**Geomechanics and Engineering**, *Vol. 9*, *No. 2* (*2015*) *145-167*
DOI: http://dx.doi.org/10.12989/gae.2015.9.2.145


# Dynamic simulation models for seismic behavior of soil systems – Part I: Block diagrams

Abdurrahman Sahin*

*Department of Civil Engineering*, *Yıldız Technical University*, *34220*, *Istanbul*, *Turkey*

(*Received April 23*, *2014*, *Revised December 17*, *2014*, *Accepted April 06*, *2015*)

**Abstract.** Digital simulation has recently become the preferred method for designing complex and dynamic systems. Simulation packages provide interactive, block-diagram environment for modeling and simulating dynamic models. The block diagrams in simulation models are flowcharts which describe the components of dynamic systems and their interaction. This paper is the first part of the study for determining the seismic behavior of soil systems. The aim of this part is to present the constructed block diagrams for discrete-time analysis of seismic site amplification in layered media for vertically propagating shear waves. Detailed block diagrams are constructed for single and multiple soil layers by considering wave propagation with and without damping, respectively. The block diagrams for recursive filter to model attenuation in discrete-time form are also constructed. Finite difference method is used for strain calculation. The block diagrams are developed by utilizing Simulink which is a software add-on to Matlab.

**Keywords:** seismic site amplification; soil dynamics; digital simulation; simulink; matlab

## 1. Introduction

The seismic damages are generally larger over soft soils than on bedrock outcrops. Because, amplitudes of seismic waves increase significantly as they propagate through the soft soil layers near the surface. This phenomenon is known as site amplification, and it strongly influences the damage to structures. Site amplification must be analyzed because many urban settlements have been constructed over such young and soft surface deposits.

In literature, a large number of investigations about site amplification have been studied (Kanai 1957, Guttenberg 1957, Idriss and Seed 1968, Borcherdt 1970, 1994, Seed *et al*. 1988, Safak 1989).

Phillips *et al*. (2012) have investigated several interpolation methods used to reduce the ground motion time step and improve the calculation of linear-elastic response spectra, and linear-elastic and nonlinear site response. Bolisetti *et al*. (2014) have investigated the applicability of some industry-standard equivalent linear (SHAKE) and nonlinear (DEEPSOIL and LS-DYNA) programs. Kaklamanos *et al*. (2015) have made comparison of 1D linear, equivalent-linear, and nonlinear site response models at six KiK-net validation sites. They have used the equivalent-

*Corresponding author, Associate Professor, E-mail: abdsahin@yildiz.edu.tr


http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

linear site response program SHAKE, the nonlinear site response program DEEPSOIL, and a nonlinear site response overlay model within the general finite element program Abaqus/Explicit for numerical simulations. Johari and Momeni (2015) have proposed a probabilistic procedure for estimating the one-dimensional site response of soil deposits with uncertain properties and applied to four real sites in south of Iran.

One dimensional site response analysis is mostly used for determining site amplification (Choudhury and Savoikar 2009, Philips and Hashash 2009, Hashash *et al*. 2010, Roullé and Bernardie 2010, Rota *et al*. 2011, Boaga *et al*. 2012). Many computer programs have been developed to estimate seismic site amplification in layered soil media (Schnabel *et al*. 1972, Idriss and Sun 1992, Bardet *et al*. 2000, Yang and Yan 2006, Robinson *et al*. 2006). Most of these programs have been developed in frequency domain. Safak (1995) has investigated site amplification in layered media by using discrete-time wave propagation technique and developed theoretical formulations in time domain. He incorporated soil damping, of discrete-time wave propagation techniques to the problem of seismic site amplification. In this study, dynamic simulation models for single soil layers and multiple soil layers subjected to seismic waves by considering propagation with and without damping are presented, respectively. The recursive filter simulation model representing the damping is also presented. The block diagrams are constructed following the theoretical formulations given by Safak (1995). The models are produced by using the software Matlab-Simulink (MATLAB 2009, SIMULINK 2009). The main advantage of Simulink is updating material properties at each time step during simulation.

The main contribution of this study is giving an opportunity to researchers for observing system behavior real time by changing system parameters. The soil parameters such as density, wave velocity, thickness etc. can be changed while the system analysis is going on. By this way, the most efficient parameters for site amplification can be determined during the analysis process. The developed block diagrams solve the system in time domain and it gives an interactive media for dynamic system simulation.

## 2. Matlab/Simulink environment

With the advances in computer technologies, the computer aided design has been developed. Matlab is one of the representatives of high-performance language for the CAD. Simulink is an add-on software package to Matlab for modeling, simulating, and analyzing dynamic systems. It supports linear and non-linear systems, modeled in continuous and discrete time. Simulation is an interactive process, so the parameters may be altered while the simulation is running and the system response may be immediately monitored.

Simulink is a graphical platform developed to construct flowcharts of the proposed algorithms. It has been developed for multi domain simulation and model-based design. It can be used for dynamic and embedded systems. Simulink provides an interactive graphical environment. A customizable set of block libraries may be used. The researchers can design, simulate, implement, and test a variety of time-varying systems like signal processing by using Simulink.

### *2.1 Block diagrams*

As indicated before, dynamic systems can be simulated using Simulink. Dynamic systems are described with block diagrams. Each block diagram performs a mathematical operation and has

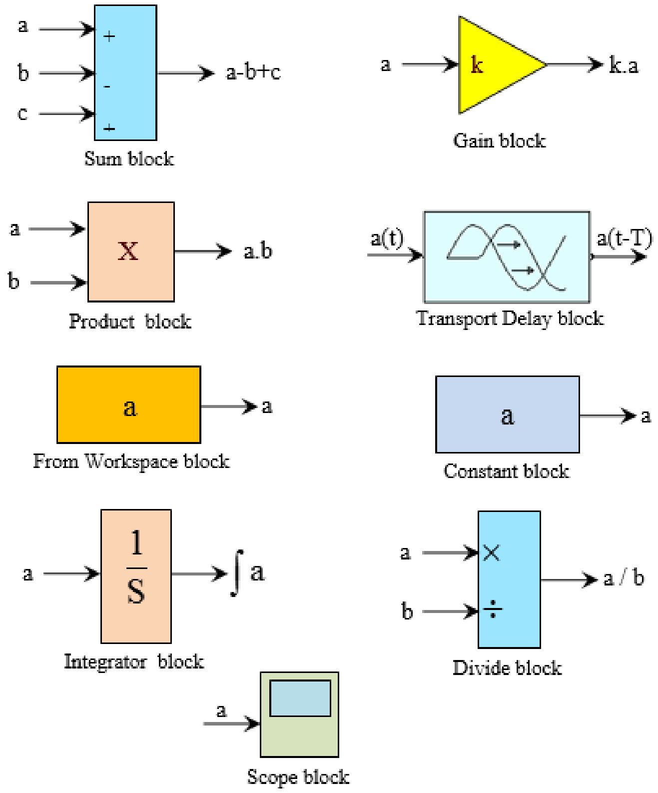


Fig. 1 Blocks used for digital simulation of seismic site amplification in time domain (SIMULINK 2009)

input, output, and some constants (i.e., the block parameters). The detailed expression of each block diagram used in this study is presented in Fig. 1. The objectives of these blocks are summarized as follows (Simulink 2009):

- The Sum block performs addition or subtraction on its inputs. This block can add or subtract scalar, vector, or matrix inputs. It can also collapse the elements of a signal.
- The Gain block multiplies the input by a constant value (gain). The input and the gain can each be a scalar, vector, or matrix.
- The Product block multiplies and divides scalars and nonscalars or multiplies and inverts matrices.
- The Transport Delay block delays the input by a specified amount of time. It can be used to simulate a time delay.
- The From Workspace block reads data from a workspace and outputs the data as a signal
- The Constant block generates a real or complex constant value.
- The Integrator block outputs the integral of its input at the current time step.
- The Scope block displays its input with respect to simulation time.

