Geomechanics and Engineering, *Vol. 8, No. 6 (2015) 783-799*
DOI: http://dx.doi.org/10.12989/gae.2015.8.6.783

In situ horizontal stress effect on plastic zone around circular underground openings excavated in elastic zones

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(*Received November 21, 2014, Revised January 29, 2015, Accepted February 02, 2015*)

Abstract. In this study, effect of horizontal in situ stress on failure mechanism around underground openings excavated in isotropic, elastic rock zones is investigated. For estimating the plastic zone occurrence, an induced stress influence area approach (Bray Equations) was modified to define critical stress ratio according to the Mohr-Coulomb failure criterion. Results obtained from modified calculations were compared with results of some other analytical solutions for plastic zone thickness estimation and the numerical modelling (finite difference method software, FLAC2D) study. Plastic zone and its geometry around tunnels were analyzed for different in situ stress conditions. The modified equations gave similar results with those obtained from the other approaches. However, safer results were calculated using the modified equations for high in situ stress conditions and excessive ratio of horizontal to vertical in situ stresses. As the outcome of this study, the modified equations are suggested to use for estimating the plastic zone occurrence and its thickness around the tunnels with circular cross-section.

Keywords: horizontal in situ stress; plastic zone; tunnel stability; stress distribution around tunnels

1. Introduction

Vertical stress in solid materials causes to occur horizontal stress and strain. The ratio of horizontal stress to vertical stress is generally named as k ratio in rock mechanics. Poisson ratio (v) is an other important parameter used to define horizontal strain arising from vertical strain, which is related with k ratio directly (Gercek 2007, Walsh 1965, Unlu and Gercek 2003). In general, Poisson ratio can’t represent the field conditions in terms of estimation of horizontal in situ stress from the vertical in situ stress. Because, in situ horizontal stress does not only occur due to gravitational stress and the poisson effect. The ratio between horizontal and vertical stresses changes in accordance with many parameters such as depth, joints and cracks, water, temperature, topographic features, surface load, tectonic stresses (active tectonic stress, remnant tectonic stress), residual stresses (like magma cooling, metamorphism, metasomatism, etc.), terrestrial stresses like seasonal variations, moon pull, diurnal stresses (Amadei and Stephansson 1997, Sheory 1994, Zamani 2011, Hudson and Harrison 1997, Saleh and Saleh 2012, Karpuz and Hindistan 2008,

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http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

Saberhosseini *et al*. 2014, Jienan *et al*. 2009).

The ratio of in situ horizontal stress to in situ vertical stress (k ratio for in-situ stress condition) is one of the most important parameters for stress distribution around underground openings and plastic zone (yield zone) geometry around underground openings. As widely known, plastic zone height increases with an increase in the k ratio. On the other hand, plastic zone width increases when the k ratio decreases (Yarali and Muftuoglu 1992, Yan and Shiao 2010, Behnam *et al*. 2014). Kirsch solutions can be used to calculate induced stresses around the circular underground openings as given in Eqs. (1)-(3) (Kirsch 1898)

$$\sigma_r = \frac{\sigma_v(1+k)}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{p_i a^2}{r^2}+\frac{\sigma_v(k-1)}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta \tag{1}$$

$$\sigma_\theta = \frac{\sigma_v(1+k)}{2}\left(1+a^2/r^2\right)-\frac{p_i a^2}{r^2}-\frac{\sigma_v(k-1)}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta \tag{2}$$

$$\tau_{r\theta} = -\frac{\sigma_v(k-1)}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta \tag{3}$$

where σ_r is radial stress (MPa), σ_θ is tangential stress (MPa), σ_r is induced shear stress (MPa) around tunnel, σ_v is vertical in situ stress (MPa), a is tunnel radius (m), r is distance from the tunnel cross-section center (m), P_i is support pressure (MPa), θ is angle with horizontal. As seen in Eq. (1), the radial stress is equal to support pressure at wall ($a = r$). If there is no support pressure, convergence continues until radial stress vanishes. Otherwise, support pressure needs to be supplied depending upon level of the induced stress and mechanical parameters of rock. Contrast to the radial stress, tangential stress varies with the change of θ angle, and can cause some failure initiation starting from the wall.

Because induced stress distribution doesn't depend on the angle of θ when k ratio is 1, the plastic zone is expected to occur with a circular boundary around the circular openings. There are some widely known plastic zone thickness estimation approaches for the hydrostatic in situ stress conditions. One of the plastic zone thickness calculations for the hydrostatic condition (k = 1) has been suggested by Hoek as follows (Hoek 2006)

$$r_p = a\left[\frac{2P_0(k_p-1)+\sigma_c}{(1+k_p)(k_p-1)P_i+\sigma_c}\right]^{\frac{1}{(k_p-1)}} \tag{4}$$

$$k_p = \frac{(1+\sin\phi)}{(1-\sin\phi)} \tag{5}$$

where r_p is the plastic zone radius, P_0 is in situ vertical stress, σ_c is uniaxial compressive strength of rock mass before yielding and ϕ is internal friction angle of rock mass before yielding.

As the induced stresses vary in accordance with the angle of θ, the plastic zone thickness cannot be considered constant around the tunnel when the k ratio is not equal to 1. In this study, plastic zone shape and dimension variations due to change of k ratio are investigated with analytical and numerical analyses. The analytical part of the study is a modification and compilation of some 2D stress distribution calculations for circular underground openings. To compare and assess the results obtained from the analytical study, the numerical modelling part was performed using the finite difference method (Flac 2D).

