Geomechanics and Engineering, *Vol. 8, No. 3 (2015) 377-390*
DOI: http://dx.doi.org/10.12989/gae.2015.8.3.377

The influence of initial stress on wave propagation and dynamic elastic coefficients

Xibing Li [1,2] and Ming Tao [*1,2]

[1] *School of Resources and Safety Engineering, Central South University, Changsha, Hunan, PR, China*
[2] *Hunan Key Laboratory of Resources Exploitation and Hazard Control for Deep Metal Mines, Changsha, China*

(*Received September 12, 2014, Revised November 12, 2014, Accepted December 10, 2014*)

Abstract. The governing equations of wave propagation in one dimension of elastic continuum materials are investigated by taking the influence of the initial stress into account. After a short review of the theory of elastic wave propagation in a rock mass with an initial stress, results indicate that the initial stress differentially influences *P*-wave and *S*-wave propagation. For example, when the initial stress is homogeneous, for the *P*-wave, the initial stress only affects the magnitude of the elastic coefficients, but for the *S*-wave, the initial stress not only influences the elastic coefficients but also changes the governing equation of wave propagation. In addition, the *P*-wave and *S*-wave velocities were measured for granite samples at a low initial stress state; the results indicate that the seismic velocities increase with the initial stress. The analysis of the previous data of seismic velocities and elastic coefficients in rocks under ultra-high hydrostatic initial stress are also investigated.

Keywords: stress wave; governing equations; initial stress; seismic velocities; elastic coefficients

1. Introduction

In underground mining and civil engineering, the problems of tectonic movements involve earth masses that are initially under stress (Sharma and Garg 2006). In particular, with the increase of mining depth, the initial stress will gradually become high (Tao *et al*. 2013). The initial stress will affect the elastic coefficients and stress wave velocities of rocks. The effect of the initial stress or pre-stress on the rock elastic coefficients and wave propagation has been explored by a number of scholars. For example, Biot investigated the governing equations of wave propagation in two- and three-dimensional pre-stressed fields (Biot 1964). Ogden and Sotiropoulos demonstrated the effect of pre-stress on the propagation and reflection of plane waves in incompressible elastic solids (Ogden and Sotiropoulos 1995, 1997). However, it is well known that the classical elastic theory is restricted to small deformations and rotations. Meanwhile, most of our knowledge of the chemical composition, physical state, and structure of the Earth's interior mainly comes from seismic data (Ji *et al*. 2007). Therefore, laboratory measurements of the elastic coefficients and the velocities of elastic waves in rocks are necessary for the interpretation of seismic velocities and elastic coefficients in materials (Birch 1960). During the last fifty years, a large number of

∗Corresponding author, Ph.D., E-mail: mingtao@csu.edu.cn

http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

laboratory measurements of *P*-wave (V_p) and *S*-wave (V_s) velocities and their characteristics of anisotropy, hysteresis, and splitting have been conducted on various types of rocks. For example, Birch (Birch 1960, 1961) measured the velocities of compressional waves in rocks at pressures of up to 10 kilobars (1,000 MPa), Christensen (Christensen 1965, 1974) tested the compressional wave velocities in metamorphic rocks at pressures of up to 1,000 MPa, and Manghnani and some other scholars performed measurements on many type of rocks at ultra-high pressure (Manghnani *et al*. 1974).

With the increasing depths of underground mining and civil engineering tunneling projects in the past decade, more exact and detailed laboratory measurements of the elastic coefficients of rocks at high and ultra-high pressure are being conducted. For example, Ji *et al*. (2007) presented some of the available data on the seismic velocities and anisotropy in minerals and rocks (Ji *et al*. 2002). Rocks may be elastically nonlinear, hysteretic and splitting (Ji *et al*. 2007, Sun *et al*. 2012). When rocks are under an initial stress, their elastic properties will be changed. The elastic properties of rocks can be determined by static and dynamic tests. Based on acoustic experiments, Wang and other researchers determined the Poisson's ratio of rocks as a function of hydrostatic confining pressure (Chevrot and van der Hilst 2000, Wang and Ji 2009). Mohammad et al. studied the effect of a confining pressure on compressional and shear wave velocities and on the dynamic to static Young's modulus ratio (Asef and Najibi 2013).

In the elastic field, after review of the theory of elastic wave propagation in a material with an initial stress, this paper presented the relationships of the initial stress with the elastic governing equations. In addition, an experimental apparatus is used to investigate the relationships between seismic velocities and elastic coefficients with initial stress. Further investigation based on the published data confirms the experimental observations in this study regarding the effect of initial stress on elastic wave velocities and elastic coefficients.

