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Vehicle/bridge interactions of a rail suspension bridge considering support movements

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Abstract. This paper is intended to investigate interaction response of a train running over a suspension bridge undergoing support settlements. The suspension bridge is modeled as a single-span suspended beam with hinged ends and the train as successive moving oscillators with identical properties. To conduct this dynamic problem with non-homogeneous boundary conditions, this study first divides the total response of the suspended beam into two parts: the static and dynamic responses. Then, the coupled equations of motion for the suspended beam carrying multiple moving oscillators are transformed into a set of nonlinearly coupled generalized equations by Galerkin's method, and solved using the Newmark method with an incremental-iterative procedure including the three phases: *predictor, corrector,* and *equilibrium-checking*. Numerical investigations demonstrate that the present iterative technique is available in dealing with the dynamic interaction problem of vehicle/bridge coupling system and that the differential movements of bridge supports will significantly affect the dynamic response of the running vehicles but insignificant influence on the bridge response.

Keywords: high speed train; support settlement; resonance; suspension bridge.

1. Introduction

Suspension bridges are usually used to cross a deep valley or a wide creek for its long-span feature. Meanwhile, a rail suspension bridge should be designed so stiff that it has the ability to carry the heavy weights of running trains over it. After construction of rail suspension bridges, however, differential settlements at bridge foundations would become one of key issues being considered for running safety of trains and normal operation of railway system. The reasons of differential settlement for bridge structures are attributed to: earthquake shaking, different soil conditions below bridge foundations, loading capacity of sub-soil in construction site, compaction of earth fill, and the shocks and vibrations coming from railway traffic (Yau 2009a). For railway bridges, these factors may distort rail geometry and further result in railway track sinking, which will directly reduce the ride quality and maneuverability of trains traveling over the bridges (Yau 2009b).

Over the past several decades, there are many research works (Chatterjee *et al*. 1993, Hayashikawa and Watanabe 1982, Vellozzi 1967, Yau and Fryba 2007, Yau and Yang 2008) devoted

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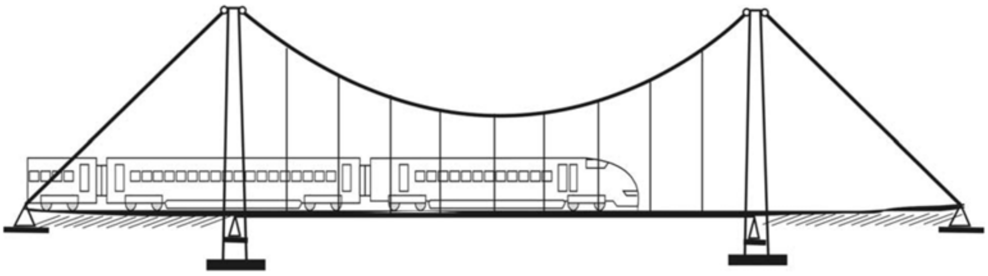

Fig. 1 Schematic diagram of a suspension bridge traveled by a train

to vehicle-induced vibration of suspension bridges. One of key findings indicated that the cable tensions of *short* or *medium span* suspension bridges would be amplified significantly when subjected to moving loads. In this study, a train is simulated as a sequence of moving sprung mass units and the suspension bridge as a single-span suspended beam with hinged supports (see Fig. 1). To resolve the dynamic problem for a suspended beam undergoing support settlement, the total response of the beam is decomposed into two parts: the *static* response due to permanent support settlement and the *dynamic* component due to inertial effect of beam vibration (Yau and Fryba 2007, Yau 2009a). An exact solution of pseudo-*static* displacement of a single suspended beam shaken by multiple support motions presented by Yau (2009b) is employed to represent the static response of the suspended beam undergoing support movements. For the purpose of numerical computation, this paper employs Galerkin's method to convert the governing equations containing sprung mass units into a set of differential equations in generalized systems, and then solve the two sets of differential equations using an iterative approach with Newmark's finite difference scheme (Newmark 1959). Numerical studies indicate that the effect of differential settlement is generally small on the bridge response, but may produce a significant amplification on the vehicle's response of a moving train. Such a fact should be taken into account in evaluating the operation performance of a railway system.

2. Formulation

In this study, the dynamic behavior of a suspension bridge carrying a moving train is limited to vertical vibration of a single-span suspended beam with hinged supports. Based on the deflection theory of small deformation (Pugsley 1957, Yau and Fryba 2007, Yau and Yang 2008), basic simplifications for the analytical mode of suspended beam and moving train are outlined as follows:

(1) The stiffening girder is modeled as a linear elastic Bernoulli-Euler beam with uniform cross section;

(2) As shown in Fig. 2, the bridge towers supporting the stiffening girder and suspension cable are assumed so rigid that their deformations during vibrations are negligible;

(3) The suspension cable is assumed to be capable of carrying all the dead loads of the stiffening girder with the aid of inextensible vertical hangers so that the suspended beam is in an unstressed state before the action of live loads and its vertical deflection is identical to the suspension cable's;

(4) The train passing over the suspended beam comprises several identical cars, and each car is modeled as two identical moving oscillators (see Fig. 2), each oscillator is used to model either the front or rear half of a carriage;

