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Analytical solutions for crack initiation on floor-strata interface during mining

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Abstract. From the related engineering principles, analytical solutions for horizontal crack initiation and propagation on a coal panel floor-underlying strata interface due to coal panel excavation are derived in this paper. Two important concepts, namely the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface and the critical panel width of vertical fracture (crack) initiation in the panel floor, have been presented. The resulting analytical solution indicates that: (1) the first criterion can be used to express the condition under which horizontal plane cracks (on the panel floor-underlying strata interface or in the panel floor because of delamination) due to the mining induced vertical stress will initiate and propagate; (2) the second criterion can be used to express the condition under which vertical plane cracks (in the panel floor) due to the mining induced horizontal stress will initiate and propagate; (3) this orthogonal set of horizontal and vertical plane cracks, once formed, will provide the necessary weak network for the flow of gas to inrush into the panel. Two characteristic equations are given to quantitatively estimate both the critical panel width of vertical fracture initiation in the panel floor and the critical panel width of horizontal crack initiation on the interface between the panel floor and its underlying strata. The significance of this study is to provide not only some theoretical bases for understanding the fundamental mechanism of a longwall floor gas inrush problem but also a benchmark solution for verifying any numerical methods that are used to deal with this kind of gas inrush problem.

Keywords: crack initiation; analytical solution; panel floor fracture; critical panel width

1. Introduction

The problem of longwall floor gas inrush from the underlying strata into a coal mining panel and its surrounding atmosphere has been given increasing attention by coal mining engineers and researchers (Noack 1995, Wang *et al*. 2012, 2013, Ye *et al*. 2014), because any occurrence of such a sudden gas inrush may cause the following serious consequences: (1) the release of a large amount of hazardous gases from the underlying strata into the coal mining panel and its surrounding atmosphere; (2) a considerable increase in the risk of explosive hazard; (3) a certain period disruption of the coal mining work; (4) the potential loss of coal miner's life and (5) significant production losses. However, owing to the complex and complicated nature of the problem, so far there have been no analytical solutions to make a clear theoretical understanding

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about the fundamental mechanism behind the occurrence of this problem. Thus, there is a definite need for deriving analytical solutions to investigate the fundamental mechanisms of: (1) vertical fracture initiation/propagation in a coal panel floor; and (2) horizontal crack initiation/propagation on the interface between a coal mining panel and its underlying strata.

Since the geotechnical aspects of this problem are very complex (see Hudson *et al*. 1993 and the references therein), such as consisting of layers of different rock types, joints, faults, bedding planes and so forth, numerical methods (Zhao and Valliappan 1995, Ali and Bradshaw 2010, Xia and Zhou 2010, Zhang *et al*. 2011, Karim *et al*. 2013) are best suited for dealing with this kind of problem. In addition, the transport of methane gas through rock masses and the feedback effect of permeability due to rock damage and crack formation are also possible to be modelled in numerical analyses. However, since numerical methods are approximate in nature, the resulting numerical solutions must be verified once a numerical method is used to solve any new kind of problem. For this reason, error estimation of the numerical solution has become a very important topic in the field of the numerical analysis. Although a considerable amount of research was done to verify numerical solutions to many scientific and engineering problems (Zhao *et al*. 2008), little, if any, work has been carried out to estimate the error of a numerical solution to longwall floor gas inrush problems. The major reason for this is that there has been no analytical solution available for this kind of problem. Therefore, the main purpose of this paper is to develop an analytical solution for crack/fracture initiation and propagation both in the panel floor and on the interface between the panel floor and its underlying strata due to underground longwall coal mining excavation.

Generally, analytical solutions, although usually derived from simplified problems, are very important from the following two points of view (Exadaktylos 2012, Bunger *et al*. 2013, Faccanoni and Mangeney 2013): (1) it can be used as a powerful means to gain a better understanding of the underlying physics behind a given problem; (2) it is often a useful or even in some circumstances a unique measure in the assessment and verification of any numerical method. However, owing to difficulties in mathematics, it is commonly assumed that the theory of elasticity is valid in the process of deriving analytical solutions to solid mechanics problems. Such an assumption was justified by Stark and Booker (1997) in their paper, where they stated that "In geotechnical engineering there is a class of problem where the application of the theory of elasticity seems reasonable, despite the fact that the real soil behaviour is far from being adequately described as linearly elastic. Certainly, results from the theory of elasticity based on the assumption of homogeneity may be reasonable for a first estimate."

It should be noted that once a numerical method is verified by the analytical solution of a benchmark problem, it can be extended to the simulation of realistic engineering problems. The resulting numerical solution obtained from simulating a realistic engineering problem can be then used to test the extent to which the simplification (that is made in the process of deriving the analytical solution) is valid.

Taking into account the above considerations, a benchmark problem (see Fig. 1), in which a beam on an elastic foundation is used to represent a coal panel floor on the underlying strata and six elastic springs at two ends of the beam are used to represent the interaction of the panel floor with its surrounding media, is established to derive the analytical solution for crack/fracture initiation and propagation in the panel floor and on the panel floor-underlying strata interface. Note that in this figure, p is the primary driving force on the top surface of the panel floor due to the mining excavation (Zhao *et al*. 2000). The resulting analytical solution indicates that: (1) the first criterion established by $f(W) \geq 0$ can be used to express the condition under which horizontal

