Moving load response in a rotating generalized thermoelastic medium

Praveen Ailawalia\textsuperscript{1*} and Naib Singh Narah\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, M.M. Engineering College, Maharishi Markandeshwar University, Mullana District Ambala, Haryana, India
\textsuperscript{2}Department of Mathematics, D.A.V College, Ambala City, Haryana, India

(Received November 20, 2009, Accepted February 13, 2010)

Abstract. The steady state response of a rotating generalized thermoelastic solid to a moving point load has been investigated. The transformed components of displacement, force stress and temperature distribution are obtained by using Fourier transformation. These components are then inverted and the results are obtained in the physical domain by applying a numerical inversion method. The numerical results are presented graphically for a particular model. A particular result is also deduced from the present investigation.

Keywords: rotation; generalized thermoelasticity; fourier transform; temperature distribution.

1. Introduction

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory thermoelasticity in which the parabolic type heat conduction equation is based on fourier’s law of heat conduction. This newly emerged theory which admits finite speed of heat propagation is now referred to as the hyperbolic thermoelasticity theory, Chandrasekhar (1998), since the heat equation for rigid conductor is hyperbolic-type differential equation.

There are two important generalized theories of thermoelasticity. The first is due to Lord Shulman (L-S) (1967). The second generalization to the coupled theory of thermoelasticity which is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity: Muller (1971), in a review of the thermodynamics of thermoelastic solid, proposed an entropy production inequality, with the help of which he consider restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (G-L) obtained another version of the constitutive equations (1972). These equations were also obtained independently and more explicitly by Suhubi (1975). This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equations. The classical Fourier law violated if the medium under consideration has a centre of symmetry. Theory of thermoelasticity without energy dissipation is

* Corresponding author, Professor, E-mail: praveen_2117@rediffmail.com
another generalized theory and was formulated by Green and Naghdi (1993). It includes the "thermal-displacement gradient" among its independent constitutive variables and differs from the previous theories in that it does not accommodate dissipation of thermal energy.

The dynamical response of solid material subjected to moving loads is of great interest to a number of engineering fields, such as civil engineering, ocean engineering, earthquake engineering and tribology. For example, ground motion and stresses are induced in saturated soils by fast moving vehicular loads or surface blast waves due to explosives.

Various researchers investigated the dynamic response of half space subjected to a moving point load. Sneddon (1951) was the first to discuss the two-dimensional problem of a line load moving with constant sub-sonic speed over the surface of a homogenous elastic half space. Some of the similar problems of the sub-sonic, transonic and supersonic were discussed by other researchers (Cole and Huth 1958, Fung 1968, Fryba 1999). A homogenous three dimensional elastic half space subjected to forces moving with a constant speed was studied by Eason (1965) using the double Fourier transformation method. Payton (1967) considered the transient problem for a line load applied suddenly and then moving with a constant speed on the surface of an elastic half space.


In the present investigation we have obtained the expressions for displacement, force stress and temperature distribution in a rotating generalized thermoelastic medium due to a moving load by using Fourier transform. Such types of moving load problems in the rotating medium are very important in many dynamical systems. A particular case has also been derived. No attempt has been made so far to study the effect of rotation due to a moving load in generalized thermoelastic medium.

2. Formulation of the problem

A homogeneous generalized thermoelastic medium rotating uniformly with angular velocity \( \Omega = \Omega \hat{n} \) is considered where \( \hat{n} \) is a unit vector representing the direction of the axis of rotation. All quantities considered are functions of the time variable \( t \) and of the coordinates \( x \) and \( z \). The
Moving load response in a rotating generalized thermoelastic medium

Displacement equation of motion in the rotating frame has two additional terms (Schoenberg and Censor 1973): centripetal acceleration, \( \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) \) due to time varying motion only and \( 2\vec{\Omega} \times \vec{u} \) where \( \vec{u} = (u_1, 0, u_3) \) is the dynamic displacement vector and angular velocity \( \vec{\Omega} = (0, \Omega, 0) \). These terms do not appear in non-rotating media.

We consider a normal point load moving in an infinite generalized thermoelastic medium. To analyze the displacement, force stresses and temperature distribution at the interface of the medium, the continuum is divided into two half-spaces defined by

i. half-space I, \( |x| < \infty \), \( 0 < z \leq 0 \), \( |\vec{y}| < \infty \)

ii. half-space II, \( |x| < \infty \), \( 0 < z \leq \infty \), \( |\vec{y}| < \infty \)

A rectangular coordinate system \((x, y, z)\) having origin on the surface \( z = 0 \) and \( z \)-axis pointing vertically into the medium is considered. We assume a pressure pulse \( P(x + Ut) \) which is moving with a constant velocity \( U \) in the negative \( x \)-direction. Since the load has constant magnitude and move with a constant speed, after a sufficiently long time the solid response may become stationary in the reference system that is fixed to the load. In this paper we study possible pattern of this stationary response. The deformation of the medium subjected to a moving point load has been studied in particular for two theories of thermoelasticity viz. L-S theory (1967) and G-L theory (1972).