2. Elastic theory and governing equations in one dimension

Consider a one-dimensional (1D) bar; the transient motion in the bar may be initiated by an impulse applied at one end (Brady 2004). A *P*-wave travels along the bar, resulting in a transient displacement at any point, and the stress-strain equation for an elastic bar is described by

$$\sigma_x = E\frac{\partial u}{\partial x} \tag{1}$$

where x is the space coordinate measured along the rod, σ_x is the stress, u is the displacement, and E is Young's modulus. The property of displacement is continuous; thus

$$\frac{\partial \upsilon}{\partial x} = \frac{\partial \varepsilon}{\partial t} \tag{2}$$

where t is the time, and ρ is the density of the solid. It is assumed here that the bar is composed of a homogeneous elastic media. Hence, E and ρ are constants, so by combing Eqs. (1), (2) and (3), it is found that

$$\frac{\partial}{\partial x}\left(E\frac{\partial u}{\partial x}\right) = \rho\frac{\partial^2 u}{\partial t^2} \tag{3}$$

Because the bar is a homogeneous elastic media, then

$$C^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t} \tag{5}$$

Eq. (5) is the well-known 1D *P*-wave equation under un-stressed condition.

Furthermore, if the bar has an initial stress, i.e., pre-stressed, then the initial stress must have an influence on elastic wave propagation. For the 1D stress condition, consider a state of initial stress such that a principal direction is always parallel to the x axis. In addition, assume that the initial stress in the body is S_{11}; X is the components of the body force per unit mass; ρ and E are the mass density and Young's modulus of the medium before deformation, respectively, when it have initial stress, ρ and E will change as ρ' and E', respectively. In this case, the initial stress component must satisfy the equilibrium condition as

$$\frac{\partial S_{11}}{\partial x} + \rho'(x) X(x) = 0 \tag{6}$$

In the absence of a change of the cross section induced by the initial stress, equation (1) can be written as

$$\sigma_x = E' \frac{\partial u}{\partial x} + S_{11} \tag{7}$$

Thus, the equation of motion has the form of

$$\frac{\partial \sigma_x}{\partial x} = \rho' \frac{\partial^2 u}{\partial t^2} \tag{8}$$

Thus, if the initial stress is homogeneous, then

$$\frac{\partial}{\partial x}\left(E' \frac{\partial u}{\partial x} \right) = \rho' \frac{\partial^2 u}{\partial t^2} \tag{9}$$

This is the 1D wave equation with homogeneous initial stress. Therefore, the pre-stressed wave equation is the same as the un-stressed one. The result indicates that the 1D stress wave theory still suits the condition of the sample having initial stress. The influence of the pre-stress appears only on the elastic coefficient of material, such as the Young's modulus and density.

3. Governing equations for the stress in two dimensions

3.1 Effect of geotextile reinforcement

In the plane x, y, we consider a position $P(x, y)$, and the initial stresses in the body are S_{11}, S_{22}, and S_{12}. In addition, it is assumed that the components of the body force per unit mass are X and Y. Under these conditions, in two dimensions, the initial stress components must satisfy the governing equations as

$$\begin{cases} \dfrac{\partial S_{11}}{\partial x} + \dfrac{\partial S_{12}}{\partial y} + \rho X(x, y) = 0 \\ \dfrac{\partial S_{12}}{\partial x} + \dfrac{\partial S_{11}}{\partial y} + \rho Y(x, y) = 0 \end{cases} \tag{10}$$

When a dynamic disturbance occurs in the pre-stressed plane, it will cause a deformation increment, assuming the position $P(x, y)$ changes as $P^{'}$ (ξ, η), and can be written as

$$\begin{cases} \xi = x + u \\ \eta = y + \upsilon \end{cases} \tag{11}$$

The displacement filed is represented by the vector of components as

$$\begin{cases} u = u(x, y) \\ \upsilon = \upsilon(x, y) \end{cases} \tag{12}$$

Then, the differential relations are as follows (Biot 1964)

$$\begin{cases} d\xi = \left(1 + \dfrac{\partial u}{\partial x}\right)dx + \dfrac{\partial u}{\partial y}dy \\ d\eta = \left(1 + \dfrac{\partial \upsilon}{\partial y}\right)dy + \dfrac{\partial \upsilon}{\partial x}dx \end{cases} \tag{13}$$

In addition, the dynamic disturbance will induce rotation, which is approximately given by

$$\omega = \frac{1}{2}\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right) \tag{14}$$

The stress components after deformation are written as

$$\begin{cases} \sigma_{11} = S_{11} + s_{11} \\ \sigma_{22} = S_{22} + s_{22} \\ \sigma_{12} = S_{12} + s_{12} \end{cases} \tag{15}$$

For elastic deformation processes, the components s_{11}, s_{22}, and s_{12} of the stress increments only depend on the strain. Thus, based on the so-called Biot's first order quantities, the relations of stress components are given as (Biot 1964)

$$\begin{cases} \sigma_{xx} = s_{11} - 2S_{11}\omega \\ \sigma_{yy} = s_{22} + 2S_{22}\omega \\ \sigma_{xy} = s_{23} + (S_{11} - S_{22})\omega \end{cases} \tag{16}$$

σ_{xx}, σ_{yy} and σ_{xy} are the stresses at the point (ξ, η) along the *x*- and *y*-directions. These stress components satisfy the dynamic equilibrium relations as (Biot 1964)

