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Coefficient charts for active earth pressures under combined loadings

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Abstract. Rankine's theory of earth pressure cannot be directly employed to c-ϕ soils backfill with a sloping ground subjected to complex loadings. In this paper, an analytical solution for active earth pressures on retaining structures of cohesive backfill with an inclined surface subjected to surcharge, pore water pressure and seismic loadings, are derived on the basis of the lower-bound theorem of limit analysis combined with Rankine's earth pressure theory and the Mohr-Coulomb yield criterion. The generalized active earth pressure coefficients (dimensionless total active thrusts) are presented for use in comprehensive design charts which eliminate the need for tedious and cumbersome graphical diagram process. Charts are developed for rigid earth retaining structures under complex environmental loadings such as the surcharge, pore water pressure and seismic inertia force. An example is presented to illustrate the practical application for the proposed coefficient charts.

Keywords: retaining walls; active earth pressure; Rankine's theory; coefficient charts; combined loadings

1. Introduction

In the design of retaining structures such as retaining walls or the stabilization of pile rows against sliding, the earth pressures or total forces from soil backfill are generally computed using analytical expressions based on Coulomb's sliding wedge theory or Rankine's earth pressure theory (Kerisel and Absi 1990, Huang *et al*. 2006, Mylonakis *et al*. 2007, Das 2008, Feng *et al*. 2008, Shukla *et al*. 2009, Yu *et al*. 2011, Yap *et al*. 2012, Iskander *et al*. 2013a, b, c, Peng and Chen 2013, Nian *et al*. 2014). The former mainly concentrated on cohesionless soils backfill with a horizontal or inclined ground surface (Fang and Chen 1995, Peng and Chen 2013, Greco 2013, 2014). The latter, Rankine earth pressure theory, mainly focused on c-ϕ soils backfill with horizontal or inclined ground surface. In this theory, a classical graphical diagram procedure using Mohr circles graphic geometry (analytical geometry) technique was widely reported by many

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researchers for obtaining the essential static or dynamic equations of lateral earth pressure in c-ϕ soils with a horizontal or sloping ground (Mazindrani and Ganjali 1997, Gnanapragasam 2000, Lancellotta 2007, Yap *et al*. 2012, Iskander *et al*. 2013a, b, c). It seems that the graphical method is very popular in the current academic and practical field. However, it becomes rather tedious for solving the practical retaining-structure problems, since several Mohr's circles of stress need to be drawn for several points along the back of the retaining walls to determine the lateral active earth pressures profiles. Moreover, it is also complicated for the practitioners and not easy to understand the procedure under seismic loadings.

In this paper, a theoretical solution of active earth pressure on retaining structures with c-ϕ soil backfill on slopes subjected to uniform surface surcharge, pore water pressure and horizontal seismic loading is presented, based on the lower-bound approach of limit analysis combined with Rankine's earth pressure theory and the Mohr-Coulomb yield criterion. The analytical procedure is described, straightforward coefficient charts for various combinations of backfill inclination β, cohesion $c/\gamma H$ and internal friction angle ϕ of soils with surface surcharge $q/\gamma H$, pore pressure ratio r_u or horizontal seismic coefficient k_h, are presented to rapidly estimate the generalized active earth pressure coefficients or to determine the dimensionless active thrusts on retaining structures for practical applications.

2. Analytical formulation

Fig. 1(a) shows a typical soil slice element, ABCD, with height z and unit width 1, cut from an inclined infinite slope with angle β. The bottom of the soil slice is designed to be parallel to the slope surface, and the water table can vary in location between the bottom of the soil slice and the slope surface. It is assumed that the effect of the left-hand soil mass of any cross section (i.e., AB)

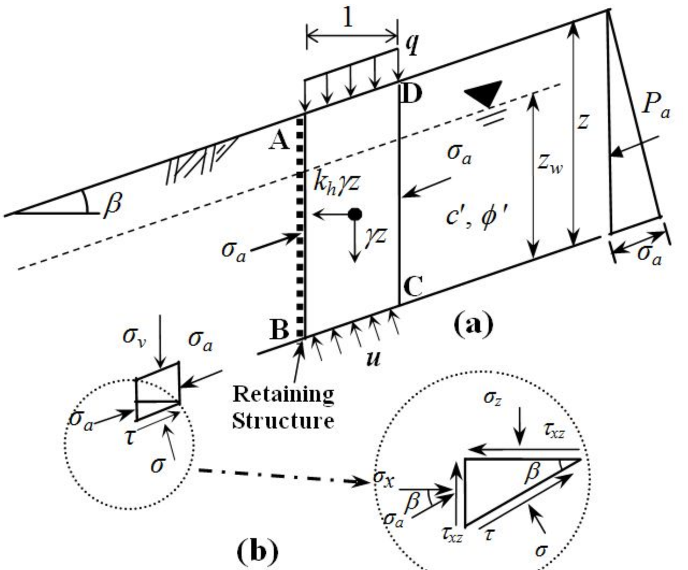


Fig. 1 Model of soil slice on an infinite slope subjected to surcharge, pore pressure and seismic forces: (a) analysis of forces in soil slice element; (b) analysis of element stress and wedge stress

in an infinite slope on the right-hand side of the soil slice is replaced by a rigid retaining structure (i.e., a retaining wall or a row of stabilizing piles against sliding, as in Fig. 1(a)) with a vertical interface, corresponding to the stress conditions analyzed in the limit-equilibrium state based on self-weight, surcharge, pore-water pressure and seismic inertia forces in a semi-infinite mass of c-ϕ soil. Here, the soil-wall mobilized friction angle at the interface is a function of inertial force, slope of backfill, inclination of the wall and soil friction angle as pointed out by Iskander *et al.* (2013a). It implies that the soil-wall friction is rather complicated. To facilitate the derivation of solution and also to keep consistency with the classical Rankine's earth pressure theory, the soil-wall interface (AB surface, see in Fig. 1(a)) is assumed to be smooth, none of the soil-wall friction (δ = 0) is considered. The active earth pressure on retaining wall under complex loadings is assumed to be parallel to the inclined backfill surface. Thus, the inclined lateral pressure value obtained by analyzing the soil slice element in an infinite slope is equal to the active earth pressures acting on a retaining structure with cohesive backfill on an inclined ground surface.

