Using the quadratic sub expression as that in equation 5 following matrix equation is obtained for each row with M data triples

$$Aa = Y \tag{8}$$

And

$$Y = \{y_1, \quad y_2, \quad y_3, \dots, \quad y_M\} \tag{9}$$

Where a is the vector of unknown coefficients of the quadratic equation, a_i is the coefficient of the quadratic polynomial equation, E is the mean square error, G_i is the quadratic function, i is the value considered in cumulative probability P, M is the total number of data sets involved, n is the number of input variables involved, U is the input variable vector, u_i is the input variable, Y is the measured output vector and y is the measured output.

It is observable that

$$A = \begin{bmatrix} 1 & u_{1p} & u_{1q} & u_{1p}u_{1q} & u_{1p}^2 & u_{1q}^2 \\ 1 & u_{2p} & u_{2q} & u_{2p}u_{2q} & u_{2p}^2 & u_{2q}^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & u_{Mp} & u_{Mq} & u_{Mp}u_{Mq} & u_{Mp}^2 & u_{Mq}^2 \end{bmatrix} \tag{10}$$

The above analysis gives a solution of the normal equation in the form of

$$a = (A^T \ A)^{-1} A^T Y \tag{11}$$

Eq. (11) shows a vector of the coefficients used in the equation 5 for M set of data triples. This procedure of regression coefficient analysis is carried out for other hidden layers as well, depending upon the connectivity topology of the network. However, such a formulation and solution from normal equations directly tends to round off errors and is susceptible to the singularity of normal equations. Main concepts involved in GMDH network are parametric and structural identification problem.

Genetic Algorithm (GA) is a stochastic method that removes the limitation of the GMDH network of the interconnection of the neuron layer with the adjacent layers only (Atashkari *et. al*. 2007). The Generalized Structure GMDH i.e., GS GMDH allows cross over and mutation for the whole length of the chromosome string (Nariman-zadeh 2007). It allows communication of all the neuron layers amongst each other. The restriction of maintaining the coordination with the adjacent layer only is removed. It is clearly shown in Fig. 1 that the neuron *ux* directly connects to the output layer. Also, it reduces the exhaustion of the model during the training and then prediction period. It minimizes the error too. On reducing the overtraining, the training error goes low but the prediction error hits high. To make the model more practical size is minimized and is made understandable.

The appropriate selection of input variables leads to the ease of the structure of the model. Unnecessary selection of variables increases the complexity of the operations indulged in the model.

In this study, we have trained the GS-GMDH model to predict the factor of safety of the retaining wall. The data that is fed to the network is calculated based on the sliding criterion of the soil. c, Φ, γ and δ are given as input variables (rest variables involved in the study are assumed constant) to develop a polynomial function and factor of safety is given as the actual measured output of the retaining wall. The data is then bifurcated into two sets, one as the training set and

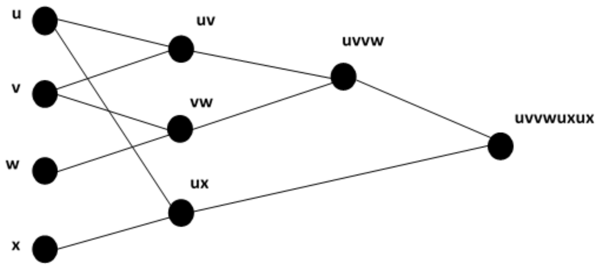

Fig. 1 GS GMDH network structure of a chromosome

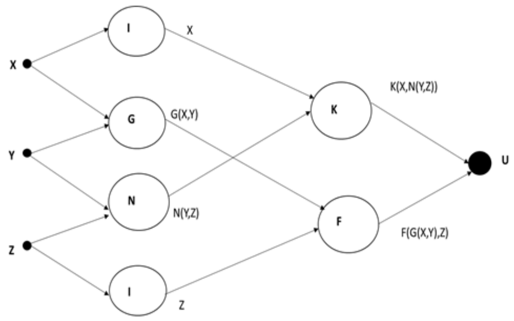

Fig. 2 Functional Network

the other as the testing set. 70 per cent of the data is used to train the model and 30 per cent is used to test the model.