2. Analytical study

The Bray stress distribution approach given in Eqs. (6) and (7), suggested for elliptical and circular underground openings was referenced in this study to estimate the plastic zone thickness (Bray 1986, Brady and Brown 2005). As seen in the equations, stress influence distance is calculated in accordance with the influence ratio (i_r) depending on the induced and in situ stresses (i_r = 100 (induced stress-in situ stress) / in situ stress).

$$W_i = H\sqrt{A_p\alpha[q(q+2)-k(3+2q)]}$$
$$W_i = H\sqrt{\alpha[A_p(k+q^2)+kq^2]} \tag{6}$$

$$H_i = H\sqrt{A_p\alpha[k(1+2q)-q(3q+2)]}$$
$$H_i = H\sqrt{\alpha[A_p(k+q^2)+1]} \tag{7}$$

$$q = \frac{W}{H} \tag{8}$$

$$A_p = \frac{100}{(2i_r)} \tag{9}$$

where H is tunnel cross-section height, W is tunnel cross-section width, H_i is influence area height and W_i is influence area width. q is the ratio of tunnel cross-section width to tunnel cross-section height, hence it is 1 for the tunnels with circular cross-section. α is 1 when k is smaller than 1, and is equal to $1/k$ when the k ratio is greater than 1. Among the double equations given in Eqs. (6) and (7), equations suggesting greater results should be considered for both H_i and W_i calculations.

To estimate plastic zone thickness by using Bray equations, critical influence ratio and critical tangential stress need to be known. Critical tangential stress is minimum tangential stress for failure occurrence, which can be calculated with Mohr-Coulomb failure criterion as shown in the following equation

$$\sigma_\theta = \sigma_c + k_p\sigma_r \tag{10}$$

Critical influence ratio (i_{rc}) can be calculated with critical tangential stress as shown in Eq. (11)

$$i_{rc} = \left[\frac{(\sigma_\theta - \sigma_v)}{\sigma_v}\right]100 = \left\{\frac{(\sigma_c + k_p\sigma_r) - \sigma_v}{\sigma_v}\right\}100 \tag{11}$$

According to the Kirsch equations, sum of tangential and radial stresses is equal to 2 times of the vertical in situ stress when the k ratio is 1

$$\sigma_\theta = \sigma_v\left(1+\frac{a^2}{r^2}\right) \tag{12}$$

$$\sigma_r = \sigma_v\left(1-\frac{a^2}{r^2}\right) \tag{13}$$

$$\sigma_\theta + \sigma_r = 2\sigma_v \tag{14}$$

Therefore, radial stress can be defined with tangential stress as follows

$$\sigma_\theta = \sigma_c + k_p(2\sigma_v - \sigma_\theta) = \frac{(\sigma_c + 2\sigma_v k_p)}{1+k_p} \tag{15}$$

Thus, critical influence ratio becomes as follows

$$i_{rc} = \frac{\left\{\dfrac{(\sigma_c + 2\sigma_v k_p)}{(1+k_p)}\right\} - \sigma_v}{\sigma_v} 100 \tag{16}$$

In this way, critical A_p (A_{pc}) for failure becomes as follows

$$A_{pc} = \frac{100}{2i_{rc}} = \frac{\sigma_v(1+k_p)}{2[\sigma_c + \sigma_v(k_p - 1)]} \tag{17}$$

The choice of the equations in Eqs. (6) and (7) depends on the critical influence ratio and k ratio. As an example, second equation of Eq. (6) should be used to calculate greater results than those of the first equation when k ratio is higher than $2A_{pc} / (6A_{pc} + 1)$. Therefore, the second equation should be considered in the hydrostatic pressure condition ($k = 1$). Also, second equation of Eq. (7) should be considered for the hydrostatic pressure conditions. According to the results of Eqs. (6) and (7), H_i and W_i are same under the hydrostatic stress condition. The plastic zone diameter (D_p) calculation for the hydrostatic pressure condition can be derived from both Eqs. (6) and (7) as given in Eq. (18)

$$D_p = D\sqrt{\alpha[2A_{pc}+1]} = D\sqrt{\frac{\sigma_v(1+k_p)}{\sigma_c + \sigma_v(k_p-1)}+1} \tag{18}$$

Behind the failure zone the elastic behaviour doesn't start directly. There is a transition zone between the failed and elastic zone; some researchers define this zone as "strain softened zone" (Jiang et al. 2001, Komurlu and Kesimal 2011). In this plastic zone study, only failure zone was investigated in accordance to the Mohr-Coloumb failure criteration.

Because critical influence ratio increases with an increase in the rock strength, plastic zone thickness is expected to decrease with an increase in the critical influence ratio. As seen from Eq.

(2), the tangential stress is equal to two times of the vertical stress ($2\sigma_v$) for the condition of unsupported tunnel wall and the hydrostatic pressure. Therefore, the plastic zone is estimated to occur for smaller critical influence ratios than 100% when k ratio is 1. In the case of smaller compressive strength than in situ stress, the area for excavation is already failed and plastic (underground confining pressure is neglected). The critical influence ratio is 0% when the vertical in situ stress is equal to the compressive strength. In this case, the plastic zone thickness cannot be estimated.

The plastic zone thickness calculation using Eq. (18) shows the need for some regulations; k_m and k_f factors explained in the further part of this study were suggested by Komurlu (2012) to use in case of the k ratios differing from 0.25 to 3.