3. Basic equations

The field equations and constitutive relations in generalized linear thermoelasticity with rotation and without body forces and heat sources are given by

\[
(\lambda + \mu)\nabla (\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} - \nu \left( 1 + \partial_0 \frac{\partial}{\partial t} \right) \nabla T = \rho \left[ \frac{\partial^2 \vec{u}}{\partial t^2} + \vec{\Omega} \times (\vec{\Omega} \times \vec{u}) + 2 \vec{\Omega} \frac{\partial \vec{u}}{\partial t} \right]
\]

(1)

\[
K^* \left( n^* + t_i \frac{\partial}{\partial t} \right) \nabla^2 T = \rho C_e \left( n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + \nu T_0 \left( n_1 \frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2} \right) (\nabla \cdot \vec{u})
\]

(2)

\[
t_{ij} = \lambda \epsilon_{ij} + 2 \mu e_{ij} - \nu \left( 1 + \partial_0 \frac{\partial}{\partial t} \right) T \delta_{ij}
\]

(3)

where

\( \lambda, \mu \) are Lamé’s constants, \( \rho \) is the density, \( \vec{u} \) is the displacement vector, \( t_{ij} \) is stress tensor, \( \tau_0, \partial_0 \) are thermal relaxation times and \( \nu = (3\lambda + 2\mu)\alpha_n \), \( e = div \vec{u} \).

4. Solution of equations

For two dimensional problem \((xz\text{-plane})\) all quantities depends only on space coordinates \( x, z \) and time \( t \), so the equations of motion (1) and (2) reduces to

\[
\rho \left[ \frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 + 2\Omega \frac{\partial u_1}{\partial t} \right] = (\lambda + \mu) \frac{\partial^2 \tilde{u}}{\partial x^2} + \mu \nabla^2 u_1 - \nu \left( 1 + \partial_0 \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x}
\]

(4)
Following Fung (1968), a Galilean transformation

\[ x^* = x + Ut, \quad z^* = z, \quad t^* = t \]

is introduced, then the boundary conditions would be independent of \( t^* \) and assuming the dimensionless variables defined by

\[ x_i' = \frac{\omega^*_i}{c_0 x_i}, \quad u_i' = \frac{\rho c_0 \alpha^*_i u_i}{\nu T_0}, \quad t' = \frac{\omega^*_i t}{\nu T_0}, \quad \tau_0' = \omega^*_i \tau_0, \quad \vartheta_0' = \omega^*_i \vartheta_0 \]

where

\[ \omega^*_i = \rho c_0^2 \vartheta_i K^*, \quad \rho c_0^2 = \lambda + 2 \mu \]

in Eqs. (4)-(6), we obtain the equations of motion in dimensionless form.

Introducing displacement potentials \( q \) and \( \psi \) which are related to displacement components \( u_1 \) and \( u_3 \) as

\[ u_1 = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \]

in the resulting dimensionless equations and applying the Fourier transform defined by

\[ \hat{f} (\xi, z) = \int f (x, z) e^{i \xi x} dx \]

we get

\[ \left[ \frac{d^2}{dz^2} - \xi^2 + \Omega^2 + \varepsilon^2 M^2_1 \right] \hat{q} + 2 \Omega i \xi M_1 \hat{\psi} - (1 - \vartheta_0 i \xi M_1) \hat{T} = 0 \] (11)

\[ \left[ \frac{d^2}{dz^2} - \xi^2 + \alpha_i \Omega^2 + \alpha_1 \xi^2 M_1 \right] \hat{\psi} - 2 \Omega \alpha_i \xi M_1 \hat{q} = 0 \] (12)

\[ \left[ \frac{d^2}{dz^2} - \xi^2 + i \xi M_1 \left( \frac{n_1 - i \xi \tau_0 M_1}{n' - i \xi \tau_1 M_1} \right) \right] \hat{T} + \frac{n_1 - \tau_0 i \xi M_1}{n' - i \xi \tau_1 M_1} (\xi \xi M_1) \left[ \frac{d}{dz^2} - \xi^2 \right] \hat{q} = 0 \] (13)

