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A mathematical model to recover missing monitoring data of foundation pit

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Abstract. A new method is presented to recover missing deformation data of lateral walls of foundation pit when the monitoring is interrupted; the method is called Dynamic Mathematical Model – Parameter Interpolation. The deformation of lateral walls of foundation pit is mainly affected by the type of supporting structure and the situation of constraints, therefore, this paper mainly studies the two different kinds of variation law of deep horizontal displacement when the lateral walls are constrained or not, proposes two dynamic curve models of normal distribution type and logarithmic type, deals with model parameters by interpolating and obtains the parameters of missing data, then missing monitoring data could be Figured out by these parameters. Compared with the result from the common average method which is used to recover missing data, in the upper 2/3 of the inclinometer tube, the result by using this method is closer to the actual monitoring data, in the lower 1/3 part of the inclinometer tube, the result from the common average method is closer to the actual monitoring data.

Keywords: foundation pit; deep horizontal displacement; dynamic mathematical model; parameter interpolation; data recovery

1. Introduction

Deep horizontal displacement of foundation pit is the comprehensive result of interaction between supporting structure and rock and soil mass, it can reflect the deformation and failure features of the pit slope and be important to judge the stability (Shi *et al*. 2002, Liu *et al*. 2012, Zhang *et al*. 2014, Huang 2004) of foundation pit. It is usual to lay inclinometer tube and get accurate continuous monitoring data by using inclinometer (Gao *et al*. 1998, Cai 2009, Liu *et al*. 2011, Simeoni and Mongiovi 2007, Stark and Choi 2008, Finno and Calvello 2005) to monitor deep horizontal displacement. In the process of actual monitoring, the monitoring is often interrupted by many factors such as the malfunction of inclinometer, bad weather, interference of on-site construction and so on, the short-term missing of monitoring data will affect the analysis of the change trend of deep horizontal displacement with time and even have a bad effect on detecting timely the abnormal sliding, determining the position of the sliding surface and analyzing the development trend of the sliding surface. Therefore, it is necessary to propose a reasonable method to recover the missing data of deep horizontal displacement when the monitoring is interrupted.

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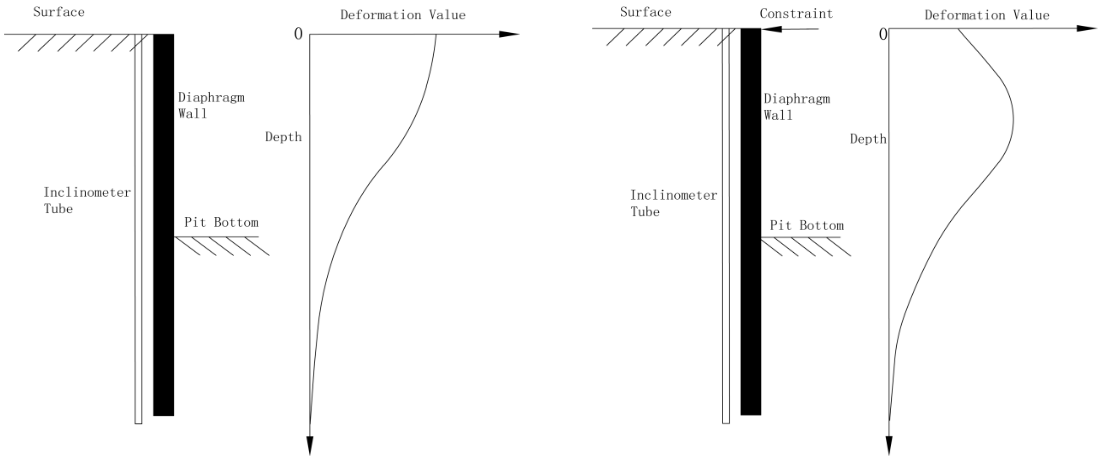

(a) No constraint at the top (b) Having constraint at the top

Fig. 1 Curves of deep horizontal displacement in the two different supporting ways

There are many methods (Pan *et al*. 2010, Peng *et al*. 2005, Wu *et al*. 2012, Tseng *et al*. 2003, Abdella and Marwala 2005, Brooks *et al*. 2012, Nakagawa and Freckleton 2008) recovering missing data at home and abroad, these methods can be divided into two types: the method based on statistics and the one based on data mining (Li *et al*. 2002, Nedic *et al*. 2014, Kwon *et al*. 2014). The former is based on statistical hypothesis, the latter requires a huge dataset, data processing is very complex and it takes a long time. Each method has its own specific suitability, it is necessary to use different methods to recover missing data for different kinds of data missing.