Building on this idea, the forces per unit length of the slice on a slope due to surcharge, pore water and seismic inertia forces, as shown in Fig.1 (a), are described as follows:

(1) Self-weight γz of the soil slice element, where γ is the unit weight of the soils.
(2) Uniformly distributed load or surcharge q located at the slope surface, which is induced by construction loading.
(3) Pore water pressure u produced at the bottom of the slice, represented by

$$u = r_u \cdot \gamma z = \gamma_w z_w \tag{1}$$

where r_u is known as pore pressure ratio, γ_w is unit weight of water, and z_w is the height the water table as shown in Fig. 1(a). Because of the difficulty and uncertainty involved in determining the earthquake-induced excess pore water pressure, this effect is not included in the analysis to maintain the simplicity of the pseudo-static method.

(4) Horizontal seismic inertia force (towards the wall) of the soil slice element, $k_h\gamma z$, where k_h is the horizontal seismic coefficient. Because it is very common to see only the horizontal acceleration coefficient considered in a retaining wall design, the vertical seismic coefficient is not involved in the present analysis.

According to the equilibrium conditions of the forces on the bottom surface of the soil element (Fig. 1(a)), the normal stress σ and shear stress τ along the bottom surface can be expressed as follows

$$\sigma = (\gamma z + q)\cos^2\beta - \gamma_u \gamma z \cos^2\beta - k_h \gamma z \sin\beta\cos\beta \tag{2}$$

$$\tau = (\gamma z + q)\sin\beta\cos\beta + k_h \gamma z \cos^2\beta \tag{3}$$

According to wedge stress analysis, the normal stress σ and shear stress τ along the inclined surface in a soil element can be expressed by combining the lateral stress σ_x, vertical stress σ_z and shear stress τ_{xz} of the soil in the slope in the limit-equilibrium state (see Fig. 1(b)). The following relationships can be established based on the stresses on the triangular element

$$\sigma = \frac{1}{2}(\sigma_z + \sigma_x) + \frac{1}{2}(\sigma_z - \sigma_x)\cos 2\beta - \tau_{xz}\sin 2\beta \tag{3a}$$

$$\tau = \frac{1}{2}(\sigma_z - \sigma_x)\sin 2\beta + \tau_{xz}\cos 2\beta \tag{3b}$$

Therefore, substituting the expressions of *normal stress σ and shear stress τ* obtained from Eq. (2) into Eq. (3), the vertical stress σ_z and shear stress τ_{xz} satisfying the equilibrium conditions can be expressed as follows

$$\sigma_z = \sigma_x \tan^2\beta + \gamma z(1 + q/\gamma z - r_u \cos 2\beta) + k_h \gamma z \tan\beta \tag{4a}$$

$$\tau_{xz} = \sigma_x \tan\beta + r_u \gamma z \sin\beta\cos\beta + k_h \gamma z \tag{4b}$$

where σ_x is an unknown variable.

Further, substituting the vertical stress σ_z and shear stress τ_{xz} into the following equation of the major and minor principal stresses gives

$$\begin{pmatrix}\sigma_1\\ \sigma_3\end{pmatrix} = \frac{1}{2}(\sigma_z + \sigma_x) \pm \sqrt{\left[\frac{1}{2}(\sigma_z - \sigma_x)\right]^2 + \tau_{xz}^2} \tag{5}$$

The stress field (σ_1, σ_3) obtained in this way, a function of σ_x, is an equilibrium equation that fulfils both the equilibrium conditions within the soil domain of the slope and the stress boundary conditions. In accordance with the lower-bound limit analysis approach (Chen 2007), this stress field will be a statically allowable stress field if it does not violate yield conditions such as the Mohr-Coulomb criteria. Thus, substituting the stress field (σ_1, σ_3) obtained with an unknown variable σ_x into the following Mohr-Coulomb failure criteria

$$\frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}(\sigma_1 + \sigma_3)\sin\phi + c\cos\phi \tag{6}$$

A quadratic equation based on σ_x can be built, in which c and ϕ are the cohesion and internal friction angle of the soil, respectively. Solving this equation yields two principal values of horizontal lateral stress σ_x in the statically allowable stress field, where the small-value one will be the generalized active earth pressures (σ_{ag}) for retaining structures using c-ϕ soil backfill on an inclined surface. Rearranging the forms of σ_x / cos β, the analytical expression can be rewritten as

$$\sigma_{ag} = \frac{\sigma_x}{\cos\beta} = \gamma z \cdot K_{ag} = \gamma z \cdot \cos\beta\left[J - \sqrt{J^2 - (1+A)^2 - \frac{4C^2}{\cos^2\phi} + 4D\tan\phi(1+A) + 4D^2}\right] \tag{7a}$$

where

$$K_{ag} = \cos\beta\left[J - \sqrt{J^2 - (1+A)^2 - \frac{4C^2}{\cos^2\phi} + 4D\tan\phi(1+A) + 4D^2}\right] \tag{7b}$$

in which

$$J = (1 + A)\left(\frac{2\cos^2\beta}{\cos^2\phi} \cdot \frac{1+B}{1+A} - 1\right) + 2D\tan\phi \tag{7c}$$

$$A = \frac{q}{\gamma z} - r_u \cos 2\beta + k_h \tan\beta \tag{7d}$$

$$B = \frac{q}{\gamma z} - r_u - k_h \tan\beta \tag{7e}$$

$$C = r_u \sin\beta \cos\beta + k_h \tag{7f}$$

$$D = \frac{c}{\gamma z}, \quad z \neq 0 \tag{7g}$$

$$E = \frac{q}{\gamma z}, \quad z \neq 0 \tag{7h}$$

where K_{ag} is the generalized active earth pressure coefficient under complex circumstances, including c-ϕ soil backfill on an inclined ground surface and combined loadings involving surcharge, pore water pressure and seismic inertia forces, the other parameters are defined earlier. In particular, analytical expressions for generalized active earth pressure (σ_{ag}) at $z = 0$ can be rewritten as