## 3. Dynamic simulation models for single soil layers

### *3.1 Single soil layer without damping over bedrock*

A single soil layer over bedrock is schematically presented in Fig. 2. The nomenclature used in formulations is as follows (Safak 1995):

- $u(t)$ : Upgoing wave at the top of the layer
- $d(t)$ : Downgoing wave at the bottom of the layer
- $\tau$ : One-way travel time of the waves in the layer
- $x(t)$ : Recorded ground motion at the surfaces of a rock outcrop
- $y(t)$ : Recorded ground motion at the surfaces of a soil site
- $r$ : Reflection coefficient of the rock-soil interface for upgoing waves
- $\rho_r$ : Mass density in the rock
- $v_r$ : Shear-wave velocity in the rock
- $\rho_s$ : Mass density in the soil
- $v_s$ : Shear-wave velocity in the soil

The reflection coefficient r for upgoing waves is calculated as follows (Aki and Richards 1980, Safak 1995)

$$r = \frac{\rho_r v_r - \rho_s v_s}{\rho_r v_r + \rho_s v_s} \tag{1}$$

This expression applies to displacement, velocity, and acceleration waves (for stress waves, the reflection coefficient is $-r$). Then, the transmission coefficient for upgoing waves is $1+r$, the free-surface reflection coefficient is one and the reflection coefficient for downgoing waves is $-r$. By using these rules, the following equations may be obtained (Safak 1995)

$$u(t) = -rd(t-\tau) + (1+r)\frac{x(t-\tau)}{2} \tag{2a}$$

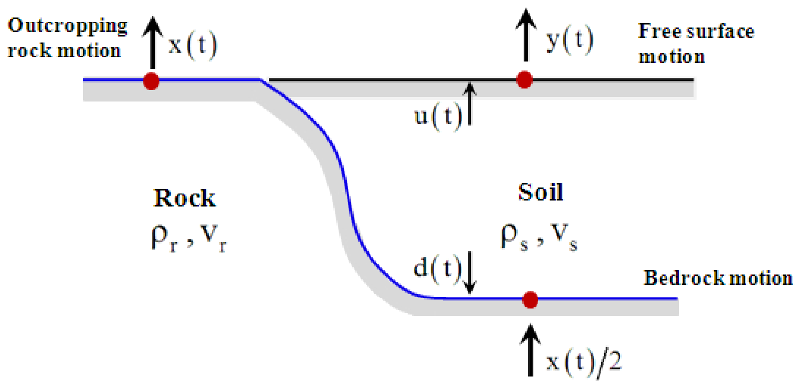


Fig. 2 Single layer over bedrock with rock- and soil- site recordings, $x(t)$ and $y(t)$, and upgoing and downgoing waves, $u(t)$ and $d(t)$

$$d(t) = u(t - \tau); \tag{2b}$$

$$y(t) = 2u(t); \tag{2c}$$

The Simulink model for these equations is presented in Fig. 3. In block diagrams, ($\tau$) is named as tau. To execute this model, firstly the reflection coefficient ($r$) and the one way travel time of the layer ($\tau$) must be calculated and must be loaded to workspace. The functions produced for these calculations are presented in Appendix I.

If Eqs. (2b), (2c) is substituted in Eq. (2a), the surface motion equation is directly obtained depending on the surface motion of the rock site as follows (Safak 1995)

$$y(t) = -ry(t - 2\tau) + (1 + r) \times (t - \tau) \tag{3}$$

As it can be seen, Eq. (3) is a recursive filter for calculating the surface motions of the rock site, assuming no damping in the soil. The block diagram for this recursive filter is presented in Fig. 4.

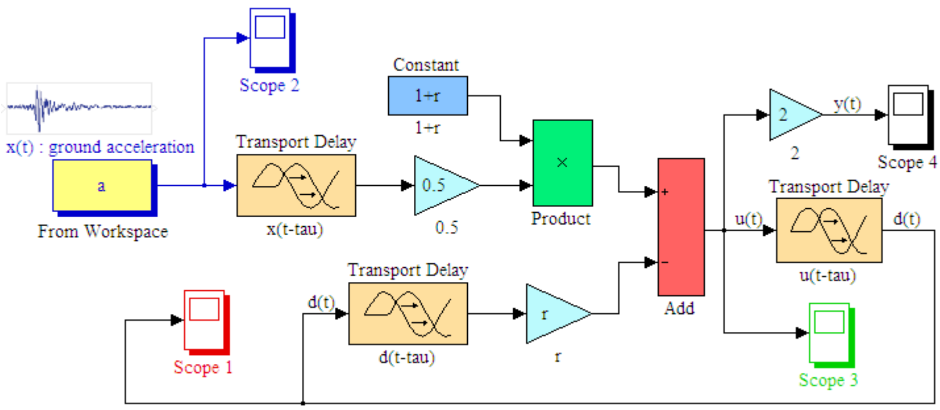


Fig. 3 Block diagram created for calculating surface motion, upgoing and downgoing wave of the single layer without damping subjected to earthquake motion

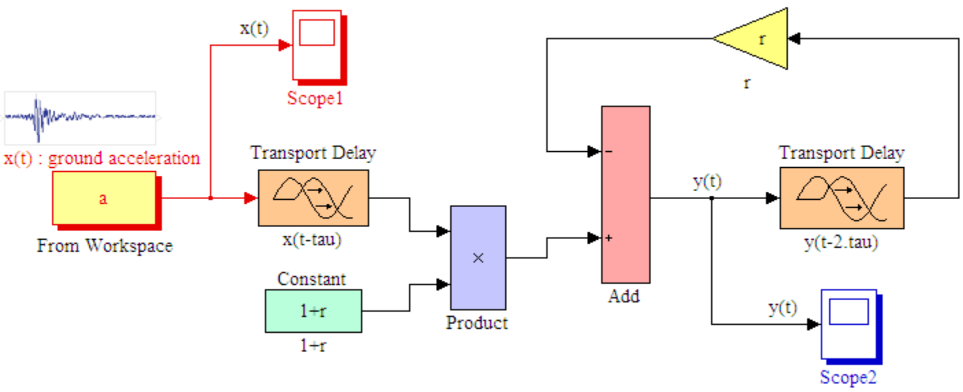


Fig. 4 Block diagram created for calculating surface motion of the single layer without damping directly from the earthquake motion

### 3.2 Single soil layer with damping over bedrock

Seismic waves attenuate as they propagate through soil layers, because of the damping in the soil. The transfer function is converted into a time domain recursive filter to incorporate attenuation in the time domain formulation of wave propagation. Since transfer function has the form of a low-pass filter, it can be approximated by a recursive filter of the following form (Cadzow 1973, Safak 1995)

$$u_o(t) = \alpha u_o(t-T) + 0.5(1-\alpha)\left[u_i(t) + u_i(t-T)\right] \tag{4}$$

where $u_i(t)$ and $u_o(t)$ are the input and output series of the filter and $T$ is the time interval in the series. The filter parameter $\alpha$ is determined from the following equations (Safak 1995)

$$\begin{aligned} &\alpha = \frac{1-\sqrt{1-\cos^2\theta}}{\cos\theta} \quad \text{with} \quad \theta = \ln 2\frac{QT}{\tau} \\ &T \le \frac{\pi\tau}{Q\ln 2} \end{aligned} \tag{5}$$

As it can be seen above, the attenuation as defined by the parameter $Q$, known as the quality factor (Knopoff 1964). $Q$ is a measure of the energy loss per cycle during a sinusoidal deformation (O'Connell and Budiansky 1978). It can be expressed as follows (Safak 1995)

$$\zeta = \frac{1}{2}Q \tag{6}$$

The upgoing and downgoing waves shown in Fig. 2 should pass through the recursive filter every time they cross the layer. By considering this filtering equation and applying some mathematical operations, the damped values of $u(t)$ and $d(t)$ (i.e., $\bar{u}(t)$ and $\bar{d}(t)$) is obtained as follows (Safak 1995)

$$\bar{u}(t) = \lambda\left[-r\bar{d}(t-\tau) + (1+r)\frac{x(t-\tau)}{2}\right] \tag{7a}$$

$$\bar{d}(t) = \lambda\bar{u}(t-\tau); \tag{7b}$$

$$y(t) = 2\bar{u}(t) \tag{7c}$$

where $\lambda$ is the filter representing the damping, and is defined as follows (Safak 1995)

$$\lambda = \frac{1-\alpha}{2}\frac{1+q^{-1}}{1-\alpha q^{-1}} \tag{8}$$

The parameter $q$ denotes the backward time-shift operator, such that $q^{-j}u(t) = u(t-jT)$. Since this recursive filter is used in many times, a simpler and more efficient way would be to construct a subsystem and to use this subsystem block in other Simulink models. A subsystem may be imagined as a function in object oriented programming. If a function about any mathematical progress is developed and added into the library, it may be called by other functions and there is no need to write it again. Subsystem logic is very similar to that. There are input and output channels in subsystem and it is used by other blocks. The subsystem block diagram of this damping recursive filter is presented in Fig. 5. In block diagrams, ($\alpha$) is named as alfa. As shown in Fig. 5, there is one input and one output signal ($u_i(t)$, $u_o(t)$) as given in Eq. (4). The input signal passes through the filter and goes out through the output channel. Once the subsystem's inputs and outputs are identified, the block diagram can be condensed to a single block, i.e., the damping filter subsystem block, as shown in Fig. 6. This block is used whenever damping filter is needed in simulation modeling.