This paper presents a method called Dynamic Mathematical Model - Parameter Interpolation, and the result by using this method to recover the missing data of deep horizontal displacement of a foundation pit is closer to the real monitoring values.

2. The vertical variation of the deep horizontal displacement

The vertical variation of deep horizontal displacement of the foundation pit lateral walls changes as the type of supporting structure changes, the variation could be divided into two categories, shown in Fig. 1. From the top to the bottom of the inclinometer tube, the horizontal displacement in Fig. 1(a) decreases gradually and the maximum horizontal displacement appears at the top; in Fig. 1(b) the horizontal displacement increases at first then decreases and the maximum horizontal displacement appears in the middle-upper part of the inclinometer tube. In this paper, missing data recovery is based on the two monitoring curves (Fig. 1) of horizontal displacement in the different supporting ways.

3. Recovery of missing data of deep horizontal displacement with no constraint at the top

3.1 Mathematical model

Feed the monitoring data of deep horizontal displacement and the corresponding depth into the

Excel table, draw the curve of the data and add the trend line of this curve, then choose the logarithmic function to be the mathematical model of deep horizontal displacement of the lateral wall by the correlation coefficient of this trend line, the mathematical model is as follows

$$s = a \ln h + c \tag{1}$$

Considering the deep horizontal displacement in Fig. 1(a) decreases with the depth increasing, Eq. (1) is a monotone function about the depth h. In order to make the mathematical model be closer to the monitoring curve (Fig. 1(a)), *bh* (also monotone function) is added into Formula 1, as following

$$s = a \ln h + bh + c \tag{2}$$

Where s is the horizontal displacement (mm); h is the depth (m); a, b, c are undetermined parameters.

3.2 Calculation method

The three parameters a, b, and c are different for the monitoring data in different days. Input the monitoring data of several days before the appearance of missing data into the mathematical software Maple (Yan and Huang, 2002), then fit the data of each day in the form of Eq. (2), the three parameters a_i, b_i, c_i (i = 1, 2,…,n) are obtained from the fitting result.

The parameters could be a_1, b_1, and b_1 in the first day, the parameters are a_i, b_i, c_i in the ith day. Input a_1, a_2, a_3,… to the Excel in the form of (1, a_1), (2, a_2)...(i, a_i)...(n, a_n), draw the trend line, and then gain the function of the parameter a about i, as following

$$a = f\ (i) \tag{3}$$

The parameter a_q of missing data could be figured out by using Eq. (3), similarly, the other parameters b_q and c_q could be Figured out. Substitute the three parameters a_q, b_q, c_q in Eq. (2), then the function of deep horizontal displacement about depth h at the time of data missing could be obtained as follows

$$s = a_q \ln h + b_q h + c_q \tag{4}$$

The missing monitoring data of horizontal displacement at various depths could be figured out from Eq. (4).

3.3 Engineering example

The excavation depth of a foundation pit called No. 1 in Nanjing was 3.5~6.5 m, lateral walls of this pit were supported by high-pressure rotary jet grouting pile. There was mainly mucky silty clay within the supporting depth, the supporting type belongs to the type of no constraint at the top like Fig. 1(a), and the supporting structure is shown in Fig. 2.

The buried depth of one inclinometer tube (denoted as CX1) was 14 m, the precision of the used inclinometer (the model number was XB30) was 0.02 mm and the horizontal displacement variation of this tube began to be relatively large for several days from June 12, 2013 and the maximum horizontal displacement appeared at the top each day. The lateral wall of the foundation pit was reinforced by using bamboo nail on June 16. Monitoring data lines are shown in Fig. 3.