4.2 Functional network

Neural networks have received great attention in the field of engineering and other fields as well. In these networks, the neurons in different layers are interconnected using links. Neurons in the network are associated with a scalar function *f*, which is fixed and the weight is altered and learned using well-known algorithms. These types of neural networks are known as *standard neural network* as opposed to a functional network. The functional network is explained as an extension over the artificial neural network (ANN). The functional network is a neural paradigm in which function *f* is allowed to be learned with each proceeding of the network (Castillo 1998). Poor generalization and certain limitations levied on ANN have given rise to FN which is a mathematical analysis and has a data-driven approach i.e., it is based on topology selection. It works on real-life problems and takes into account both domain knowledge and data knowledge which further estimates neuron functions. This model was introduced by Enrique Castillo with the ideology that every existential thing is a function of something. The key features of the functional network are:

- Topology is selected based on both domain and data knowledge.
- Functions associated with the neuron are learnt in both the stages: structural learning (for obtaining simplified network) and parametric learning (for obtaining optimal neuron function).

• Arbitrary functions can be assigned to neurons. On the contrary, ANN has fixed sigmoidal function.

Functions can be multi argument and multivariate.

4.2.1 Definition of a functional network

A functional network is a pair (X, U). Here, X is a set of nodes and U= {(A_i, G_i, F_i, Y_i); i=1, 2, …, n} is a set of functional units over X, which satisfies the condition that Xi € X exists an input or an output node of at least one functional unit in U (Castillo *et al*. 2012).

Elements of functional network are as follows

• A network layer consisting of input units.

• A network layer consisting of output u

• One or several layers of neurons or computing units.

Computing units are connected to each other in a way that an output from one unit serves as an input for another unit or directly connects to the unit in the output layer.

• Directing links.

It connects the input layer to other neuron layer, other neuron among each other and neuron layer to the output units. These are represented by arrows indicating direction of flow of information.

Altogether, these elements build a network architecture. In multilayers networks, units are arranged in several layers. Information flows in one direction only i.e., from input to output. As an example, we have Fig. 2.

In Fig. 2, the input layer has three variables i.e., x, y and z. The first layer of neurons contains I, G, N and F followed by the second layer of neurons which has K and F units and output has reduced to one computing unit, u. I, here depicts identity transformation. In this analysis, the degree of freedom of a multidimensional functional is reduced considerably. It channelizes the neurons and gives unique output i.e., it ensures connectivity of a large number of neurons to the output unit, reducing the degree of freedom of the network.

4.2.2 Generalize associativity functional network

The equation below is the representation of the functional network in Fig. 2.

$$F[G(x,y),z] = K[x,N(y,z)] \tag{12}$$

Solution of equation 12 is given as follows (Castillo *et al*. 1999)

Theorem 1: Generalized Associativity equation

For G invertible in both variables; F, for the fixed value of second variable, is invertible in first variable, K and N invertible in the second variable for a constant value of first variable, the general solution which is continuous on a rectangle of Eq. (12), is

$$\begin{gathered} F(x,y) = k[f(x)+g(y)], G(x,y) = f^{-1}[p(x)+q(y)], \\ K(x,y) = k[p(x)+n(y)],\ N(x,y) = n^{-1}[q(x)+g(y)] \end{gathered} \tag{13}$$

Where f, g, k, n, p *and* q are arbitrary continuous functions which have monotonous behavior with relations as listed below

$$\begin{aligned} f_2(x) &= cf_1(x)+a \quad , \quad g_2(x) = cg_1(x)+b \\ n_2(x) &= cn_1(x)+b+d \quad , \quad p_2(x) = cp_1(x)+a-d \\ k_2(x) &= k_1\left(\frac{x-a-b}{c}\right) \quad , \quad q_2(x) = cq_1(x)+d \end{aligned} \tag{14}$$

On solving further, the two sides of Eq. (12) can be written as

$$k[p(x) + q(y) + g(z)] \tag{15}$$

Above equation can give a simplified network of Fig. 2 which is shown in Fig. 4.