$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial \xi} + \dfrac{\partial \sigma_{xy}}{\partial \eta} + \mu(\xi,\eta)X(\xi,\eta) = \mu(\xi,\eta)\dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial \sigma_{xy}}{\partial \xi} + \dfrac{\partial \sigma_{yy}}{\partial \eta} + \mu(\xi,\eta)Y(\xi,\eta) = \mu(\xi,\eta)\dfrac{\partial^2 \upsilon}{\partial t^2} \end{cases} \tag{17}$$

Substituting the values of Eq. (6) for σ_{xx}, σ_{yy}, and σ_{xy} and using the coordinate transformation, the dynamic governing equation in two dimensions is written as (Biot 1964)

$$\begin{cases} \dfrac{\partial s_{11}}{\partial x} + \dfrac{\partial s_{12}}{\partial y} + \rho u \dfrac{\partial X}{\partial x} + \rho \upsilon \dfrac{\partial X}{\partial y} + \rho \omega Y - 2S_{12}\dfrac{\partial \omega}{\partial x} \\ + \left(S_{11} - S_{22}\right)\dfrac{\partial \omega}{\partial y} - \dfrac{\partial S_{11}}{\partial x} e_{xx} - \dfrac{\partial S_{12}}{\partial y} e_{yy} - \left(\dfrac{\partial S_{11}}{\partial y} + \dfrac{\partial S_{12}}{\partial x}\right) e_{xy} = \rho \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial s_{12}}{\partial x} + \dfrac{\partial s_{22}}{\partial y} + \rho u \dfrac{\partial Y}{\partial x} + \rho \upsilon \dfrac{\partial XY}{\partial y} + \rho \omega X + 2S_{12}\dfrac{\partial \omega}{\partial y} \\ + \left(S_{11} - S_{22}\right)\dfrac{\partial \omega}{\partial x} - \dfrac{\partial S_{22}}{\partial y} e_{yy} - \dfrac{\partial S_{12}}{\partial x} e_{xx} - \left(\dfrac{\partial S_{22}}{\partial x} + \dfrac{\partial S_{12}}{\partial y}\right) e_{xy} = \rho \dfrac{\partial^2 \upsilon}{\partial t^2} \end{cases} \tag{18}$$

where, the notation *e* is introduced as

$$\begin{cases} e_{xx} = \dfrac{\partial u}{\partial x}, \quad e_{yy} = \dfrac{\partial \upsilon}{\partial y} \\ e_{xy} = e_{yx} = \dfrac{1}{2}\left(\dfrac{\partial \upsilon}{\partial x} + \dfrac{\partial u}{\partial y}\right) \end{cases} \tag{19}$$

Alternatively, if the initial stress is assumed to be homogeneous, then there is no body force increment and the dynamic Eq. (18) for plane strain propagation becomes

$$\begin{cases} \dfrac{\partial s_{11}}{\partial x} + \dfrac{\partial s_{12}}{\partial y} + \left(S_{11} - S_{22}\right)\dfrac{\partial \omega}{\partial y} = \rho \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial s_{12}}{\partial x} + \dfrac{\partial s_{22}}{\partial y} + \left(S_{11} - S_{22}\right)\dfrac{\partial \omega}{\partial x} = \rho \dfrac{\partial^2 \upsilon}{\partial t^2} \end{cases} \tag{20}$$

Furthermore, if the initial stress is hydrostatic, then

$$S_{11} = S_{22} = S_{12} = 0 \tag{21}$$

As a result, Eq. (20) is written as

$$\begin{cases} \dfrac{\partial s_{11}}{\partial x} + \dfrac{\partial s_{12}}{\partial y} = \rho \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial s_{12}}{\partial x} + \dfrac{\partial s_{22}}{\partial y} = \rho \dfrac{\partial^2 \upsilon}{\partial t^2} \end{cases} \tag{22}$$

Obviously, these equations are the same as the classical ones for a body in an unstressed state. Thus, the influence of the pressure appears only on the elastic coefficients of the material. In addition, because the incremental stress-strain relation is of orthotropic symmetry, we can make the following assumption

$$\begin{cases} s_{11} = B_{11} e_{xx} + B_{12} e_{yy} \\ s_{22} = B_{21} e_{xx} + B_{22} e_{yy} \\ s_{12} = 2Q e_{xy} \end{cases} \tag{23}$$