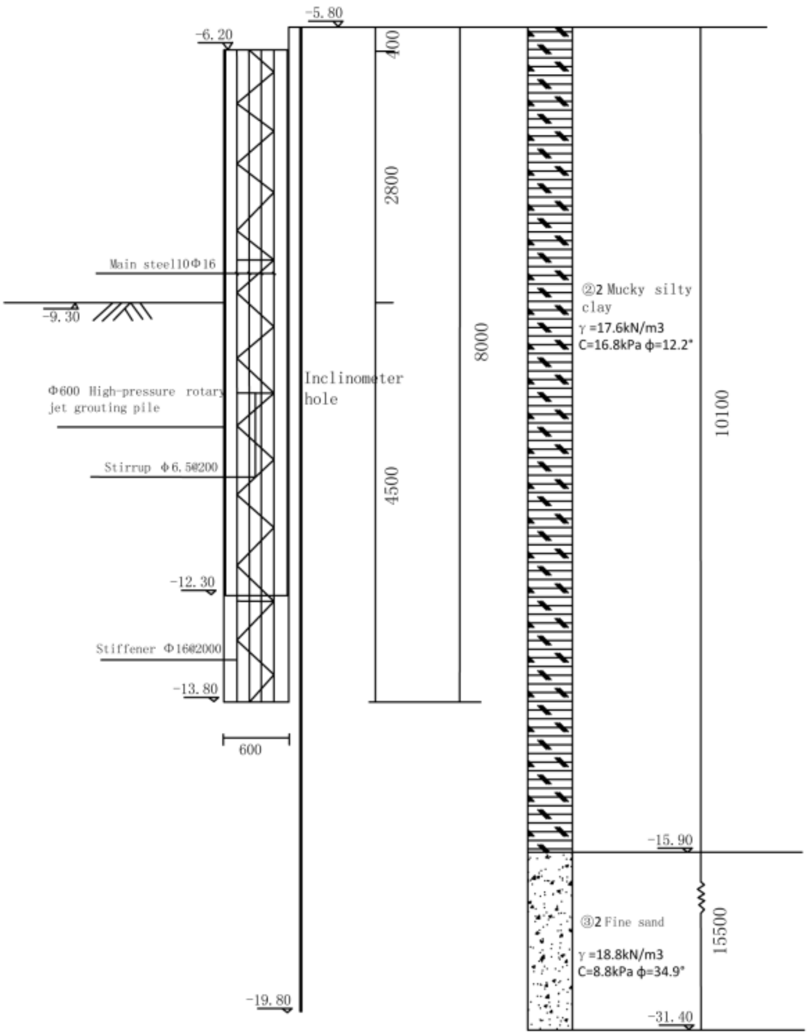

Fig. 2 Supporting structure profile of the foundation pit (called NO.1) in Nanjing

From Fig. 3, the inclinometer tube deformed away from the pit rather than drawing near the pit on June 16, if the monitoring data on June 15 is unknown, it will be easy to mistake that the inclinometer tube deform to the pit all the time, which will make it hard to judge that bamboo nail can prevent the foundation pit deforming too much or can't. Therefore, it is important to get the accurate monitoring data of June 15.

Suppose that the inclinometer tube was not monitored because of the inclinometer failure on June 15, monitoring was recovered on June 16. Input the monitoring data and corresponding depth from June 11 to June 14 into the mathematical software Maple, fit them in the form of Eq. (2) and compare these real monitoring values of each day with those values Figured out by using the fitting equation, use maximum absolute deviation (maximum absolute value of the difference between the real monitoring value and the fitted value) to reflect the fitting result. The parameters *a*, *b*, and *c* obtained by fitting are shown in Table 1.