The Simulink model for determining the surface motion, upgoing and downgoing wave of the single layer with damping subjected to earthquake motion is presented in Fig. 7. Eqs. (7a)-(7c) is taken into account to construct this block diagram. To execute this model, firstly the one way travel time of the layer ($\tau$), time interval in the series ($T$) and the filter parameter ($\alpha$) must be calculated and must be loaded to workspace. The functions produced for these calculations are presented in Appendix II.

If Eqs. (7b) and (7c) is substituted in Eq. (7a), the surface motion equation is directly obtained depending on the surface motion of the rock site as follows (Safak 1995)

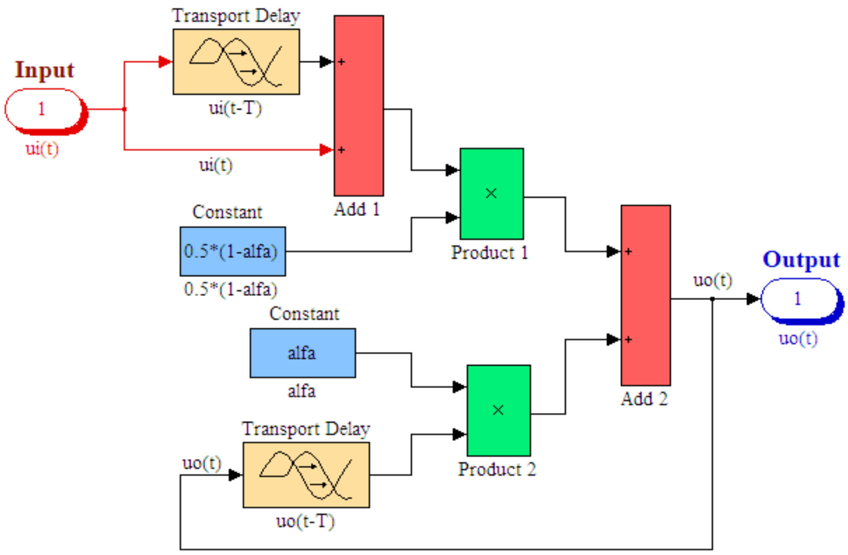


Fig. 5 Inside of the subsystem block diagram of recursive filter representing the damping

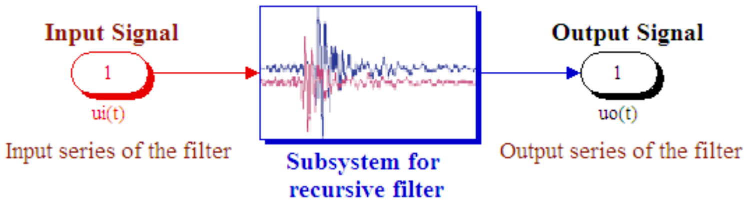


Fig. 6 Subsystem block diagram of recursive filter which may be called in any model

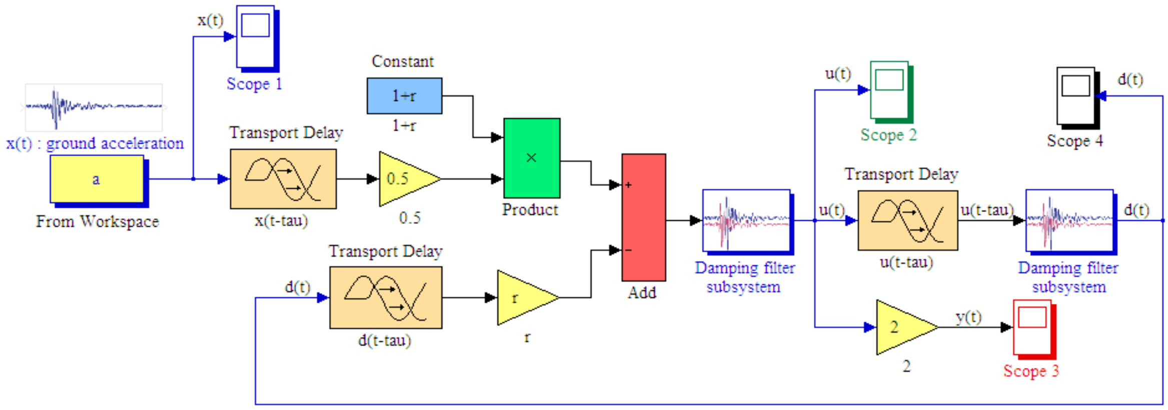


Fig. 7 Block diagram for calculating surface motion, upgoing and downgoing wave of the single layer with damping subjected to earthquake motion

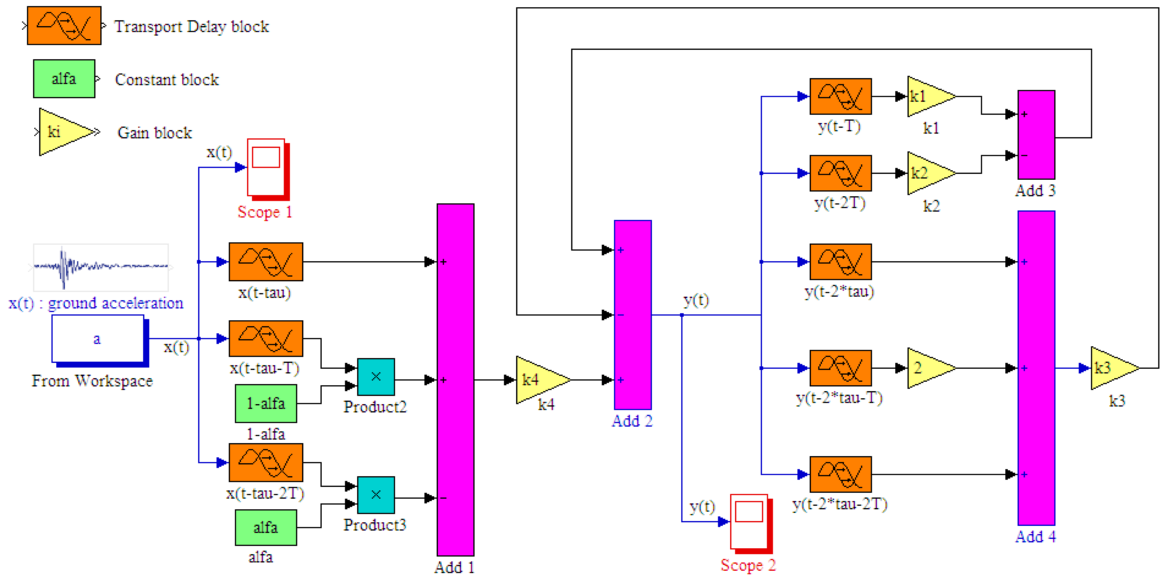


Fig. 8 Block diagram for calculating surface motion of the single layer with damping directly from the earthquake motion

As it can be seen, Eq. (9) is a recursive filter for calculating the surface motion, considering damping in the soil. This equation is updated for a programming tool as follows

$$\begin{aligned} y(t) = {} & k_1 y(t-T) - k_2 y\,(t-2T) \\ & - k_3[y(t-2\tau) + 2y(t-2\tau-T) + y(t-2\tau-2T)] \\ & + k_4[x(t-\tau) + (1-\alpha)\times(t-\tau-T) - \alpha x(t-\tau-2T)] \end{aligned} \tag{10}$$

where

$$k_1 = 2\alpha$$
$$k_2 = \alpha^2$$
$$k_3 = 0.25(1-\alpha)^2 r$$
$$k_4 = 0.5(1-\alpha)(1+r)$$

The block diagram for this recursive filter is presented in Fig. 8. To execute this model, firstly the one way travel time of the layer ($\tau$), time interval in the series ($T$), the filter parameter ($\alpha$) and $k_1$, $k_2$, $k_3$ and $k_4$ coefficients must be calculated and must be loaded to workspace. The functions produced for these calculations are presented in Appendix III.

If the bedrock motion instead of the surface rock motion is given, then a gain block which has parameter 0.5 is added to the model and ground acceleration line is connected to that.

## 4. Dynamic simulation models for multiple soil layers

### 4.1 Multiple soil layers without damping over bedrock

For an $m$-layer soil media given in Fig. 9, the following equations starting with the $m$'th layer directly above the bedrock may be obtained as follows (Safak 1995)

$$u_m(t) = -r_m d_m(t-\tau_m) + (1+r_m)\frac{x(t-\tau_m)}{2} \tag{11a}$$

$$d_m(t) = (1-r_{m-1})d_{m-1}(t-\tau_m) + r_{m-1}u_m(t-\tau_m) \tag{11b}$$

$$\cdots$$

$$u_j(t) = -r_j d_j(t-\tau_j) + (1+r_j)u_{j+1}(t-\tau_j) \tag{11c}$$

$$d_j(t) = (1-r_{j-1})d_{j-1}(t-\tau_j) + r_{j-1}u_j(t-\tau_j) \tag{11d}$$

$$u_1(t) = -r_1 d_1(t-\tau_1) + (1+r_1)u_2(t-\tau_1) \tag{11e}$$

$$d_1(t) = u_1(t-\tau_1) \tag{11f}$$

where the subscripts indicate the layer number. The motion at the free surface of the soil is $y(t) = 2u_1(t)$.