For different k ratios than 1, the distance between plastic zone boundaries in the direction of the horizontal diameter of tunnel (W_p) and the distance between plastic zone boundaries in the direction of vertical diameter of tunnel (H_p) can be calculated for circular tunnels (q = 1) according to the Bray influence ratio approach as follows

$$W_p = D\sqrt{\alpha\left[\frac{100}{2i_{rc}}(k+1)+k\right]} \tag{19}$$

$$H_p = D\sqrt{\alpha\left[\frac{100}{2i_{rc}}(k+1)+1\right]} \tag{20}$$

Sum of radial stress and tangential stress calculated in accordance with the Kirsch equations is given in Eq. (21) as $\ddot{o}$

$$\sigma_\theta + \sigma_r = \sigma_v(1+k) - \frac{\sigma_v(k-1)}{2}\cos 2\theta\left(\frac{4a^2}{r^2}\right) = \ddot{o} \tag{21}$$

Therefore, tangential stress at failure limit according to Mohr-Coloumb failure envelope and the critical influence ratio can be written with $\ddot{o}$ parameter as follows

$$\sigma_\theta = \sigma_c + k_p(\ddot{o} - \sigma_\theta) = (\sigma_c + k_p\,\ddot{o})/(k_p+1) \tag{22}$$

$$i_{rc} = \frac{\left(\dfrac{\sigma_c + k_p\ddot{o}}{k_p+1} - \sigma_v\right)}{\sigma_v} \tag{23}$$

To simplify the approach and use same formula to calculate both H_p and W_p, it was suggested to rotate locations by angle of $\pi/2$ clockwise, and consider the k ratio as $1/k$ ratio for H_p calculations (Komurlu 2012). By this way, θ increases by $+\pi/2$ for H_p calculations ($\theta = 0°$ for the floor), vertical and horizontal stresses are exchanged, and $\ddot{o}$ becomes as given in Eq. (24)

$$\ddot{o}_h = \sigma_h\left[(1+k^{-1}) - \frac{(k^{-1}-1)}{2}\cos 2\left(\theta+\frac{\pi}{2}\right)\left(\frac{4a^2}{r^2}\right)\right] \tag{24}$$

Therefore, the critical influence ratio for H_p calculations becomes as follows

$$i_{rc} = \frac{\left(\dfrac{\sigma_c + k_p \ddot{o}_h}{k_p + 1} - \sigma_h\right)}{\sigma_h} \tag{25}$$

And, W_p and H_p calculations become as follows

$$W_p = D\sqrt{\alpha\left[\frac{\sigma_v(1+k)}{2\left(\dfrac{\sigma_c + k_p \ddot{o}}{k_p + 1} - \sigma_v\right)} + k\right]} \tag{26}$$

$$H_p = D\sqrt{\Gamma\left[\frac{\sigma_h\left(1+1/k\right)}{2\left(\dfrac{\sigma_c + k_p \ddot{o}_h}{k_p + 1} - \sigma_h\right)} + 1/k\right]} \tag{27}$$

where Γ is the substitution parameter of α. As the k ratio is considered as $1/k$ for H_p calculations, Γ is needed to use instead of α. When k ratio is greater than 1, $1/k$ is smaller than 1 and Γ parameter is equal to 1 for this situation (when $k < 1$, $\alpha = 1$, when $k > 1$, $\alpha = 1/k$). If k ratio is smaller than 1, $1/k$ is greater than 1 and Γ parameter is equal to k (when $k > 1$, $\Gamma = 1$, when $k < 1$, $\Gamma = k$).

Eq. (26) becomes as given in Eq. (28) when the $\ddot{o}$ parameter is clearly written for the sidewalls θ angle is 0° and 180°). For floor and roof (θ is respectively 90° and 270°), Eq. (29) can be derived writting $\ddot{o}_h$ parameters clearly in Eq. (27). As r is the distance of investigated point on the boundary of plastic zone from the center of circular tunnel crossection, W_p/D and H_p/D are equal to r/a in $\ddot{o}$ and $\ddot{o}_h$ parameters, respectively. Therefore, Eqs. (28) and (29) have only one unknown parameter which is W_p/D or H_p/D.

$$W_p = D\sqrt{\alpha\left(\left[\sigma_v(1+k)/\left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \dfrac{\sigma_v(k-1)}{2}\left(\dfrac{4D^2}{W_p^2}\right)\right]}{k_p+1} - \sigma_v\right)\right\}\right] + k\right)} \tag{28}$$

$$H_p = D\sqrt{\Gamma\left(\left[\sigma_h(1+k^{-1})/\left\{2\left(\frac{\sigma_c + k_p\left[\sigma_h(1+k^{-1}) - \dfrac{\sigma_h(k^{-1}-1)}{2}\left(\dfrac{4D^2}{H_p^2}\right)\right]}{k_p+1} - \sigma_h\right)\right\}\right] + k^{-1}\right)} \tag{29}$$

Eq. (30) can be derived from Eq. (29) writing σ_h as $\sigma_v k$

$$H_p = D\sqrt{\left(\Gamma\left[\left[\sigma_v(1+k)/\left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(1-k)}{2}\left(\frac{4D^2}{H_p^2}\right)\right]}{k_p+1} - \sigma_v k\right)\right\}\right]\right] + k^{-1}\right)} \tag{30}$$

As mentioned before, regulator parameters are needed to use for using Bray stress distribution formulas to estimate plastic zone thickness around the circular openings (q = 1). k_f factor is suggested to use instead of $+k$ which is the last parameter in square root of Eq. (19). Because of the position rotating, Eq. (19) is used for both W_p and H_p calculations. Therefore, k_f parameter is used to regulate only one analytical solution. In hydrostatic pressure condition, k_f parameter must be zero since plastic zone occurs for the condition of $2\sigma_v > \sigma_c$ (i_{rc} = 100%, A_{pc} = 1 when $2\sigma_v = \sigma_c$). As seen in Eq. (19), plastic zone is estimated to occur when the uniaxial compressive strength (σ_c) is two times of the vertical in situ stress (σ_v) and k_f parameter is zero for the hydrostatic pressure condition. This situation is parallel with the Kirsch equation results. Because, the maximum tangential stress is calculated as two times of the vertical in situ stress according to the Kirsch equations.