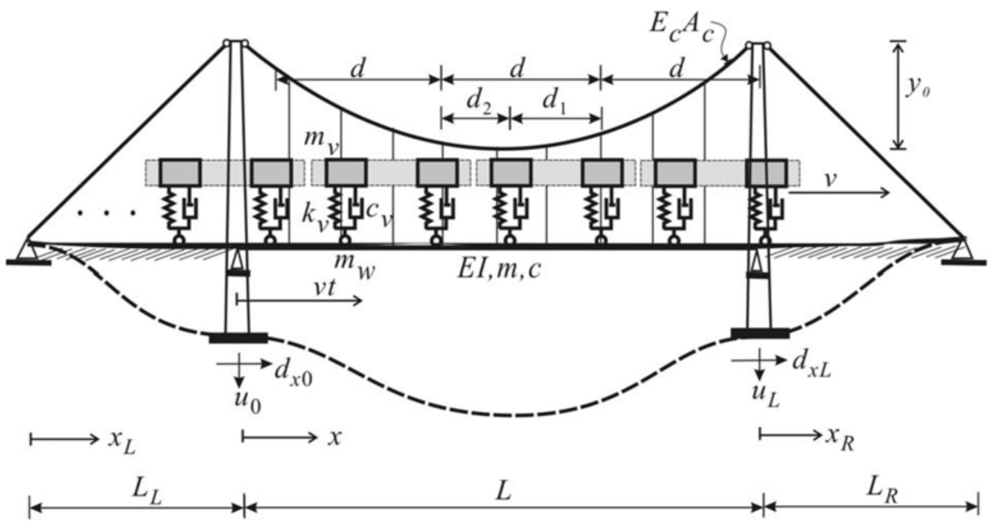

Fig. 2 A series of sprung masses cross a suspended beam with support movements

(5) According to the design specification written by THSR (1999), allowable angular distortion between any two points along a bridge span due to ground settlement should not exceed 1/1000.

2.1 Governing equations of motion

For a simply supported beam suspended by a parabolic cable with a cable sag y_0 (Irvine 1981), the governing equation of motion for the suspended beam undergoing differential support movements can be described as (Yau 2009b)

$$m\ddot{u} + c\dot{u} + EIu'''' - (T + \Delta T_s)u'' + \alpha\int_0^L u dx = p(x,t) + \frac{\alpha L}{2}[u_0 + u_L] - \kappa(d_{xL} - d_{x0}) \tag{1}$$

where $(\bullet)' = \partial(\bullet)/\partial x$, $(\dot{\bullet}) = \partial(\bullet)/\partial t$, m = mass of the beam and cable per unit length along x-axis, c = damping coefficient, $u(x,t)$ = vertical deflection of the beam, EI = flexural rigidity of the beam, T = horizontal component in the initial cable tension (due to dead loads), $p(x,t)$ = loading function of moving oscillators, and

$$T = \frac{-mg}{y''} = \frac{mgL^2}{8y_0}, \Delta T_s = \frac{E_cA_c}{L_c}\left[(d_{xL} - d_{x0}) - \frac{4y_0}{L}(u_0 + u_L)\right]$$

$$\alpha = \left(\frac{8y_0}{L^2}\right)^2 \frac{E_cA_c}{L_c}, \kappa = \left(\frac{8y_0}{L^2}\right)\frac{E_cA_c}{L_c}, L_c = \int_0^L \left(\frac{ds}{dx}\right)^3 dx = \int_0^L (\sqrt{1+y'^2})^3 dx \tag{2}$$

with E_c = elastic modulus of the cable, A_c = area of the cable, L_c = the effective length of the cable. Meanwhile, (u_0, u_L) and (d_{x0}, d_{xL}) represent the vertical and horizontal support movements at the left and right bridge supports, respectively. Consider the differential movements at bridge supports depicted in Fig. 2, the non-homogeneous boundary conditions for the suspended beam with hinged ends are given as follows:

$$u(0,t) = u_0, u(L,t) = u_L; u_x(0,t) = d_{x0}, u_x(L,t) = d_{xL}$$
$$EIu''(0,t) = EIu''(L,t) = 0 \tag{3}$$

Since the horizontal component of cable force in Eqs. (1) and (2) is dependent on both the beam deflection $u(x,t)$ and support movements $(u_0, u_L, d_{xL}, d_{x0})$, the governing equation of motion in Eq. (1) is non-linear in nature.

2.2 Equations of moving sprung mass units

As shown in Fig. 2, a sequence of identical sprung masses is crossing a single-span suspended beam at constant speed v. In this study, each sprung mass unit is used to model either the front or rear half of a carriage, which is composed of a lumped mass supported by a spring-dashpot system. Let the oscillator model has the following properties: m_w = mass of wheel-set, m_v = lumped mass, c_v = damping coefficient, and k_v = stiffness coefficient. Consider the regular nature of sprung mass units shown in Fig. 2, the load function $p(x,t)$ is given as [Yau 2009b]:

$$p(x,t) = \sum_{k=1}^{N} (P - m_v\ddot{u}_{vk} - m_w\ddot{u})\delta(x - x_k)\left[H(t-t_k) - H\left(t - t_k - \frac{L}{v}\right)\right]$$
$$m_v\ddot{u}_{vk} + c_v\dot{u}_{vk} + k_v u_{vk} = f_{vk}$$
$$f_{vk} = \begin{cases} k_v[u(x_k,t) + \gamma(x_k)] + c_v\dot{u}(x_k,t) & 0 \le x_k (= v(t - t_g - t_k)) \le L \\ k_v[\gamma(x_k) + g_s(x_k)] & x_k < 0, x_k > L \end{cases} \tag{4a-c}$$

in which, $P = -(m_v + m_w)g$ = lumped weight of a moving oscillator, δ = Dirac's delta function, $H(t)$ = unit step function, $k = 1, 2, 3, \ldots, N$-th moving load on the beam, t_k = arrival time of the k-th oscillator entering the beam, u_{vk} = vertical displacement of the k-th lumped mass, f_{vk} = interaction force existing between the beam and the wheel mass of the k-th moving oscillator, $\gamma(x_k)$ = track irregularity (vertical profile), $g_s(x_k)$ = vertical ground settlement profile, and x_k = position of the k-th load along the rail line, as defined in Eq. (4b). As shown in Eq. (4c), $x_k < 0$ represents the k-th oscillator is entering to the suspended beam, $0 \le x_k (= v(t - t_k)) \le L$ running on the beam, and $x_k > L$ departing the beam.