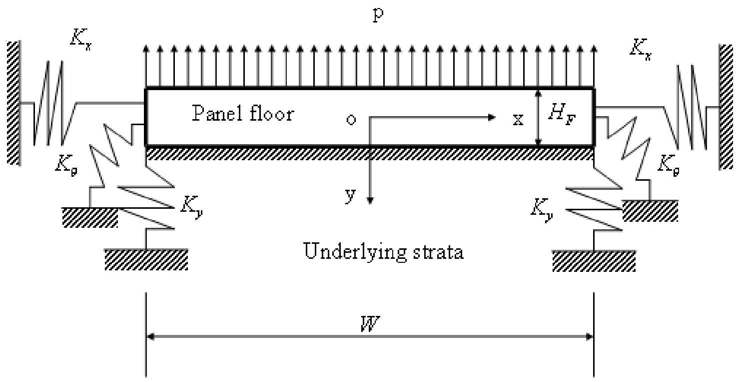


Fig. 1 The mechanical model of a panel floor-underlying strata system (Full contact stage)

plane cracks (on the panel floor-underlying strata interface or in the panel floor because of delamination) due to the mining induced vertical stress $\left(\sigma_y^{induced}\right)$ will initiate/propagate; (2) the second criterion established by $g(W) \geq 0$ (or $g_1(W) \geq 0$) can be used to express the condition under which vertical plane cracks (in the panel floor) due to the mining induced horizontal stress $\left(\sigma_x^{induced}\right)$ will initiate/propagate; (3) This orthogonal set of horizontal and vertical plane cracks, once formed, will provide the necessary weak network for the flow of gas to inrush into the panel.

2. Statement of the problem and two related criteria

The occurrence of a sudden gas inrush into the coal mining panel may be induced by the following two general modes. First, the initiation of a vertical fracture in the panel floor occurs prior to or at the same time as the initiation of a horizontal crack on the interface between the panel floor and its underlying strata. Second, the initiation of a horizontal crack on the interface between the panel floor and its underlying strata takes place before the initiation of a vertical fracture in the panel floor.

The mechanism behind the occurrence of a sudden gas inrush problem induced by the first general mode can be described as follows. At the early stage of a longwall coal panel excavation, the panel width is very small so that the primary driving force on the top surface of the panel floor is undertaken by both the panel floor and its underlying strata. As the coal panel excavation goes on, the panel width can increase to such an extent that a vertical fracture in the panel floor may firstly occur.

On the other hand, the mechanism behind the occurrence of a sudden gas inrush problem induced by the second general mode is described below. During a longwall coal panel excavation, the panel width can increase to such an extent that the horizontal crack/delamination may be firstly initiated on the interface between the coal mining panel floor and its underlying strata because this surface is a weak surface of small tensile strength, from the geomechanics point of view. Once the horizontal crack/delamination is initiated, the proportion of the primary driving force undertaken by a part of the underlying strata needs to be undertaken by the panel floor. This may lead to both the horizontal crack propagation on the floor-strata interface and an increase in the horizontal tensile stress within the coal panel floor. As a result, the vertical fracture of the panel floor may occur under the same panel width as the horizontal crack on the floor-strata interface is initiated. If the above is not the case, with further increases in the panel width, the horizontal crack on the panel floor-underlying strata interface may propagate along this interface so that more and more proportion of the primary driving force on the top surface of a coal mining panel floor is undertaken by the panel floor alone. This may enable the mining induced horizontal stress in the panel floor to significantly increase to such an extent that the vertical fracture of the panel floor may occur during the horizontal crack propagation on the panel floor-underlying strata interface.

From the above recognition, the whole process of the horizontal crack initiation and propagation on the interface between the panel floor and its underlying strata involves the following four stages: a full contact stage (see Fig. 1), an interface (horizontal) crack initiation stage (see Fig. 2), a (horizontal) crack propagation stage (see Fig. 3) and a floor (vertical) fracture initiation stage (Fig. 4). At the full contact stage, the floor width is usually small and the total vertical contact stress on the interface between the panel floor and its underlying strata is compressive stress due to the existence of the vertical virgin stress on this surface. However, at the interface (horizontal) crack initiation stage, the total contact vertical stress, which is the summation

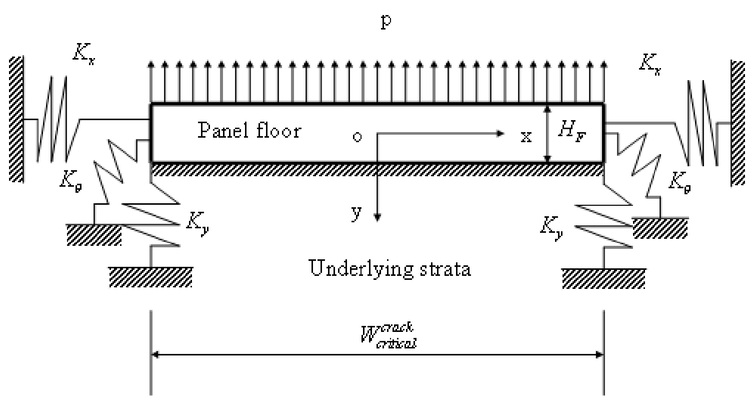


Fig. 2 The mechanical model of a panel floor-underlying strata system (Interface crack initiation stage)

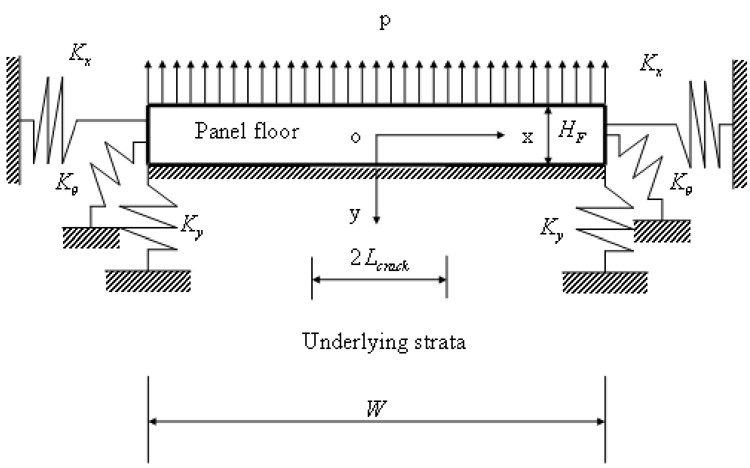


Fig. 3 The mechanical model of a panel floor-underlying strata system (Interface crack propagation stage)