k_f parameter varies with the change of k ratio. Eq. (33) was suggested to relate the k ratio to k_f (Komurlu 2012). In the original equations, $-\ i_{rc}$% defines W_p when k ratio is higher than 1, and H_p when k ratio is smaller than 1. The relation between stress influence contours and elliptical approximation of the Bray approach is shown in Fig. 1. For instance, 5 MPa stress contour is considered for −50% influence ratio and the in situ vertical stress of 10 MPa. In this case, lower stress than crtical stress level defines the W_i which is also important for H_p estimations because of

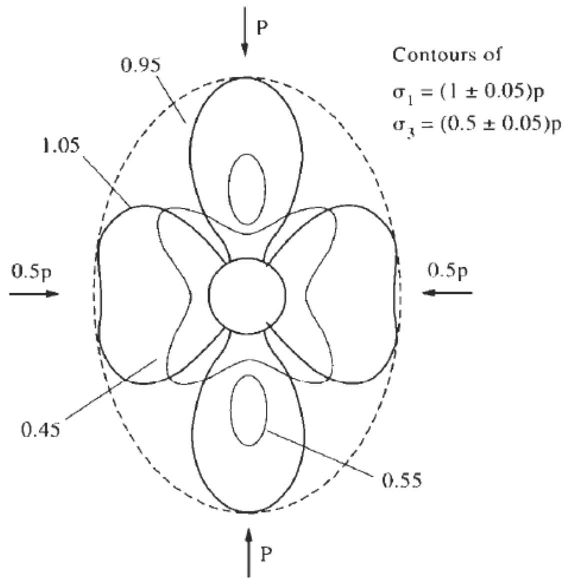

Fig. 1 Zone of influence and stress contours (from Hudson and Harrison 1997)

the position rotating. This situation can cause to consider unfailed rock as in the plastic zone boundary. Therefore, k_f regulator parameter is suggested to use with H_p for $k < 1$ situation, and it is suggested to use with W_p calculation when k is greater than 1. In the original approach, $+\ i_{rc}\%$ defines H_i when k is higher than 1, and W_i is defined by $+\ i_{rc}$ % for $k < 1$ situation. Therefore, there is no need to use k_f with H_p estimation formula for $k > 1$ situation. Also, k_f is zero for W_p calculations when k is smaller than 1.

k_m is general regulator factor that varies for H_p and W_p calculations. It is necessary to use k_m for estimation of plastic zone for critical k and σ_v / σ_c ratios. However, k_m is not usable in the Eq. (40) suggested to use for smaller k ratios than $2A_{pc}$ / ($6A_{pc}$ + 1). k_m factor is 1 for the hydrostatic condition. As given in Eqs. (34) and (35), k_m varies with the change of vertical in situ stress, the uniaxial compressive strength of rock and the k ratio (Komurlu 2012). k_m is a regulator for H_p (k < 1) and W_p (k > 1), which can limit misleading excessivity with safe plastic zone thickness estimation. Original $\ddot{o}$ parameter should be used instead of $\ddot{o}_h$ when the regulator factors are used.

$$H_p = D\sqrt{\left(\Gamma\left[\sigma_v(1+k)\Big/\left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) + \frac{\sigma_v(1-k)}{2}\left(\frac{4D^2}{H_p^2}\right)\right]}{k_p+1} - \sigma_v\right)\right\}\right] + k_f\right) k_m} \quad (31)$$

$$W_p = D\sqrt{\left(\alpha\left[\sigma_v(1+k)\Big/\left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(k-1)}{2}\left(\frac{4D^2}{W_p^2}\right)\right]}{k_p+1} - \sigma_v\right)\right\}\right] + k_f\right) k_m} \quad (32)$$

$$\begin{aligned} &k_f = \sqrt{1-k^{-1}}, && \text{for} \quad W_p, \quad k>1 \\ &k_f = \sqrt{1-k}, && \text{for} \quad H_p, \quad k<1 \\ &k_f = 0, && \text{for} \quad W_p, \quad k<1 \quad \text{and for} \quad H_p, \quad k>1 \end{aligned} \quad (33)$$

$$k_m = k^{-\left(\left[2\sigma_v/\sigma_c\right]^{\alpha-2}\right)}, \quad \text{for} \quad W_p \quad (34)$$

$$k_m = \sqrt{k}^{\left(\left[2\sigma_v/\sigma_c\right]^{\alpha-1}\right)}, \quad \text{for} \quad H_p \quad (35)$$

Critical influence ratio and k ratio change the equation choice as it was mentioned before. For instance, the second equation of Eq. (6) mustn't be used when k ratio is smaller than $2A_{pc}/(6A_{pc}+1)$ since the first equation gives higher results in comparison with the second equation for the situation. If W_p is calculated with the first equation in Eq. (6), calculation becomes as shown in Eq.

(36), and Eq. (37) can be formed when critical influence ratio and the *ö* parameter are clearly written in Eq. (36).

$$\frac{W_p}{D} = \sqrt{A_{pc}\alpha(3-5k)} = \sqrt{\left(\frac{100}{2i_{rc}}\right)\alpha(3-5k)} \tag{36}$$

$$W_p = D\sqrt{\left(\left[\sigma_v \Big/ \left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(k-1)}{2}\left(\frac{4D^2}{H_p^2}\right)\right]}{k_p+1} - \sigma_v\right)\right\}\right]\alpha\right)(3-5k)} \tag{37}$$

H_p mustn't be estimated in accordance with Eq. (31) derived from the second equation in Eq. (6) by position rotating when the *k* ratio is greater than $(6A_{pc}+1)/2A_{pc}$. Because, $1/k$ is smaller than $2A_{pc}/(6A_{pc}+1)$ when the *k* ratio is greater than $(6A_{pc}+1)/2A_{pc}$. Therefore, it can be concluded that Eq. (31) is usable for the *k* ratio from $1/3$ to $(6A_{pc}+1)/2A_{pc}$. Because of tensile stress occurrence, the plastic zone thickness along roof and floor cannot be calculated in accordance with compressive strength of rock when the *k* ratio is smaller than $1/3$.