The block diagram for the above set of equations can be constructed directly by developing the block diagram for each equation (similar to that for a single layer system), and connecting them appropriately to account for the coupling between layers. As indicated before, in such repeated cases, subsystem option should be taken into account for fast and easy programming.

Firstly, the block diagram for $j$. soil layer without damping is developed as shown in Fig. 10. As it can be seen from Fig. 10, the constants of the block diagram are, $\{r(j)\}$, $\{1-r(j-1)\}$, $\{r(j-1)\}$ and $\{1+r(j)\}$, respectively. The $r(j)$ and $\tau(j)$ must be calculated and loaded to workspace before

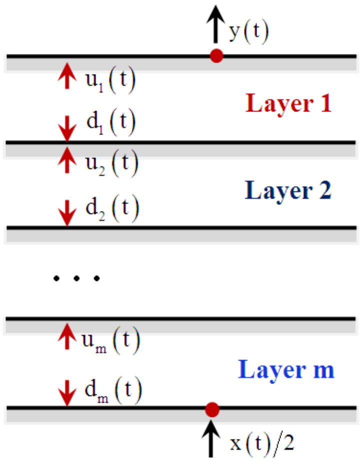


Fig. 9 Multiple soil layers over bedrock with rock- and soil- site recordings, *x*(*t*) and *y*(*t*), and upgoing and downgoing waves, *u*(*t*) and *d*(*t*)

running the model. In Fig. 10, a gain block which has the parameter of a_coef(*j*) is seen. This block is placed there to give chance the users to apply ground acceleration as bedrock motion or rock outcrop motion. If it is bedrock motion, the gain parameter for *m*'th layer is equal to 0.5 (i.e., a_coef(*n*) = 0.5) and if it is a rock outcrop motion, the gain parameter for *m*'th layer is equal to 1 (i.e., a_coef(*n*) = 1). For other layers, it's value would be one to make it ineffective (i.e., a_coef(*n*) = 1).

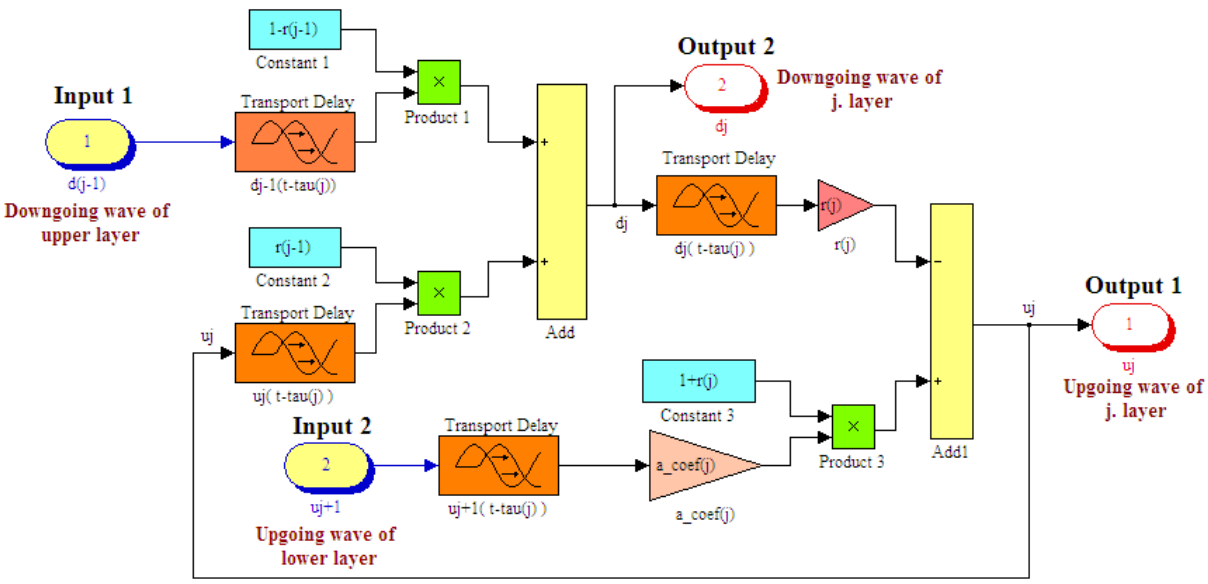


Fig. 10 Inside of the subsystem block diagram for *j*'th layer of a multi-layered soil media subjected to earthquake motion

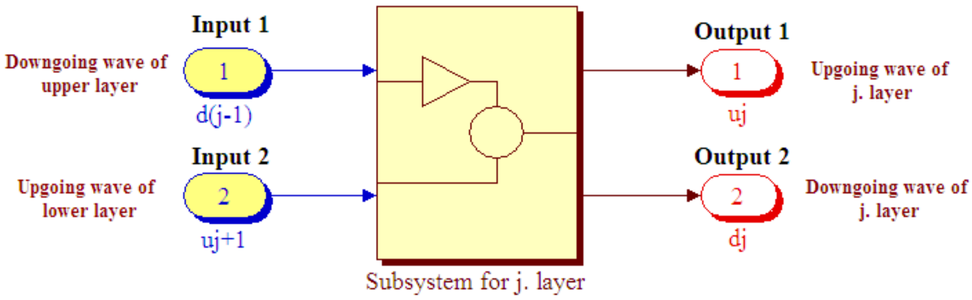


Fig. 11 Subsystem block diagram for j'th layer of a multi-layered soil media subjected to earthquake motion

The equations for the *j*'th layer are coupled to the equations for the layers above through the downgoing wave $\{d(j-1)\}$ and below through the upgoing wave $\{u(j+1)\}$. These downgoing wave from upper layer and upgoing wave from lower layer are needed at every time step in order to solve the equation for this layer; hence they represent the two inputs for the subsystem representing this layer. For *m*'th soil layer, upgoing wave of below layer would be ground acceleration. The two inputs are marked in Fig. 10. The two outputs of the subsystem are upgoing and downgoing waves of the layer, which are also marked in Fig. 10. These two outputs will be the inputs for the subsystems for the layers above and below. Once the subsystem's inputs and outputs are identified, the block diagram can be condensed into a single block, i.e., the layer subsystem block, as shown in Fig. 11. Each layer of the soil may be modeled by using this single block.

For the first soil layer, there is not a downgoing wave from upper layer; therefore, the subsystem block for this layer should be modified as shown in Fig. 12. This block diagram is constructed depending on the Eqs. (19e) and (19f) The equations for the first layer are coupled to the equations for the second layer upgoing wave $\{u(2)\}$. This upgoing wave from lower layer 2 is needed at every time step in order to solve the equation for first layer; hence this represents the input for the subsystem representing first layer. The input is marked in Fig. 12. The two outputs of the subsystem are upgoing and downgoing waves of the layer, which are also marked in Fig. 12.

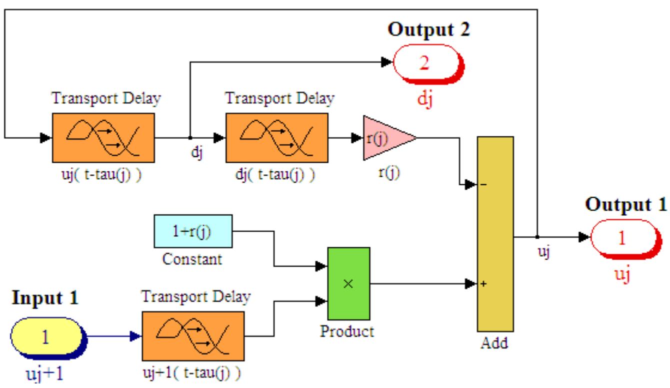


Fig. 12 Inside of the subsystem block diagram for first layer of a multi-layered soil media subjected to earthquake motion

Once the subsystem's inputs and outputs are identified, the block diagram can be condensed into a single block, i.e., the first layer subsystem block, as shown in Fig. 13.