3. Method of solution

As indicated in Eqs. (1) and (3), it is a partial integro-differential equation with time-dependent boundary condition. For this problem, this study divides the total deflection response $u(x,t)$ of the suspended beam into two parts: the *static* displacement $U(x)$ and the *dynamic* deflection $u_d(x, t)$ (Yau and Fryba 2007, Yau 2009b), or

$$u(x,t) = U(x) + u_d(x,t) \tag{5}$$

Here, $U(x)$ represents the structure deformation caused by the *relative* support displacements applied statically (Yau and Fryba 2007), and $u_d(x,t)$ the *dynamic* deflection due to inertia effect of the structure. By this concept, substituting Eq. (5) into Eq. (1) and discarding all the dynamic terms and external loads, the static equation of motion in terms of the static displacement $U(x)$ is written as follows:

$$EI\frac{\partial^4 U}{\partial x^4} - (T + \Delta T_s)\frac{\partial^2 U}{\partial x^2} + \alpha\int_0^L U dx = \frac{\alpha L}{2}[u_0 + u_L] - \kappa(d_{xL} - d_{x0}) \tag{6}$$

And the static response $U(x)$ in Eq. (6) has to satisfy the following non-homogeneous boundary conditions (Yau and Fryba 2007) :

$$U(0) = u_0,\ U(L) = u_L,\ EIU''(0) = EIU''(L) = 0 \tag{7}$$

As the closed form solution solved by Yau (2009b), the static displacement is:

$$U(x, t) = \left[u_0 + (u_L - u_0)\frac{x}{L}\right] + \left[\frac{d_{xL} - d_{x0}}{8y_0/L} - (u_L + u_0)\right]\frac{\beta(x)}{\chi} \tag{8}$$

where

$$\lambda^2 = \frac{T + \Delta T}{EI} > 0,\ \beta(x) = 1 + \frac{\lambda^2 x(x-L)}{2} - \frac{\cosh\lambda(x - L/2)}{\cosh(\lambda L/2)}$$

$$\chi = \frac{(T + \Delta T_s)(\lambda L)^2}{\alpha L^3} + \frac{(\lambda L)^2}{12} + \frac{\tanh(\lambda L/2)}{\lambda L/2} - 1 \tag{9}$$

The static displacement shown in Eq. (8) reveals that the first term represents the *rigid body* displacement due to *vertical* support movements, and the second term means the *static* natural deformation caused by the relative support movements of horizontal and vertical components. It is emphasized that the non-uniform horizontal support movements may affect the increments of cable force. On the other hand, introducing Eqs. (5) and (6) into Eq. (1), the equation of motion for the *dynamic* deflection $u_d(x, t)$ of the suspended beam is converted into the following equation:

$$m\ddot{u}_d + c\dot{u}_d + EIu_d'''' - Tu_d'' + \alpha\int_0^L u_d dx = p(x, t) \tag{10}$$

Since the static displacement $U(x)$ has satisfied the non-homogeneous boundary conditions shown in Eq. (7), introducing Eqs. (5) and (7) into Eq. (3) yields the following *homogeneous* boundary conditions for the dynamic deflection $u_d(x,t)$:

$$\begin{aligned} u_d(0, t) &= u_d(L, t) = 0 \\ EIu_d''(0, t) &= EIu_d''(L, t) = 0 \end{aligned} \tag{11}$$

Next, the dynamic deflection (u_d) can be approximated by a series of sinusoidal functions (Yau and Yang 2008):

$$u_d(x, t) = \sum_{n=1} q_n(t)\sin\frac{n\pi x}{L} \tag{12}$$

where $q_n(t)$ means the generalized coordinate associated with the n-th assumed mode of the suspended beam. By Galerkin's method, the following generalized equation of motion for the n-th *dynamic* system of the suspended beam is given:

$$m\ddot{q}_n + \frac{2m_w}{L}\Big[\sum_{n=1} \ddot{q}_n(t)\sin\frac{n\pi x_k}{L}\Big]\psi_n(\varpi_n, t) + c\dot{q}_n + k_n q_n + \Pi_n$$

$$= \sum_{k=1}^{N} F_k(\varpi_n, v, t) - \sum_{k=1}^{N} F_{vk}(\varpi_n, v, t) \tag{13}$$

$$k_n = \left(\frac{n\pi}{L}\right)^2\left[\left(\frac{n\pi}{L}\right)^2 EI + (T + \Delta T_s)\right]$$

$$\Pi_n = \frac{2\alpha L}{n\pi^2}(1 - \cos n\pi)\Big[\sum_{k=1} \frac{1}{k}(1 - \cos k\pi) q_k\Big] \tag{14}$$

with the coupled equation of the k-th sprung mass unit in Eqs. (4). Here, the generalized forces of $F_k(\varpi_n, v, t)$ and $F_{vk}(\varpi_n, v, t)$ with respect to the k-th sprung mass unit are respectively expressed as

$$F_k(\varpi_n, v, t) = \frac{2P}{L}\psi_n(\varpi_n, t),\ F_{vk}(\varpi_n, v, t) = \frac{2m_v\ddot{u}_{v,k}}{L}\psi_n(\varpi_n, t)$$

