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Stochastic cost optimization of ground improvement with prefabricated vertical drains and surcharge preloading

Hyeong-Joo Kim [1a], Kwang-Hyung Lee [1b], Jay C. Jamin [1c] and Jose Leo C. Mission [*2]

[1] *Department of Civil and Environmental Engineering, Kunsan National University, Gunsan 573-701, Republic of Korea*
[2] *SK Engineering and Construction (SK E&C), Seoul 100-192, Republic of Korea*

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Abstract. The typical design of ground improvement with prefabricated vertical drains (PVD) and surcharge preloading involves a series of deterministic analyses using averaged or mean soil properties for the various combination of the PVD spacing and surcharge preloading height that would meet the criteria for minimum consolidation time and required degree of consolidation. The optimum design combination is then selected in which the total cost of ground improvement is a minimum. Considering the variability and uncertainties of the soil consolidation parameters, as well as considering the effects of soil disturbance (smear zone) and drain resistance in the analysis, this study presents a stochastic cost optimization of ground improvement with PVD and surcharge preloading. Direct Monte Carlo (MC) simulation and importance sampling (IS) technique is used in the stochastic analysis by limiting the sampled random soil parameters within the range from a minimum to maximum value while considering their statistical distribution. The method has been verified in a case study of PVD improved ground with preloading, in which average results of the stochastic analysis showed a good agreement with field monitoring data.

Keywords: ground improvement; prefabricated vertical drain (PVD); surcharge preloading; stochastic cost optimization; direct Monte Carlo (MC) simulation; importance sampling (IS)

1. Introduction

Surcharge preloading is employed for ground improvement when the required degree of consolidation cannot be achieved by prefabricated vertical drain (PVD) alone under limited construction time and schedule. PVD has been generally used to decrease the overall time required for completion of primary consolidation by shortening the drainage path length. PVD has largely replaced other drainage techniques due to its advantages of economic competitiveness, less disturbance of the soil mass, speed and simplicity of installation (Rixner *et al*. 1986). PVD is often used in conjunction with surcharge preloading to eliminate all or portion of the anticipated post-construction settlements caused by primary consolidation due to fill and imposed surface

*Corresponding author, Ph.D., Geotechnical Engineer, E-mail: joseleo_mission@yahoo.com
[a] Ph.D., Professor, E-mail: kimhj@kunsan.ac.kr
[b] Ph.D. Student, E-mail: kwanghyung@naver.com
[c] Graduate Student, E-mail: jaminjc@kunsan.ac.k

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loads.

The typical design of the optimum combination of PVD spacing and preloading height is usually determined from deterministic analyses using averaged or mean soil properties, which is usually performed by trial and iteration procedure (Chai *et al*. 2009). Mission *et al*. (2012) presented a ground improvement cost optimization method with PVD and preloading using direct Monte Carlo (MC) simulation under ideal conditions, where smear effects and well resistance is neglected based on Barron's (1948) simplified theory. This study has advanced the method in a stochastic cost analysis and optimization considering the combined effects of soil disturbance (smear zone) and drain resistance of the PVD. Furthermore, in order to reduce the number of iteration steps and solution time for analysis, importance sampling (IS) technique is used by limiting the sampled data for calculation within the range of the minimum and maximum value of the random soil parameter, while considering their statistical distribution in order to account for the variability and uncertainties of soil consolidation parameters. The optimum design can then be selected from the range of the minimum, maximum, and most probable cost of ground improvement from the results of the stochastic analysis.

2. Stochastic consolidation analysis and cost estimation with PVD and preloading considering variability of soil consolidation parameters

2.1 Theory of consolidation with prefabricated vertical drain (*PVD*)

The effects of vertical drain on the consolidation are generally analyzed using an idealized model shown in Fig. 1(a). In this model, the vertical drain is idealized as an equivalent circular drain. An annular zone, called a smear zone, is considered in the soil surrounding the drain to account for the disturbance caused by the installation of the drain. The permeability of the smear zone in the vicinity of the drain is reduced compared to the native soil due to installation disturbance.