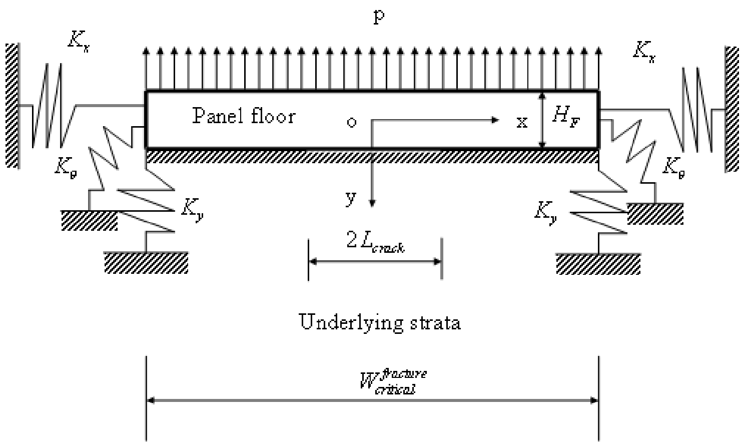


Fig. 4 The mechanical model of a panel floor-underlying strata system (Floor fracture initiation stage)

of the mining induced vertical stress and the vertical virgin stress on the interface between the panel floor and its underlying strata, becomes zero at some point on this interface. As a result, the horizontal crack is initiated at this point because the tensile strength on the interface between the panel floor and its underlying strata is usually negligible, from the geomechanics point of view. The corresponding panel width at this stage is a very important parameter for the design and management of a longwall coal mining system and therefore, is defined as the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface. At the (horizontal) crack propagation stage, the interface between the panel floor and its underlying strata is divided into a contact portion and a non-contact portion. In the non-contact portion, the primary driving force on the top surface of the panel floor is undertaken by the panel floor alone so that the mining induced horizontal stress in this portion of the panel floor may increase significantly. This will lead to the floor (vertical) fracture initiation stage once the panel width reaches a certain value. The panel width, at which the vertical fracture is initiated in the panel floor, is another very important parameter and is defined as the critical panel width of vertical fracture initiation in the panel floor. Clearly enough, both the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface and the critical panel width of vertical fracture initiation in the panel floor are of great scientific and practical significance in the design and management of a longwall coal mining system. For example, if the panel width exceeds the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface, measures should be taken to prevent the horizontal crack from further propagation. Otherwise, the vertical fracture may be initiated in the panel floor so that a longwall floor gas inrush accident will occur.

In order to calculate the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface, the following criterion is established.

$$\sigma_y^{induced} - \sigma_y^{virgin} - \overline{\sigma}^{interface} = 0 \tag{1}$$

where $\sigma_y^{induced}$ is the mining induced vertical stress on the interface between the panel floor and its underlying strata; σ_y^{virgin} is the vertical virgin stress on this interface; $\overline{\sigma}^{interface}$ is the tensile strength of this interface. Generally, this tensile strength is very small because the interface between the panel floor and its underlying strata is a weak surface, from the geomechanics point of view.

Similarly, in order to calculate the critical panel width of vertical fracture initiation in the panel floor, the related criterion is established as follows:

$$\sigma_x^{induced} - \sigma_x^{virgin} - \overline{\sigma}^{floor} = 0 \tag{2}$$

where $\sigma_x^{induced}$ is the mining induced horizontal stress in the panel floor; σ_x^{virgin} is the horizontal virgin stress in the panel floor; $\overline{\sigma}^{floor}$ is the tensile strength of the panel floor.

It is noted that for the purpose of determining the critical panel width of horizontal crack initiation on a panel floor-underlying strata interface and the critical panel width of vertical fracture initiation in a panel floor, both the mining induced vertical stress on the panel floor-underlying strata interface and the mining induced horizontal stress in the panel floor need to be calculated. This will be carried out in the following sections of this paper.

3. Analysis of horizontal crack initiation on the panel floor-underlying strata interface

At the full contact stage (see Fig. 1), the governing equation of deflection of the panel floor can be expressed as

$$EI\frac{d^4 v}{dx^4} = q - p \tag{3}$$

where v is the deflection of the panel floor; E is the elastic modulus; I is the moment of inertia of the cross-section of the panel floor with respect to the centroidal z axis, which is perpendicular to the x-y plane; p is the primary driving force on the top surface of the panel floor due to the mining excavation; q is the mining induced reaction stress on the panel floor-underlying strata interface and can be expressed as

$$q = \sigma_y^{induced} = -\beta v \tag{4}$$

where $\beta = kb$; b is the breadth of the panel floor in the direction perpendicular to the x-y plane; k is the underlying strata modulus to represent the normal reaction stress induced by per unit deflection of the underlying strata.

Owing to the symmetric feature of the problem, the corresponding boundary conditions are expressed as follows:

$$\begin{aligned}
&\frac{dv}{dx} = 0 && (\text{at } x = 0)\\
&\frac{d^3 v}{dx^3} = 0 && (\text{at } x = 0)\\
&EI\frac{d^2 v}{dx^2} = -K_\theta \frac{dv}{dx} && (\text{at } x = \frac{W}{2})\\
&EI\frac{d^3 v}{dx^3} = K_y v && (\text{at } x = \frac{W}{2})
\end{aligned} \tag{5}$$