$$\psi_n(\varpi_n, t) = \sin\varpi_n(t - t_k)[H(t - t_k) - H(t - t_k - L/v)] \tag{15}$$

and $\varpi_n (= n\pi v/L)$. Combining Eqs. (4) and (13) yields the following equation of motion for the vehicle/bridge interaction model

$$[m_b(t)]\{\ddot{q}\} + [c_b]\{\dot{q}\} + [k_b]\{q\} = \{p\}$$

$$[m_v]\{\ddot{u}_v\} + [c_v]\{\dot{u}_v\} + [k_v]\{u_v\} = \{f_v\} \tag{16}$$

where $\{q\}$ = generalized coordinate vector of the suspended beam, $[m_b]$ = generalized beam mass matrix including the sprung masses moving on the suspended beam, $[c_b]$ = generalized beam damping matrix, $[k_b]$ = generalized beam stiffness matrix, $\{p\}$ = generalized force vector acting on the generalized beam system; $\{u_v\}$ = vehicle displacement vector, $\{f_v\}$ = exciting force vector, and $([k_c],[c_c],[m_c])$ = structural matrices of the vehicles corresponding to mass, damping, and stiffness. To compute the dynamic response of vehicle-bridge interactions for a suspended beam undergoing support movements, an incremental-iterative procedure needs to be carried out in Section 5.

4. Simulation of vertical ground settlement profile

When a train crosses a suspension bridge subject to multi support settlements, the train may encounter simulation problem of “falling” or “jumping” if the transition length (see Fig. 2) of ground surface settlement is assumed to be zero in analysis. Because of this, as shown in Fig. 2, the following cubic functions will be adopted to simulate the vertical profile of ground settlement at both the left and right sides of the bridge:

$$g_s = \begin{cases} u_0[3(x_L/L_L)^2 - 2(x_L/L_L)^3] & 0 \le x_L/L_L \le 1 \\ u_L[1-3(x_R/L_R)^2 + 2(x_R/L_R)^3] & 0 \le x_R/L_R \le 1 \end{cases} \tag{17}$$

Here, x_L = the distance measured from the left reference point, L_L = the length from the left reference point to the left tower support of the bridge, and x_R = the distance measured from the right tower support, L_R = the length from the right tower support of the bridge to the right reference point, u_0 = vertical support settlement at the left tower, and u_L = vertical support settlement at the right tower.

5. Strategy for incremental-iterative dynamic interaction analysis

In conducting the dynamic response analysis of a structure with support settlements, two sets of structural responses have to be solved; one is the static response due to support settlements and the other the dynamic component caused by the vibration of the structure. By adopting the discretized process based on the Newmark method (Newmark 1959), we can transform the vehicle-bridge system shown in Eq. (16) into equivalent stiffness equations tailored for the ith iteration of the incremental step at time $t + \Delta t$ as

$$\begin{aligned} [K_{b,\,eq}]\{\Delta q^i_{t+\Delta t}\} &= \{\Delta p^i_{t+\Delta t}\} \\ [K_{v,\,eq}]\{\Delta u^i_{v,\,t+\Delta t}\} &= \{\Delta f^i_{v,\,t+\Delta t}\} \end{aligned} \tag{18}$$

where the equivalent stiffness matrices are given as follows:

$$\begin{aligned} [K_{eq}] &= a_0[m_b] + a_1[c_b] + [k_b] \\ [K_{v,\,eq}] &= a_0[m_v] + a_1[c_v] + [k_v] \end{aligned} \tag{19}$$

and $(\{\Delta q^i_{t+\Delta t}\}, \{\Delta u^i_{v,\,t+\Delta t}\})$ = displacement increments generated at the incremental step of the ith iteration, $(\{\Delta p^{i-1}_{t+\Delta t}\}, \{\Delta f^{i-1}_{v,\,t+\Delta t}\})$ = *unbalanced forces* resulting from the last iterative step, and $a_k|_{k=0,1,2,\ldots 7}$ denote the Newmark coefficients. For the first iteration ($i = 1$), $\{\Delta p^i_{t+\Delta t}\} \equiv \{\Delta p_{t+\Delta t}\}$ and $\{\Delta f^i_{v,\,t+\Delta t}\} \equiv \{\Delta f_{v,\,t+\Delta t}\}$ represent the *load increments* of the suspended beam and moving oscillators at the beginning of the incremental step, respectively. For the following iterations, the unbalanced force $\{\Delta p^{i-1}_{t+\Delta t}\}$ is equal to the difference between the external force $\{p_{t+\Delta t}\}$ and the effective internal forces $\{f^{i-1}_{t+\Delta t}\}$ for all the generalized system of the suspended beam at time $t + \Delta t$, i.e.,

$$\{\Delta p^{i-1}_{t+\Delta t}\} = \{p_{t+\Delta t}\} - \{R^{i-1}_{t+\Delta t}\}$$

$$[R^{i-1}_{t+\Delta t}] = \begin{cases} [k_b]\{q^{i-1}_{t+\Delta t}\} - [m_b](a_2\{\dot{q}^{i-1}_{t+\Delta t}\} + a_3\{\ddot{q}^{i-1}_{t+\Delta t}\})\ for\ i = 1 \\ \quad -[c_b](a_4\{\dot{q}^{i-1}_{t+\Delta t}\} + a_5\{\ddot{q}^{i-1}_{t+\Delta t}\}) \\ [k_b]\{q^{i-1}_{t+\Delta t}\} + [m_b]\{\ddot{q}^{i-1}_{t+\Delta t}\} + [c_b]\{\dot{q}^{i-1}_{t+\Delta t}\} \quad for\ i > 1 \end{cases} \tag{20}$$