The block diagram for the entire soil media is developed by combining the layer blocks according to the interaction between the layers, as defined by Eq. (19). The block diagram for three-layered soil media is presented in Fig. 14. As it can be seen from this figure, the interaction between the layers are illustrated by the fact that the outputs from one layer become inputs for the layers above and below. For the first layer, there is only one input, because there are no layers

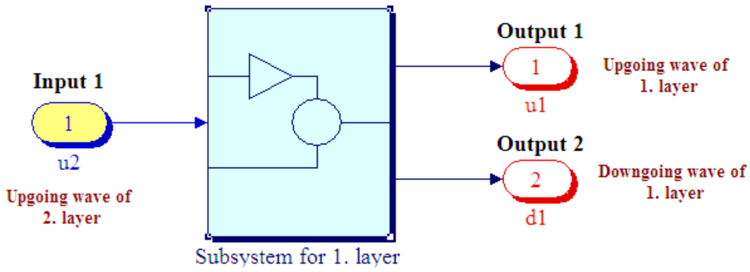


Fig. 13 Subsystem block diagram for first layer of a multi-layered soil media subjected to earthquake motion

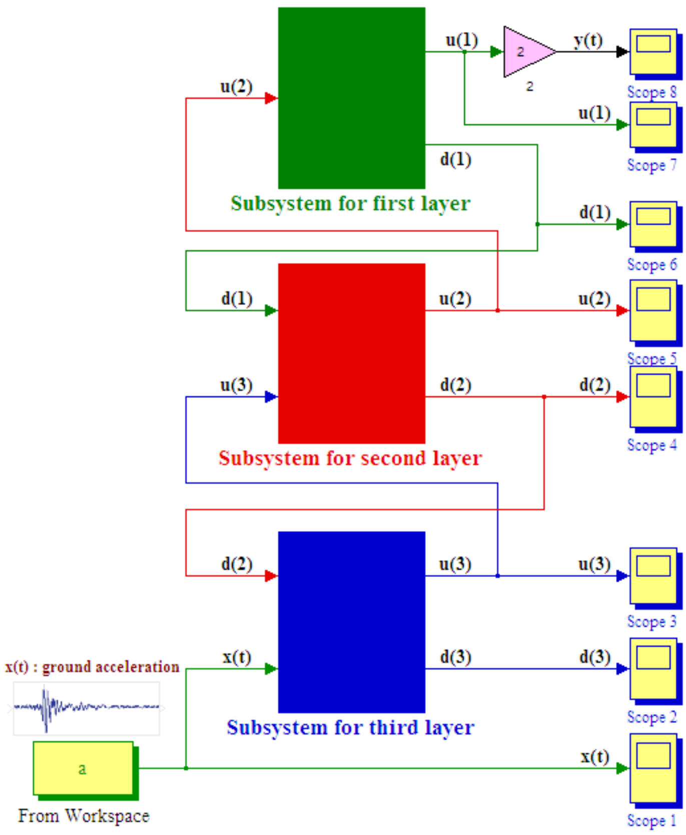


Fig. 14 Block diagram for a 3-layered soil media subjected to earthquake motion

above. The upgoing wave from the second layer is the input of the first layer. This illustration is a general presentation. If the soil media consists of $n$ layers, the subsystems for $j$. layers may be duplicated and the Simulink model for $n$-layered soil media may be easily created and analyzed.

For making analysis more effective, subsystems may be masked. The Simulink Mask Editor enables users to create a mask for any subsystem. A mask is a custom user interface for a subsystem that hides the subsystem's contents, making it appear to the user as an atomic block with its own icon and parameter dialog box (SIMULINK 2009). In this study, the layer number $j$ is defined as a mask parameter for layer subsystems. In this way, all layer subsystems may use their constant parameters by assigning the layer number into the subsystems. This assignment may be carried out from commands in any functions, therefore the blocks may easily be controlled by any graphical user interfaced (GUI) program written in Matlab.

### *4.2 Multiple soil layers with damping over bedrock*

For damped soil layers, damping is incorporated in each layer by using recursive damping filter. The only difference between damped and undamped soil layers is the recursive damping filter as it can be seen from the following equations (Safak 1995)

$$u_m(t) = \lambda_m \left[ -r_m d_m(t-\tau_m) + (1+r_m)\frac{x(t-\tau_m)}{2} \right] \tag{12a}$$

$$d_m(t) = \lambda_m \left[ (1-r_{m-1}) d_{m-1}(t-\tau_m) + r_{m-1} u_m(t-\tau_m) \right] \tag{12b}$$

$$\dots$$

$$u_j(t) = \lambda_j \left[ -r_j d_j(t-\tau_j) + (1+r_j) u_{j+1}(t-\tau_j) \right] \tag{12c}$$

$$d_j(t) = \lambda_j \left[ (1-r_{j-1}) d_{j-1}(t-\tau_j) + r_{j-1} u_j(t-\tau_j) \right] \tag{12d}$$

$$u_1(t) = \lambda_1 \left[ -r_1 d_1(t-\tau_1) + (1+r_1) u_2(t-\tau_1) \right] \tag{12e}$$

$$d_1(t) = \lambda_1 \left[ u_1(t-\tau_1) \right] \tag{12f}$$

The downgoing and upgoing waves pass through the recursive damping filter every time they cross their layer. In Eq. (12), $\lambda_j$ is defined as follows (Safak 1995)

$$\lambda_j = \frac{1-\alpha_j}{2} \frac{1+q^{-1}}{1-\alpha_j q^{-1}} \tag{13}$$

As it can be seen from Figs. 5 and 6, a subsystem has been created for damping recursive filter. It should be updated to a general $j$. layer and may be used in subsystems for each soil layer. The block diagram for the recursive damping filter updated for $j$'th layer is presented in Fig. 15. The block diagram for $j$. soil layer with damping is developed as shown in Fig. 16. This block diagram is very similar to Fig. 10 which was developed for $j$. soil layer without damping. The only difference between these block diagrams is the damping filter.

After considering damping by using the recursive damping filter subsystem, all other procedure is the same with previous section applied for multilayered soil with no damping. If the soil media consists of *n* layers, the subsystem for *j*. layer may be used for representing other layers and they are linked as indicated above. After that, the Simulink model for n-layered soil media with damping may be easily created and analyzed.

To execute the soil simulation model, firstly the reflection coefficient of each layer ($r_j$), the one way travel time of each layer ($\tau_j$), time interval in the series ($T_j$), the filter parameter for each layer ($\alpha_j$) must be calculated and must be loaded to workspace. The functions produced for these calculations are presented in Appendix IV.

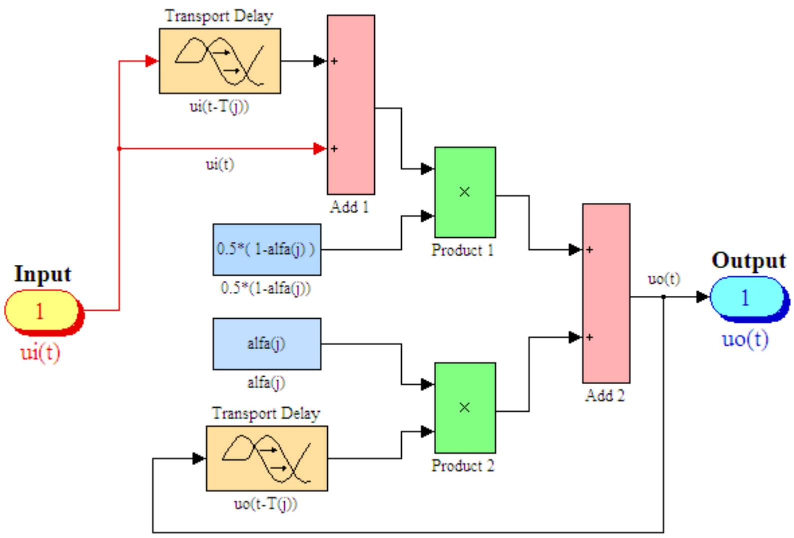


Fig. 15 Block diagram of recursive filter representing the damping for *j*'th layer

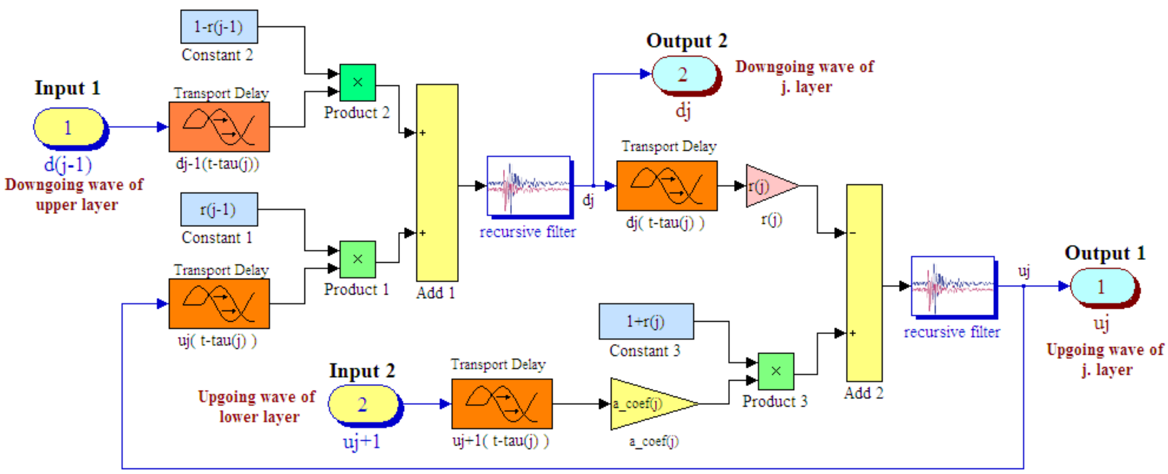


Fig. 16 Block diagram for *j*'th layer of a multi-layered soil media with damping subjected to earthquake motion