$$\begin{aligned}
\sigma_{ag}\Big|_{z=0} &= \cos\beta\left[\begin{array}{l} q\left(\dfrac{2\cos^2\beta}{\cos^2\phi} - 1\right) + 2c\tan\phi \\ -\dfrac{2\cos\beta}{\cos\phi}\sqrt{q^2\cos^2\beta\left(\dfrac{\cos^2\beta}{\cos^2\phi} - 1\right) + 2qc\tan\phi\cos^2\beta + c^2} \end{array}\right] \\
&= \gamma H \cdot \cos\beta\left[\begin{array}{l} E\left(\dfrac{2\cos^2\beta}{\cos^2\phi} - 1\right) + 2D\tan\phi \\ -\dfrac{2\cos\beta}{\cos\phi}\sqrt{(E)^2\cos^2\beta\left(\dfrac{\cos^2\beta}{\cos^2\phi} - 1\right) + 2DE\tan\phi\cos^2\beta + (D)^2} \end{array}\right] \\
&= \gamma H \cdot K_{ag}\Big|_{z=0}
\end{aligned} \tag{8a}$$

where

$$K_{ag}\Big|_{z=0} = \cos\beta\left[\begin{array}{l} E\left(\dfrac{2\cos^2\beta}{\cos^2\phi} - 1\right) + 2D\tan\phi \\ -\dfrac{2\cos\beta}{\cos\phi}\sqrt{(E)^2\cos^2\beta\left(\dfrac{\cos^2\beta}{\cos^2\phi} - 1\right) + 2DE\tan\phi\cos^2\beta + (D)^2} \end{array}\right] \tag{8b}$$

The parameter z in D and E (defined in Eq. (7)) is replaced by the height H of the retaining wall. When $q = 0$, Eqs. (8a) and (8b) can be reduced to the following form

$$\sigma_{ag}\Big|_{z=0} = \left(2c\tan\phi - \sqrt{(2c\tan\phi)^2 + 4c^2}\right)\cos\beta = 2c\frac{\sin\phi - 1}{\cos\phi}\cos\beta \tag{8c}$$

To verify the analytical solution, a special case involving backfill composed of cohesive soils on a sloping surface without surcharge, pore water or horizontal seismic loading is adopted: $\beta \neq 0$, $c \neq 0$, $\phi \neq 0$, $q = 0$, $r_u = 0$ and $k_h = 0$. From Eqs. (7), $q = 0$, $r_u = 0$ and $k_h = 0$. Eq. (7b) reduces to

$$K_{ag} = \frac{\cos\beta}{\cos^2\phi}\left\{2\cos^2\beta + 2D\cos\phi\sin\phi \pm \begin{bmatrix} 4\cos^2\beta\left(\cos^2\beta - \cos^2\phi\right) \\ +4D^2\cos^2\phi + 8D\cos^2\beta\sin\phi\cos\phi \end{bmatrix}^{1/2}\right\} - \cos\beta \tag{9}$$

Eq. (9) is identical to that presented by Mazindrani and Ganjali (1997) using the graphic geometry procedure.

If $\beta = 0$, Eq. (9) can be rewritten as

$$K_{ag} = \frac{(1 - \sin\phi)^2}{(1 - \sin\phi)(1 + \sin\phi)} - 2D\sqrt{\frac{(1 - \sin\phi)^2}{(1 - \sin\phi)(1 + \sin\phi)}} = K_a - 2D\sqrt{K_a} \tag{10}$$

In which K_a is the active earth pressure coefficient based on Rankine's theory, and is conventionally expressed as $K_a = \dfrac{1 - \sin\phi}{1 + \sin\phi}$. Hence, Eq. (7a) at $\beta = 0$ with a static loading condition can be transformed into

$$\sigma_{ag} = K_{ag} \cdot \gamma z = \gamma z K_a - 2c\sqrt{K_a} \tag{11}$$

Eq. (11) is the classical Rankine solution of active earth pressures for the case of cohesive (c-ϕ soils) backfill with a horizontal ground surface, vertical wall and static loading (Das 2008). Similarly, if $\beta = 0$ and $c = 0$, Eq. (7a) can also be rewritten as

$$\sigma_{ag} = K_{ag} \cdot \gamma z = \gamma z K_a \tag{12}$$

Eq. (12) is the classical Rankine solution of active earth pressures for the case of cohesionless ($c = 0$) backfill with a horizontal ground surface, vertical wall and static loading (Das 2008).

3. Distribution of generalized active earth pressure under combined loadings

Consider the following details for a rigid retaining structure supporting c-ϕ soil backfill on a sloping ground subjected to surcharge, pore water pressure and seismic inertia forces. The material properties and loading parameters are listed in Table 1.

Table 1 Material properties and loading parameters for a typical retaining structure

γ (kN/m^3)	c (kPa)	ϕ (°)	H (m)	β (°)	q (kPa)	r_u	k_h
Soil unit weight	Soil cohesion	Soil friction angle	Wall height	Slope angle	Surcharge	Pore pressure ratio	Horizontal seismic coefficient
18.0	21.6	35	12	10	43.2	0.25	0.2