and the unbalanced force vector $\{\Delta f^{i-1}_{v,\,t+\Delta t}\}$ for the vehicle at time $t + \Delta t$ is expressed as

$$\{\Delta f^{i-1}_{v,\,t+\Delta t}\} = \{f^i_{v,\,t+\Delta t}\} - \{r^{i-1}_{v,\,t+\Delta t}\}$$

$$[r^{i-1}_{v,t+\Delta t}] = \begin{cases} [k_c]\{u^{i-1}_{v,t+\Delta t}\} - [m_c](a_2\{\dot{u}^{i-1}_{v,t+\Delta t}\} + a_3\{\ddot{u}^{i-1}_{v,t+\Delta t}\}) \\ \quad -[c_c](a_4\{\dot{u}^{i-1}_{v,t+\Delta t}\} + a_5\{\ddot{u}^{i-1}_{v,t+\Delta t}\}) & for\ i = 1 \\ [k_c]\{u^{i-1}_{v,t+\Delta t}\} + [m_c]\{\ddot{u}^{i-1}_{v,t+\Delta t}\} + [c_c]\{\dot{u}^{i-1}_{v,t+\Delta t}\} & for\ i > 1 \end{cases} \tag{21}$$

where $(\{R^{i-1}_{t+\Delta t}\}, \{r^{i-1}_{v,t+\Delta t}\})$ = effective resistant force vectors of the generalized beam system and the vehicles. Let us define the root mean square of all the sum of unbalanced forces as

$$\tau_{tol} = [\Sigma_{k=1\ldots}(\Delta f^{i-1}_{vk,t+\Delta t})^2 + \Sigma_{n=1\ldots}(\Delta p^{i-1}_{n,t+\Delta t})^2]^{1/2} \tag{22}$$

When τ_{tol} is larger than a preset tolerance (say 10^{-3}), iteration for removing the unbalanced forces involving the predictor and corrector should be repeated. With the flowchart shown in Fig. 3, an incremental-iterative procedure for nonlinear analysis of the vehicle-bridge system considering support settlements is summarized as follows (Yau 2009b):

(1) Solve the static displacement $U(x)$ given in Section 3. In this stage, the sprung masses that have not yet entered the suspended beam are only subjected to rail irregularities and transition ground settlements shown in Section 4;

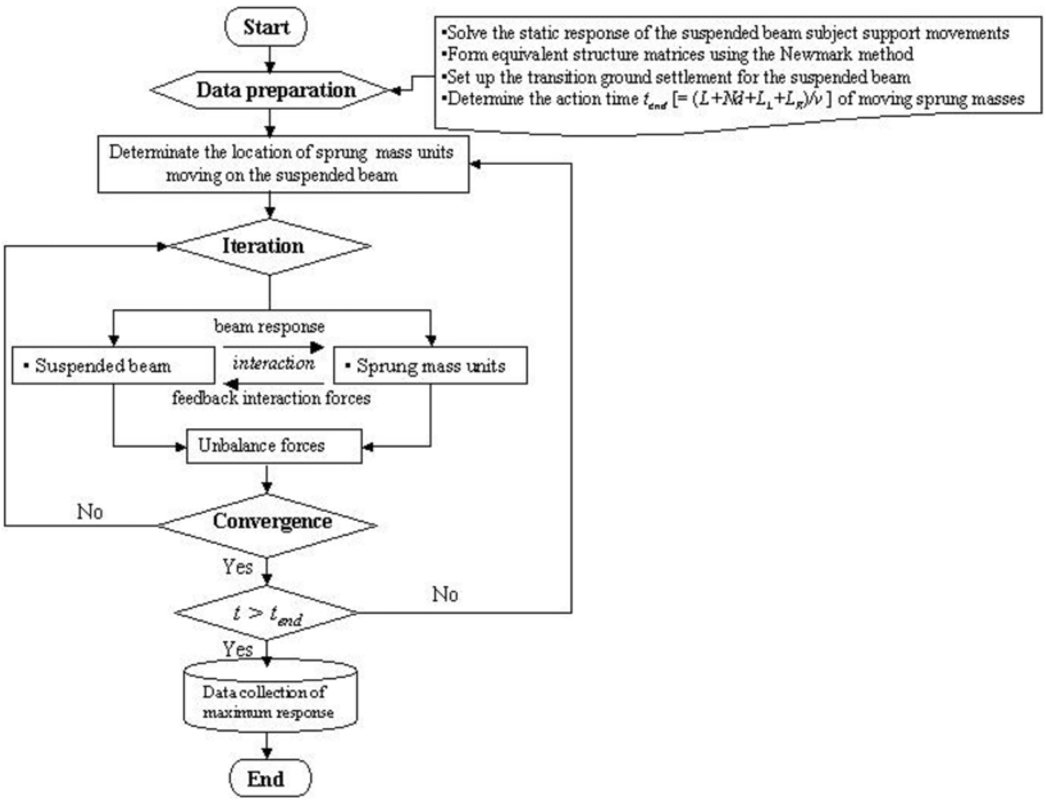

Fig. 3 Flow chart of incremental-iterative procedure

Table 1 Properties and natural frequencies of the suspended beam

L (m)	EI (kN-m^2)	E_cA_c (kN)	m (t/m)	c (kN-s/m/m)	y_0 (m) [y_0/L]	E_cA_c / L_e (kN/m)	f_1 (Hz) ( )	f_2 (Hz) ( )
150	3.3×10^8	10^8	16	3.84	15.0 [0.10]	6.16×10^5	1.31	1.8

(2) Transform the governing differential equation in Eq. (10) into a set of coupled equations in generalized coordinates as in Eq. (13);
(3) Discretize the vehicle-bridge interaction equations into a set of equivalent stiffness equations using Newmark's method (see Eqs. (18) and (19));
(4) Perform the iterative procedure with vehicle-bridge interaction given in the flowchart of Fig. 3 to compute the response of the suspended beam and sprung mass units.
(5) Update all the structural matrices in Eq. (16) at each increment;
(6) Check the unbalanced forces to see if they are smaller than preset tolerances. When the root mean square of the sum of the generalized unbalanced forces is larger than the preset tolerance, go to step (4) to precede the next iteration for removing the unbalanced forces;
(7) Repeat the steps (4)-(6) for another time increment once the condition of convergence is satisfied.