Eliminating \( \hat{T} \) and \( \hat{\psi} \) from Eqs. (11) - (13) we obtain

\[ [\Delta^6 - A \Delta^4 + B \Delta^2 - C] \hat{q} = 0 \] (14)
where

\[ \Delta = \frac{d^2}{dz^2}, \quad a_1 = \frac{\rho c_0^2}{\mu}, \quad M_1 = \frac{U}{c_0} \]

\[ \epsilon = \frac{\nu^2 T_0}{\rho K^* \omega^4} \quad c_1 = \xi^2 - i \xi M_1 \left( \frac{n_1 - i \xi \tau_0 M_1}{n^* - i \xi \tau_1 M_1} \right) \]

\[ c_2 = \xi^2 - \Omega^2 - \xi^2 M_1^2, \quad c_3 = -i \xi \tau_0 M_1 \left( \frac{n_1 - i \xi \tau_0 i \xi M_1}{n^* - i \xi \tau_1 M_1} \right) \]

\[ c_4 = \xi^2 - \alpha \Omega^2 - \alpha \xi^2 M_1^2 \]

\[ A = c_1 + c_2 + c_3 + c_4 \]

\[ B = c_4 (c_1 + c_2 + c_3) + c_2 c_2 + c_3 \xi^2 - 4 \alpha \Omega^2 \xi^2 M_1^2 \]

\[ C = c_4 (c_1 + c_2 + c_3 \xi^2) - 4 \alpha \xi^2 \Omega^2 \xi^2 M_1^2 \]

(15)

The solutions of Eq. (14) are

\[ \tilde{q} = A_1 e^{q_1 z} + A_2 e^{q_2 z} + A_3 e^{q_3 z} + A_4 e^{q_4 z} + A_5 e^{q_5 z} + A_6 e^{q_6 z} \]

(16)

\[ \tilde{\nu} = a_1^* A_1 e^{q_1 z} + a_2^* A_2 e^{q_2 z} + a_3^* A_3 e^{q_3 z} + a_4^* A_4 e^{q_4 z} + a_5^* A_5 e^{q_5 z} + a_6^* A_6 e^{q_6 z} \]

(17)

\[ \tilde{T} = b_1^* A_1 e^{q_1 z} + b_2^* A_2 e^{q_2 z} + b_3^* A_3 e^{q_3 z} + b_4^* A_4 e^{q_4 z} + b_5^* A_5 e^{q_5 z} + b_6^* A_6 e^{q_6 z} \]

(18)

where \( q_i^2 \) are the roots of Eq. (14) and \( a_1^*, b_1^* \) are coupling constants defined by

\[ a_1^* = \frac{q_i^2 - (c_1 + c_2 + c_3) q_i^2 + (c_1 c_2 + c_1 c_3 \xi^2)}{2 i \xi^2 M_1 (c_1 - q_i^2)} \]

\[ b_1^* = i \xi^2 M_1 \left( \frac{n_1 - i \xi n_0 \tau_0 M_1}{n^* - i \xi \tau_1 M_1} \right) \left( \frac{\xi^2 - q_i^2}{q_i^2 - c_i} \right), \quad i = 1, 2, 3 \]

(19)

5. Boundary conditions

For a concentrated point force, we take \( P(x + Ut) = F \delta(x^+) \), where \( \delta(x^+) \) is Dirac-delta function and \( F \) is the magnitude of force applied along the interface of two media. In moving coordinates the boundary conditions at the interface \( z = 0 \) are,

(i) \( t_{33}(x, 0^+, t) = t_{33}(x, 0^-, t) - F \delta(x^+) \), \quad (ii) \( t_{33}(x, 0^+, t) = t_{33}(x, 0^-, t) \)

(iii) \( u_1(x, 0^+, t) = u_1(x, 0^-, t) \), \quad (iv) \( u_3(x, 0^+, t) = u_3(x, 0^-, t), \quad T = 0 \)

(20)

Using Eqs. (3), (8), and (9) in the boundary conditions (20), we obtain the boundary conditions in the dimensionless form. On suppressing the primes and applying the Fourier transform defined by
Praveen Ailawalia and Naib Singh Narah

(10) on the dimensionless boundary conditions and using (16) - (18) in the resulting transformed boundary conditions, we get the transformed expressions for displacement, force stress, and temperature distribution in a rotating generalized thermoelastic medium as

\[ \tilde{u}_1 = \tilde{F} \left( \sum_{m=1}^{3} b_m D_m e^{-q_m z} + \sum_{w=4}^{6} b_w D_w e^{q_w z} \right) \] (21)

\[ \tilde{u}_3 = \tilde{F} \left( \sum_{m=1}^{3} p_m D_m e^{-q_m z} + \sum_{w=4}^{6} p_w D_w e^{q_w z} \right) \] (22)

\[ \tilde{t}_{31} = \tilde{F} \left( \sum_{m=1}^{3} s_m D_m e^{-q_m z} + \sum_{w=4}^{6} s_w D_w e^{q_w z} \right) \] (23)

\[ \tilde{t}_{33} = \tilde{F} \left( \sum_{m=1}^{3} r_m D_m e^{-q_m z} + \sum_{w=4}^{6} r_w D_w e^{q_w z} \right) \] (24)

\[ \tilde{T} = \tilde{F} \left( \sum_{m=1}^{3} b'_m D_m e^{-q'_m z} + \sum_{w=4}^{6} b'_w D_w e^{q'_w z} \right) \] (25)