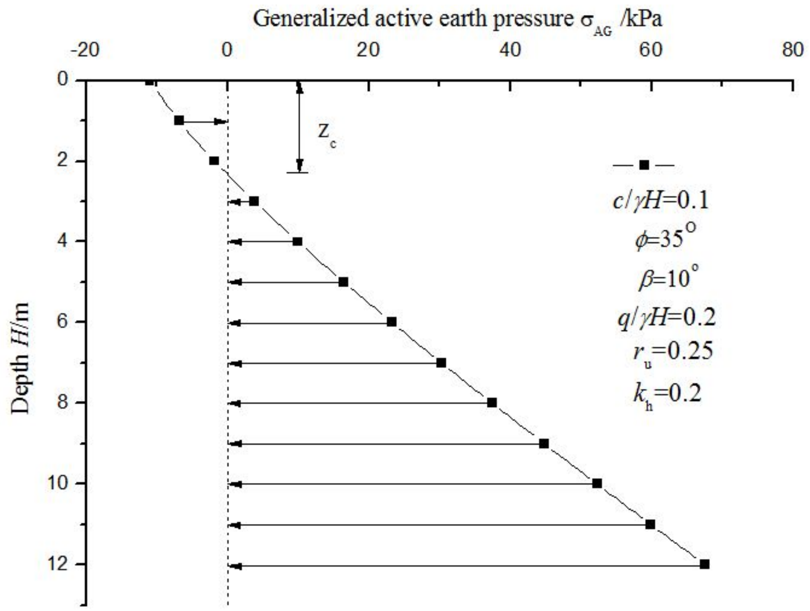


Fig. 2 Distribution of generalized active earth pressure along the wall depth

Using Eq. (7), calculations are performed to obtain the generalized active earth pressures on retaining structures subjected to surcharge, pore-water and pseudo-static seismic loading. The distribution of generalized active earth pressure along the wall depth is shown in Fig. 2. It can be observed from Fig. 2 that the generalized active earth pressure has a triangular (linear) distribution, and a tension crack zone (negative active earth pressure) exists within the critical depth z_c of the retaining wall.

4. Total active thrusts acting on retaining structures

Fig. 2 shows a triangular active earth pressure distribution with a tension crack in the range of the critical depth z_c. It can be observed from Fig. 2 that the critical depth z_c can be obtained when the generalized active earth pressure $\sigma_{ag} = 0$. A quadratic equation involving z can be built using Eq. (7a) with $\sigma_{ag} = \dfrac{\sigma_x}{\cos\beta} = 0$ and rearranging the formula after opening the brackets and embedding into the square root. By solving this quadratic equation, the critical depth value z_c of the tension crack can be given as follows

$$z_c = \frac{2HD\left[\left(\sin\phi - \lambda\cos\phi\right)\left(1 + A - E\right) + \sqrt{\left(1 + A - E\right)^2 + 4C^2\left(1 + 2\lambda\tan\phi - \lambda^2\right)}\right]}{\cos\phi\left[\left(1 + A - E\right)^2 + \dfrac{4C^2}{\cos^2\phi}\right]} \tag{13}$$

where $\lambda = E / 2D$ and the parameter z in A~E (defined in Eq. (7)) is replaced by the height H of the retaining structure.

When combined with Eq. (1), the static water pressure induced by groundwater, P_w, acting on a retaining structure with height H can be written as follows

$$P_w = \frac{1}{2}\gamma_w z_w \cdot z_w = \frac{1}{2}\gamma H^2 \frac{\gamma}{\gamma_w} r_u^2 \tag{14}$$

Thus, the generalized total active thrust, P_{ag}, acting on the retaining wall per unit length can be expressed as

$$\begin{aligned} P_{ag} &= \frac{1}{2}\sigma_{ag}\big|_{z=H}\left(H - z_c\right) + P_w = \frac{1}{2}\gamma H^2 \cdot K_{ag}\big|_{z=H}\left(1 - \frac{z_c}{H}\right) + \frac{1}{2}\gamma H^2 \cdot \frac{\gamma}{\gamma_w} r_u^2 \\ &= \frac{1}{2}\gamma H^2\left[K_{ag}\big|_{z=H}\left(1 - \frac{z_c}{H}\right) + 2r_u^2\right] \end{aligned} \tag{15}$$

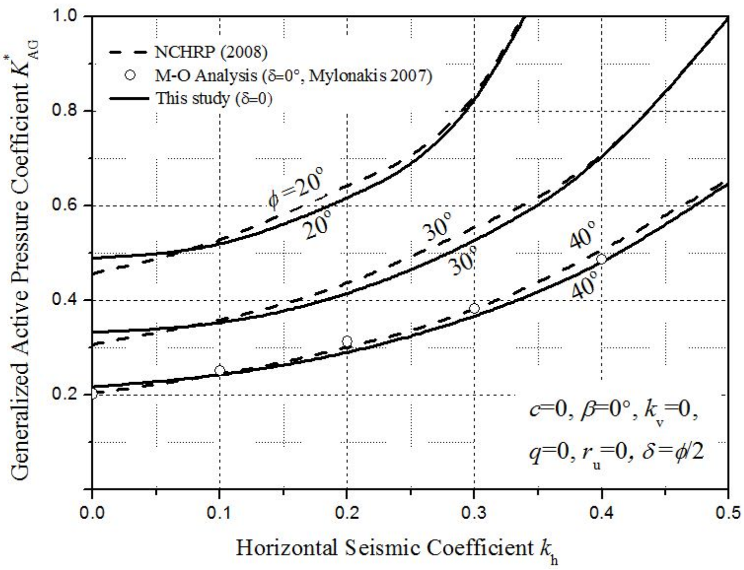
where γ / γ_w is approximately equal to 2.0. Considering the dimensionless parameter (called the generalized active earth pressure coefficients), $K_{ag}^* = \dfrac{P_{ag}}{(\frac{1}{2}\gamma H^2)}$, Eq. (15) can be represented in the following form

$$K_{ag}^* = \frac{P_{ag}}{(\frac{1}{2}\gamma H^2)} = K_{ag}\big|_{z=H}\left(1 - \frac{z_c}{H}\right) + 2r_u^2 \tag{16}$$

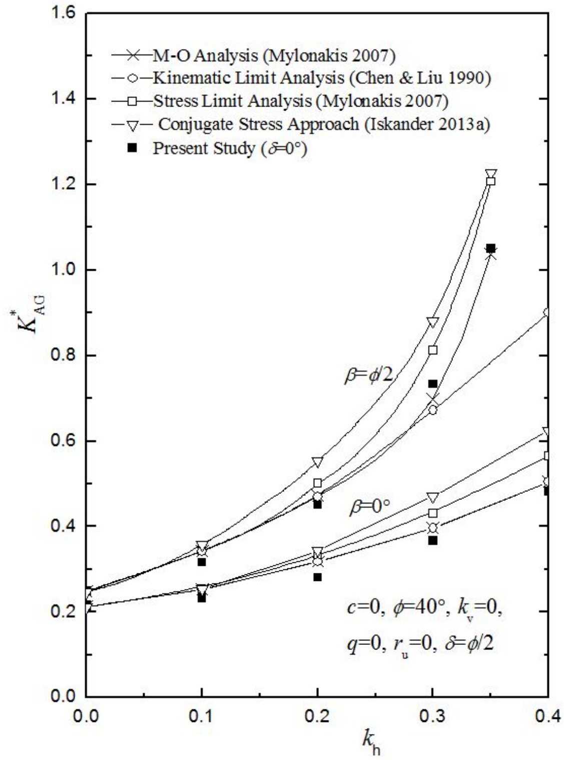
5. Comparison of solutions and discussions