This process should be carried in such a way that in the end, argument in the simplified network reduces to one.

Following theorem helps in learning the functions given in Eq. (13).

Theorem 2: uniqueness of representation of $F(x, y) = f(g(x) + h(y))$

If $F(x, y)$ can be represented in two ways i.e.,

$$F(x, y) = f_3^{-1}[f_1(x) + f_2(y)] = g_3^{-1}[g_1(x) + g_2(y)] \tag{16}$$

$$x, y \in \boldsymbol{R} \ or \ [\alpha, \beta] \ with \ \alpha, \beta \in \boldsymbol{R}$$

where functions f_i, g_i $(i = 1, 2, 3)$ are monotonic and continuous, then

$$g_3^{-1} = f_3^{-1}\left(\frac{x - a - b}{c}\right); \ g_{1(x)} = cf_1(x) + a; \ g_2(y) = cf_2(y) + b \tag{17}$$

a, b and c here, are arbitrary constants are not identifiable and not needed as well because any set of a, b, c gives same $F(x, y)$ as in Eq. (13).

Also, it is seen that functional equations play a vital role in functional network; therefore, researchers must be well equipped with the knowledge of functional equations. These help to establish a connection amidst physical, engineering or economic problems.

Functional network has proved to be an enhancement in the computing field. It has also extended its scope to the medical and engineering fields and has been appropriately used by civil engineers. It has been used in structural engineering such as problems of deflection of the beam, multimodal function, weight of space trusses, forced vibration of the spring-mass system (Rajasekaran 2004). Also, geotechnical engineers have used this to contemplate the uncertain behavior of the soil and reduce complexity. The first application of this computing technique was to obtain time series prediction (Castillo and Guitierrez 1998). Thereafter, it was used to solve regression problems. In geotechnical engineering, it was used to model problems like the prediction of lateral load capacity, prediction of the factor of safety of slope, the uplift capacity of suction caisson in clay, swelling etc.

As done in the previous model, 4 variables c, Φ, γ and δ are fed to this model along with the calculated factor of safety. It is modelled with 70 per cent of the data and 30 per cent of the data is used for testing the model. Then the performance of the model is measured accordingly based on several criterions. It is expected to give a general and unique solution and better result as compared to ANN and ANFIS.

4.3 Genetic programming

Biological existence, this far, has proved to be an element of surprise for all. From the introduction of neural networks to genetic algorithms science has spread its wing in all the areas. Engineering is one among them. Computer science engineers with their apt knowledge were able to build software and adaptive programs working on the theory of biology solving big problems in all the fields with enormous datasets or big population. Working on the principles of natural selection and genetic combination has glorified the field of engineering. Researchers, over the years, have worked on the

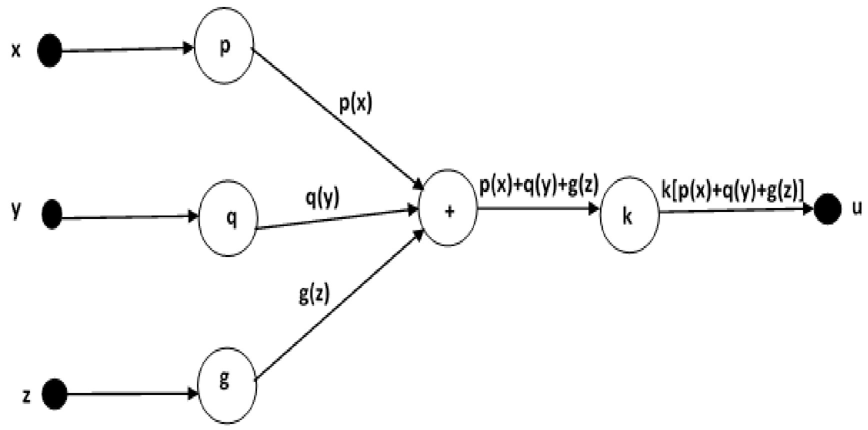

Fig. 3 Equivalent simplified network

evolutionary computing techniques which have given rise to five independent approaches (Babovic 1996):