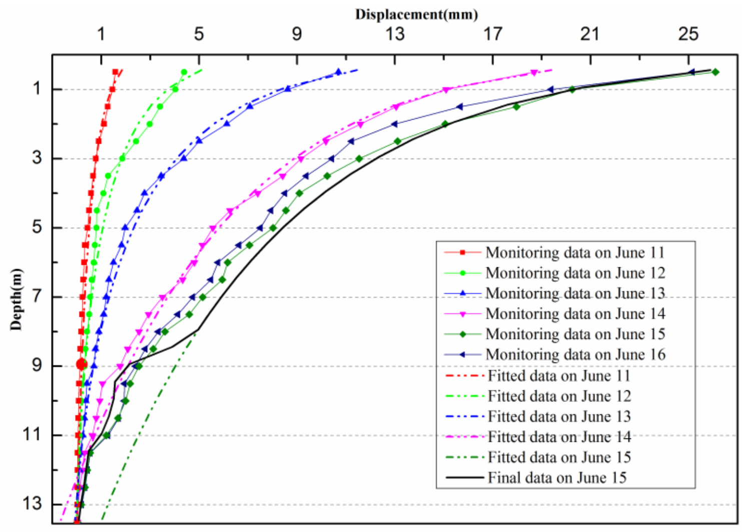

Fig. 3 Monitoring data from June 11 to June 16 and fitted data from June 11 to June 15

Table 1 Parameters obtained by fitting from June 11 to June 14

The *i*th day	Date	*a*	*b*	*c*
1	June 11	-0.5870	0.0087	1.3646
2	June 12	-1.9165	0.0983	3.6055
3	June 13	-4.3451	0.2255	8.1518
4	June 14	-5.6007	-0.1055	15.3078

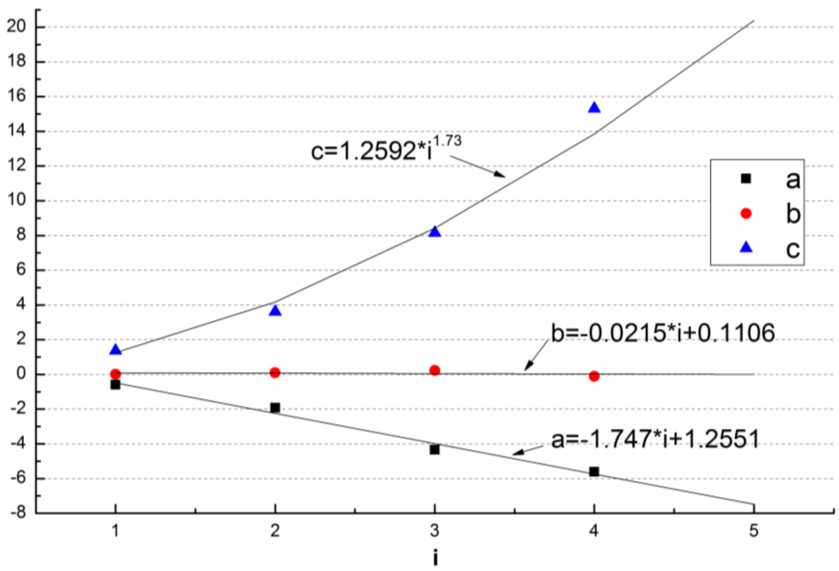

Fig. 4 Parameters and trend lines

According to these parameters and Eq. (4), the fitted data from June 11 to June 14 can be obtained in Fig. 3. The maximum absolute value of difference between the monitoring data and fitted data is 0.72 mm and the mean absolute difference is 0.16 mm, which indicates that Eq. (4) can be used to fit the monitoring data and this mathematical function is adequate.

Feed *a*, *b*, *c* into the Excel table and get each function between *a* and *i*, *b* and *i*, *c* and *i* by drawing the trend line in Excel.

The parameters *a*, *b*, and *c* and the trend lines are demonstrated in Fig. 4.

Table 2 Comparison among the values (the fitted values, average values and real monitoring values) (CX1)