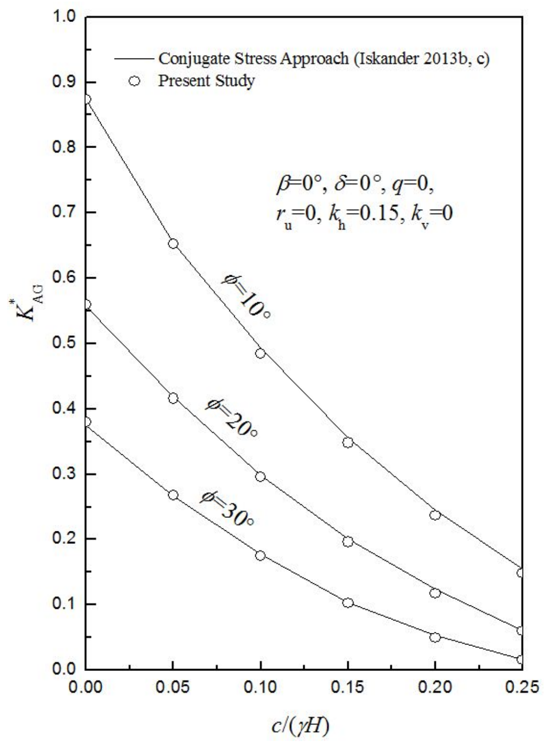
A series of comparisons between the present analytical results and those available in the published work (NCHRP report 611 by Anderson *et al*. 2008, Mylonakis *et al*. 2007, Iskander *et al*. 2013a, b, c) have been made to validate the proposed analytical approach.

Based on Eqs. (7) and (16), the variations of generalized active earth pressure coefficient (K_{ag}^*) with the horizontal seismic coefficient (k_h) under different soil friction angles (ϕ), and dimensionless cohesions ($c / \gamma H$) are presented in Fig. 3(a). In order to validate the analytical approach, the results from the new approach in certain situations are also compared with those from the Mononob-Okabe (M-O) equations (NCHRP report 611, Anderson *et al*. 2008). It can be seen from Fig. 3(a) that the K_{ag}^* values obtained using the present solution without considering the wall friction ($\delta = 0$) are generally lower than those obtained using the M-O approach considering the wall friction ($\delta = \phi / 2$), but the differences of the calculated results by these two approaches are within 5%.



(a) Variations of K^*_{ag} with k_h under different soil friction angle ϕ





(b) Variations of K^*_{ag} with k_h under different backfill inclination β

(c) Variations of K^*_{ag} with $c / \gamma H$ under different soil friction angle ϕ

Fig. 3 Comparisons of the present analytical approach and the existed methods

To further verify the present solutions, the representative data from M-O solutions without considering the wall friction ($\delta = 0$) for the cohesionless soils with internal friction angle $\phi = 40°$ under horizontal seismic loading (Mylonakis *et al*. 2007) are also plotted in Fig. 3(a). It can be seen that given the existence of a uniform Rankine stress field, the derived solutions by the present approach are almost identical to those from M-O solutions for the earth pressure on the wall, when the same soil wall friction angle $\delta = 0$ is used in both solutions. Therefore, the new analytical approach is valid for practical use.

In order to investigate the variations of generalized active earth pressure coefficient K_{ag}^* with horizontal seismic coefficient k_h under different backfill inclination β, the present analytical approach and some existed methods, such as the M-O analysis (Mylonakis *et al*. 2007), kinematical limit analysis (Chen and Liu 1990), stress limit analysis or alternative M-O analysis (Mylonakis *et al*. 2007) and conjugate stress limit analysis (Iskander *et al*. 2013a), are employed to achieve the values of K_{ag}^* under different backfill inclination β. The results from the present solutions in certain situations ($\phi = 40°$, $\delta = 0$) are compared with those from the existed methods for the case of $\phi = 40°$ and $\delta = \phi/2$ under different inclined backfill ($\beta = 0°$ and $\beta = \phi/2$), and the charts of comparative analysis by the several methods can be found in Fig. 3 (b). It can be shown that the K_{ag}^* values obtained using the present solution without considering the soil-wall friction are generally less than those obtained using the other approaches considering the soil-wall friction ($\delta = \phi/2$) under different horizontal seismic coefficients k_h and lower inclined backfill (e.g., $\beta <$ 10°). However, under higher horizontal seismic coefficients k_h (e.g., $k_h > 0.2$) and rather larger inclined backfill (e.g., $\beta = 20°$) the obtained results using the present approach without considering the wall friction is evidently higher than those using the other two methods (M-O analysis and kinematic limit analysis) considering the wall frictions, but the differences of the calculated results by these several approaches are in general within 5%. In short, the proposed solutions based on the conventional Rankine's earth pressure theory have a relatively good agreement with those based on the existed methods under the same backfill inclination.

Comparison between the proposed method and the conjugate stress limit analysis solutions (Iskander *et al*. 2013b, c) for c-ϕ soils is also made for different dimensionless cohesion $c/\gamma H$ (from 0 to 0.25 with an interval of 0.05) and soil friction angle ϕ (from 10° to 30° with an interval of 10°) under a horizontal seismic coefficient of $k_h = 0.15$ in Fig. 3(c). It can be seen that the present results agree well with those from the previously published work. Particularly, Fig. 3(c) also shows that the generalized active earth pressure coefficient values decrease sharply with the increase of dimensionless cohesion $c/\gamma H$ or soil friction angle ϕ when the environmental loadings are given.