Based on Biot's order, the existence of a potential strain energy leads to conditions of a general type to be satisfied by the coefficients, i.e.,

$$B_{12} - B_{21} = S_{22} - S_{11} \tag{24}$$

Substituting the stresses in Eq. (23) into the dynamical Eq. (20) results in the following (Biot 1964)

$$\begin{cases} B_{11} \dfrac{\partial^2 \upsilon}{\partial x^2} + \left[B_{12} + Q - \dfrac{1}{2}(S_{22} - S_{11}) \right] \dfrac{\partial^2 \upsilon}{\partial x \partial y} + \left[Q + \dfrac{1}{2}(S_{22} - S_{11}) \right] \dfrac{\partial^2 u}{\partial y^2} = \rho \dfrac{\partial^2 u}{\partial t^2} \\ B_{22} \dfrac{\partial^2 \upsilon}{\partial y^2} + \left[B_{21} + Q + \dfrac{1}{2}(S_{22} - S_{11}) \right] \dfrac{\partial^2 u}{\partial x \partial y} + \left[Q - \dfrac{1}{2}(S_{22} - S_{11}) \right] \dfrac{\partial^2 \upsilon}{\partial x^2} = \rho \dfrac{\partial^2 \upsilon}{\partial t^2} \end{cases} \tag{25}$$

Generally, the dynamic disturbance causes two types of deformations: one is anti-symmetric corresponding to a bending, i.e., so-called longitudinal wave, *P*-wave; the other is symmetric and called transverse wave, *S*-wave.

First, we consider plane *P*-waves propagating in the elastic medium. For example, consider a *P*-wave propagating in the *x* direction.

$$u = u_0 \cos(lx - \alpha t) \tag{26}$$

Substituting Eqs. (26) into (25) results in the following

$$B_{11} \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{27}$$

Similarly, a *P*-wave propagating in the *y* direction is governed by the equation

$$B_{22} \frac{\partial^2 \upsilon}{\partial y^2} = \rho \frac{\partial^2 \upsilon}{\partial t^2} \tag{28}$$

and the velocity of *P*-wave is

$$V_p^2 = \frac{B_{11}}{\rho} \quad \text{or} \quad V_p^2 = \frac{B_{22}}{\rho} \tag{29}$$

Obviously, these governing equations have the same form as those for the unstressed medium. The initial stress influences the propagation only through its effect on the magnitudes of the elastic coefficients B_{11} and B_{22} and the density ρ.

Second, we consider *S*-waves. An *S*-wave propagating in the *x* direction is represented by

$$\begin{cases} u = 0 \\ \upsilon = \upsilon_0 \cos(lx - \alpha t) \end{cases} \tag{30}$$

This equation satisfies identically the first of Eq. (25), and the second equation reduces to

$$\left[Q - \frac{1}{2}(S_{22} - S_{11})\right]\frac{\partial^2 \upsilon}{\partial x^2} = \rho \frac{\partial^2 \upsilon}{\partial t^2} \tag{31}$$

Therefore, the velocity V_x is

$$V_x^2 = \frac{Q - \frac{1}{2}(S_{22} - S_{11})}{\rho} \tag{32}$$

Similarly, an *S*-wave propagating in the *y* direction is represented by

$$\begin{cases} u = \upsilon_0 \cos(ly - \alpha t) \\ \upsilon = 0 \end{cases} \tag{33}$$

Substituting Eq. (33) into Eq. (25) results in equation as

$$\left[Q + \frac{1}{2}(S_{22} - S_{11})\right]\frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{34}$$

The velocity V_y is given by

$$V_y^2 = \frac{Q + \frac{1}{2}(S_{22} - S_{11})}{\rho} \tag{35}$$

Therefore, for *S*-waves, the results indicate that the important factor of the acoustic propagation under initial stress is fundamentally different from the stress-free case and cannot be represented by simply introducing into classical theory stress-dependent elastic coefficients.

4. Dynamic elastic coefficients

The elastic coefficients include Young's modulus E, shear modulus G, bulk modulus K, Poisson's Ratio v, and others. If any two elastic coefficients are determined, then the rest of them can be determined. Meanwhile, the previous theory indicated that the elastic coefficients have relationships with the initial stress, which can be determined from acoustic experiments. For example, the Poisson's Ratio and dynamic Young's modulus can be determined by the elastic wave velocity as follows

$$v = \frac{1}{2}\left[1 - \frac{V_s^2}{V_p^2 - V_s^2}\right] \tag{36}$$

$$E_d = \rho V_s^2 \frac{3V_p^2 - 4V_s^2}{V_p^2 - V_s^2} \tag{37}$$

In addition, the relationship of density and initial stress can be approximated by using the following equations in Birch's paper (Birch 1961).

$$\rho = \rho_0 + 0.395p \tag{38}$$

where wave velocities V_s and V_p and the density ρ can be obtained by experiment. Thus, experiments will further verify the wave velocities and elastic coefficients in the following sections.

5. Experimental study of stress wave propagation in the low pre-stressed rock sample

The previous theoretical derivations are based on the continuous and elastic mechanics, but rock is typically discontinuous, nonlinear and inelastic. Therefore, to more directly and reasonably verify this problem, the experimental method is used to reveal the relationship between the initial stress and the stress wave velocities.