As the tangential stress is equal to $\sigma_v(3-k)$ at the sidewalls, tensile stress occurs at sidewalls when the *k* ratio is greater than 3. Therefore, plastic zone thickness can't be estimated for sidewalls by considering the compressive strength of rock when *k* ratio is greater than 3. A new position rotating was suggested to calculate W_p using Eq. (40). The first equation in Eq. (7) gives higher results when the *k* ratio is greater than $(6A_{pc}+1)/2A_{pc}$. Therefore, H_p calculation formula becomes as shown in Eq. (38), and can be formed as Eq. (39) when critical influence ratio and the *ö* parameter are clearly written. θ is 90° and 270° for roof and floor, respectively.

$$H_p/D = \sqrt{A_{pc}\alpha(3k-5)} = \sqrt{(100/2i_{rc})\alpha(3k-5)} \tag{38}$$

$$H_p = D\sqrt{\left(\left[\sigma_v \Big/ \left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) + \frac{\sigma_v(k-1)}{2}\left(\frac{4D^2}{H_p^2}\right)\right]}{k_p+1} - \sigma_v\right)\right\}\right]\alpha\right)(3k-5)} \tag{39}$$

k ratio is considered as k^{-1} for calculating W_p with Eq. (39). Therefore, Γ is used instead of *α* parameter. Vertical in situ stress is considered as horizontal in situ stress by this way. Therefore, the function of cos2(*θ* + 90°) is considered instead of cos2*θ*, and the Eq. (40) is derived as follows

$$W_p = D\sqrt{\left(\left[\sigma_v k \Big/ \left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(1-k)}{2}\left(\frac{4D^2}{W_p^2}\right)\right]}{k_p+1} - \sigma_v k\right)\right\}\right]\Gamma\right)(3k^{-1}-5)} \tag{40}$$

As mentioned before, Eq. (40) is usable when k ratio is smaller than $2A_{pc} / (6A_{pc} + 1)$. To choose suitable W_p equation, A_{pc} can be practically considered as follows

$$A_{pc} = \frac{100}{2i_{rc}} \approx \frac{\sigma_v}{2(\sigma_c - \sigma_v)} \tag{41}$$

Eq. (40) is to be used for low k ratios (like 0.2, 0.25) and high $\sigma_v\sigma_c$ ratios. A new regulator parameter, k_w defined by both vertical and horizontal in situ stresses and rock strength parameter is suggested to use with the Eq. (40). By this way, Eq. (40) becomes as shown in Eq. (42). k_w is given in Eq. (43) and causes safer W_p results when σ_v / σ_c increases and k ratio decreases excessively.

$$W_p = D\sqrt{\left(\left[\sigma_v k \Big/ \left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(1-k)}{2}\left(\frac{4D^2}{W_p^2}\right)\right]}{k_p+1} - \sigma_v k\right)\right\}\right]\Gamma\right)(3k^{-1} - 5)k_w} \tag{42}$$

$$k_w = \left[\sigma_v / 2(\sigma_c - \sigma_v)\right]k^{-1}\left(\sigma_v / \sigma_c + 1\right) \approx A_{pc}k^{-1}\left(\sigma_v / \sigma_c + 1\right) \tag{43}$$

As summary, Eqs. (31), (32) and (42) were suggested to use for plastic zone thickness estimation as modified Bray stress influence formulas. Eq. (42) is suggested to use for W_p calculations when the k ratio is smaller than $2A_{pc} / (6A_{pc} + 1)$. On the other hand, Eq. (32) is usable when k ratio is smaller than 3. Eq. (31) is suggested for k ratios between $1/3$ and 3 ($0.33 \leq k \leq 3$). If k ratio is smaller than 1/3, tensile strength of rock has to be known for estimating H_p.

To compare with the results of the modified equations, another analytical and numerical models were used in this study. Yan and Shihao have suggested an approach also derived in accordance with Mohr-Coulomb criterion and Kirsch equations to estimate plastic zone thickness as follows (Yan and Shiao 2010)

$$\left[(1+k)\sigma_v - 2(k-1)\frac{\sigma_v a^2}{r^2}\cos 2\theta\right]\sin\phi - \sigma_v\beta + 2c\cos\phi = 0 \tag{44}$$

$$\beta = \sqrt{\left[(1+k)\frac{a^2}{r_p^2} - (k-1)\left(1 - \frac{2a^2}{r_p^2} + \frac{3a^4}{r_p^4}\right)\cos 2\theta\right]^2 + \left[(k-1)\left(1 + \frac{2a^2}{r_p^2} - \frac{3a^4}{r_p^4}\right)\sin 2\theta\right]^2} \tag{45}$$

where r_p is the plastic zone boundary distance from the center of circular tunnel cross-section.

3. Numerical analysis

To investigate the effect of k ratio and in situ stress distribution on yielding around tunnels with circular cross-sections, series of finite difference analyses were performed by using Flac 2D software. In this part, different rock strength values, k ratios and in situ vertical stress conditions were considered to estimate the plastic zone occurring and its thickness around the underground openings excavated in isotropic, homogeneous and elastic rock zones. The results obtained from numerical analyses was compared with those of the analytical approaches (modified Bray equations and Yan&Shihao equations). As uniaxial compressive strength parameter for different rock zones modelled with the software, 10.45 MPa, 14.70 MPa, 20.25 MPa values were considered. In situ vertical stress parameter for the numerical analysis was selected as 2.7 MPa, 5.4 MPa and 8.1 MPa in order to represent the weight at 100 meters, 200 meters and 300 meters depths of the rock mass having density of 2.7 t/m^3. To see the effect of horizontal stress variations, k ratio was considered within a large range, between 0.25 and 3 as same in the analytical part.