6. Numerical investigations

Fig. 2 shows a series of moving oscillators with non-uniformly regular intervals crossing a single-span suspended beam at constant speed v. The properties of the suspended beam and sprung mass unit are listed in Tables 1 and 2, respectively. In Table 1, the symbol of f_i represents the ith modal frequency. It is noted that the first natural frequency of anti-symmetric mode of the suspended beam is lower than that one of symmetric bending mode due to a strengthening effect of cable tension on the first symmetric bending mode.

The acceleration response of moving vehicle is usually used to evaluate the running safety of a train traveling over railway bridges (Yau and Yang 2006, Yau 2006a, Yau 2006b, Yau 2007). From the computed results of trained-induced response of a suspended beam by Yau and Yang (2008), the use of 16 modes is sufficient to describe the dynamic behavior of the suspended beam. For this reason, the same number of modes will be used in all the examples to follow.

To account for the random nature and characteristics of track irregularity in practice, the *power spectrum density* (PSD) function for track class 6 designed by *Federal Railroad Administration* (USA) (Yang *et al.* 2004) will be adopted to simulate the vertical profile of track geometry variations:

$$S(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)}$$

where Ω = spatial frequency, and A_v, (= 1.5×10^{-6} m) Ω_r (= 2.06×10^{-6} rad/m), and Ω_c (= 0.825 rad/m) are relevant parameters. Fig. 4 shows the vertical profile of track irregularity for simulation of track geometry variations in this study.

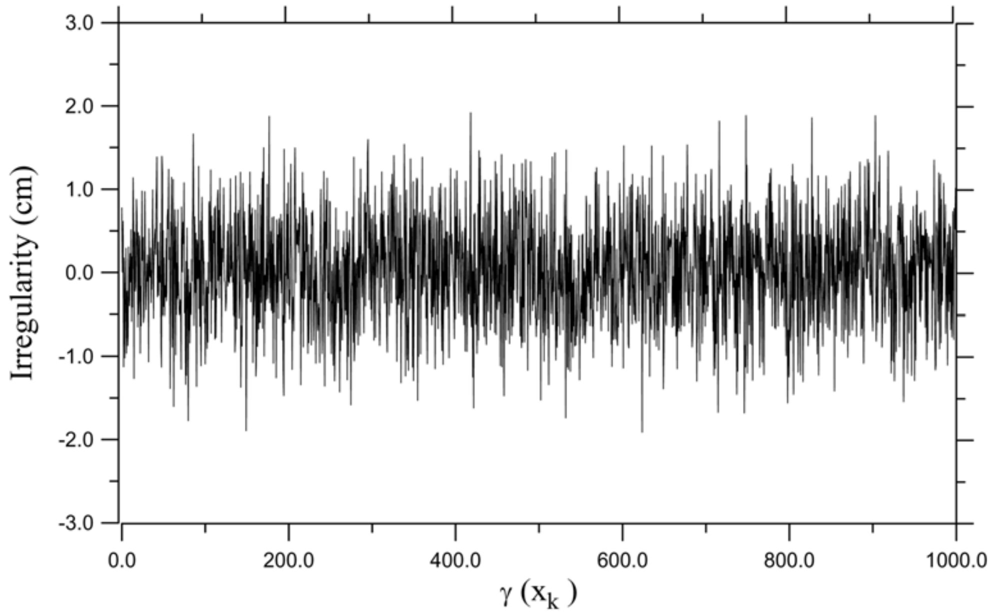

Fig. 4 Vertical profile of track irregularities

6.1 Resonance of acceleration response

Whenever the *passage frequency* (= v/d) of train loadings with identical intervals d matches any of the natural frequencies (f_i) of the bridge, the bridge will be set in resonance as there are more train loads pass through the bridge (Yang *et al.* 1997). The corresponding resonant speed of the train is denoted as $v_{res,i} = f_i d$ (Yau and Yang 2006). This is the so called *resonance phenomenon* for train-induced response of the railway bridge. The resonance phenomenon of a suspended beam induced by moving loads with identical intervals has been demonstrated in Refs. (Yau 2006a, Yau and Fryba 2007, Yau and Yang 2008). In this study, we shall let the moving oscillators pass through the suspended beam with the first two resonant speeds, i.e., $v_{res,1} = f_1 d = 40.6$ m/s (= 146 km/h) and $v_{res,2} = f_2 d = 45$ m/s (= 162 km/h).

Fig. 5 shows the time-history response acceleration responses at the first quarter and midpoint of the suspended beam. The results indicate the response amplitude at the mid-span due to the train loads traveling at the second resonant speed $v_{\mathrm{res},2}$ (= 162 km/h) is generally smaller than that at the first quarter-point with the first resonant speed $v_{\mathrm{res},1}$ (= 146 km/h). One reason for this is that as the train loads with a repetitive car length (d = 25 m) far less than the bridge span length (L = 150 m) move over the bridge, the simultaneous presence of multi train loads on the bridge deck may exert a *suppression action* on the first symmetric mode (i.e. the second bending mode), which may cause the mid-span acceleration of the bridge deck to be less severe compared with the other resonant case involving the anti-symmetric mode. In addition, Fig. 6 depicts the maximum acceleration ($a_{v,max}$) of the sprung masses versus the vehicle speed (v). Such a plot is called $a_{v,max} - v$ plot in the following examples. The results indicate that the maximum acceleration of the vehicle increases along with the increase of moving speeds.