For a *P*-wave, it is easy to measure the velocity by using two perpendiculars *P*-wave probes. The velocity of an *S*-wave can be measured with the probes for a *P*-wave in a rock block with a large plane. The two probes for a *P*-wave are coupled in parallel over a distance on the same plane of the rock block via grease. The transmitting probe produces a pulse that causes a *P*-wave in the normal direction of the plane and simultaneously an *S*-wave in its shear direction. Therefore, three *P*-wave probes can be used to measure the *P*-wave velocity and the *S*-wave velocity. Fig. 1 shows the experimental set-up, where the size of the specimen is 200 × 100 × 100 mm.

The experiments are conducted using an Instron 1346 hydraulic servo-controlled machine. The experimental system is controlled by a computer, and the load-deformation data are acquired automatically. The several groups of uniaxial pre-stressed specimens are examined using the Instron 1346 machine, which has a load capacity of 2,000 kN. The experimental system is shown in Fig. 2.

Uniaxial compression test is first conducted to obtain the basic mechanical properties of granite in the un-stressed state, which provided some rock properties, as presented in Table 1.

The initial stresses of 5 MPa, 10 MPa, 15 MPa, 20 MPa, 25 MPa and 30 MP are set to measure the relationship of the velocities of *P*-waves and *S*-waves; the results are presented in Fig. 3.

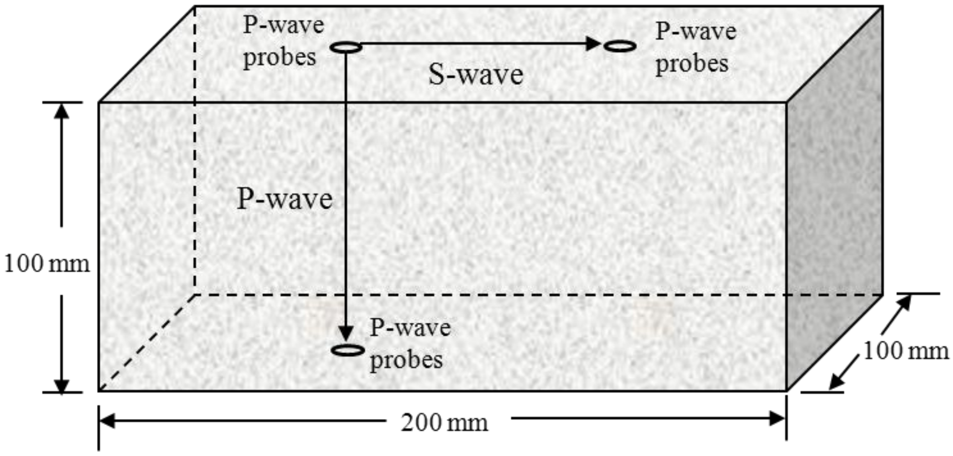


Fig. 1 Determination of the velocities of *P*-waves and *S*-waves using *P*-wave probes

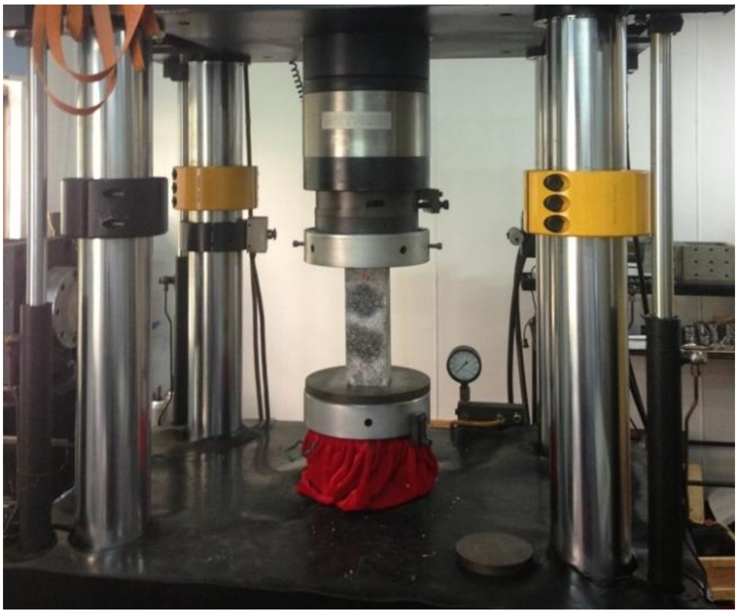
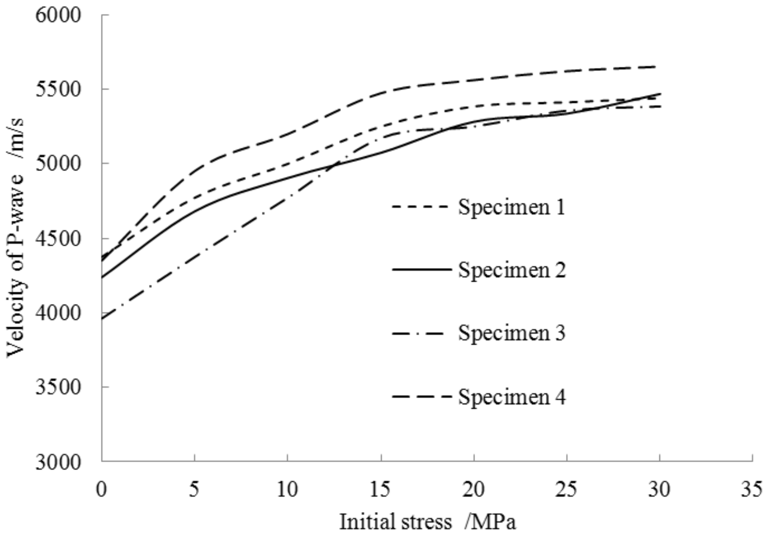