Depth (m)	Monitoring values on June 15	Monitoring values on June 16	Fitted values on June 15	Averages of the monitoring values on June14 and June 16	Absolute value of the difference between the monitoring values and the fitted values	Absolute value of the difference between the monitoring values and the average values	Final values of missing data
0.5	26.10	25.16	25.57	21.92	0.53	4.18	25.57
1	20.25	19.39	20.39	17.24	0.14	3.01	20.39
1.5	17.97	15.68	17.36	14.37	0.61	3.60	17.36
2	15.06	13.01	15.21	12.30	0.15	2.76	15.21
2.5	13.11	11.23	13.54	10.70	0.43	2.40	13.54
3	11.54	10.43	12.18	9.80	0.64	1.74	12.18
3.5	10.23	9.38	11.03	8.89	0.80	1.33	11.03
4	9.09	8.51	10.03	7.95	0.94	1.13	10.03
4.5	8.54	7.94	9.15	7.10	0.61	1.44	9.15
5	8.02	7.50	8.36	6.53	0.34	1.49	8.36
5.5	7.05	6.63	7.65	5.88	0.60	1.17	7.65
6	6.16	5.77	7.00	5.29	0.84	0.87	7.00
6.5	5.94	5.48	6.40	4.91	0.47	1.03	6.40
7	5.14	4.74	5.85	4.12	0.71	1.02	5.85
7.5	4.59	4.13	5.34	3.53	0.74	1.06	5.34
8	3.60	3.33	4.86	2.93	1.26	0.67	4.86
8.5	3.12	2.80	4.40	2.44	1.29	0.68	4.40
9	2.54	2.40	3.98	2.08	1.44	0.46	2.08
9.5	2.18	1.95	3.58	1.50	1.39	0.68	1.50
10	1.99	1.94	3.19	1.44	1.20	0.55	1.44
10.5	1.68	1.71	2.83	1.25	1.15	0.43	1.25
11	1.20	1.28	2.48	0.96	1.28	0.24	0.96
11.5	0.54	0.55	2.15	0.43	1.62	0.11	0.43
12	0.42	0.44	1.84	0.33	1.42	0.08	0.33
12.5	0.33	0.34	1.53	0.24	1.20	0.09	0.24
13	0.17	0.20	1.24	0.12	1.07	0.05	0.12
13.5	0.03	0.03	0.96	0.03	0.93	0.00	0.03

Make i = 5, the parameters of deep horizontal displacement on June 15 could be figured out (a_q = -7.4799, b_q = 0.0031, c_q = 20.3851), then the horizontal displacement of the inclinometer tube on June 15 could be obtained by using Eq. (4). The fitted values, real monitoring values and average values (average of monitoring values of the two days before and after the day of data missing) are compared in Table 2.

From Table 2, in the upper of the inclinometer tube, the fitted values are closer to the real monitoring values, in the lower part of the inclinometer tube, the average values are closer to the real monitoring values, the horizontal displacement variation is relatively small in the lower part of the inclinometer tube, sometimes it even doesn't change which results in the average values being closer to the real monitoring values in the lower part of the inclinometer tube. If choose the average values of the monitoring data on June 14 and June 16 to be the final values on June 15, then the horizontal displacements gradually increase, and it will be difficult to assess if the bamboo nail was effective. Although the deviation between the fitted values and the real monitoring values is big in the lower part of the inclinometer tube, the deformation condition of horizontal displacement in the upper part of the inclinometer tube is undoubtedly more important for the inclinometer tube which has very small horizontal displacement in the bottom. Above all, the fitted values could be the missing data in the upper 2/3 part of the inclinometer tube and the average values could be the missing data in the lower 1/3 part of the inclinometer tube. Besides, the maximum absolute value of difference between the monitoring values on June 16 and the proposed final values is 2.31 mm and the mean absolute difference is 0.95 mm, which are values significantly greater than the differences between the monitoring data on June 15 and the final values for June 15.

4. Recovery of missing data of deep horizontal displacement with constraint at the top

4.1 Mathematical model

The variation of deep horizontal displacement when the maximum horizontal displacement appears in the middle-upper part of the inclinometer tube is shown in Fig. 1(b), according to the feature of the curve, the normal probability density function is chosen to be the mathematical model

$$s = f(h) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(h-\mu)^2}{2\sigma^2}\right) \tag{5}$$

Simplify Eq. (5), as following

$$s = f(h) = \frac{a * exp^{b*(h+c)^2}}{\sqrt{\pi}} \tag{6}$$

In fact, the curve in Fig. 1(b) is not a complete normal distribution model, the symmetry axis of the curve lies in the middle-upper part of the inclinometer tube.

4.2 Calculation method

Same with Section 3.2 above.