6. Particular case

Neglecting angular velocity (i.e., \( \tilde{\Omega} = 0 \)) in Eq. (1), we obtain the transformed components of displacement, force stress and temperature distribution in a generalized thermoelastic medium due to moving load at the interface as

\[ \tilde{u}_1 = \tilde{F} \left( \sum_{m=1}^{3} b''_m D_m^{(1)} e^{-q''_m z} + \sum_{w=4}^{6} b''_w D_w^{(1)} e^{q''_w z} \right) \] (26)

\[ \tilde{u}_3 = \tilde{F} \left( \sum_{m=1}^{3} p''_m D_m^{(1)} e^{-q''_m z} + \sum_{w=4}^{6} p''_w D_w^{(1)} e^{q''_w z} \right) \] (27)

\[ \tilde{t}_{31} = \tilde{F} \left( \sum_{m=1}^{3} s''_m D_m^{(1)} e^{-q''_m z} + \sum_{w=4}^{6} s''_w D_w^{(1)} e^{q''_w z} \right) \] (28)

\[ \tilde{t}_{33} = \tilde{F} \left( \sum_{m=1}^{3} r''_m D_m^{(1)} e^{-q''_m z} + \sum_{w=4}^{6} r''_w D_w^{(1)} e^{q''_w z} \right) \] (29)

\[ \tilde{T} = \tilde{F} \left( \sum_{m=1}^{2} b'''_m D_m^{(1)} e^{-q'''_m z} + \sum_{w=4}^{6} b'''_w D_w^{(1)} e^{q'''_w z} \right) \] (30)

In Eqs. (21)-(25) the transformed displacement, force stress and temperature distribution components for the region \(-\infty < z \leq 0\) are obtained by inserting \( D_4 = D_5 = D_6 = 0 \) and in Eqs. (26)-(30) by inserting \( D_4^{(1)} = D_5^{(1)} = D_6^{(1)} \). Similarly, for the region \( 0 \leq z < \infty \), the components are obtained by
Moving load response in a rotating generalized thermoelastic medium

7. Numerical results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of temperature, displacement and stress fields in the context of L-S and G-S theories. For this purpose magnesium crystal like material is taken as the thermoelastic material for which we take the following values of physical constants (Dhilliwal and Singh (1980)) at \( T_0 = 298K \)

\[
\begin{align*}
\lambda &= 2.17 \times 10^{10} \text{Nm}^{-2}, \\
\mu &= 3.278 \times 10^{10} \text{Nm}^{-2}, \\
\rho &= 1.74 \times 10^{3} \text{Km}^{-3}, \\
C_e &= 1.04 \times 10^3 \text{J/Kg}^{-1} \text{deg}^{-1}, \\
\nu &= 2.68 \times 10^6 \text{Nm}^{-2} \text{deg}^{-1}, \\
K^* &= 1.7 \times 10^2 \text{Wm}^{-1} \text{s}^{-1} \text{deg}^{-1}.
\end{align*}
\]

The computations are carried out for \( U < c_0 \) on the surface \( z = 1.0 \) at \( t = 1.0 \). The graphical results for normal displacement \( u_3 \), normal force stress \( t_{33} \) and temperature distribution \( T \) for \( \Omega = 0.3 \) and non dimensional thermal relaxation times \( \tau_0 = 0.1 \) and \( \tau_0 = 0.2 \) are shown in Figs. (1)-(3), for

(i) thermoelastic solid with rotation (L-S theory) by solid line (-----)

(ii) thermoelastic solid without rotation (L-S theory) by dashed line (-------------)

(iii) thermoelastic solid with rotation (G-L theory) by solid lines with centered symbols (**-**-**)

(iv) thermoelastic solid without rotation (G-L theory) by dashed lines with centered symbols (**-**-**-**-**).

8. Special cases of thermoelastic theory

8.1 The equations of the coupled thermoelasticity (C-T theory) for a rotating media are obtained when

\[
n^* = n_1 = 1, \quad t_1 = \tau_0 = \tau_0 = 0
\]

Eqs. (1) and (2) has the form

\[
(\lambda + \mu) \nabla (\nabla \cdot \hat{u}) + \mu \nabla^2 \hat{u} - \nu \nabla T = \rho \left[ \frac{\partial^2 \hat{u}}{\partial t^2} + \lambda \nabla \times (\nabla \times \hat{u}) + 2 \mu \frac{\partial \hat{u}}{\partial t} \right]
\]

\[
K^* \nabla^2 T = \rho C_e \frac{\partial T}{\partial t} + \nu \frac{\partial \varepsilon}{\partial t}
\]