6. Design charts

A computer code is developed to calculate the generalized active earth pressure coefficients K_{ag}^* or dimensionless active thrusts $\dfrac{P_{ag}}{(\frac{1}{2}\gamma H^2)}$ on retaining structures based on Eqs. (7), (15) and (16). However, from a practical viewpoint, it would be inconvenient for engineers to obtain these calculated results in scenarios where they do not have access to computers, and it would also be difficult to examine the distribution characteristics of active earth pressures or lateral active thrusts under various loadings. Therefore, a design procedure based on coefficient charts is becoming more necessary in practice.

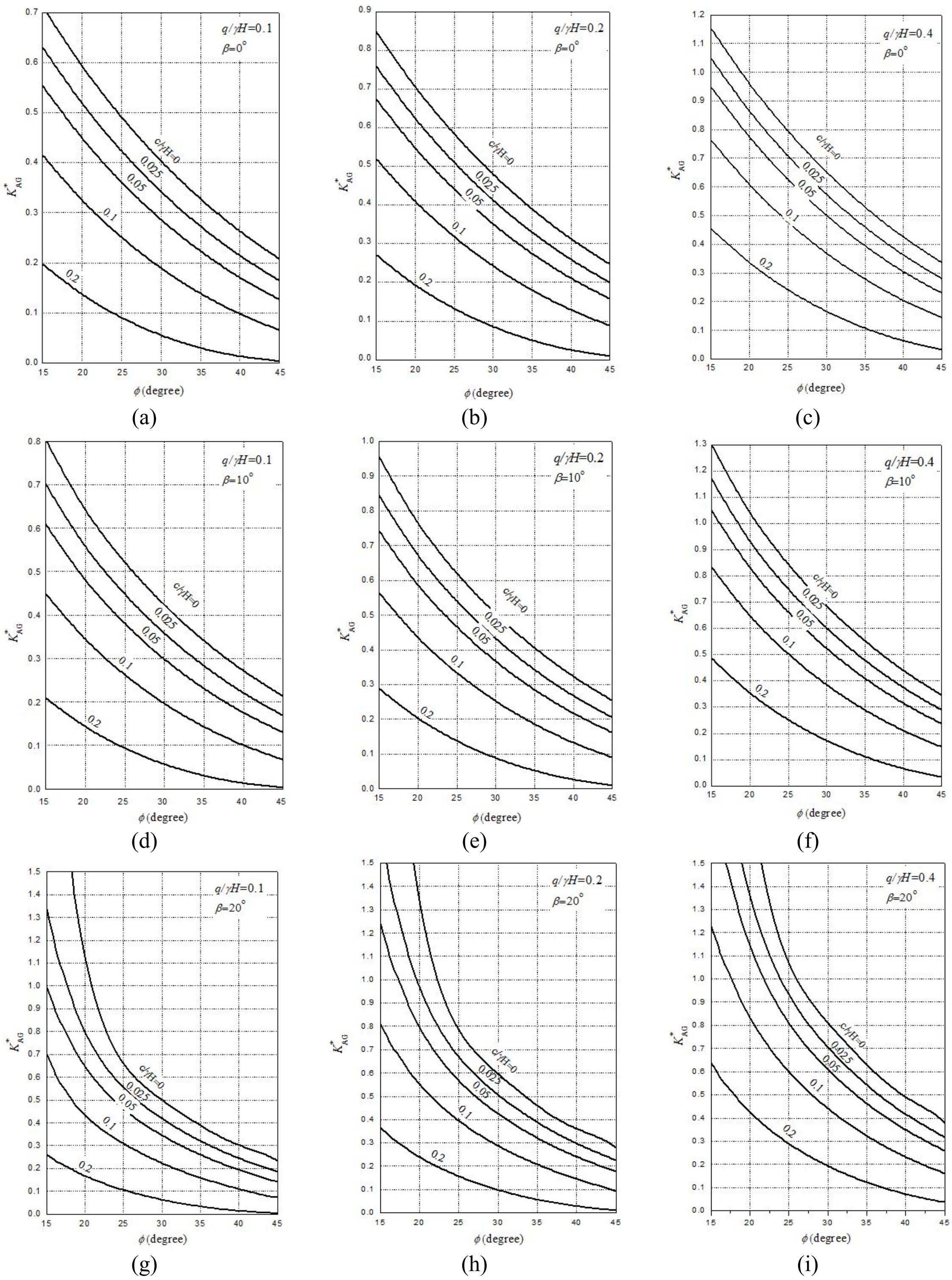