4.3 Engineering example

The excavation depth of a foundation pit called No. 2 also in Nanjing was 6.8 m, the lateral walls of this foundation pit were supported by continuous walls which consisted of deep mixing piles, there was ring beam on the top of mixing piles, and the profile is shown in Fig. 5.

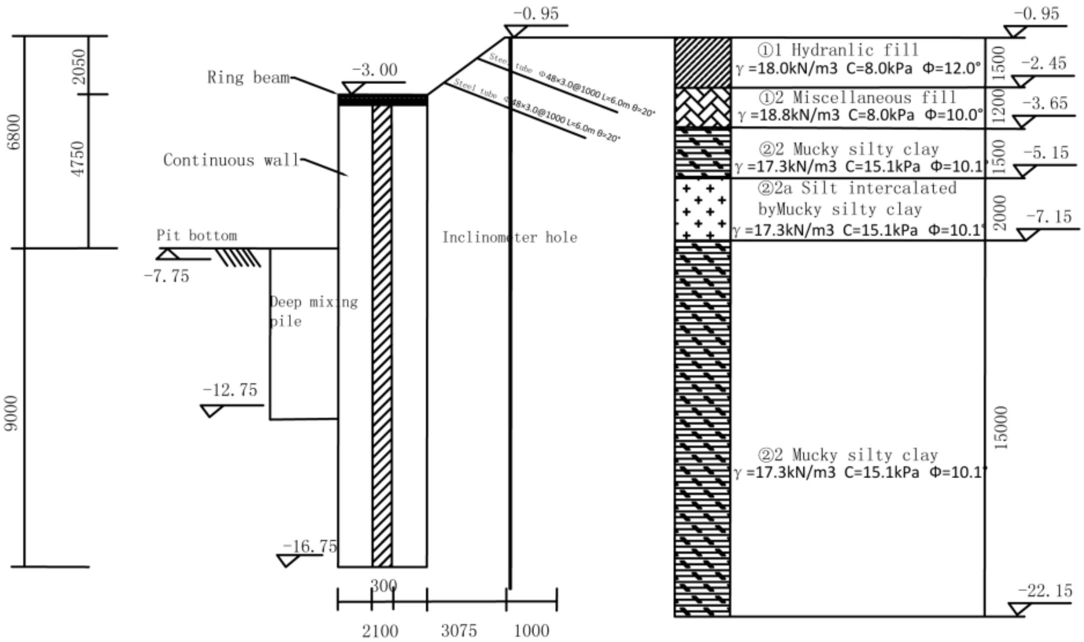

Fig. 5 Supporting structure profile of the No. 2 foundation pit in Nanjing

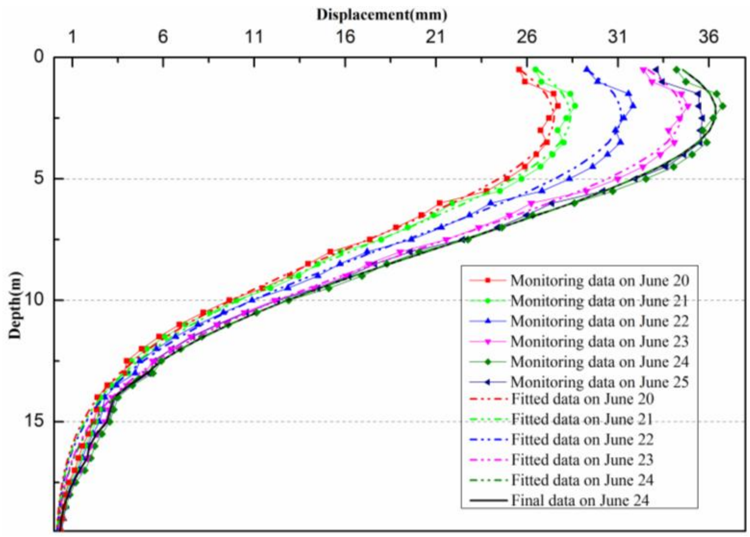

Fig. 6 Monitoring data from July 20 to July 25 and fitted data from July 20 to July 24

The buried depth of one inclinometer tube (denoted as CX2) was 20 m; the monitoring data is shown in Fig. 6. On June 25, the lateral wall of the foundation pit was reinforced by using soil nail and the work of concrete pouring of foundation mat had begun.