8.2 For Lord-Shulman (L-S theory), when

\[
n^* = n_1 = n_0 = 1, \quad t_1 = \tau_0 = 0, \quad \tau_0 > 0
\]
where \( \tau_0 \) is the relaxation time. Eq. (1) is the same as Eq. (32) and Eq. (2) has the form

\[
K\nabla^2 T = \rho C_e \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + \nu T_0 \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) e
\]  

\[(34)\]

8.3 For Green–Lindsay (G-L theory)

\[
 n^* = n_1 = 1, \quad n_0 = 0, \quad t_1 = 0, \quad \theta_0 \geq \tau_0 > 0
\]  

\[(35)\]
where \( \beta_0, \tau_0 \) are the two relaxation times. Eq. (1) remains unchanged and Eq. (2) takes the form

\[
K* \nabla^2 T = \rho C_e \left( 1 + \frac{\tau_0}{\tau} \right) \left( \frac{\partial T}{\partial \tau} + \nu T_0 \frac{\partial^2 e}{\partial \tau^2} \right)
\]

(36)

8.4 The equations of the generalized thermoelasticity for a rotating medium, without energy dissipation (the linearized GN theory of type II) are obtained when

\[
n' = n_1 = 0, \quad n_0 = 1, \quad \beta_0 = 0, \quad \tau_0 = 1
\]

(37)

Eq. (1) is the same as Eq. (32) and Eq. (2) takes the form

\[
K* \nabla^2 T = \rho C_e \frac{\partial^2 T}{\partial \tau^2} + \nu T_0 \frac{\partial^2 e}{\partial \tau^2}
\]

(38)

where \( n^* \) is constant which has the dimension of (1/sec) and

\[n^* \kappa' = K' = C_e (\lambda + 2\mu) / 4\]

is a characteristic constant of this theory.

9. Discussions

The values of all the quantities i.e., normal displacement, normal force stress and temperature distribution are very close for L-S and G-L theories. These variations of normal displacement and normal force stress under the effect of rotation (\( \Omega \neq 0 \)) are oscillatory to a large extent. When the rotation effect is neglected (\( \Omega = 0 \)), the variations of normal displacement for both L-S and G-L theories increases linearly in the range \( 0 \leq x \leq 10 \). Similarly in the absence of rotation the values of
normal force stress lie in a very short range and are close to zero in the range $2 \leq x \leq 10$. These variations of normal displacement and normal force stress are shown in Figs. 1 and 2 respectively.

When the medium is rotating with some angular velocity, the values of temperature distribution are very less in magnitude. To compare the results between both the mediums, these values of temperature distribution have been multiplied by $10^4$. The variations of temperature distribution are shown in Fig. 3.

10. Conclusions

The variations of all the quantities are similar in nature for L-S and G-L theories. As observed from the graphical results, rotation plays an important role on the deformation of the body.

References


Moving load response in a rotating generalized thermoelastic medium

Sci. 5, 49-79.

Appendix A

The field equations and constitutive relations for Lord Shulman (L-S) (1967) theory are

\[(\lambda + \mu)\nabla (\nabla \cdot \dot{\mathbf{u}}) + \mu \nabla^2 \dot{\mathbf{u}} - \nu \nabla T = \rho \left[ \frac{\partial^2 \mathbf{u}}{\partial t^2} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \dot{\mathbf{u}}) + 2 \mathbf{\Omega} \frac{\partial \dot{\mathbf{u}}}{\partial t} \right] \]

\[K^* \nabla^2 T = \rho C_T \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \right) T + \nu T_0 \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \right) (\nabla \cdot \dot{\mathbf{u}}) \]

\[t_{ij} = \lambda \epsilon_{ij} + 2 \mu \epsilon_{ij} - \nu T \delta_{ij} \]

The field equations and constitutive relations for Green-Lindsay (G-L) (1972) theory are

\[(\lambda + \mu)\nabla (\nabla \cdot \dot{\mathbf{u}}) + \mu \nabla^2 \dot{\mathbf{u}} - \nu \left( 1 + \delta_{ij} \frac{\partial}{\partial t} \right) \nabla T = \rho \left[ \frac{\partial^2 \mathbf{u}}{\partial t^2} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \dot{\mathbf{u}}) + 2 \mathbf{\Omega} \frac{\partial \dot{\mathbf{u}}}{\partial t} \right] \]

\[K^* \nabla^2 T = \rho C_T \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \right) T + \nu T_0 \frac{\partial}{\partial t} (\nabla \cdot \dot{\mathbf{u}}) \]

\[t_{ij} = \lambda \epsilon_{ij} + 2 \mu \epsilon_{ij} - \nu \left( 1 + \delta_{ij} \right) \mathbf{T} \delta_{ij} \]

\[D_m = \frac{\Delta_m}{\Delta}, D_w = \frac{\Delta_w}{\Delta}, m = 1, 2, 3, \text{ and } w = 4, 5, 6 \]