From Eqs. (3) to (5), the analytical solution for the deflection of the panel floor at the full contact stage can be derived and expressed as

$$v = -\frac{p}{\beta} - \frac{K_y p}{2EI\alpha^3 \beta (D_3 - D_4 D_5)} [\sin(\alpha x)\sinh(\alpha x) + D_5 \cos(\alpha x)\cosh(\alpha x)] \tag{6}$$

where

$$\begin{aligned}
& p = \gamma_R H_R + \gamma_C H_C \\
& D_1 = \cosh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) + \sinh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) \\
& D_2 = \cosh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) - \sinh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) \\
& D_3 = \frac{-K_y}{2EI\alpha^3}\sinh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) - \cosh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) + \sinh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) \\
& D_4 = \frac{K_y}{2EI\alpha^3}\cosh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) + \cosh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) + \sinh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) \\
& D_5 = \frac{\cosh(\alpha \frac{W}{2})\cos(\alpha \frac{W}{2}) + \frac{K_\theta}{2EI\alpha} D_1}{\sinh(\alpha \frac{W}{2})\sin(\alpha \frac{W}{2}) + \frac{K_\theta}{2EI\alpha} D_2}
\end{aligned} \tag{7}$$

where γ_R and γ_C are the unit weights of the roof rock mass and the coal seam; H_R and H_C are the heights of the roof and the coal seam; W is the penal width; K_y and K_θ are the vertical translation stiffness and the rotation stiffness at both the left and right ends of the panel floor; α is a characteristic parameter of the panel floor-underlying strata system and can be expressed as follows:

$$\alpha = \sqrt[4]{\frac{\beta}{4EI}} \tag{8}$$

For the panel floor-underlying strata system shown in Fig. 1, the maximum deflection of the panel floor occurs at the middle span of the panel floor so that the interface (horizontal) crack will be initiated at this point on the interface between the panel floor and its underlying strata. This leads to the following characteristic equation for the critical panel width of horizontal crack initiation on the interface between the panel floor and its underlying strata:

$$f(W_{critical}^{crack}) = p + \frac{K_y p D_5 (W_{critical}^{crack})}{2EI\alpha^3 [D_3(W_{critical}^{crack}) - D_4(W_{critical}^{crack}) D_5(W_{critical}^{crack})]} - \sigma_y^{virgin} - \overline{\sigma}^{interface} = 0 \tag{9}$$

where $W_{critical}^{crack}$ is the critical panel width of horizontal crack initiation on the interface between the panel floor and its underlying strata.

Since D_3, D_4 and D_5 are the functions of a panel floor width, the solution to the panel floor width in Eq. (9) is the critical panel width of horizontal crack initiation on the interface between the panel floor and its underlying strata. However, owing to the nonlinear nature of Eq. (9), it is difficult to express the critical panel width of horizontal crack initiation on the interface between the panel floor and its underlying strata in an explicit manner but it is easy to solve it using some conventional algebraic techniques.

It needs to be pointed out that for a given panel floor width, W, Eq. (9) can be used to check whether a horizontal crack occurs on the interface between the panel floor and its underlying strata by simply replacing $W_{critical}^{crack}$ with W. If $f(W)$ is less than zero, it means that no horizontal crack occurs on the panel floor-underlying strata interface. However, if $f(W)$ is greater than zero, it means that a horizontal crack occurs on the panel floor-underlying strata interface. This judgment may enable $W_{critical}^{crack}$ to be found out from Eq. (9) easily. From this point of view, $f(W) \geq 0$ can be used as the first criterion to judge whether or not a horizontal crack occurs on the interface between the panel floor and its underlying strata.

Similarly, using the criterion expressed in Eq. (2) and the related solution, the characteristic equation for the critical panel width of vertical fracture initiation in the panel floor can be derived and expressed as

$$g(W_{critical}^{fracture}) = \frac{pK_y H_F}{2\alpha\beta I[D_3(W_{critical}^{fracture}) - D_4(W_{critical}^{fracture})D_5(W_{critical}^{fracture})]} - \sigma_x^{virgin} - \overline{\sigma}^{floor} = 0 \tag{10}$$

where H_F is the height of the panel floor; $W_{critical}^{fracture}$ is the critical panel width of vertical fracture initiation in the panel floor. Other symbols are of the same meanings as before.

Using Eqs. (9) and (10), the general mode, which induces the occurrence of a sudden gas inrush problem in a longwall coal mining system, can be determined for a given panel width, W. For example, if $g(W)$ is greater than zero but $f(W)$ is less than zero, it means that the initiation of a vertical fracture in the panel floor occurs before the initiation of a horizontal crack on the panel floor-strata interface. If $g(W)$ and $f(W)$ are equal to zero, it implies that the initiation of a vertical fracture in the panel floor occurs at the same time as the initiation of a horizontal crack on the panel floor-strata interface. If $g(W)$ is less than zero but $f(W)$ is greater than zero, it indicates that the initiation of a horizontal crack on the floor-strata interface occurs before the initiation of a vertical fracture in the panel floor. In this case, the analysis of vertical fracture initiation needs to be carried out in the next section. However, if both $g(W)$ and $f(W)$ are greater than zero or less than zero simultaneously, the panel width needs to be reduced or increased so that new $g(W)$ and $f(W)$ can be calculated and compared repeatedly until a judgment is made to determine which general mode will induce the occurrence of a sudden gas inrush problem in a given longwall coal mining system. From this point of view, $g(W) \geq 0$ can be used as the second criterion to judge whether or not a vertical crack occurs in the panel floor.

4. Analysis of vertical fracture initiation in the panel floor during the horizontal crack propagation on the panel floor-underlying strata interface

Once a horizontal crack is initiated on the interface between the panel floor and its underlying strata, the length of the horizontal crack needs to be considered for the analysis of the horizontal crack propagation on this interface. As shown in Fig. 3, due to the existence of the horizontal crack, the panel floor-underlying strata interface is divided into the contact portion and non-contact portion. In order to determine the critical panel width of vertical fracture initiation in the panel floor, the deflection of both the contact portion and the non-contact portion of the panel floor needs to be calculated so that the mining induced horizontal stress in the panel floor can be evaluated.

4.1 Solution for the non-contact portion of a panel floor

Considering the equilibrium of a segment of the panel floor shown in Fig. 5(b), the governing equation for the deflection of the non-contact portion of the panel floor can be derived as

$$EI\frac{d^4 v_1}{dx^4} = -p \tag{11}$$

where v_1 is the deflection of the non-contact portion of the panel floor; E is the elastic modulus of the panel floor; I is the moment of inertia of the cross section of the panel floor; p is the primary driving force on the top surface of the panel floor due to the mining excavation.