Fig. 4 Coefficient charts for generalized active earth pressure under surcharge

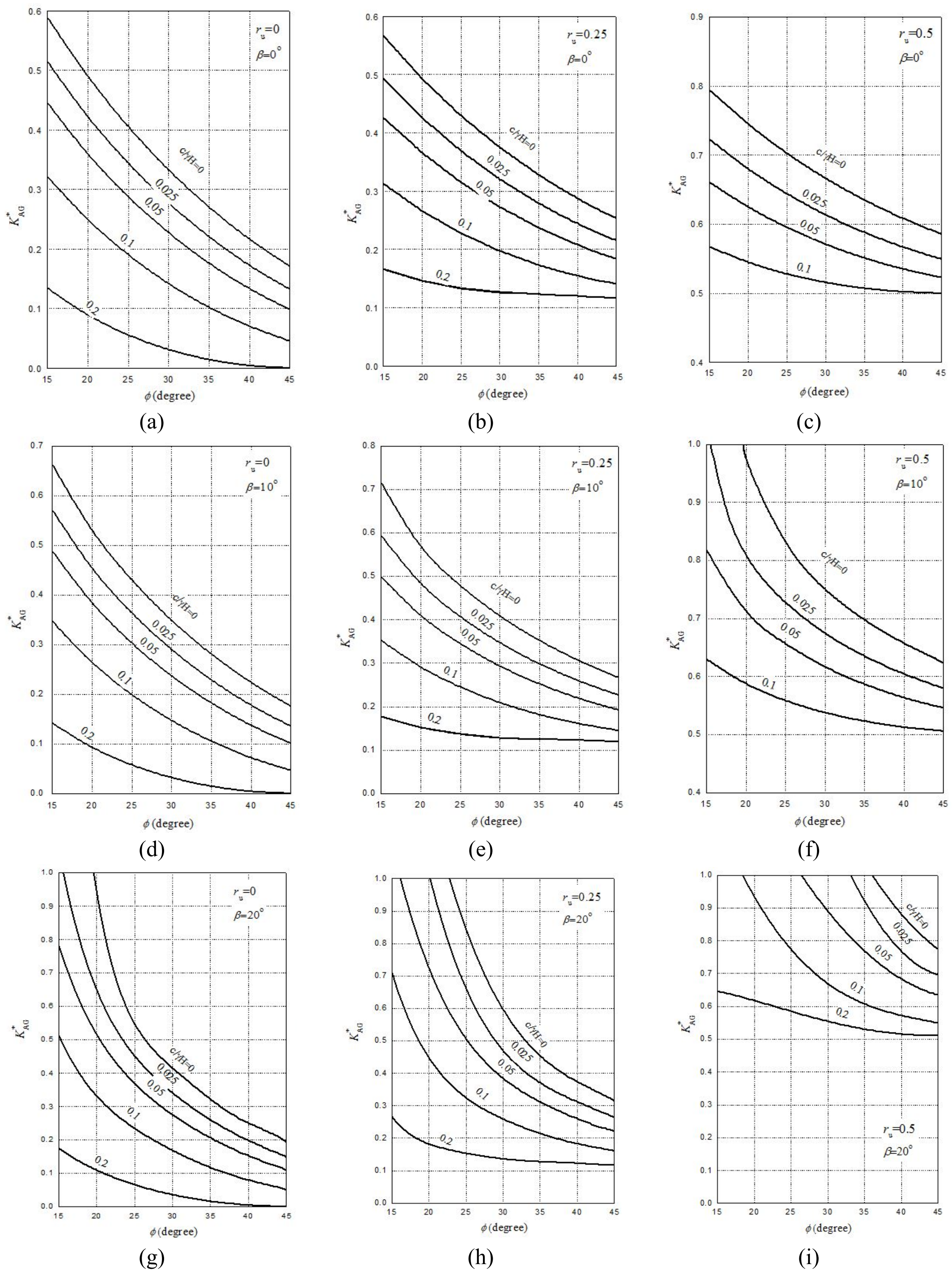


Fig. 5 Coefficient charts for generalized active earth pressure considering pore pressure

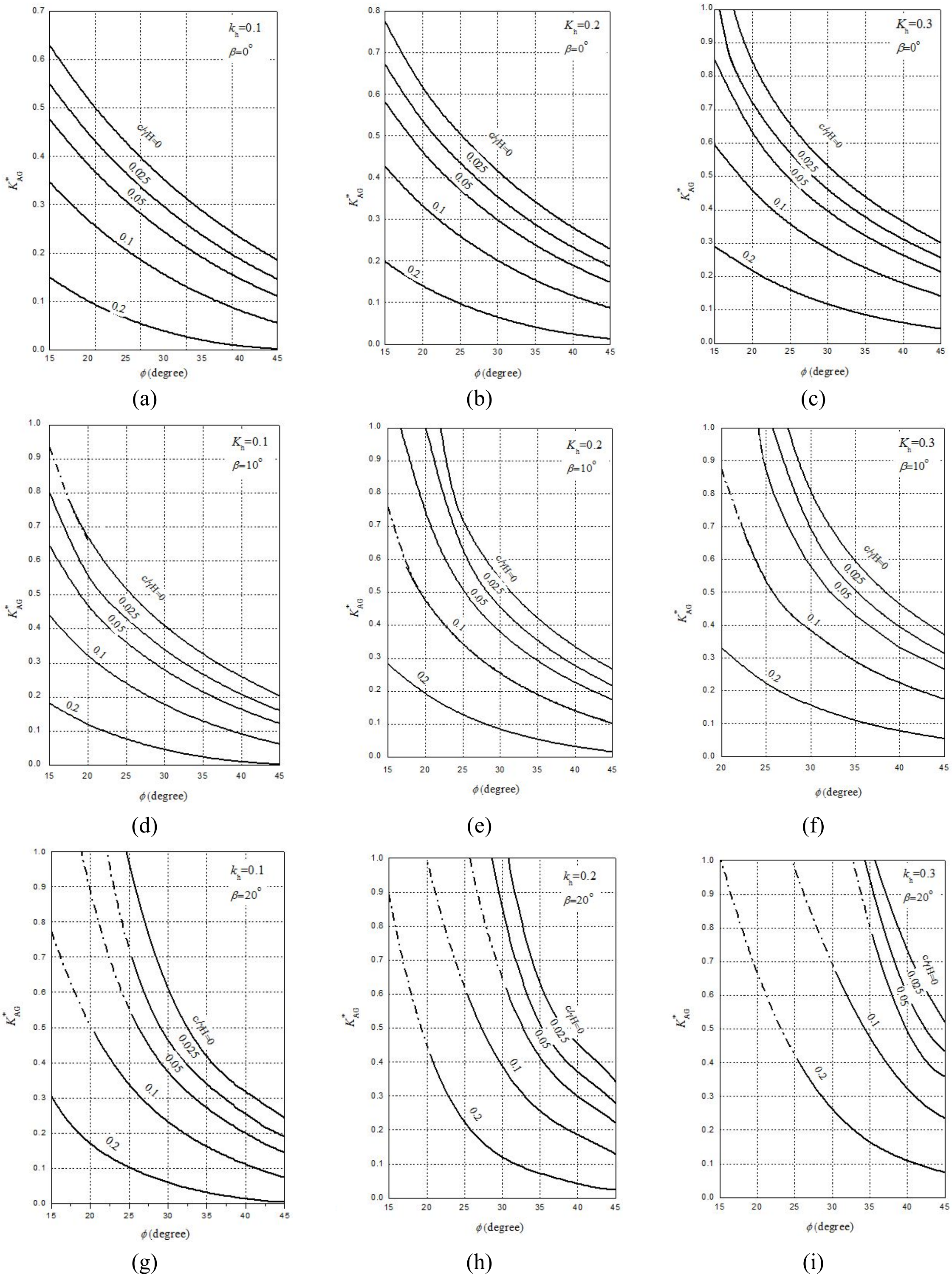


Fig. 6 Coefficient charts for generalized active earth pressure under seismic forces

A set of coefficient charts for generalized active earth pressures or total active thrusts have been created and are shown in Figs. 4, 5 and 6 for three slope inclinations ($\beta = 0°$, $10°$ and $20°$) under uniform surface surcharge ($q / \gamma H$), pore water pressure (r_u) and horizontal seismic forces (k_h), respectively.