Because the compressive strength value is not directly used in Yan&Shihao equations, cohesion and internal friction angle parameters have to be known to obtain results. Internal friction angle of rock material was considered as 30° in numerical and analytical studies. According to Mohr-Coulomb failure criterion, cohesion was considered as 3 MPa, 4.24 MPa, 5.85 MPa for 10.45 MPa, 14.70 MPa, 20.25 MPa strength values (the uniaxial strength values used in numerical analysis), respectively.

Results obtained from the numerical and analytical studies are given in Table 1. Because there is no plastic zone occurrence when W_p / D and H_p / D are smaller than 1, the case is written as 0 in the table. H_p / D for 0.25 k ratio is signed as "-" due to the tensile stress occurrence which is not considered in the analytical study. As seen in Table 1, Flac^{2D} generally gave results between those of Yan&Shihao and modified Bray equations.

Table 1 Plastic zone thickness variations according to different stress and strength conditions

σ_v (MPa)	k	Yan & Shihao		Bray (modified)		Flac 2D		σ_c (MPa)
		Wp / D	Hp / D	Wp / D	Hp / D	Wp / D	Hp / D	
2.7	0.25	0	-	0	-	0	0	10.45
2.7	0.5	0	0	0	0	0	0	10.45
2.7	1	0	0	0	0	0	0	10.45
2.7	2	0	1.05	0	1.18	0	1.12	10.45
2.7	3	0	1.24	0	1.34	0	1.28	10.45
5.4	0.25	1.10	-	1.25	-	1.14	0	10.45
5.4	0.5	1.06	0	1.11	0	1.04	0	10.45
5.4	1	1.02	1.02	1.01	1.01	0	0	10.45
5.4	2	0	1.20	0	1.45	0	1.28	10.45
5.4	3	0	1.30	0	1.63	0	1.52	10.45
8.1	0.25	1.21	-	1.93	-	1.28	0	10.45
8.1	0.5	1.14	0	1.74	0	1.20	0	10.45
8.1	1	1.10	1.10	1.22	1.22	1.16	1.16	10.45
8.1	2	0	1.66	0	1.77	0	1.44	10.45
8.1	3	0	2.24	0	2.82	0	1.72	10.45
2.7	0.25	0	-	0	-	0	0	14.70
2.7	0.5	0	0	0	0	0	0	14.70
2.7	1	0	0	0	0	0	0	14.70

Table 1 Continued

σ_v (MPa)	k	Yan & Shihao		Bray (modified)		Flac 2D		σ_c (MPa)
		Wp/D	Hp/D	Wp/D	Hp/D	Wp/D	Hp/D	
2.7	2	0	0	0	1.08	0	0	14.70
2.7	3	0	1.08	0	1.23	0	1.12	14.70
5.4	0.25	1.01	-	1.11	-	0	0	14.70
5.4	0.5	0	0	0	0	0	0	14.70
5.4	1	0	0	0	0	0	0	14.70
5.4	2	0	1.13	0	1.31	0	1.20	14.70
5.4	3	0	1.22	0	1.43	0	1.36	14.70
8.1	0.25	1.11	-	1.31	-	1.12	0	14.70
8.1	0.5	1.07	0	1.22	0	1.08	0	14.70
8.1	1	1.03	1.03	1.02	1.02	0	0	14.70
8.1	2	0	1.21	0	1.49	0	1.28	14.70
8.1	3	0	1.32	0	1.71	0	1.52	14.70
2.7	0.25	0	-	0	-	0	0	20.25
2.7	0.5	0	0	0	0	0	0	20.25
2.7	1	0	0	0	0	0	0	20.25
2.7	2	0	0	0	0	0	0	20.25
2.7	3	0	1.02	0	1.14	0	0	20.25
5.4	0.25	0	-	0	-	0	0	20.25
5.4	0.5	0	0	0	0	0	0	20.25
5.4	1	0	0	0	0	0	0	20.25
5.4	2	0	1.06	0	1.20	0	1.08	20.25
5.4	3	0	1.15	0	1.33	0	1.20	20.25
8.1	0.25	1.02	-	1.02	-	1.04	0	20.25
8.1	0.5	0	0	0	0	0	0	20.25
8.1	1	0	0	0	0	0	0	20.25
8.1	2	0	1.15	0	1.36	0	1.20	20.25
8.1	3	0	1.24	0	1.47	0	1.36	20.25

The ratio of Elasticity modulus to uniaxial compressive strength (MR value) is considered as 550 for the models whose results are given in Table 1. Because the Elasticity modulus value is not considered in both Yan&Shihao and modified Bray equations, different Elasticity modulus values were used in numerical analyses to investigate its effect on the failure around tunnel cross-section. Totally, 5 different MR values from 250 to 1000 were used in the numerical analyses. Even though the change was not big, the plastic zone thickness was found to increase with a decrease in the MR value according to Flac^{2D} results. As seen from Table 2, most of the change in W_p/D or H_p/D didn't exceed 0.16 for a wide MR range between 250 and 1000. Fig. 2 shows the yielding zone around the circular opening with the diameter of 6 meters. The failed rock zone is shown as plastic zone around tunnel in red colour.

