Interaction and Multiscale Mechanics, *Vol. 6, No. 2 (2013) 83-105*
DOI: http://dx.doi.org/10.12989/imm.2013.6.2.083

Multi-scale finite element analysis of acoustic waves using global residual-free meshfree enrichments

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(Received March 1, 2013, Revised April 8, 2013, Accepted May 5, 2013)

Abstract. In this paper, a multi-scale meshfree-enriched finite element formulation is presented for the analysis of acoustic wave propagation problem. The scale splitting in this formulation is based on the Variational Multi-scale (VMS) method. While the standard finite element polynomials are used to represent the coarse scales, the approximation of fine-scale solution is defined globally using the meshfree enrichments generated from the Generalized Meshfree (GMF) approximation. The resultant fine-scale approximations satisfy the homogenous Dirichlet boundary conditions and behave as the "global residual-free" bubbles for the enrichments in the oscillatory type of Helmholtz solutions. Numerical examples in one dimension and two dimensional cases are analyzed to demonstrate the accuracy of the present formulation and comparison is made to the analytical and two finite element solutions.

Keywords: acoustic; multi-scale; finite element; Helmholtz; meshfree

1. Introduction

There are many important scientific and engineering problems that involve dealing with information on multiple spatial and temporal scales. The time-harmonic acoustics governed by the Helmholtz equation in structural acoustics and electromagnetic applications is one of such problems which require simultaneous resolution of different scales spanned over a wide frequency spectrum. In particular, evanescent waves play a significant role in the design of various nano-optical and nano-biological sensors, and thus demand advanced modeling technique because of their multi-scale nature. Numerical analysis by the standard finite element method of such problems in the medium or higher frequency regimes is either computationally unfeasible or simply unreliable, particularly in the presence of evanescent waves. This is because the standard low-order continuous Galerkin finite element method fails to adequately control numerical dispersion errors (Belytschko and Mullen 1978) when the wave number increases. As a consequence, the finite element method not only inaccurately approximates the oscillatory part of the solution, but also experiences a notorious pollution error (Ihlenburg and Babuska 1995), a numerical error related to the phase difference between the exact and finite element solutions. To minimize the pollution error and obtain an accurate solution in the finite element analysis of Helmholtz equation, the resolution of mesh should be adjusted to the wave number according to

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http://www.techno-press.org/?journal=imm&subpage=7 ISSN: 1976-0426 (Print), 2092-6200 (Online)

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