Input the monitoring data and corresponding depth in the form of (h, s) from July 20 to July 23 into the mathematical software Maple, and fit them in the form of Eq. (5), the result of fitting equation will be shown in the form of Eq. (6) in Maple. Compare these real monitoring values of each day with those values Figured out by using the fitting equation, use maximum absolute deviation to reflect the fitting result. The parameters a, b, and c obtained by fitting are shown in Table 3.

According to these parameters and Eq. (6), the fitted data from July 20 to July 23 can be obtained in Fig. 6. The maximum absolute value of difference between the monitoring data and fitted data is 0.80 mm, and the mean absolute difference is 0.37 mm, which indicates that Eq. (6) can be used to fit the monitoring data.

Feed a, b, and c into the Excel table and get each function between a and i, b and i, c and i.

Make i = 5, the parameters of deep horizontal displacement on July 24 could be figured out (a_q = 0.645119, b_q = -0.017304, c_q = 2.244110), then the horizontal displacement of the inclinometer tube on July 24 could be obtained by using Eq. (6). The fitted values, real monitoring values and average values are compared in Table 4.

Table 3 Parameters of the inclinometer tube (CX2) obtained by fitting from July 20 to July 23

The *i*th day	Date	*a*	*b*	*c*
1	July 20	0.487213	-0.018301	2.464851
2	July 21	0.503386	-0.017968	2.429701
3	July 22	0.552923	-0.018037	2.362971
4	July 23	0.612264	-0.017436	2.295050

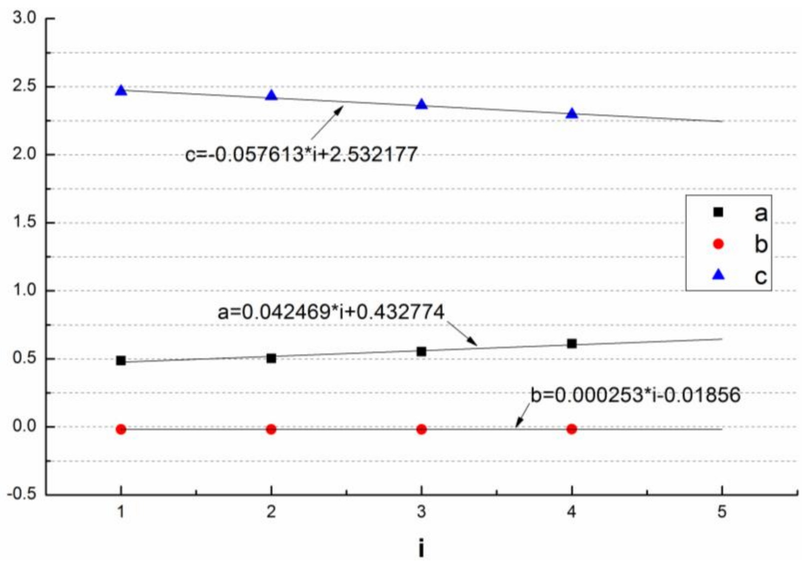

Fig. 7 Parameters and trend lines

Table 4 Comparison among the values (the fitted values, average values and real monitoring values) (CX2)