\[\Delta = -(f_1 h_1 + f_2 h_2 + f_3 h_3 + f_4 h_4 + f_5 h_5 + f_6 h_6 + f_7 h_7 + f_8 h_8 + f_9 h_9 + f_{10} h_{10} + f_{11} h_{11} + f_{12} h_{12} + f_{13} h_{13} + f_{14} h_{14} + f_{15} h_{15}) \]
\[\Delta_1 = \tilde{F}(E_1 + E_2), \quad \Delta_2 = \tilde{F}(E_3 + E_4), \quad \Delta_3 = \tilde{F}(E_5 + E_6) \]
\[ f_1 = k_2 d_6 - d_2 k_2, \quad f_2 = l_2 d_6 - d_2 l_6, \quad f_3 = l_2 k_2 - k_2 l_6, \quad f_4 = b_2 d_6 - d_2 b_6 \]
\[ f_5 = b_2 k_2 - k_2 b_6, \quad f_6 = b_2 l_6 - l_2 b_6, \quad f_7 = s_2 d_6 - d_2 s_6, \quad f_8 = s_2 k_2 - k_2 s_6 \]
\[ f_9 = s_2 l_6 - l_2 s_6, \quad f_{10} = s_2 b_6 - b_2 s_6, \quad f_{11} = r_2 d_6 - d_2 r_6, \quad f_{12} = r_2 k_2 - k_2 r_6 \]
\[ f_{13} = r_2 l_6 - l_2 r_6, \quad f_{14} = r_2 b_6 - b_2 r_6, \quad f_{15} = r_2 s_6 - s_2 r_6 \]
\[ g_1 = r_2 s_2 - s_2 r_2, \quad g_2 = b_2 r_1 - r_2 b_1, \quad g_3 = r_1 l_2 - l_2 r_3, \quad g_4 = r_1 k_2 - k_2 r_2 \]
\[ g_5 = r_1 d_2 - d_2 r_5, \quad g_6 = b_2 s_1 - s_2 b_1, \quad g_7 = l_2 s_1 - s_2 l_1, \quad g_8 = s_1 k_2 - k_1 s_2 \]
\[ g_9 = s_1 d_2 - d_2 s_9, \quad g_{10} = b_1 l_2 - l_2 b_2, \quad g_{11} = b_1 k_2 - k_1 b_2, \quad f_{12} = b_1 d_2 - d_2 b_2 \]
\[ g_{13} = l_1 k_2 - k_1 l_2, \quad g_{14} = l_1 d_2 - d_1 l_2, \quad g_{15} = k_1 d_2 - d_1 k_2 \]
\[ p_1 = s_2 k_3 - k_2 s_3, \quad p_2 = s_2 l_3 - l_2 s_3, \quad p_3 = s_2 k_3 - k_2 s_3, \quad p_4 = s_2 d_3 - d_2 s_3 \]
\[ p_5 = b_2 l_3 - l_2 b_3, \quad p_6 = b_2 k_3 - k_2 b_3, \quad p_7 = b_2 d_3 - d_2 b_3, \quad p_8 = l_2 k_3 - k_2 l_3 \]
\[ p_9 = l_2 d_3 - d_2 l_3, \quad p_{10} = k_2 d_3 - d_2 k_3, \quad p_{11} = s_1 b_3 - b_1 s_3, \quad p_{12} = s_1 l_3 - l_3 s_3 \]
\[ p_{13} = s_1 k_3 - k_1 s_3, \quad p_{14} = s_1 d_3 - d_1 s_3, \quad p_{15} = b_1 l_3 - l_3 b_3, \quad p_{16} = b_1 k_3 - k_3 b_3 \]
\[ p_{17} = b_1 d_3 - d_3 b_3, \quad p_{18} = l_1 k_3 - k_3 l_3, \quad p_{19} = l_1 d_3 - d_3 l_3, \quad p_{20} = k_3 d_3 - d_3 k_3 \]
\[ h_1 = y_4 l_4 - y_2 b_4 + y_9 s_4 - y_4 r_4, \quad h_2 = y_3 b_4 - y_1 k_4 - y_6 s_4 + y_7 r_4 \]
\[ h_3 = y_1 d_4 - y_8 b_4 + y_9 s_4 - y_10 r_4, \quad h_4 = y_2 k_4 - y_3 l_4 + y_{11} s_4 - y_{12} r_4 \]
\[ h_5 = y_8 l_4 - y_4 d_4 - y_13 s_4 + y_{14} r_4, \quad h_6 = y_5 d_4 - y_8 k_4 + y_{15} s_4 - y_{16} r_4 \]
\[ h_7 = y_6 l_4 - y_3 k_4 - y_{11} l_4 + y_{17} r_4, \quad h_8 = y_3 d_4 - y_9 l_4 + y_{13} b_4 - y_{18} r_4 \]
\[ h_9 = y_9 k_4 - y_6 d_4 - y_{15} b_4 + y_{19} r_4, \quad h_{10} = y_{11} d_4 - y_{13} k_4 + y_{15} l_4 - y_{20} r_4 \]
\[ h_{11} = y_4 k_4 - y_7 l_4 + y_{12} b_4 - y_{17} s_4, \quad h_{12} = y_{10} l_4 - y_4 d_4 - y_{14} b_4 + y_{18} s_4 \]
\[ h_{13} = y_5 d_4 - y_{10} k_4 + y_{16} b_4 - y_{19} s_4, \quad h_{14} = y_{14} k_4 - y_{15} d_4 - y_{16} s_4 + y_{20} s_4 \]
\[ h_{15} = y_{17} l_4 - y_{18} k_4 + y_{19} l_4 - y_{20} b_4, \quad y_1 = g_1 b_3 - g_2 s_3 + g_6 r_3 \]
\[ y_2 = g_1 l_3 - g_3 s_3 + g_7 r_3, \quad y_3 = g_2 l_3 - g_4 b_3 + g_6 r_3, \quad y_4 = g_6 l_3 - g_7 b_3 - g_{10} s_3 \]
\[ y_5 = g_1 k_3 - g_4 s_3 + g_8 r_3, \quad y_6 = g_2 k_3 - g_4 b_3 + g_{11} r_3, \quad y_7 = g_6 k_3 - g_7 b_3 + g_{11} s_3 \]
\[ y_8 = g_2 d_3 - g_5 s_3 + g_9 r_3, \quad y_9 = g_2 d_3 - g_5 b_3 + g_{12} r_3, \quad y_{10} = g_6 d_3 - g_9 b_3 + g_{12} s_3 \]
\[ y_{11} = g_3 l_3 - g_4 l_3 + g_{13} r_3, \quad y_{12} = g_7 k_3 - g_8 l_3 + g_{13} s_3, \quad y_{13} = g_3 d_3 - g_4 l_3 + g_{14} r_3 \]
Moving load response in a rotating generalized thermoelastic medium