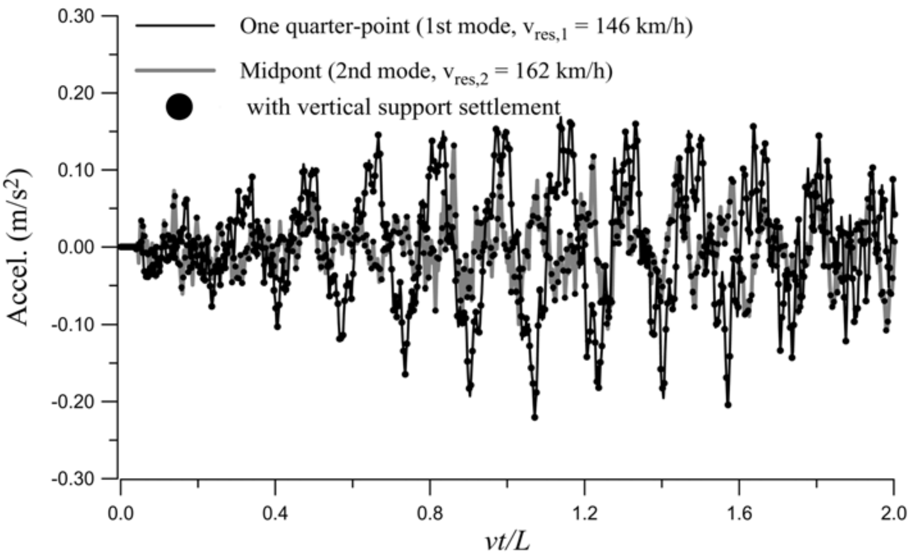

Fig. 5 Time history of acceleration of the suspended beam

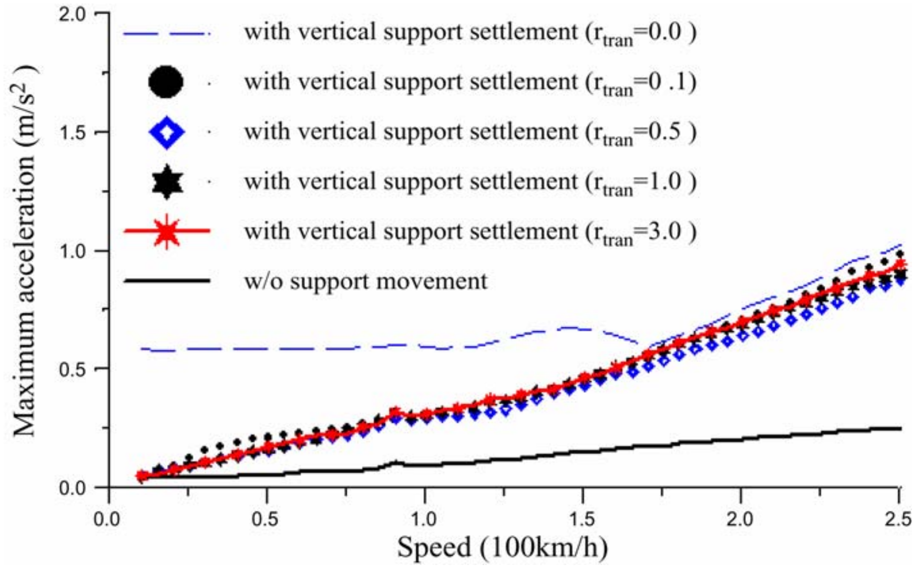

Fig. 6 Effect of transition length

6.2 Simulation of transition length for support settlement

Adopt the same data used in Example 6.1 and add the conditions of vertical support settlements with ($u_0 = -2.5$ cm, $u_L = -3.75$ m) to the left and right bridge supports, respectively. To simulate various transition lengths for a train entering the suspended bridge with support settlement, five sets of transition length are considered, that is, r_{tran} $(= L_L / L = L_R / L) = 0$, 0.1, 0.5, 1.0, 3. The analysis results of $a_{v,max} - v$ plot have been plotted in Fig. 6 as well. Obviously, the inclusion of ground

Table 2 Properties of moving oscillator and resonant speeds

N	d_1 (m)	d_2 (m)	$d = d_1 + d_2$ (m)	P (kN)	m_w (t)	m_v (t)	c_v (kN-s/m)	k_v (kN/m)	$v_{res,1}$ (km/h)	$v_{res,2}$ (km/h)
10	17.5	7.5	25	200	4.0	16.4	45	250	117	162

Table 3 Types of support movements

Support movements/ Type	u_0 (cm)	u_L (cm)	d_{x0} (cm)	d_{xL} (cm)
Type 1	−2.5	−3.75	0	0
Type 2	−2.5	−3.75	0	3
Type 3	−2.5	−3.75	0	−3

settlement will amplify the vehicle's response significantly. However, as the transition length ratio of r_{tran} is zero, the acceleration response amplitudes at lower speeds appear abnormally amplified since the train enters ot departs the suspended beam in a way of *falling* or *jumping*. Because of this, the larger one of transition length for settlement profile ($L_L = L_R = 3L$) is selected in the following example. On the other hand, for the same moving resonant speeds of moving sprung masses used in Example 6.1, Fig. 5 has also drawn the time-history acceleration responses of the suspended beam undergoing the present vertical support settlement. From the analysis results, the influence of differential settlement on bridge response is generally insignificant since the inertial forces induced by the running sprung mass units acting on the suspended beam are much smaller than the *static* weights of sprung masses. This effect will be further discussed in the following example.