While the simulation is running, the model parameters may be altered simultaneously. The simulation is in real time and interactive, which means that the system's response can be observed as it develops, and the systems parameters can be changed abruptly (i.e., by pausing and changing their numerical values) or gradually (i.e., by using a graphical slider control while the simulation is in progress). The soil parameters such as thickness, mass density, shear wave velocity and quality factor for each soil layer may be changed by using slider or edit boxes while simulation is running as given in Fig. 17. The simulation of a given soil media with a parameter changing slider window is presented in Fig. 18. The soil parameters may be changed in real time and the system response may be observed simultaneously.

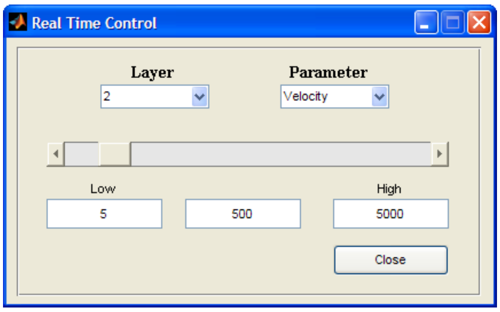


Fig. 17 Real time parameter altering window

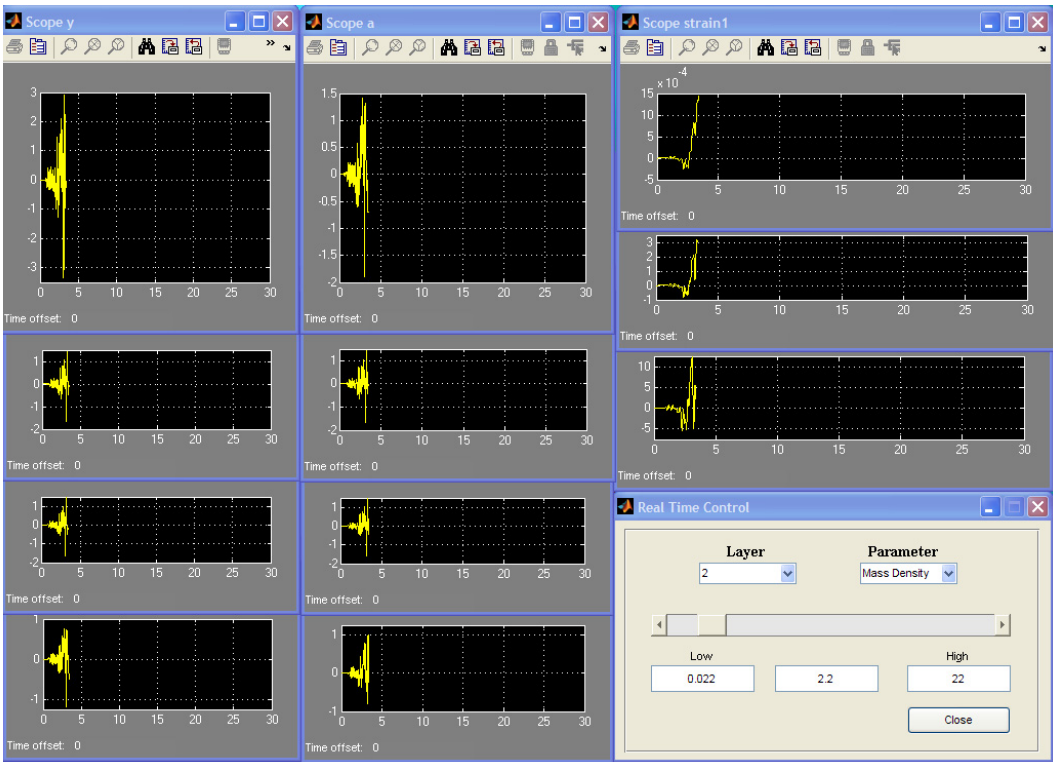


Fig. 18 A view of the simulation of a given soil media with a parameter changing slider window

## 5. Strain calculation of soil layers

The time history of shear strain $\gamma$ and strain values at each time steps are used in site response analysis and calculated as follows

$$\gamma(z,t) = \frac{\partial u(z,t)}{\partial z} \tag{14}$$

The solution of differential equations is approximated by replacing derivative expressions with approximately equivalent difference quotients with Finite-difference methods (FDM). The FDM is very common technique and consists of transforming the partial derivatives in difference equations over a small interval. First order derivative may be approximated using a forward finite-difference approximation

$$\frac{df(z_i)}{dz} = \lim_{\Delta z_i \to 0} \frac{f(z_{i+1}) - f(z_i)}{z_{i+1} - z_i} \tag{15}$$

The strain for $i$'th layer is calculated using finite difference formulation as follows

$$\gamma_i = \frac{\partial d}{\partial z} = \frac{d_{i+1} - d_i}{z_{i+1} - z_i} = \frac{d_{i+1} - d_i}{\Delta z_i} \tag{16}$$

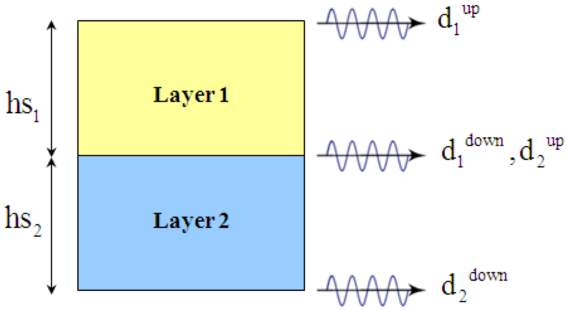


Fig. 19 “Model A” representation with a two layered soil media

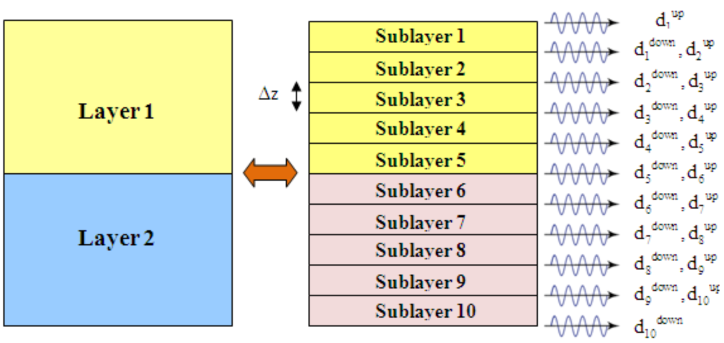


Fig. 20 “Model B” representation with a two layered soil media which is divided into equivalent sublayers for strain calculation

Where; $d_i$ is $i$'th layer displacement. The accuracy of finite difference formulations depend on the mesh frequency between layers. The distance should be very small as it can be seen in Eq. (16). To supply such a suitable mesh, different solution alternatives may be considered. In this study, two alternative models are taken into account. In the first model, the finite difference formulation is applied by considering the upper and lower displacement values of each layer. The difference value of upper and lower displacement values of the layers are divided to layer heights and the strain time history of corresponding layer is calculated. The graphical representation of "Model A" for two layered soil media is presented in Fig. 19. The strain time history for these two layers is calculated as follows

$$\gamma_1 = \frac{d_1^{\text{up}} - d_1^{\text{down}}}{hs_1} \tag{17}$$

$$\gamma_2 = \frac{d_2^{\text{up}} - d_2^{\text{down}}}{hs_2} \tag{18}$$

In the second model, layers are divided into same sublayers which have the same geotechnical parameters of corresponding layers. The strain distributions of each sublayer are calculated. The graphical representation of "Model B" for two layered soil media is presented in Fig. 20. The strain time history for each layer is calculated as follows

$$\gamma_i = \frac{d_i^{\text{up}} - d_i^{\text{down}}}{\Delta z} \tag{19}$$

## 6. Block diagrams constructed for strain calculation

In the previous section, two different models are given for strain calculation of soil layers. In this part, block diagrams for these models are constructed. In Figs. 21-23, the block diagrams developed to calculate strain time histories and site response of soil layers for "Model A" are presented. As shown in these figures, the strain time histories of the layers are calculated with Finite Difference Method by following the methodology described in previous section. The sub-layer displacements are calculated by using integrator blocks in the model. The Integrator block outputs the integral of its input at the current time step. The input data of integrator blocks are acceleration time history, and the two times integral of acceleration gives the displacement.