• Evolution strategies,
• Evolutionary programming
• Genetic algorithms
• Classifier systems
• Genetic programming

Basic cycle for each of the above-mentioned approaches is:

• Creating a new mating pool from the already existing population using solutions based on their fitness. (calculated using objective/evaluation function).
• Genetic operators (crossover and mutation) are applied to the pairs selected from the pool created above to build offspring to be used as a member of the new population.
• Existing population is then replaced with the new population.

All the computing methods differ substantially although they are all broadly based on the simulation of natural evolution.

Genetic Programming (GP), a recently developed technique has its basic structure programmed following the more elementary evolutionary computing technique i.e. Genetic Algorithm (Holland 1992). But GP can be stated as an enhanced version of GA. GP uses algebraic expressions or hierarchical computer programs in contrary with GA in which model parameters are generally binary. The GP technique increases the efficacy of a GA paradigm by making the adapting structures more complex. In GP, large population is bred genetically. Darwinian principle of survival and reproduction of the fittest along with genetic operations (crossover and mutation) apt for computer programs are used for breeding. This technique approximately solves the problems using Darwinian natural selection and genetic operations (Koza 1992, Soh and Yang 2000).

4.3.1 Representation

Parse trees are used for representation of hierarchical programs instead of lines of code. For instance, algebraic expression '(m/5) +(6.7*n)' can be represented as in Fig. 5. The first and most important element that holds the tree together is called the *root node*. Nodes with the functions are called *interior nodes* and those with constants and/or variables are called *leaf nodes*. Tree structures can