6.3 Effect of differential support movements

To investigate the influence of horizontal support movements on the interaction response of the vehicle/bridge system, let us suppose the right bridge support in Fig. 2 subject to a permanent horizontal movement due to earthquakes. Table 3 lists three types of support movements, that is, vertical support settlements (type 1), and vertical/horizontal support movements (type 2 and 3). Here type 2 means the *positive* horizontal support movement at the right tower tends to elongate the suspension cable and further increases the cable tension. By contrast, the *negative* horizontal support movement of type 3 would decrease the strengthening effect of cable tension on the suspended beam.

With the inclusion of horizontal support movements, the maximum acceleration responses of sprung mass units and midpoint of the suspended beam against speed have been drawn in Figs. 7 and 8, respectively. The results indicate that considering the horizontal support movement of type 3 results in a significant amplification on the $a_{v,max} - v$ plot compared with the support movements of types 1 and 2. This can be attributed to the decrease of cable tension due to the *negative* horizontal support movement of type 3, which will reduce the bending stiffness of the suspend beam. On the other hand, for the support movement of type 2, the girder has been stiffened by the additional cable tension due to *positive* horizontal movement. Because of this, the response of the moving sprung masses traveling over the suspended beam is reduced. Besides, as shown in Fig. 8, the effect of differential ground settlements on the beam response is insignificant, since the inertial forces induced by the moving oscillators are much smaller than the static weight of these oscillators.

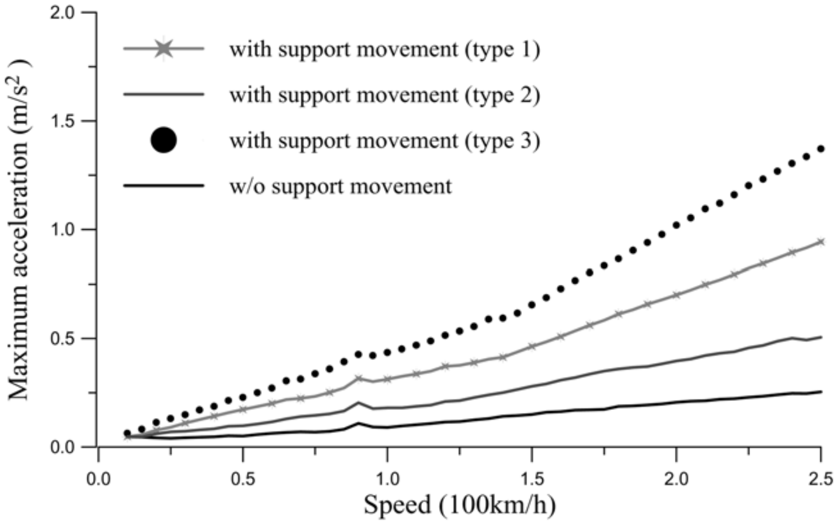

Fig. 7 Effect of support movements on $a_{v,max} - v$ plots

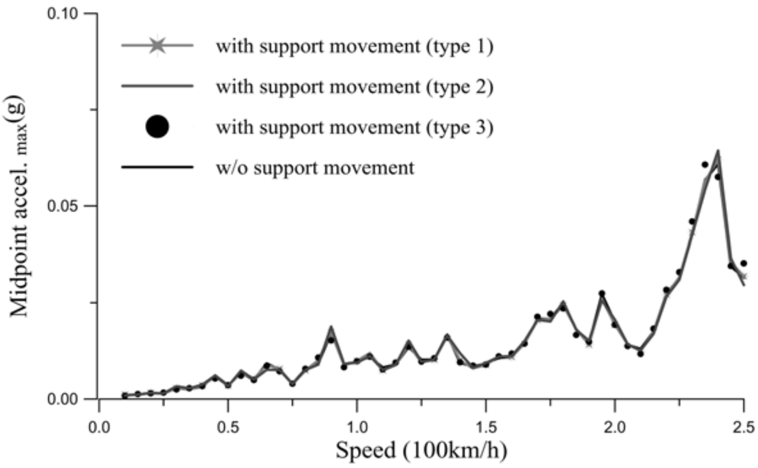

Fig. 8 Effect of support movements on beam response

7. Conclusions

By taking into account the effect of multi support settlements, the dynamic analysis of a single-span suspended beam traveled by successive moving sprung masses have been carried out. The *decomposition* concept was adopted for dealing with the static and dynamic response of the suspended beam. The system equations were solved by an incremental-iterative procedure involving the three phases of *predictor, corrector,* and *equilibrium-checking*. From the numerical study, the following conclusions are drawn: (1) The simulation of support settlements for a suspension bridge should include the transition length on the two side of the bridge; (2) Differential support settlements will result in significant amplification on the acceleration response of the moving vehicles, but not for the stiffening girder; (3) As the horizontal support movement tends to decrease the cable tension of a suspension bridge, the amplified extent of acceleration response of the running vehicles over the bridge is particularly noticeable; (4) Generally, as a train passes through a region with potential ground settlement, it means that the moving speed of the train may exceed the shear wave velocity of soft soil (Yang *et al*. 2007). For this reason, bridge-soil interaction will become significant and a further study should be conducted to introduce the soil-structure interaction into the vehicle/bridge coupling system.

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