A subsystem is constructed to compute strain time history from surface accelerations as shown in Fig. 21. The general presentation of this subsystem is given in Fig. 22. This subsystem is used for strain calculation of each layer. The block diagram for the entire soil media is developed by combining the total accelerations above and below the sublayers to the strain subsystem. The block diagram including strain calculation for three-layered soil media is presented in Fig. 23. This illustration is a general presentation. If the soil media consists of n layers, the subsystems for $j$. layers may be duplicated and the Simulink model for $n$-layered soil media may be easily created and analyzed.

"Model B" block diagrams are constructed by using same methodology with the "Model A" block diagrams. The only difference is block parameters and block numbers. In "Model B", the soil layers are divided into sublayers. Therefore, the layer number is increased in this model and the block diagram uses updated block parameters.

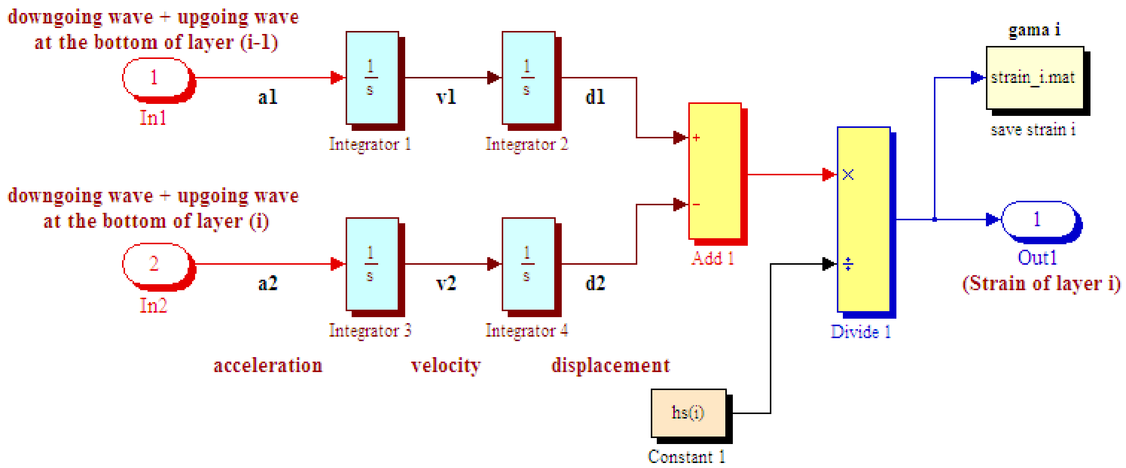


Fig. 21 Inside of the subsystem block diagram developed for "Model A" strain calculation of i. layer

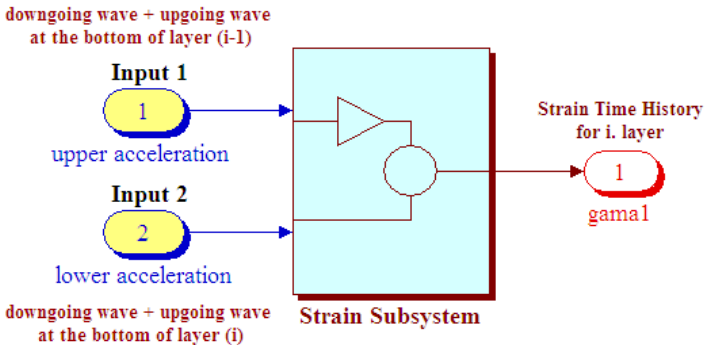


Fig. 22 Outside of the subsystem block diagram developed for "Model A" strain calculation of i. layer

## 7. Conclusions

Digital simulation provides superior tools for design, sensitivity analysis, system identification, and model calibration in real time. In this paper, digital simulation models formed for discrete time analysis of seismic site amplification in layered media for vertically propagating shear waves are presented. The block diagrams are developed for single and multi-layered soils, with and without damping. The simulation software Matlab-Simulink is used for dynamic simulations. Developed block diagrams in this study are not restricted to be used in just Matlab-Simulink. They are global charts and may be constructed in any simulation program and may be executed and tested.

The key advantages of digital simulation are that the simulations are in real time and interactive; the development of models is intuitive and graphical; and the properties of the system can be changed and visualized while the response is in progress. By developing digital simulation models for seismic site amplification, the possibility of monitoring the soil behavior in real time is created. If ground acceleration affecting bedrock in the models is connected to system in real time, the behavior of the soil layer may be simultaneously observed by using developed block diagrams.

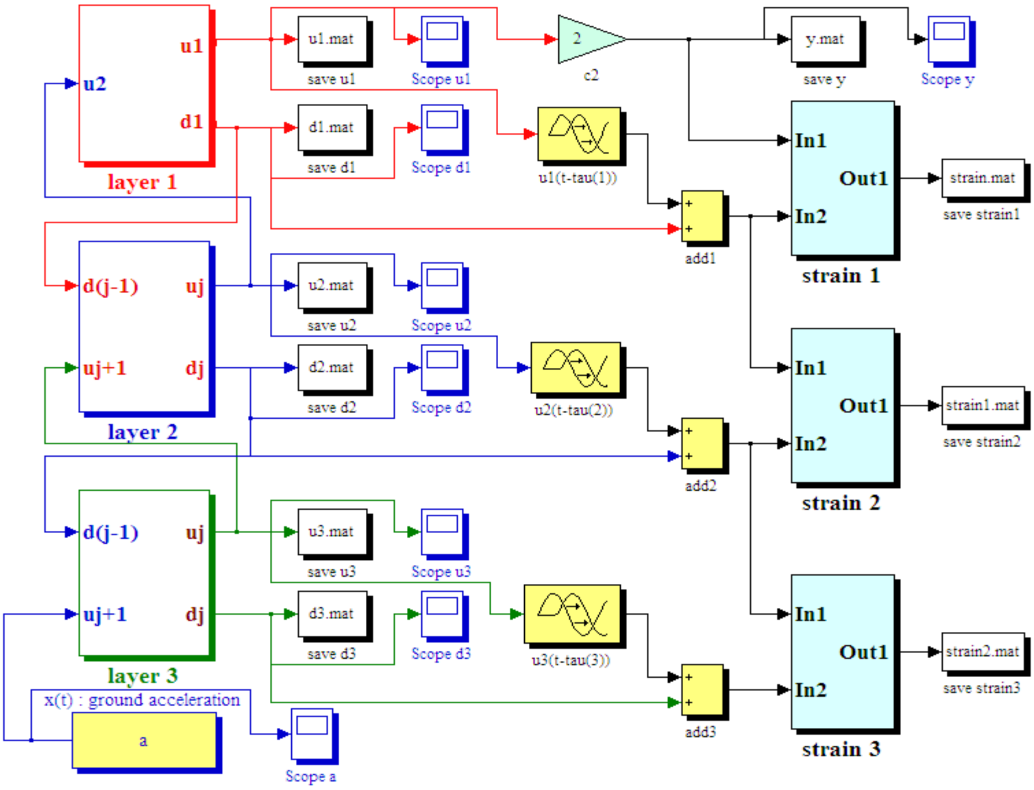


Fig. 23 "Model A" block diagram for a three-layered soil media developed for strain calculation of layers

There are many site amplification tools developed for seismic site amplification purpose. Most of these programs have been developed in frequency domain. In this study, Simulink block diagrams have been developed to calculate site amplification in layered media by using discrete-time wave propagation techniques. The site amplification is calculated in time domain and the soil parameters can be changed real time during the analysis process by using the advanced Simulink tools. The numerical applications have been carried out to check the reliability of developed algorithm in the second part of the paper. The results of the proposed Simulink diagrams are compared with SUA, EERA and NERA programs for the particular example problems and it can be said that there is a good harmony between the obtained results.

A new graphical user interfaced (GUI) program called DTASSA standing for Discrete-Time Analysis of Seismic Site Amplification is developed. In this software, automatic block diagram producing system is developed and seismic site amplification for multiple soil layers may easily be investigated in real time. The general description of the developed program, data input and numerical verifications are given in the second part of the study.

## Acknowledgments

I would like to thank to Prof. Dr. E. Safak from Bogazici University for his unique advices, comments and support.