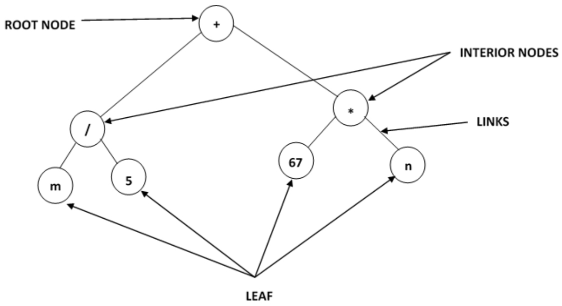

Fig. 4 Tree representation in GP

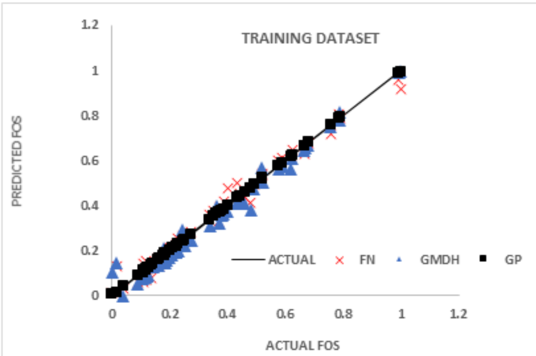

Fig. 5 Actual vs Predicted FOS for training datasets

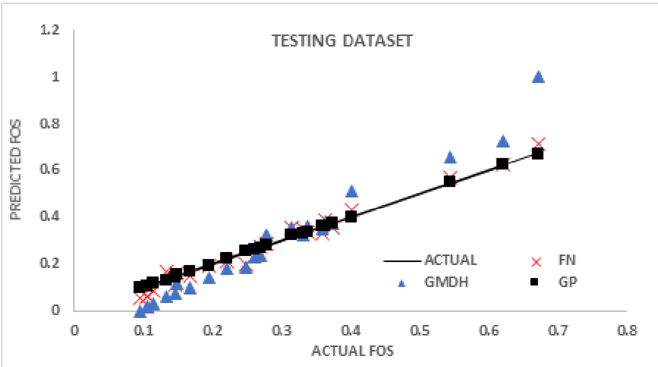

Fig. 6 Actual vs Predicted FOS for testing datasets

represent any complex hierarchical program if they are made rich with the functions. Functions may include mathematical functions, arithmetic operators, logical expressions, Boolean algebra or any other user defined functions. This process results in a set of trees representing programmed structures of different shapes and sizes each of them with varying fitness.

4.3.2 Crossover

In GP, crossover operator takes genetic material belonging to two parents and combine them to

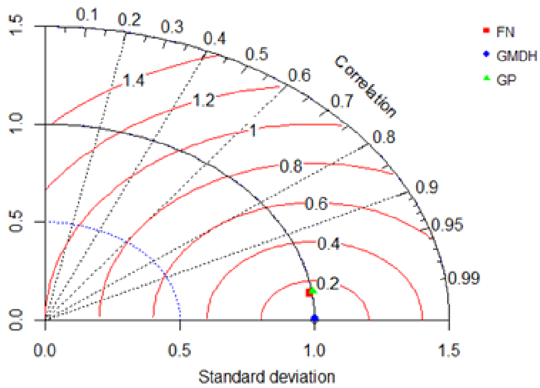

Fig. 7 Taylor diagram for training dataset

create new offspring as was the case in GA. Also, GP was able to create two new solutions from a single solution and this was an improvement over GA. GP allows identical parents to produce different offspring unlike GA (Holland 1992) which created identical offspring from identical parents.

4.3.3 Mutation

In GA mutation operator changes each element of the coded string with a small probability, p_m. whereas, in GP a node is randomly exchanged with another node or a sub-tree. This is called *allele mutation* (Savic *et al*. 1999).

It has gained light in several branches of civil engineering for example in water resource, geotechnical and structural engineering. In this study, GP is used to model reliability of retaining wall using the Factor of safety data generated by sliding criteria of the retaining wall. We have used normalized values of c, Φ, δ and γ as input variables and FOS as output and then we have predicted FOS and compared the results with the predicted values of other models used in this study and the actual observed values.

The expression used by Genetic Programming to evaluate the problem with 4 variables Φ, c, γ and δ as x_1, x_2, x_3 and x_4 respectively is as follows

$$0.0968x_2 - 0.0968x_3 + 0.2873x_4 + 0.01933(x_4x_1^2)^2 + 0.0594x_3^2 + 0.4483x_4(x_2 + x_4^2) + 0.05207x_2x_3x_4(x_2 - 5.469) - 0.04859 \quad (18)$$

5. Discussions

This section of the paper summarizes the result generated by all the models and compares the generated factor of safety by all three models (FN, GMDH, GP) depending upon their distribution and squared mean errors. In Figs. 6 and 7 actual measured values are compared with the predicted outputs from different models. One is for the training dataset and the other is for testing dataset respectively. In figure 6 almost all the points converge to the line of actual output which describes perfection but when data is tested in the models it is observed that many points went off the track

Table 1 Asymptotic p-value and Asymptotic significance value for different models in their training and testing periods

MODELS	p value	q value
FN (training)	0.9982	0.9884
GMDH (training)	0.9490	0.5984
GP (training)	1	0.9791
FN (testing)	0.8695	0.8944
GMDH (testing)	0.2808	0.5601
GP (testing)	1	0.9919