Depth (m)	Monitoring values on July 24	Monitoring values on July 25	Fitted values on July 24	Averages of the monitoring values on July 23 and July 25	Absolute value of the difference between the monitoring values and the fitted values	Absolute value of the difference between the monitoring values and the average values	Final values of missing data
0.5	34.22	33.12	34.56	32.76	0.34	1.46	34.56
1	34.74	33.45	35.45	33.16	0.71	1.58	35.45
1.5	36.42	35.43	36.06	34.96	0.35	1.46	36.06
2	36.75	35.46	36.36	35.15	0.38	1.60	36.36
2.5	36.23	35.65	36.35	35.01	0.12	1.21	36.35
3	35.67	35.54	36.03	34.66	0.35	1.01	36.03
3.5	35.89	35.52	35.40	34.81	0.49	1.08	35.40
4	35.08	34.63	34.48	33.97	0.60	1.10	34.48
4.5	34.06	33.64	33.29	33.00	0.76	1.05	33.29
5	32.54	31.95	31.87	31.47	0.66	1.07	31.87
5.5	30.71	30.18	30.25	29.72	0.46	0.99	30.25
6	28.60	27.37	28.47	26.80	0.14	1.80	28.47
6.5	26.31	25.98	26.55	25.50	0.24	0.81	26.55
7	24.62	24.40	24.56	23.87	0.06	0.74	24.56
7.5	22.75	22.47	22.51	22.01	0.24	0.74	22.51
8	20.08	19.58	20.46	19.30	0.38	0.78	20.46
8.5	18.30	17.64	18.44	17.46	0.14	0.84	18.44
9	16.93	16.21	16.47	16.13	0.46	0.81	16.47
9.5	15.10	14.51	14.59	14.41	0.51	0.69	14.59
10	12.89	12.30	12.81	12.22	0.08	0.67	12.81
10.5	11.12	10.58	11.15	10.52	0.03	0.60	11.15
11	9.56	9.01	9.62	8.98	0.06	0.58	9.62
11.5	8.12	7.61	8.23	7.56	0.11	0.55	8.23
12	6.95	6.49	6.98	6.47	0.03	0.48	6.98
12.5	5.88	5.50	5.87	5.46	0.01	0.42	5.87
13	5.43	5.21	4.89	5.10	0.54	0.33	5.10
13.5	4.30	4.23	4.04	4.10	0.26	0.20	4.10
14	3.50	3.37	3.31	3.28	0.19	0.22	3.28
14.5	3.27	3.16	2.69	3.08	0.58	0.19	3.08
15	3.05	2.99	2.17	2.89	0.88	0.15	2.89
15.5	2.62	2.33	1.73	2.33	0.89	0.29	2.33
16	2.23	1.88	1.37	1.94	0.86	0.29	1.94
16.5	2.01	1.79	1.07	1.81	0.93	0.20	1.81
17	1.68	1.35	0.84	1.39	0.84	0.29	1.39
17.5	1.18	1.07	0.64	1.04	0.54	0.15	1.04

Table 4 Continued

Depth (m)	Monitoring values on July 24	Monitoring values on July 25	Fitted values on July 24	Averages of the monitoring values on July 23 and July 25	Absolute value of the difference between the monitoring values and the fitted values	Absolute value of the difference between the monitoring values and the average values	Final values of missing data
18	0.86	0.79	0.49	0.77	0.37	0.10	0.77
18.5	0.63	0.53	0.37	0.51	0.26	0.12	0.51
19	0.46	0.38	0.28	0.39	0.18	0.07	0.39
19.5	0.35	0.30	0.21	0.30	0.14	0.05	0.30

From Table 4, in the upper part of the inclinometer tube, the fitted values are closer to the real monitoring values, in the lower part of the inclinometer tube, the average values are closer to the real monitoring values. Therefore, the fitted values could be the missing data in the upper 2/3 part of the inclinometer tube and the average values could be the missing data in the lower 1/3 part of the inclinometer tube. Besides, the maximum absolute value of difference between the monitoring values on July 25 and the proposed final values is 2.00 mm, and the mean absolute difference is 0.38 mm. It indicates that the final values are significantly different from values of July 25.

5. Conclusions

According to the different supporting structures and the different deformation law of foundation pit lateral walls, establish the dynamic mathematical model of deep horizontal displacement about the depth. Through the fitting and interpolation of those mathematical model parameters, a feasible method called Dynamic Mathematical Model – Parameter Interpolation is proposed to recover the missing monitoring data which result from the short-time monitoring interruption. Considering the two different kinds of typical deformation law of foundation pit lateral walls, this paper presents two different calculation methods based on two different mathematical models to recover the missing data. The comparison between the fitted values and the average values shows that: in the upper 2/3 of the inclinometer tube, the result by using this method is closer to the real monitoring data, in the lower 1/3 part of the inclinometer tube, and the result from the common average method is closer to the real monitoring data.

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