\begin{align*}
y_{14} &= g_7 d_3 - g_9 l_3 + g_{14} s_3, y_{15} = g_4 d_3 - g_3 k_3 + g_{12} r_3, y_{16} = g_8 d_3 - g_6 k_3 + g_{15} s_3 \\
y_{17} &= g_{10} k_3 - g_{11} l_3 + g_1 b_3, y_{18} = g_{10} d_4 - g_{13} l_3 + g_{14} b_3, y_{19} = g_{11} d_3 - g_{12} k_3 + g_{13} b_3 \\
y_{20} &= g_{12} d_3 - g_{14} k_3 + g_{13} l_3, n_1 = f_1 l_4 - f_2 k_1 + f_3 d_4, n_2 = f_1 b_4 - f_4 k_1 + f_5 l_4 \\
n_3 &= f_2 b_4 - f_4 l_4 + f_5 d_4, n_4 = f_2 b_4 - f_5 l_4 + f_6 k_1, n_5 = f_1 s_4 - f_7 l_4 + f_8 d_4 \\
n_6 &= f_2 s_4 - f_7 l_4 + f_9 d_4, n_7 = f_3 s_4 - f_8 l_4 + f_9 b_1, n_8 = f_4 s_4 - f_9 b_4 + f_10 l_4 \\
n_9 &= f_5 s_4 - f_9 b_4 + f_{10} l_4, n_{10} = f_6 s_4 - f_9 b_4 + f_{10} l_4 \\
E_1 &= p_n c_n - p_{n2} n_2 + p_{n3} n_3 + p_{n4} n_4 + p_{n5} n_5 - p_{n6} n_6 \\
E_2 &= p_n c_n - p_{n8} n_8 - p_{n9} n_9 + p_{n10} n_{10} \\
E_3 &= -p_{n1} n_1 + p_{n2} n_2 - p_{n3} n_3 + p_{n4} n_4 - p_{n5} n_5 + p_{n6} n_6 \\
E_4 &= -p_{n7} n_7 + p_{n8} n_8 + p_{n9} n_9 - p_{n10} n_{10} \\
E_5 &= g_{9} n_1 - g_{7} n_3 + g_{8} n_4 - g_{10} n_5 - g_{11} n_6 \\
E_6 &= g_{12} n_7 - g_{13} n_8 - g_{14} n_9 + g_{15} n_{10} \\
l_i &= i \xi q_i^\ast \xi - q_i , l_{4.5.6} = i \xi a_i^\ast \xi + q_i \ , s_i = \frac{\mu}{\rho c_0^2} \left( 2i \xi q_i + (q_i^2 + \xi^2) a_i^\ast \right) \\
s_{4.5.6} &= \frac{\mu}{\rho c_0^2} \left( (q_i^2 + \xi^2) a_i^\ast - 2i \xi q_i \right), b_i = -(i \xi + a_i^\ast \xi) \ , b_{4.5.6} = a_i^\ast q_i - i \xi \\
r_i &= q_i^2 - \frac{\lambda \xi^2}{\rho c_0^2} - 2i \mu \xi a_i^\ast \xi - \left( 1 - i \xi \mathbf{c}_0 \mathbf{M}_1 \right) b_i^\ast \ , k_i = k_{4.5.6} = b_i^\ast \ , d_i = -q_i b_i^\ast \\
r_{4.5.6} &= q_i^2 - \frac{\lambda \xi^2}{\rho c_0^2} + 2i \mu \xi a_i^\ast \xi q_i - \left( 1 - i \xi \mathbf{c}_0 \mathbf{M}_1 \right) b_i^\ast \ , d_{4.5.6} = q_i b_i^\ast \\
\end{align*}