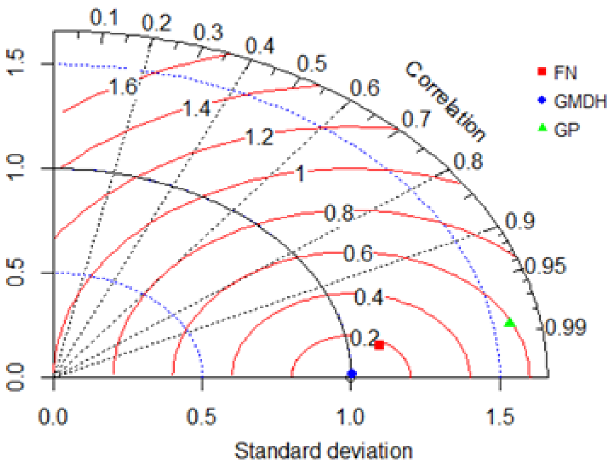

Fig. 8 Taylor diagram for testing dataset

(Fig. 7). It is visible that FN and GP outperformed and GMDH didn't predict FOS well as from the learning of the structure of different models it is known that models establish relation between inputs while training the data and generate outputs accordingly. Inability to develop relation between different inputs might be a reason behind not predicting an agreeable result.

5.1 Taylor diagram

Taylor diagram is a graphical representation of different patterns (Taylor 2001). It statistically summarizes the overlapping of different patterns based upon their correlation, root mean square error and standard deviations. This study creates a framework that builds a relation between the reference data (observed data) and modelled data. Figs. 8 and 9 contain Taylor diagrams for both training and testing datasets respectively for FN, GMDH and GP. For training dataset, we can see that GMDH has exquisitely outperformed GP and FN and has generated better results with the same standard deviation as of the observed. GP and FN have performed equally well with less correlation and higher RMS error. Whereas, in Fig. 9 it is visible that for testing data GP modelled FOS doesn't have good correlation with observed data as compared to other two. Also, it has higher standard deviation than the observed data. In parallel, GMDH has predicted the output quite

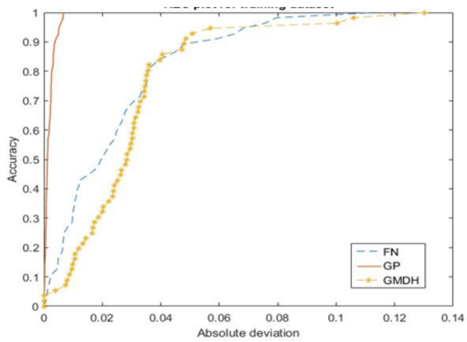

Fig. 9 REC plot for training dataset

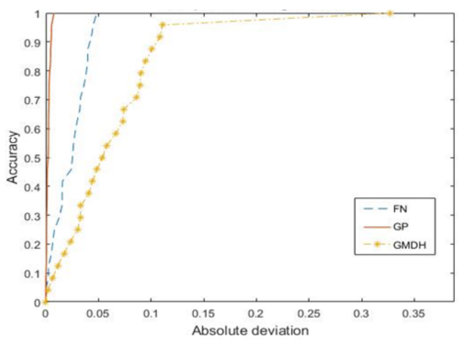

Fig. 10 REC plot for testing dataset

well and has generated data that overlapped the observed data. Also, FN model lies near to the observed data but with a slight shift towards left which shows an increase in standard deviation along with less correlation with the observed data.

5.2 AOC-REC curve

Regression Error Characteristic Curve is a solution for regression problems (Bi and Bennett 2003). In this graph tolerance is plotted on the x-axis and accuracy of the model is plotted on the y-axis. By accuracy here it means percentage of predicted points that fit within the tolerance limit. To approximately calculate expected error for regression problem we take AOC (Area Over REC Curve) value into consideration. AOC can also be used for estimating other statistical values for regression models. As AOC values quantify the error in the model, lower the AOC value better is the model. Figs. 10 and 11 have showcased the REC curves for all the three models (FN, GMDH, GP) in their training and testing phase respectively.

Table 1 clarifies that AOC values for all the models are quite less but here GP has proved to be better than rest two models in training period and most importantly in the testing period.