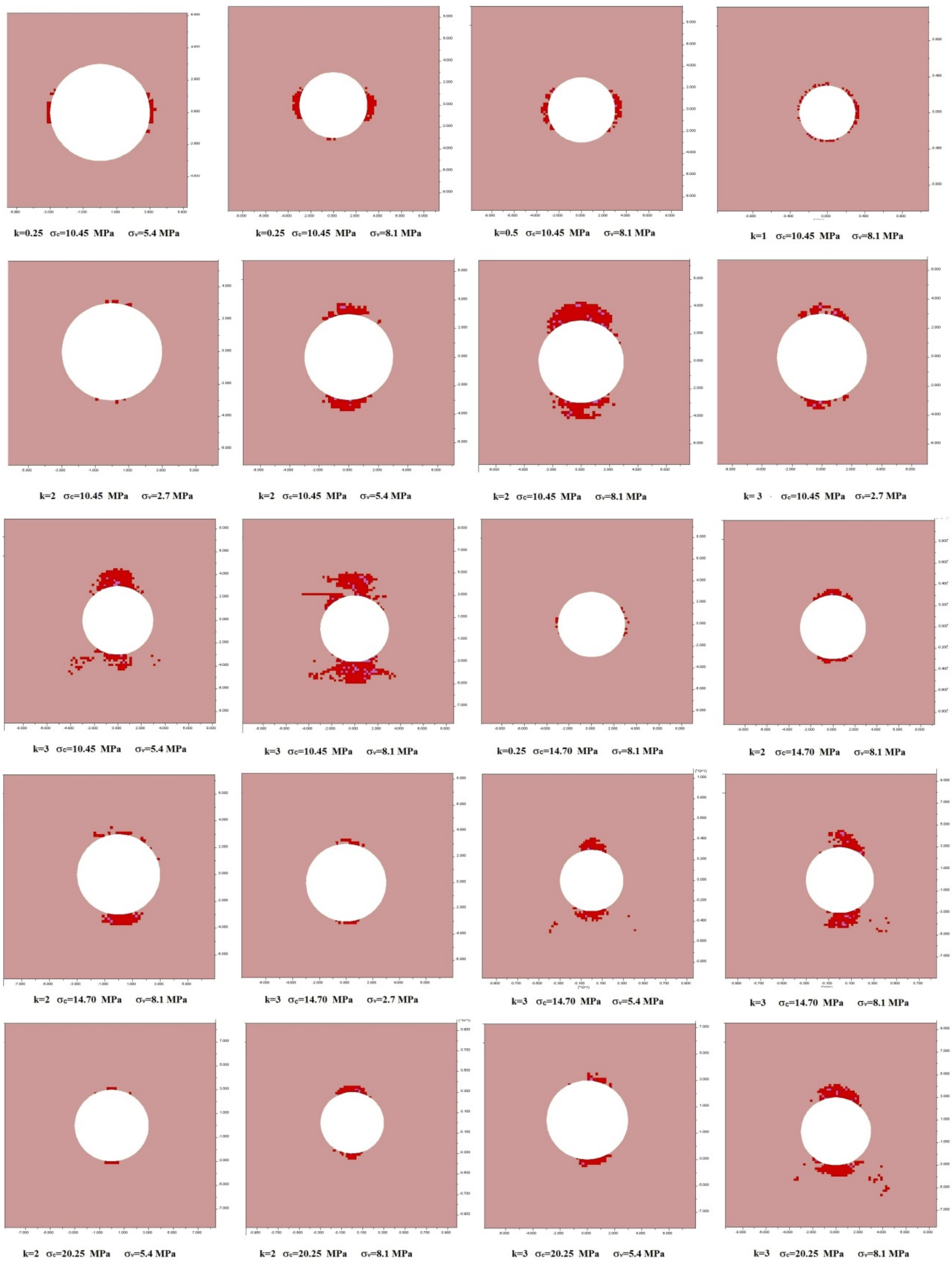

Fig. 2 Plastic zone occurring according to Flac^{2D}

Table 2 Plastic zone variations with the change of Elasticity Modulus (Situations for no plastic zone occurrence are not given)

$k - \sigma_v$ (MPa) $- \sigma_c$ (MPa)	MR = 250		MR = 400		MR = 550		MR = 800		MR = 1000	
	H_p / D	W_p / D	H_p / D	W_p / D	H_p / D	W_p / D	H_p / D	W_p / D	H_p / D	W_p / D
k0.25-5.4-10.45	0	1.12	0	1.12	0	1.12	0	1.12	0	1.12
k0.25-8.1-10.45	0	1.32	0	1.28	0	1.28	0	1.24	0	1.20
k0.25-8.1-14.70	0	1.16	0	1.16	0	1.12	0	1.08	0	1.08
k0.25-8.1-20.25	0	1.04	0	1.04	0	0	0	0	0	0
k0.50-5.4-10.45	0	1.04	0	1.04	0	1.04	0	0	0	0
k0.50-8.1-10.45	0	1.24	0	1.24	0	1.20	0	1.16	0	1.12
k0.50-8.1-14.70	0	1.12	0	1.08	0	1.08	0	0	0	0
k1.00-5.4-10.45	1.04	1.04	0	0	0	0	0	0	0	0
k1.00-8.1-10.45	1.16	1.16	1.16	1.16	1.12	1.12	1.08	1.08	0	0
k2.00-2.7-10.45	1.08	0	1.08	0	1.08	0	1.04	0	0	0
k2.00-5.4-10.45	1.32	0	1.28	0	1.28	0	1.24	0	1.20	0
k2.00-5.4-14.70	1.24	0	1.20	0	1.20	0	1.16	0	1.12	0
k2.00-5.4-20.25	1.08	0	1.08	0	1.08	0	1.04	0	1.04	0
k2.00-8.1-10.45	1.48	0	1.48	0	1.44	0	1.44	0	1.40	0
k2.00-8.1-14.70	1.32	0	1.28	0	1.28	0	1.24	0	1.16	0
k2.00-8.1-20.25	1.20	0	1.20	0	1.20	0	1.16	0	1.16	0
k3.00-2.7-10.45	1.32	0	1.28	0	1.24	0	1.20	0	1.20	0
k3.00-2.7-14.70	1.16	0	1.16	0	1.12	0	1.08	0	1.08	0
k3.00-5.4-10.45	1.56	0	1.56	0	1.52	0	1.48	0	1.44	0
k3.00-5.4-14.70	1.48	0	1.44	0	1.36	0	1.32	0	1.24	0
k3.00-5.4-20.25	1.28	0	1.24	0	1.20	0	1.16	0	1.12	0
k3.00-8.1-10.45	1.76	0	1.76	0	1.72	0	1.68	0	1.60	0
k3.00-8.1-14.70	1.56	0	1.52	0	1.52	0	1.48	0	1.44	0
k3.00-8.1-20.25	1.44	0	1.40	0	1.36	0	1.32	0	1.32	0