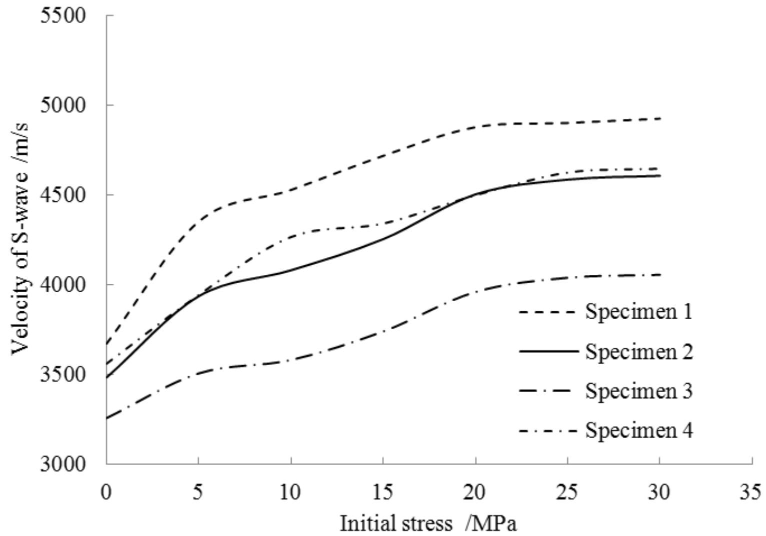
Fig. 2 The velocity determination system

Table 1 Material properties of rock

No.	Poisson's ratio	Density / (Kg·m^{-3})	Uniaxial compression strength / MPa
T1	0.19	2685	152.7
T2	0.18	2669	151.1
T3	0.21	2608	148.7
T4	0.20	2589	147.3



(a) *P*-wave variation with initial stress



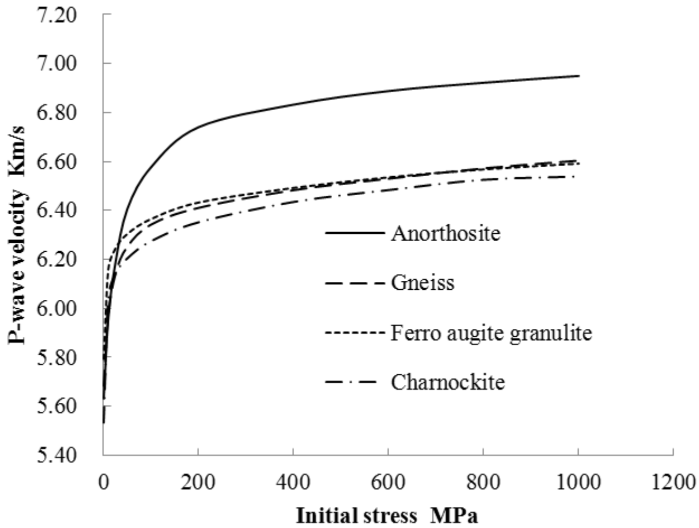
(b) *S*-wave variation with initial stress

Fig. 3 The relationship between initial stresses and the wave velocity

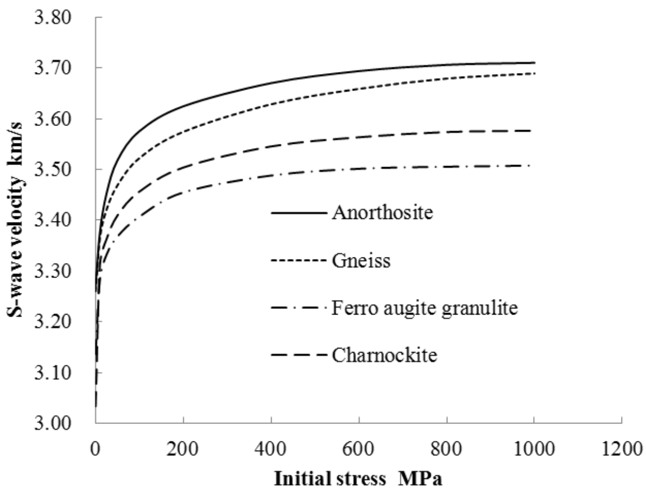
The results indicate that the velocities of *P*-waves and *S*-waves rapidly increase with the initial stress; thus, the experimental work indicated that the initial stress affects the material properties of rock. For the *P*-wave velocity increase with the initial stress, considering Eq. (29), the increase verifies that the initial stress has more effect on the elastic coefficients B_{11} or B_{22} than on the density ρ. In addition, for the *S*-wave, Eqs. of (32) and (35) indicate that the wave velocity not only has a relationship with the elastic coefficients but also depends on the initial stress because the *S*-wave velocity increases with the initial stress; thus, the increment of the elastic coefficients *Q* increase is greater than the increments of the initial stress increment and density.