## References


Aki, K. and Richards, P.G. (1980), *Quantitative Seismology Theory and Methods*, W.H. Freeman and Co. San Francisco, CA, USA.

Bardet, J.P., Ichii, K. and Lin, C.H. (2000), "EERA: A computer program for equivalent-linear earthquake site response analyses of layered soil deposits", Department of Civil Engineering, University of Southern California.

Boaga, J., Renzi, S., Vignoli, G., Deiana, R. and Cassiani, G. (2012), "From surface wave inversion to seismic site response prediction: Beyond the 1D approach", *Soil Dyn. Earthq. Eng.*, **36**, 38-51.

Bolisetti, C., Whittaker, A.S., Mason, H.B., Almufti, I. and Willford, M. (2014), "Equivalent linear and nonlinear site response analysis for design andrisk assessment of safety-related nuclear structures", *Nucl. Eng. Des.*, **275**, 107-121.

Borcherdt, R.D. (1970), "Effects of local geology on ground motion near San Francisco Bay", *Bull. Seism. Soc. Am.*, **60**(1), 29-61.

Borcherdt, R.D. (1994), "Estimates of site-dependent response spectra for design (methodology and justification)", *Earthq. Spectra*, **10**, 617-654.

Cadzow, A.J. (1973), *Discrete-Time Systems*, Prentice-Hall, Inc., Englewood Cliffs, NJ, USA.

Choudhury, D. and Savoikar, P. (2009), "Equivalent-linear seismic analyses of MSW landfills using DEEPSOIL", *Eng. Geol.*, **107**(3-4), 98-108.

Guttenberg, B. (1957), "Effects of ground on earthquake motion", *Bull. Seism. Soc. Am.*, **47**(3), 221-250.

Hashash, Y.M.A., Phillips, C. and Groholski, D. (2010), "Recent advances in non-linear site response analysis", *Proceedings of the 5th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, San Diego, CA, USA, May, Paper No. OSP 4.

Idriss, I.M. and Seed, H.B. (1968), "Seismic response of horizontal soil layers", *J. Soil Mech. Found. Div. ASCE*, **94**(SM4), 1003-1031.

Idriss, I.M. and Sun, J.I. (1992), "User's manual for SHAKE91", Department of Civil and Environment Engineering, University of California.

Johari, A. and Momeni, M. (2015), "Stochastic analysis of ground response using non-recursive algorithm", *Soil Dyn. Earthq. Eng.*, **69**, 57-82.

Kaklamanos, J., Baise, L.G., Thompson, E.M. and Dorfmann, L. (2015), "Comparison of 1D linear, equivalent-linear, and nonlinear site response models at six KiK-net validation sites", *Soil Dyn. Earthq. Eng.*, **69**, 207-219.

Kanai, K. (1957), "Semi-empirical formula for the seismic characteristics of the ground", *Bull. Earthq. Res. Inst. – Tokyo Japan*, **35**(2), 308-325.

Knopoff, L.Q. (1964), *Reviews of Geophysics*, Volume 2, pp. 625-660.

MATLAB (2009), The MathWorks Inc., Natick, MA, USA.

O'Connell, R.J. and Budiansky, B. (1978), "Measures of dissipation in viscoelastic media", *Geophys. Res. Lett.*, **5**(1), 5-8.

Philips, C. and Hashash, Y.M.A. (2009), "Damping formulation for nonlinear 1D site response analyses", *Soil Dyn. Earthq. Eng.*, **29**(7), 1143-1158.

Phillips, C., Kottke, A.R., Hashash, Y.M.A. and Rathje, E.M. (2012), "Significance of ground motion time step in one dimensional site response analysis", *Soil Dyn. Earthq. Eng.*, **43**, 202-217.

Robinson, D., Dhu, T. and Schneider, J. (2006), "SUA: A computer program to compute regolith site-response and estimate uncertainty for probabilistic seismic hazard analyses", *Comput. Geosci.*, **32**(1), 109-123.

Rota, M., Lai, C.G. and Strobbia, C.L. (2011), "Stochastic 1D site response analysis at a site in central Italy", *Soil Dyn. Earthq. Eng.*, **31**(4), 626-639.

Roullé, A. and Bernardie, S. (2010), "Comparison of 1D non-linear simulations to strong-motion observations: A case study in a swampy site of French Antilles (Pointe-à-Pitre, Guadeloupe)", S*oil Dyn. Earthq. Eng.*, **30**(5), 286-298.

Safak, E. (1989), "Optimal-adaptive filters for modeling spectral shape, site amplification, and source

scaling", *Soil Dyn. Earthq. Eng.*, **8**(2), 75-95.
Safak, E. (1995), "Discrete-time analysis of seismic site amplification", *J. Eng. Mech.*, **121**(7), 801-809.
Schnabel, P.B., Lysmer, J. and Seed, H.B. (1972), "SHAKE: A computer program for earthquake response analysis of horizontally layered sites", Report EERC 72-12; Earthquake Engineering Research Center, University of California, Berkeley, CA, USA.
Seed, H.B., Romo, M.P., Sun, J.I., Jaime, A. and Lysmer, L. (1988), "The Mexico earthquake of September 19, 1985: Relationships between soil conditions and earthquake ground motions", *Earthq. Spectra*, **4**(4), 687-729.
SIMULINK – Dynamic System Simulation for MATLAB (2009), The MathWorks Inc., Natick, MA, USA.
Yang, J. and Yan, X.R. (2006), "PASS: A computer program for practical analysis of layered soil–rock systems", Department of Civil Engineering, The University of Hong Kong, Hong Kong.

*CC*

## Appendix I

```
function tau = travel_time(h,vs)
tau = h/vs;
function r = f_ref_coef(vs , qs, vr , qr)
pay = qr*vr-qs*vs;
pyd = qr*vr+qs*vs;
r = pay/pyd;

function block_parameters (h, vs, vr, qs, qr)
tau = travel_time(h,vs)
r = f_ref_coef(vs , qs, vr , qr)
assignin('base','r',r);            % assign r into Matlab Workspace
assignin('base','tau',tau);        % assign tau into Matlab Workspace
```

## Appendix II

```
function [T,teta, alfa] = f_filter_param(tau,Q)
T = pi*tau/(Q*log(2));
T = T*0.1;
teta = log(2)*Q*T/tau;
cs = cos(teta);
pay = 1-sqrt(1-cs^2);
pyd = cs;
alfa = pay/pyd;

function block_parameters(h,vs,qs,vr,qr,Q)
tau = travel_time(h,vs);
[T,teta, alfa] = f_filter_param(tau,Q);
assignin('base','tau',tau);       % assign tau into Matlab Workspace
assignin('base',T,T);             % assign T into Matlab Workspace
assignin('base','alfa',alfa);     % assign alfa into Matlab Workspace
```

## Appendix III

```
function block_parameters (h,vs,qs,vr,qr,Q)
tau = travel_time(h,vs);
[T,teta, alfa] = f_filter_param(tau,Q);
k1 = 2*alfa;
k2 = alfa^2;
k3 = 0.25*(1-alfa)^2*r;
k4 = 0.5*(1-alfa)*(1+r);
assignin('base','tau',tau);       % assign tau into Matlab Workspace
assignin('base',T,T);             % assign T into Matlab Workspace
assignin('base','alfa',alfa);     % assign alfa into Matlab Workspace
assignin('base','k1',k1);  % assign k1 into Matlab Workspace
assignin('base','k2',k2);  % assign k2 into Matlab Workspace
assignin('base','k3',k3);  % assign k3 into Matlab Workspace
assignin('base','k4',k4);  % assign k4 into Matlab Workspace
```

## Appendix IV

```
function [alfa]=f_alfa(teta)
cs = cos(teta);
pay = 1-sqrt(1-cs^2);
pyd = cs;
alfa = pay/pyd;

function block_parameters (h,vs,qs,vr,qr,Q,n)
% n: number of layers
for j = 1 : n-1
      r(j) = f_ ref_coef (vs(j) , qs(j), vs(j+1) , qs(j+1) );
end
      r(n) = f_ ref_coef (vs(n) , qs(n), vr , qr);
assignin('base','r',r);              % assign r into Matlab Workspace
for j = 1:n
     tau(j) = travel_time (hs(j),vs(j));
end
assignin('base','tau',tau);         % assign tau into Matlab Workspace
for j = 1:n
     T(j) = pi*tau(j)/(Q(j)*log(2));
end
tmin = min(T);
T(1:n) = tmin;
assignin('base','T',T);             % assign T into Matlab Workspace
for j = 1:n
     teta(j) = log(2)*Q(j)*T(j)/tau(j);
end
for j = 1:n
     [alfa(j)] = f_alfa(teta(j));
end
assignin('base','alfa',alfa);       % assign alfa into Matlab Workspace
```