4. Conclusions

The results obtained from the modified and other approaches can be considered as similar. However, difference between results of the modified equation and other approaches increases when the ratio of σ_v / σ_c and k ratio increase; the modified approach gives higher plastic zone thickness in comparison with the other approaches for excessive k and σ_v / σ_c ratios. Because in situ stress distribution and k ratio can immediately change underground, safe yielding zone estimation is preferable for the rock engineering applications (Aydan and Genis 2006, Komurlu and Kesimal 2012, Kavvadas 2005, Do *et al*. 2014). A significant advantage of the modified equations was found to be increasing safety factor effect resulted from the regulator factors for high in situ stress and k ratio values.

If in situ stress is equal to uniaxial compressive strength of rock, the critical ifluence ratio (i_{rc}) in the modified equation is zero. For a safe approach, the underground confining effect can be neglected and the excavation zone is considered as already plastic under the condition. According to the Kirsch equations, tensile stress occurs at the roof and floor in case of smaller k ratios than 1/3. The tangential stress is compressive for all positions on the wall when k ratio is between $1/3$ and 3. Tensile tangential stress is also induced at sidewalls when the k ratio is higher than 3. In this study, only compressive tangential stress is considered whether it causes a plastic zone around the circular openings.

According to the both analytical and numerical studies, the plastic zone thickness increases along the roof and floor with an increase in k ratio, and the thickness increases along the sidewalls with a decrease in k ratio as parrallel with literature (Carranza-Torres and Fairhust 2000, Wu *et al.* 2009, Aydan and Genis 2010, Komurlu and Kesimal 2011). The numerical model for k ratio of 3, σ_v of 8.1 MPa and σ_c of 10.45 MPa confirmed that the plastic zone thickness can maximize between roof/floor and sidewalls when k ratio and in situ stress are quite high (Behnam *et al.* 2014). The original elliptical stress influence zone boundary approach was changed in the modified approach to consider $+i_{rc}\%$ instead of the $-i_{rc}\%$. Therefore, it is prevented to consider elastically deformed zones to be in the plastic zone near sidewalls and roof/floor.

In this study, a new approach for plastic zone thickness estimation is suggested for isotropic, homogeneous, elastic rock zones. Suggested H_p calculation formula is given in Eq. (31), and W_p calculation is given with Eq. (32). The Eq. (31) and Eq. (32) are usable for the higher k ratios than $1/3$ and $2A_{pc}/(6A_{pc}+1)$, respectively. Additionally, Eq. (42) is suggested to use for W_p calculations when the k ratio is smaller than $2A_{pc}/(6A_{pc}+1)$. To choose suitable equation, A_{pc} can be practically considered as given in Eq. (41).

When k ratio is higher than 3, suggested W_p calculations are not usable because of the tensile

$$H_p = D\sqrt{\Gamma\left(\left[\sigma_v(1+k)/\left\{2\left(\frac{\sigma_c+k_p\left[\sigma_v(1+k)+\dfrac{\sigma_v(1-k)}{2}\left(\dfrac{4D^2}{H_p^2}\right)\right]}{k_p+1}-\sigma_v\right)\right\}\right]+k_f\right)k_m} \tag{31}$$

$$W_p = D\sqrt{\alpha\left(\left[\sigma_v(1+k)/\left\{2\left(\frac{\sigma_c+k_p\left[\sigma_v(1+k)-\dfrac{\sigma_v(k-1)}{2}\left(\dfrac{4D^2}{W_p^2}\right)\right]}{k_p+1}-\sigma_v\right)\right\}\right]+k_f\right)k_m} \tag{32}$$

$$\begin{aligned} k_f &= \sqrt{1-k^{-1}}, && \text{for } W_p,\ k>1 \\ k_f &= \sqrt{1-k}, && \text{for } H_p,\ k<1 \\ k_f &= 0, && \text{for } W_p,\ k<1 \text{ and for } H_p,\ k>1 \end{aligned} \tag{33}$$

$$k_m = k^{-\left([2\sigma_v/\sigma_c]^{\alpha-2}\right)}, \quad \text{for } W_p \tag{34}$$

$$k_m = \sqrt{k}^{\left([2\sigma_v/\sigma_c]^{\alpha-1}\right)}, \quad \text{for} \quad H_p \tag{35}$$

$$W_p = D\sqrt{\left(\left[\sigma_v k \Big/ \left\{2\left(\frac{\sigma_c + k_p\left[\sigma_v(1+k) - \frac{\sigma_v(1-k)}{2}\left(\frac{4D^2}{W_p^2}\right)\right]}{k_p+1} - \sigma_v k\right)\right\}\right]\Gamma\right)(3k^{-1}-5)k_w} \tag{42}$$

$$k_w = [\sigma_v / 2(\sigma_c - \sigma_v)]k^{-1}(\sigma_v/\sigma_c + 1) \approx A_{pc}k^{-1}(\sigma_v/\sigma_c + 1) \tag{43}$$

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