Appendix B

\[
D_\beta^{(0)} = \frac{D_\alpha^{(1)}}{\Delta_\beta^{(0)}}, \quad D_\alpha^{(0)} = \frac{D_\alpha^{(1)}}{\Delta_\beta^{(0)}} \\
\Delta_\beta^{(0)} = 8(d_i^2 E_i^\ast - d_i^\ast E_i^\ast), \quad \Delta_\alpha^{(1)} = 4 \tilde{F} k_i^\ast b_i^\ast (s_i^\ast g_i^\ast - l_i^\ast g_i^\ast) \\
\Delta_\alpha^{(2)} = -\frac{k_i^\ast}{k_i^\ast} \Delta_\alpha^{(0)}, \quad \Delta_\alpha^{(3)} = 4 \tilde{F} g_i^\ast (d_i^\ast p_i^\ast - d_i^\ast p_i^\ast)
\]
\[ E'_1 = l'_1 (g'_1 p'_1 + g'_2 p'_2) + s'_1 (g'_1 p'_1 - g'_2 p'_2) \]
\[ E'_2 = s'_1 (g'_2 p'_6 - g'_1 p'_1) - l'_1 (g'_1 p'_1 + g'_2 p'_6) \]
\[ p'_1 = s'_1 b'_1 - b'_1 s'_1, \quad p'_2 = r'_1 s'_1 - s'_1 r'_1, \quad p'_3 = b'_1 l'_1 - l'_1 b'_1, \quad p'_4 = r'_1 l'_1 - l'_1 r'_1 \]
\[ p'_5 = b'_2 l'_1 - l'_2 b'_1, \quad p'_6 = r'_2 l'_1 - l'_2 r'_1, \quad p'_7 = s'_1 b'_1 - b'_1 s'_1, \quad p'_8 = r'_1 s'_1 - s'_1 r'_1 \]
\[ p'_{10} = s'_1 l'_1 - l'_1 s'_1, \quad g'_1 = k' r'_2 - r'_2 k'_2, \quad g'_2 = b' k'_2 - k'_2 b' \]
\[ g'_3 = l'_1 d'_2 - d'_1 l'_2, \quad g'_4 = s'_1 d'_2 - d'_1 s'_2, \quad r'_{1,2} = q'_{1,2} - \frac{\lambda \xi^2}{\rho c_0^2} - (1 - i \xi M_1) \beta'_{1,2} \]
\[ r'_3 = -\frac{2i \xi \mu q'_{1,2}}{\rho c_0^2}, \quad s'_{1,2} = \frac{2i \xi \mu q'_{1,2}}{\rho c_0^2}, \quad s'_3 = \frac{\mu}{\rho c_0^2} (q'_3 - \xi^2), \quad b'_{1,2} - i \xi, \quad b'_3 = -q'_3 \]
\[ l'_{1,2} = -q'_{1,2}, \quad l'_3 = i \xi, \quad k'_{1,2} = k'_{4,5} = b'_{1,2}, \quad d'_{1,2} = -q'_{1,2} h'_{1,2}, \quad b'^*_{1,2} = \frac{q'^2_{1,2} - \epsilon'^2_{1,2}}{1 - i \xi M_1} \]
\[ r'_{4,5} = r'_{1,2}, \quad r'_6 = -r'_3, \quad s'_{4,5} = -s'_{1,2}, \quad s'_6 = s'_1, \quad b'_{4,5} = b'_{1,2}, \quad b'_6 = -b'_3, \quad d'_{4,5} = -d'_{1,2} \]
\[ q'^2_{1,2} = \frac{A_4 \pm \sqrt{A_4^2 - 4B_1}}{2}, \quad q'^2_{1,2} = \xi^2 \left( 1 - \frac{\rho U^2}{\mu} \right), \quad A_4 = e_1 + \epsilon_2 + e_3, \quad B_1 = e_1 \epsilon_2 + e_3 \xi^2 \]