It can be observed in Fig. 4 that generalized active earth pressure coefficients K_{ag}^* decreases nonlinearly with the increase of soil friction angle ϕ under a given dimensionless cohesion $c / \gamma H$, slope inclination β and uniform surface surcharge $q/\gamma H$. Similar changes are also observed between K_{ag}^* and $c / \gamma H$ when soil friction angle ϕ, slope inclination β and surface surcharge $q/\gamma H$ are fixed. On the contrary, the values of K_{ag}^* increase with the increase in surcharge $q / \gamma H$. It can also be observed by comparison with Figs. 4(a), (d) and (g), or Figs. 4(b), (e) and (h), or Figs. 4(c), (f) and (i) that K_{ag}^* increases with the increase in slope inclination β.

Fig. 5 shows the relationship between K_{ag}^* and soil friction angle ϕ under three slope inclinations ($\beta = 0°$, $10°$ and $20°$) and three pore pressure ratios (r_u = 0, 0.25, 0.50). It can be observed by comparing with Figs. 5(a), (b) and (c), or Figs. 5(d), (e) and (f), or Figs. 5(g), (h) and (i) that the values of K_{ag}^* increase significantly with the increase in pore pressure ratio r_u. When the pore pressure ratio r_u is not equal to zero (assuming the existence of groundwater), the cohesion c and soil friction angle ϕ can be replaced by the effective cohesion c' and the effective internal friction angle ϕ' of soils, respectively, and a higher active thrust can be achieved. This will lead to a more realistic solution for active thrust on retaining structures.

Fig. 6 shows the design charts for dimensionless active thrust on retaining structures subjected to horizontal seismic loadings with three horizontal seismic coefficients (k_h = 0.1, 0.2, 0.3) under different slope inclinations ($\beta = 0°$, $10°$ and $20°$). It can be observed by comparing with Figs. 6(a), (b) and (c), or Figs. 6(d), (e) and (f), or Figs. 6(g), (h) and (i) that K_{ag}^* increases significantly with the increase in the horizontal seismic coefficient k_{h}. When the backfill slope inclination reaches 20° (see the dashdotted line in Figs. 6(g), (h) and (i)), or a combination of rather bigger slope inclination of $\beta = 10°$ and higher horizontal seismic coefficient of k_h = 0.3 is achieved (see the dashdotted line in Fig. 6(f)), a lower soil friction angle ϕ is incapable of producing a reasonable active thrust to resist the failure (sliding or overturning) of retaining structures. In other words, a design using a lower friction angle ϕ in a seismic active zone is not feasible.

The design charts achieved above are applied to a representative reinforced concrete retaining structure with a vertical and smooth back-face. The wall has a height of H = 12 m, and the width of the base and top are B = 3 m and b = 1.5 m, respectively. It is built of a uniform soil characterised by $\phi = 35°$, c = 21.6 kPa, and γ = 18 kN/m^3. The soil backfill is inclined at $\beta = 10°$ to the horizontal ground (Table 1) and the friction coefficient at the contact surface between the wall base and soil is $\mu = 0.4$. What would the safety factor of this retaining wall against sliding under an active state be if the magnitude of the horizontal acceleration were 0.2 of the gravity acceleration? First, calculate $c / \gamma H = 0.1$; next, read K_{ag}^* from Fig. 6(e) ($K_{ag}^* = 0.2$). It should be noted that when the backfill inclination β is not equal to 0°, 10° or 20° (i.e., $0° < \beta < 10°, 10° < \beta < 20°$), the interpolation method in mathematics can be employed to achieve the generalized active earth pressure coefficient under a specified backfill inclination β. Then, calculate the gravity of the retaining wall, G = 594 kN, and finally, solve for the safety factor against sliding: $F_s = \dfrac{\mu\left(G + P_{ag}\sin\beta\right)}{P_{ag}\cos\beta} = 1.0$.

Consequently, the retaining wall is at the critical active state against seismic loading. Without considering the external loading, the generalized active earth pressure coefficient of this retaining wall can be found from Fig. 5(d), $K_{ag}^* = 0.11$. The safety factor of the retaining wall against sliding under these conditions has a high value of $F_s = 1.7$.

However, this retaining wall would also be on the verge of sliding failure (the factor of safety against sliding being about $F_s = 1.0$) if it were subjected to either a uniform construction loading equivalent to those described by $q = 43.2$ kPa or pore water pressure equivalent to $r_u = 0.25$. However, it should be noted that when the effective cohesion c' and the effective internal friction angle ϕ' of a soil are adopted under pore water pressure ($r_u = 0.25$), a lower safety factor against sliding ($F_s < 1.0$) will be yielded. In other words, the present solution, without considering the effective shear strength, is a rather unconservative estimate of the safety factor.

7. Conclusions

A set of charts was produced for estimating the generalized active earth pressure coefficients or dimensionless total active thrusts on the rigid frictionless retaining structures. The data were obtained from calculations based on the lower bound approach of limit analysis. These charts can be used for the design of retaining structures using c-ϕ soil backfill on the sloping ground subjected to surcharge, pore water pressure and horizontal seismic forces. The charts are in a convenient-to-use format for direct application in engineering work, and evaluating the generalized earth pressures does not require a tedious calculation process. However, these charts are not intended for soil backfills with a rather steep slope and a low frictional component of strength.

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