5.3 Anderson-Darling Normality test (AD test)

This test is used to analyze if data comes from a particular type of probability distribution or

Table 2 Asymptotic p-value for different models using AD test

MODEL	p value
FN (training)	0.9982
GMDH (training)	0.9490
GP (training)	1
FN (testing)	0.8695
GMDH (testing)	0.2808
GP (testing)	1

Table 3 Asymptotic significance value for different models using MWW test

MODEL	q value
FN (training)	0.9884
GMDH (training)	0.5984
GP (training)	0.9791
FN (testing)	0.8944
GMDH (testing)	0.5601
GP (testing)	0.9919

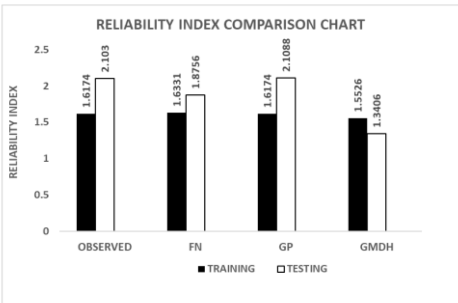

Fig. 11 Reliability Index comparison chart

not. This test was developed by Theodore Wilbur Anderson and Donald A. Darling. This test calculates Anderson statistical value that helps in comparing the observed value and the predicted value so as to know if they fit in the normal distribution or not (Scholz *et al*. 2019, Ceryan *et al*. 2012). The AD test rejects the hypothesis of normality when the p value is $\leq$ 0.05 (level of significance).

From Table 2 it is seen that GP has performed extraordinarily well with a significance value of unity. Also, observation tells that FN has performed well in the testing stage when compared to GMDH which has a poor significance value comparatively. But as the values lie above 0.05, models are acceptable.

5.4 Mann-Whitney-Wilcoxon test (MWW test)

Homogeneity of models is examined using a statistical criterion known as Mann-Whitney-

Table 4 Table for Performance Validation

Performance validation parameters	Condition	Result					
		Training			Testing		
		FN	GMDH	GP	FN	GMDH	GP
R	0<=R<=1	0.9904	0.9894	0.9999	0.9909	0.9865	0.9998
k	0.85<k<1.15	0.9974	0.9792	1.0000	1.0000	1.0343	1.0003
k'	0.85<k'<1.15	1.0000	1.0182	1.0000	0.9976	0.9452	0.9997
R_o^2	Value near to 1	1.0000	0.9997	1.0000	1.0000	0.9927	1.0000
$R_o'^2$	Value near to 1	1.0000	0.9996	1.0000	0.9999	0.9590	1.0000
m	m<0.1	-0.0195	-0.0213	-0.0002	-0.0185	-0.0201	-0.0004
n	n<0.1	-0.0195	-0.0211	-0.0002	-0.0184	0.0145	-0.0004

Wilcoxon test developed by Mann and Whitney. It is a non-parametric test that helps in determining whether two independent models follow the same distribution or not (Ceryan *et al*. 2013). The test static value is calculated and its asymptotic significance (taken as 'q' in this paper) is taken into consideration for comparing the models. In table 3 it is visible that none of the models can be rejected as all the q values are >0.05. also, it can be seen that GP has performed well when compared to other models and GMDH here has low significance value which shows its inability to produce results which overlap observed pattern.

5.5 Reliability index

As this paper is the study of reliability analysis of a retaining wall therefore we calculate reliability index, β for different models which emphasizes on the probability of failure (Samui *et al*. 2011). Universally, value 3 and 4 is adopted for β. Patterns with reliability index value 3 or 4 are considered good. Here, the value of β for different models are compared with the β for the observed pattern. Fig. 12 compares the different model based on the β value. We can see that the result generated by GP almost coincides that of the observed data and β of FN is near to the β of the observed dataset. This study concludes that GP is a better model among three.