The corresponding boundary conditions for the non-contact portion of the panel floor are expressed as follows:

$$\frac{dv_1}{dx} = 0 \qquad (\text{at } x = 0) \tag{12}$$

$$\frac{d^3 v_1}{dx^3} = 0 \qquad (\text{at } x = 0) \tag{13}$$

$$v_1 = v_A \qquad (\text{at } x = L_{crack}) \tag{14}$$

$$\frac{dv_1}{dx} = \theta_A \qquad (\text{at } x = L_{crack}) \tag{15}$$

$$-EI\frac{d^2 v_1}{dx^2} = M_A \qquad (\text{at } x = L_{crack}) \tag{16}$$

$$-EI\frac{d^3 v_1}{dx^3} = Q_A \qquad (\text{at } x = L_{crack}) \tag{17}$$

Integrating Eq. (11) with respect to x four times yields the general solution for the deflection of the non-contact portion of the panel floor as follows:

$$v_1 = -\frac{1}{24EI}px^4 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4 \tag{18}$$

From Eqs. (12) and (13), C_1 and C_3 are determied as

$$C_1 = 0, \qquad C_3 = 0 \tag{19}$$

Substituting Eq. (19) into Eq. (18) yields the following equation:

$$v_1 = -\frac{1}{24EI} p x^4 + \frac{1}{2} C_2 x^2 + C_4 \tag{20}$$

It is noted that C_2 and C_4 can be determined by considering the continuity conditions at the panel floor cross-section of $x = L_{crack}$.

4.2 Solution for the contact portion of a panel floor

Considering the equilibrium of a segment of the panel floor shown in Fig. 5(a), the governing equation for the deflection of the contact portion of the panel floor can be derived as

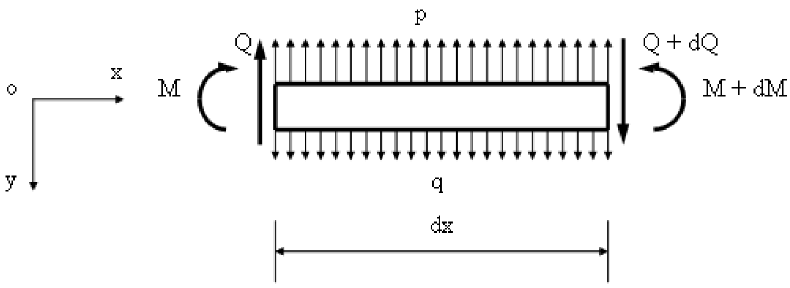
$$EI \frac{d^4 v_2}{dx^4} + \beta v_2 + p = 0 \tag{21}$$

The corresponding boundary conditions for the non-contact portion of the panel floor are expressed as follows:

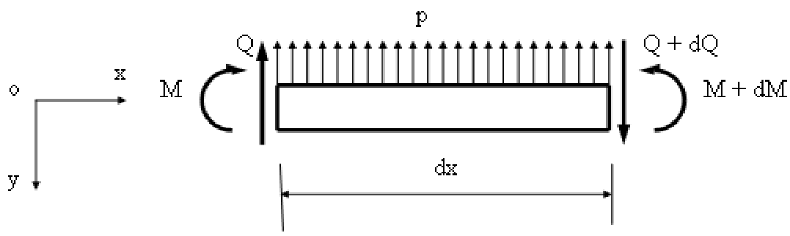
$$v_2 = v_A \qquad (\text{at } x = L_{crack}) \tag{22}$$

$$\frac{dv_2}{dx} = \theta_A \qquad (\text{at } x = L_{crack}) \tag{23}$$

$$-EI \frac{d^2 v_2}{dx^2} = M_A \qquad (\text{at } x = L_{crack}) \tag{24}$$



(a) Contact portion



(b) Non-contact portion

Fig. 5 Equilibrium of a segment of the panel floor

$$-EI\frac{d^3v_2}{dx^3}=Q_A \qquad (\text{at } x=L_{crack}) \tag{25}$$

$$EI\frac{d^2v_2}{dx^2}=-K_\theta\frac{dv_2}{dx} \qquad (\text{at } x=\frac{W}{2}) \tag{26}$$

$$EI\frac{d^3v_2}{dx^3}=K_y v_2 \qquad (\text{at } x=\frac{W}{2}) \tag{27}$$

The general solution for the deflection of the contact portion of the panel floor can be derived as follows:

$$v_2=-\frac{p}{\beta}+e^{\alpha x}[C_5\sin(\alpha x)+C_6\cos(\alpha x)]+e^{-\alpha x}[C_7\sin(\alpha x)+C_8\cos(\alpha x)] \tag{28}$$

Substituting the general solution in Eq. (28) into Eq. (26) yields the following equation:

$$\begin{aligned}
&e^{\alpha\frac{W}{2}}\{\cos(\alpha\frac{W}{2})-\alpha_\theta[\sin(\alpha\frac{W}{2})+\cos(\alpha\frac{W}{2})]\}C_5\\
&-e^{\alpha\frac{W}{2}}\{\sin(\alpha\frac{W}{2})+\alpha_\theta[\cos(\alpha\frac{W}{2})-\sin(\alpha\frac{W}{2})]\}C_6\\
&-e^{-\alpha\frac{W}{2}}\{\cos(\alpha\frac{W}{2})+\alpha_\theta[\cos(\alpha\frac{W}{2})-\sin(\alpha\frac{W}{2})]\}C_7\\
&+e^{-\alpha\frac{W}{2}}\{\sin(\alpha\frac{W}{2})+\alpha_\theta[\sin(\alpha\frac{W}{2})+\cos(\alpha\frac{W}{2})]\}C_8=0
\end{aligned} \tag{29}$$

where

$$\alpha_\theta=-\frac{K_\theta}{2EI\alpha} \tag{30}$$

Similarly, substituting Eq. (28) into Eq. (27) yields the following equation:

$$\begin{aligned}
&e^{\alpha\frac{W}{2}}[\cos(\alpha\frac{W}{2})-\sin(\alpha\frac{W}{2})-\alpha_y\sin(\alpha\frac{W}{2})]C_5\\
&-e^{\alpha\frac{W}{2}}[\sin(\alpha\frac{W}{2})+\cos(\alpha\frac{W}{2})+\alpha_y\cos(\alpha\frac{W}{2})]C_6\\
&+e^{-\alpha\frac{W}{2}}[\sin(\alpha\frac{W}{2})+\cos(\alpha\frac{W}{2})-\alpha_y\sin(\alpha\frac{W}{2})]C_7\\
&+e^{-\alpha\frac{W}{2}}[\cos(\alpha\frac{W}{2})-\sin(\alpha\frac{W}{2})-\alpha_y\cos(\alpha\frac{W}{2})]C_8=-\frac{p\alpha_y}{\beta}
\end{aligned} \tag{31}$$

where

$$\alpha_y=\frac{K_y}{2EI\alpha^3} \tag{32}$$

The continuity conditions at the cross-section between the contact portion and non-contact portion of the panel floor can be expressed as

$$v_2 = v_1 \qquad (\text{at } x = L_C) \tag{33}$$

$$\frac{dv_2}{dx} = \frac{dv_1}{dx} \qquad (\text{at } x = L_C) \tag{34}$$

$$\frac{d^2 v_2}{dx^2} = \frac{d^2 v_1}{dx^2} \qquad (\text{at } x = L_C) \tag{35}$$

$$\frac{d^3 v_2}{dx^3} = \frac{d^3 v_1}{dx^3} \qquad (\text{at } x = L_C) \tag{36}$$

where

$$L_C = L_{crack} \tag{37}$$

Substituting Eqs. (20) and (28) into Eq. (33) yields the following equation:

$$\begin{aligned} &\frac{1}{2} L_C^2 C_2 + C_4 - e^{\alpha L_C} \sin(\alpha L_C) C_5 - e^{\alpha L_C} \cos(\alpha L_c) C_6 \\ &- e^{-\alpha L_C} \sin(\alpha L_C) C_7 - e^{-\alpha L_C} \cos(\alpha L_C) C_8 = \frac{1}{24EI} p L_C^4 - \frac{p}{\beta} \end{aligned} \tag{38}$$

Similarly, substituting Eqs. (20) and (28) into Eq. (34) yields the following equation:

$$\begin{aligned} &L_C C_2 - \alpha e^{\alpha L_C} [\sin(\alpha L_C) + \cos(\alpha L_C)] C_5 - \alpha e^{\alpha L_C} [\cos(\alpha L_C) - \sin(\alpha L_C)] C_6 \\ &- \alpha e^{-\alpha L_C} [\cos(\alpha L_C) - \sin(\alpha L_C)] C_7 + \alpha e^{-\alpha L_C} [\sin(\alpha L_C) + \cos(\alpha L_C)] C_8 = \frac{1}{6EI} p L_C^3 \end{aligned} \tag{39}$$

From Eqs. (20), (28) and (35), the following equation can be derived:

$$\begin{aligned} &C_2 - 2\alpha^2 e^{\alpha L_C} \cos(\alpha L_C) C_5 + 2\alpha^2 e^{\alpha L_C} \sin(\alpha L_C) C_6 \\ &+ 2\alpha^2 e^{-\alpha L_C} \cos(\alpha L_C) C_7 - 2\alpha^2 e^{-\alpha L_C} \sin(\alpha L_C) C_8 = \frac{1}{2EI} p L_C^2 \end{aligned} \tag{40}$$

Similarly, from Eqs. (20), (28) and (36), the following equation can be derived:

$$\begin{aligned} &- 2\alpha^3 e^{\alpha L_C} [\cos(\alpha L_C) - \sin(\alpha L_C)] C_5 \\ &+ 2\alpha^3 e^{\alpha L_C} [\sin(\alpha L_C) + \cos(\alpha L_C)] C_6 \\ &- 2\alpha^3 e^{-\alpha L_C} [\sin(\alpha L_C) + \cos(\alpha L_C)] C_7 \\ &- 2\alpha^3 e^{-\alpha L_C} [\cos(\alpha L_C) - \sin(\alpha L_C)] C_8 = \frac{1}{EI} p L_C \end{aligned} \tag{41}$$

It is noted that from Eq. (40), C_2 can be expressed as follows:

$$
\begin{aligned}
C_2 &= 2\alpha^2 e^{\alpha L_C}\cos(\alpha L_C)C_5 - 2\alpha^2 e^{\alpha L_C}\sin(\alpha L_C)C_6 \\
&- 2\alpha^2 e^{-\alpha L_C}\cos(\alpha L_C)C_7 + 2\alpha^2 e^{-\alpha L_C}\sin(\alpha L_C)C_8 + \frac{1}{2EI}pL_C^2
\end{aligned}
\tag{42}
$$

Substituting Eq. (42) into Eq. (38), C_4 can be derived as follows:

$$
\begin{aligned}
C_4 &= e^{\alpha L_C}[\sin(\alpha L_C) - \alpha^2 L_C^2\cos(\alpha L_C)]C_5 + e^{\alpha L_C}[\cos(\alpha L_C) + \alpha^2 L_C^2\sin(\alpha L_C)]C_6 \\
&+ e^{-\alpha L_C}[\sin(\alpha L_C) + \alpha^2 L_C^2\cos(\alpha L_C)]C_7 + e^{-\alpha L_C}[\cos(\alpha L_C) - \alpha^2 L_C^2\sin(\alpha L_C)]C_8 \\
&- \frac{1}{8EI}pL_C^4 - \frac{p}{\beta}
\end{aligned}
\tag{43}
$$

Substituting Eq. (42) into Eq. (39) yields the following equation:

$$
\begin{aligned}
& e^{\alpha L_C}\{2\alpha^2 L_C\cos(\alpha L_C) - \alpha[\sin(\alpha L_C) + \cos(\alpha L_C)]\}C_5 \\
& - e^{\alpha L_C}\{2\alpha^2 L_C\sin(\alpha L_C) + \alpha[\cos(\alpha L_C) - \sin(\alpha L_C)]\}C_6 \\
& - e^{-\alpha L_C}\{2\alpha^2 L_C\cos(\alpha L_C) + \alpha[\cos(\alpha L_C) - \sin(\alpha L_C)]\}C_7 \\
& + e^{-\alpha L_C}\{2\alpha^2 L_C\sin(\alpha L_C) + \alpha[\sin(\alpha L_C) + \cos(\alpha L_C)]\}C_8 = -\frac{1}{3EI}pL_C^3
\end{aligned}
\tag{44}
$$

Thus, constants C_5, C_6, C_7 and C_8 can be determined by solving Eqs. (29), (31), (41), and (44) simultaneously (see Appendix).