6. High and ultra-high initial stress measurement procedure

High and ultra-high pressure velocity measurements using the standard pulse transmission method were performed by Birch and other researchers (Birch 1960, 1961, Shaocheng *et al*. 1993, Ji *et al*. 2007, Wang *et al*. 2009, Sun *et al*. 2012). In this experimental apparatus, a rectangular electrical pulse was applied to a barium titanate transducer, imparting a pulse to one face of the sample. The mechanical pulse was received by an identical transducer and converted into an electrical signal, which was amplified and displayed onto a dual-trace oscilloscope. The *P*-waves were generated and received by lead zirconate transducers, and *S*-waves were generated and received using lead zirconate titanate transducers (Sun *et al*. 2012). Based on this apparatus or an



(a) *P*-wave variation with initial stress



(b) *S*-wave variation with initial stress

Fig. 4 The relationship of the initial stresses with the wave velocity under ultra-high initial stress conditions

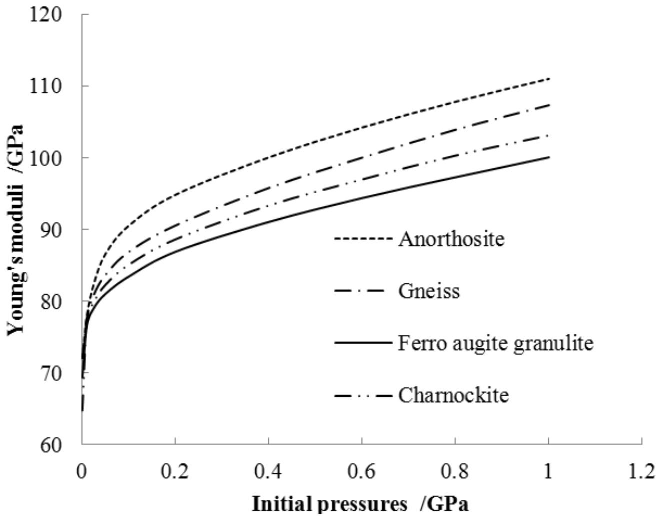


Fig. 5 Mean dynamic Young's moduli variation with the initial stress

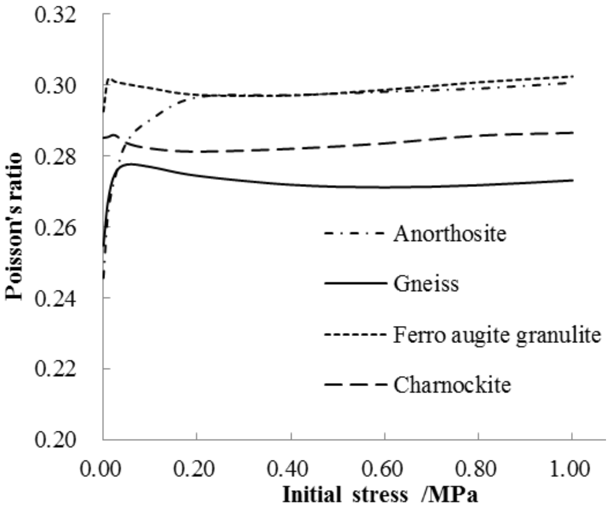


Fig. 6 Poisson's ratio as a function of the initial stress

equivalent apparatus, during the past 50 years or so, tremendous progress has been made in studying the stress wave velocity of rocks in relation to the initial stress. Many results have appeared in earlier publications, for example, the results of Manghnani *et al*. (1974).

Based on the data of Manghnani (Manghnani *et al*. 1974), the relationships of the seismic velocities with the initial stresses are verified, as shown in Fig. 4.

The results also indicated that the seismic velocities increase with the initial stress, even when the initial stress is ultra-high; the increasing trend does not change, but the increasing velocity trends are not as rapid when the initial stress is higher than 200 MPa.

Based on Eqs. (36), (37) and (38), the elastic coefficients can be calculated using the *P*-wave and *S*-wave velocities; the Young's modulus and Poisson's ratio for different values of the initial stress are shown in Figs. 5 and 6.

The results also indicated that the seismic velocities increase with the initial stress, even when the initial stress is ultra-high; the increasing trend does not change, but the increasing velocity trends are not as rapid when the initial stress is higher than 200 MPa.

Based on Eqs. (36), (37) and (38), the elastic coefficients can be calculated using the *P*-wave and *S*-wave velocities; the Young's modulus and Poisson's ratio for different values of the initial stress are shown in Figs. 5 and 6.