5.6 Performance measure (model validation)

For reliability analysis correlation coefficient 'R' is of sheer importance. It predicts the efficiency of the models used. There are several other statistical criterions which can be used to measure the performance of a models. Eqs. (18) to (24) shows the calculation of such parameters and table 4 shows the validation of each parameter along with the results obtained for different models (Roy and Roy 2008, Golbraikh and Tropsha 2002). Table 4 clarifies that all the models can be accepted and used as none of them can be rejected. But if they are compared among each other GP has outshone and has predicted the results that overlaps the actual calculated result.

$$k = \frac{\sum_{i=1}^{n} d_i y_i}{\sum_{i=1}^{n} d_i^2} \tag{19}$$

$$k' = \frac{\sum_{i=1}^{n} d_i y_i}{\sum_{i=1}^{n} y_i^2} \tag{20}$$

$$R = \frac{\sum_{i=1}^{n}(y_i - \overline{y_i})(d_i - \overline{d_i})}{\sqrt{\sum_{i=1}^{n}(y_i - \overline{y_i})^2 \sum_{i=1}^{n}(d_i - \overline{d_i})^2}} \tag{21}$$

$$R_o^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - d_i^o)^2}{\sum_{i=1}^{n}(y_i - \overline{y_i})^2}, where\ \ d_i^o = ky_i \tag{22}$$

$$R'^2_o = 1 - \frac{\sum_{i=1}^{n}(d_i - y_i^o)^2}{\sum_{i=1}^{n}(d_i - \overline{d_i})^2}, where\ \ y_i^o = k'd_i \tag{23}$$

$$m = \frac{R^2 - R_o^2}{R^2} \tag{24}$$

$$n = \frac{R^2 - R'^2_o}{R^2} \tag{25}$$

d_i and y_i are actual and predicted values respectively, R is the correlation coefficient, k and k' are the slopes for the regressions of d against y or y against d through origin, n is the total number of observations. R_o and $R'_o$are the optimum correlation coefficient for the regression lines which are different from R. m and n are the parameters showing the nearness of the value R_o and R'_o to the value R.

From the comparison graph, it is visible that the result generated by the GP almost overlaps the actual observed value. Also, the deviation of the values can be seen more in the GMDH plot. From Taylor diagram we can see that all the, models lie near to the actual data point but GP varies with quite a distinct value of standard deviation. Whereas correlation is almost same in all the three models. From REC plots and their AOC values, it can be seen that less value of AOC of GP model makes it more accurate. Also, reliability index value, β almost match the value of β of original dataset. GP has also proved itself better when p and q values were calculated from AD test and MWW test; it gave best results. Several other statistical parameters used for the measure of the performance of the models show that GP has rode past rest two and has shown the best results in all the analysis and operations performed.

6. Conclusions

This study has emphasized on the predictive capability of the models (FN, GMDH and GP) and reliability analysis of the retaining wall. Several tests and calculations have been performed to compare the performance of the models in the above section. Geotechnical engineers have been trying for years, as can be seen from the research works, to compete the lengthy calculation and make it easy for other engineers to tackle with the problems which stand stubborn and increase in number and extent each day. Here, in this paper, Functional Network, Group Method of Data Handling and Genetic Programming have been used to model the problem of retaining wall and predict factor of safety from the data used for calculating the factor of safety of the wall using sliding criterion (c, Φ, γ, and δ). Every model works on different patterns with distinct methodology. Therefore, in other terms, here different methodologies are compared based on some parameters and graphs. It can be seen and concluded that Genetic Programming has outperformed and has proved to be best among the three models used in this study. Also, the other two didn't fail and can't be rejected in any manner or on any scale. They have performed well but they didn't match the predictive capability of GP. Hence, methodology used for GP which was an extended version of GA (used in GMDH) turned out to fit in aptly in creating relations between input variables fed to the model and give an output which coincides well with the actual observation when tested. This study and its researchers have keenly worked on the models and have put their wits in analyzing and studying the structure of each model. It can be concluded that GP gave favorable results among the three models used and can be further used for extensive design works of retaining wall and other retaining structures.

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