It is noted that the horizontal crack initiation criterion expressed in Eq. (1) holds true at the point between the non-contact portion and the contact portion on the panel floor-underlying strata interface. This leads to the characteristic equation for calculating the horizontal crack length on the panel floor-underlying strata interface as follows:

$$
\frac{\beta}{24EI}pL_C^4 - \frac{\beta}{2}C_2(L_C)L_C^2 - \beta C_4(L_C) - \sigma_y^{virgin} - \overline{\sigma}^{\text{interface}} = 0 \tag{45}
$$

For a given panel width, which is greater than the critical width of horizontal crack initiation on the panel floor-underlying strata interface, the horizontal crack length on this interface can be determined using Eq. (45). Although Eq. (45) is a nonlinear one, it can be easily solved using the conventional algebraic method.

From the solutions for the deflections of both the contact portion and non-contact portion of a single panel floor, the bending moment in the panel floor can be calculated using the following equation:

$$
\begin{aligned}
M_1 &= -EI\frac{d^2 v_1}{dx^2} \qquad \text{(for the non-contact portion of the floor)} \\
M_2 &= -EI\frac{d^2 v_2}{dx^2} \qquad \text{(for the contact portion of the floor)}
\end{aligned}
\tag{46}
$$

Once the bending moment in a panel floor is evaluated, the induced horizontal stress in the panel floor due to the panel excavation can be evaluated using the following formula:

$$\sigma_x^{induced} = \frac{M_1 y}{I} \qquad \text{(for the non-contact portion of the floor)}$$
$$\sigma_x^{induced} = \frac{M_2 y}{I} \qquad \text{(for the contact portion of the floor)} \tag{47}$$

where

$$I = \frac{bH_F^3}{12} \tag{48}$$

where b is the breadth of the panel floor in the direction perpendicular to the x-y plane; H_F is the height of the panel floor.

For the panel floor-underlying strata system shown in Fig. 4, the maximum value of the mining induced horizontal stress in the panel floor usually occurs at the cross-section of the middle span ($x = 0$) of the panel floor so that the vertical fracture in the panel floor will be initiated at this cross-section. This leads to the following characteristic equation for the critical panel width of vertical fracture initiation in the panel floor.

$$g_1(W_{criticaal}^{fracture}) = \frac{EH_F}{2} C_2(W_{critical}^{fracture}) - \sigma_x^{virgin} - \overline{\sigma}^{floor} = 0 \tag{49}$$

Since C_2 is the function of a panel floor width, the solution to the panel floor width in Eq. (49) is the critical panel width of vertical fracture initiation in the panel floor. In addition, since Eq. (49) is a nonlinear algebraic equation with one unknown variable only, it can be easily solved using some conventional algebraic methods, such as the direct searching method.

It needs to be pointed out that for a given panel floor width, W, Eq. (49) can be also used to check whether a vertical fracture occurs in the panel floor by simply replacing $W_{critical}^{fracture}$ with W. If $g_1(W)$ is less than zero, it means that no vertical fracture occurs in the panel floor. However, if $g_1(W)$ is greater than zero, it means that a vertical fracture occurs in the panel floor. This judgment may enable $W_{critical}^{fracture}$ to be found out from Eq. (49) easily. Thus, $g_1(W) \geq 0$ can be also used as the second criterion to judge whether or not a vertical crack occurs in the panel floor when an existing horizontal crack propagates on the panel floor-underlying strata interface.

5. Conclusions

There are two general modes which may induce the occurrence of a sudden gas inrush problem in a longwall coal mining system. The first is that the initiation of a vertical fracture in the panel floor occurs prior to or at the same time as the initiation of a horizontal crack on the panel floor-underlying strata interface, while the second is that the initiation of a horizontal crack on the panel floor-underlying strata interface occurs before the initiation of a vertical fracture in the panel floor. The theoretical development of this paper is to consider the whole process of horizontal crack initiation and propagation on the interface between a single panel floor and its underlying strata as the following four specific stages: a full contact stage, an interface (horizontal) crack initiation stage, a (horizontal) crack propagation stage and a floor (vertical) fracture initiation stage. This leads to two important concepts, namely the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface and the critical panel width of vertical fracture initiation in the panel floor. In order to quantitatively analyse the crack/fracture initiation and propagation due to underground longwall coal mining excavation, three characteristic equations have been derived to calculate the critical panel width of horizontal crack initiation on the panel floor-underlying strata interface, the critical panel width of vertical fracture initiation in the panel floor and the horizontal crack length on the interface between the panel floor and its underlying strata. The significance of this study is to provide not only some theoretical bases for understanding the fundamental mechanism of a longwall floor gas inrush problem but also a benchmark solution for verifying any numerical method when it is used to deal with this kind of gas inrush problem in real engineering practice.