7. Conclusions

This study derived the governing equations for acoustic waves propagating in a pre-stressed one dimension elastic field, and the results indicated that the influence of the initial stress is only on the elastic coefficient of material. Meanwhile, based on the previous classic theoretical results, this paper presented that the initial stresses have different influence on *P*-waves and *S*-waves.

In addition, in the laboratory, we conducted an experiment on the wave velocity measurement for rock under a low pre-stressed state; and the experimental results demonstrated that *Vp* and *Vs* increase rapidly with the increase of the initial stress. Furthermore, using the previously published laboratory data of seismic velocity and elastic coefficients of rock under ultra-high hydrostatic, the seismic velocity and elastic coefficients of rocks under different initial stress were investigated. The results demonstrated that the initial stress affected the seismic velocities and elastic coefficients of rocks.

Acknowledgments

The research presented in this paper was jointly supported by the National Natural Science Foundation of China (Grant No. 51404303, 41272304 and 51274254).

References

Asef, M.R. and Najibi, A.R. (2013), "The effect of confining pressure on elastic wave velocities and dynamic to static Young's modulus ratio", *Geophysics*, **78**(3), D135-D142.

Biot, M.A. (1964), *Mechanics of Incremental Deformations*, Wiley, New York, NY, USA.

Birch, F. (1960), "The velocity of compressional waves in rocks to 10 kilobars, part 1", *J Geophys. Res.*, **65**(4), 1083-1102.

Birch, F. (1961), "The velocity of compressional waves in rocks to 10 kilobars: 2", *J. Geophys. Res.*, **66**(7), 2199-2224.

Brady, B.H. (2004), *Rock Mechanics*: *For Underground Mining*, Springer Science & Business Media.

Chevrot, S. and van der Hilst, R.D. (2000), "The poisson ratio of the Australian crust: Geological and geophysical implications", *Earth Planetary Sci. Lett.*, **183**(1), 121-132.

Christensen, N.I. (1965), "Compressional wave velocities in metamorphic rocks at pressures to 10 kilobars", *J. Geophys. Res.*, **70**(24), 6147-6164.

Christensen, N.I. (1974), "Compressional wave velocities in possible mantle rocks to pressures of 30 kilobars", *J. Geophys. Res.*, **79**(2), 407-412.

Ji, S., Wang, Q. and Xia, B. (2002), *Handbook of Seismic Properties of Minerals, Rocks and Ores*, Polytechnique International Press, Montréal, QC, Canada.

Ji, S., Wang, Q., Marcotte, D., Salisbury, M.H. and Xu, Z. (2007), "*P*-wave velocities, anisotropy and hysteresis in ultrahigh-pressure metamorphic rocks as a function of confining pressure", *J*. *Geophys*. *Res*.: *Solid Earth*, **112**(B9).

Manghnani, M.H., Ramananantoandro, R. and Clark, S.P. (1974), "Compressional and shear wave velocities in granulite facies rocks and eclogites to 10 kbar', *J*. *Geophys*. *Res*., **79**(35), 5427-5446.

Ogden, R. and Sotiropoulos, D. (1995), "On interfacial waves in pre-stressed layered incompressible elastic solids", *Proceedings of the Royal Society of London*: *Series A*, *Mathematical and Physical Sciences*, 450(1939), pp. 319-341.

Ogden, R. and Sotiropoulos, D. (1997), "The effect of pre-stress on the propagation and reflection of plane waves in incompressible elastic solids", *IMA J*. *Appl*. *Math*., **59**(1), 95-121.

Shaocheng, J., Salisbury, M.H. and Hanmer, S. (1993), "Petrofabric, *P*-wave anisotropy and seismic reflectivity of high-grade tectonites", *Tectonophysics*, **222**(2), 195-226.

Sharma, M. and Garg, N. (2006), "Wave velocities in a pre-stressed anisotropic elastic medium", *J*. *Earth Syst*. *Sci*., **115**(2), 257-265.

Sun, S., Ji, S., Wang, Q., Xu, Z., Salisbury, M. and Long, C. (2012), "Seismic velocities and anisotropy of core samples from the Chinese Continental Scientific Drilling borehole in the Sulu UHP terrane, Eastern China", *J*. *Geophys*. *Res*.: *Solid Earth*, **117**(B1).

Tao, M., Li, X. and Wu, C. (2013), "3D numerical model for dynamic loading-induced multiple fracture zones around underground cavity faces", *Comput*. *Geotech*., **54**, 33-45.

Wang, Q. and Ji, S. (2009), "Poisson's ratios of crystalline rocks as a function of hydrostatic confining pressure", *J*. *Geophys*. *Res*.: *Solid Earth*, **114**(B9).

Wang, Q., Ji, S., Sun, S. and Marcotte, D. (2009), "Correlations between compressional and shear wave velocities and corresponding Poisson's ratios for some common rocks and sulfide ores", *Tectonophysics*, **469**(1), 61-72.

GM