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Appendix

Eqs. (29), (31), (41) and (44) can be rewritten into the following matrix form:

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{Bmatrix} C_5 \\ C_6 \\ C_7 \\ C_8 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} \tag{A1}$$

where

$$\begin{aligned}
A_{11} &= e^{\alpha\frac{W}{2}}\left\{\cos(\alpha\frac{W}{2}) - \alpha_\theta\left[\sin\left(\alpha\frac{W}{2}\right) + \cos\left(\alpha\frac{W}{2}\right)\right]\right\} \\
A_{12} &= -e^{\alpha\frac{W}{2}}\left\{\sin\left(\alpha\frac{W}{2}\right) + \alpha_\theta\left[\cos\left(\alpha\frac{W}{2}\right) - \sin\left(\alpha\frac{W}{2}\right)\right]\right\} \\
A_{13} &= -e^{-\alpha\frac{W}{2}}\left\{\cos\left(\alpha\frac{W}{2}\right) + \alpha_\theta\left[\cos\left(\alpha\frac{W}{2}\right) - \sin\left(\alpha\frac{W}{2}\right)\right]\right\} \\
A_{14} &= e^{-\alpha\frac{W}{2}}\left\{\sin\left(\alpha\frac{W}{2}\right) + \alpha_\theta\left[\sin\left(\alpha\frac{W}{2}\right) + \cos\left(\alpha\frac{W}{2}\right)\right]\right\}
\end{aligned} \tag{A2}$$

$$\begin{aligned}
A_{21} &= e^{\alpha\frac{W}{2}}\left[\cos\left(\alpha\frac{W}{2}\right) - \sin\left(\alpha\frac{W}{2}\right) - \alpha_y\sin\left(\alpha\frac{W}{2}\right)\right] \\
A_{22} &= -e^{\alpha\frac{W}{2}}\left[\sin\left(\alpha\frac{W}{2}\right) + \cos\left(\alpha\frac{W}{2}\right) + \alpha_y\cos\left(\alpha\frac{W}{2}\right)\right] \\
A_{23} &= e^{-\alpha\frac{W}{2}}\left[\sin\left(\alpha\frac{W}{2}\right) + \cos\left(\alpha\frac{W}{2}\right) - \alpha_y\sin\left(\alpha\frac{W}{2}\right)\right] \\
A_{24} &= e^{-\alpha\frac{W}{2}}\left[\cos\left(\alpha\frac{W}{2}\right) - \sin\left(\alpha\frac{W}{2}\right) - \alpha_y\cos\left(\alpha\frac{W}{2}\right)\right]
\end{aligned} \tag{A3}$$

$$\begin{aligned} A_{31} &= -2\alpha^3 e^{\alpha L_C}\left[\cos(\alpha L_C) - \sin(\alpha L_C)\right] \\ A_{32} &= 2\alpha^3 e^{\alpha L_C}\left[\sin(\alpha L_C) + \cos(\alpha L_C)\right] \\ A_{33} &= -2\alpha^3 e^{-\alpha L_C}\left[\sin(\alpha L_C) + \cos(\alpha L_C)\right] \\ A_{34} &= -2\alpha^3 e^{-\alpha L_C}\left[\cos(\alpha L_C) - \sin(\alpha L_C)\right] \end{aligned} \tag{A4}$$

$$\begin{aligned} A_{41} &= e^{\alpha L_C}\left\{2\alpha^2 L_C \cos(\alpha L_C) - \alpha\left[\sin(\alpha L_C) + \cos(\alpha L_C)\right]\right\} \\ A_{42} &= -e^{\alpha L_C}\left\{2\alpha^2 L_C \sin(\alpha L_C) + \alpha\left[\cos(\alpha L_C) - \sin(\alpha L_C)\right]\right\} \\ A_{43} &= -e^{-\alpha L_C}\left\{2\alpha^2 L_C \cos(\alpha L_C) + \alpha\left[\cos(\alpha L_C) - \sin(\alpha L_C)\right]\right\} \\ A_{44} &= e^{-\alpha L_C}\left\{2\alpha^2 L_C \sin(\alpha L_C) + \alpha\left[\sin(\alpha L_C) + \cos(\alpha L_C)\right]\right\} \end{aligned} \tag{A5}$$

$$\begin{aligned} b_1 &= 0 \\ b_2 &= -\frac{p\alpha_y}{\beta} \\ b_3 &= \frac{1}{EI} p L_C \\ b_4 &= -\frac{1}{3EI} p L_C^3 \end{aligned} \tag{A6}$$

Through solving Eq. (A1), the related solution can be expressed as follows:

$$\begin{aligned} C_8 &= \frac{(b_4 B_{11} - b_2 B_{31})(B_{11}B_{22} - B_{12}B_{21}) - (b_3 B_{11} - b_2 B_{21})(B_{11}B_{32} - B_{12}B_{31})}{(B_{11}B_{33} - B_{13}B_{31})(B_{11}B_{22} - B_{12}B_{21}) - (B_{11}B_{23} - B_{13}B_{21})(B_{11}B_{32} - B_{12}B_{31})} \\ C_7 &= \frac{b_3 B_{11} - b_2 B_{21}}{B_{11}B_{22} - B_{12}B_{21}} - \frac{B_{23}B_{11} - B_{13}B_{21}}{B_{11}B_{22} - B_{12}B_{21}} C_8 \\ C_6 &= \frac{b_2}{B_{11}} - \frac{B_{12}}{B_{11}} C_7 - \frac{B_{13}}{B_{11}} C_8 \\ C_5 &= -\frac{A_{12}}{A_{11}} C_6 - \frac{A_{13}}{A_{11}} C_7 - \frac{A_{14}}{A_{11}} C_8 \end{aligned} \tag{A7}$$

where

$$B_{11} = A_{22} - \frac{A_{12}}{A_{11}} A_{21}$$

$$B_{12} = A_{23} - \frac{A_{13}}{A_{11}} A_{21} \tag{A8}$$

$$B_{13} = A_{24} - \frac{A_{14}}{A_{11}} A_{21}$$

$$B_{21} = A_{32} - \frac{A_{12}}{A_{11}} A_{31}$$

$$B_{22} = A_{33} - \frac{A_{13}}{A_{11}} A_{31} \tag{A9}$$

$$B_{23} = A_{34} - \frac{A_{14}}{A_{11}} A_{31}$$

$$B_{31} = A_{42} - \frac{A_{12}}{A_{11}} A_{41}$$

$$B_{32} = A_{43} - \frac{A_{13}}{A_{11}} A_{41} \tag{A10}$$

$$B_{33} = A_{44} - \frac{A_{14}}{A_{11}} A_{41}$$