Hansbo (1979) and Holtz *et al*. (1987) presented the conventional design procedures for vertical drains in which for an ideal case of radial drainage, an expression for the average degree of consolidation, U_h, at a certain depth, z is presented as

$$U_h = 1 - \exp\left(\frac{-8c_h t}{\mu D_e^2}\right) \tag{1}$$

where for one-way vertical drainage (Rixner *et al*. 1986)

$$\mu = \ln\left(\frac{D_e}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4} + \frac{2\pi}{3}L^2\frac{k_h}{q_w} \tag{2}$$

and for two-way vertical drainage

$$\mu = \ln\left(\frac{D_e}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4} + \frac{\pi}{6}L^2\frac{k_h}{q_w} \tag{3}$$

$C_h = (k_h / k_v)\, C_v$ is the coefficient of consolidation for horizontal drainage that is expressed as a

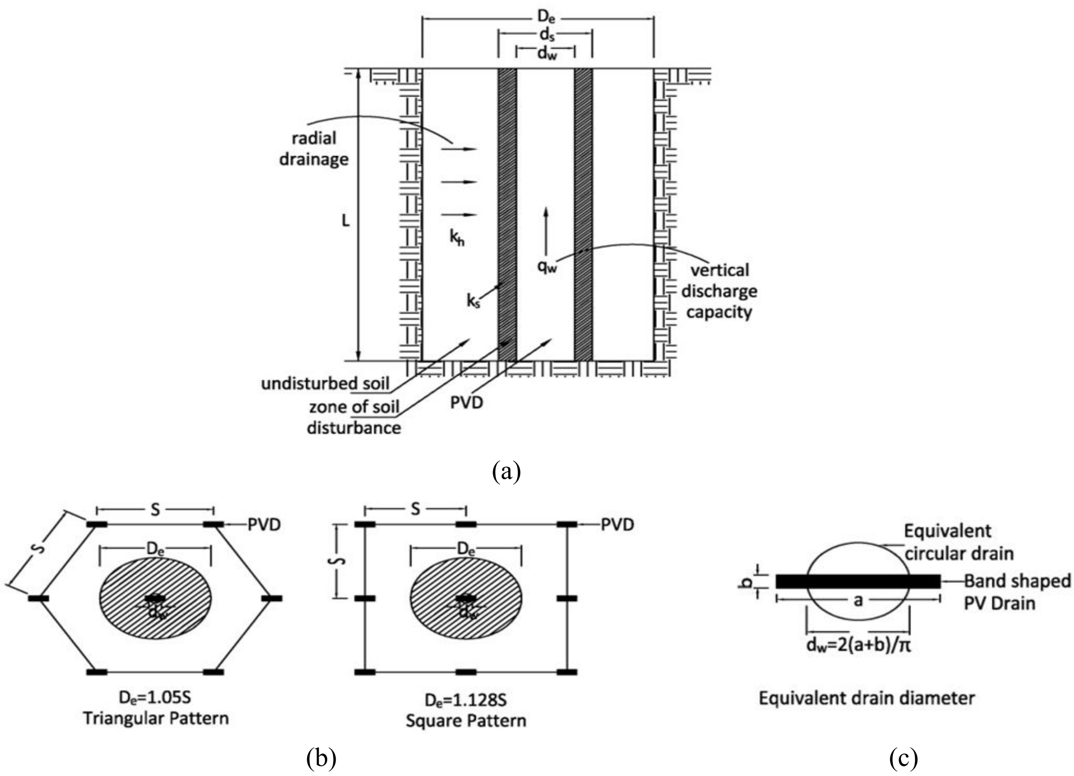


Fig. 1 (a) Schematic of PVD with drain resistance and soil disturbance; (b) equivalent diameter of soil (D_e); and (c) equivalent diameter of PVD (d_w) (Rixner *et al*. 1986)

function of the vertical consolidation coefficient (C_v), (k_h / k_v) is the ratio of horizontal to vertical permeability, t is the time of consolidation, D_e is the equivalent diameter of the soil cylinder dewatered by a drain (Fig. 1(b)), d_w is the equivalent drain diameter (Fig. 1(c)), d_s is the diameter of the smear zone, k_h is coefficient of horizontal permeability of the undisturbed soil, k_s is the permeability of the smeared soil, q_w is the discharge capacity of the drain, and L is the length of drain.

One dimensional consolidation settlement (S_c) according to the classical theory is given by (Das 2010)

$$S_c = \frac{C_c}{1+e_o} H \log\left(\frac{p_o' + \Delta p}{p_o'}\right) \tag{4}$$

where, C_c = compression index, e_o = initial void ratio; H = thickness of layer; Δp = increase in total vertical stress at the center of layer; p_o' = initial effective vertical stress at the center of layer.

2.2 Variability and uncertainty of soil consolidation parameters with PVD

Several studies were conducted for determination of the smear zone and the smear effects for consolidation with vertical drains. Hansbo (1981, 1997) estimated the diameter of smear zone, d_s

as 1.5 to 3 times the diameter of the drain, d_w. Bergado *et al.* (1991) proposed to assume smear diameter as 2 times the diameter of the drain. However, Indraratna and Redna (1998), Bo *et al.* (2000) and Xiao (2001) indicated that the smear zone diameter can be as high as 4 to 8 times the diameter of the drain. In this study, the range of the minimum and maximum diameter of smear zone $d_s = (1.5{\sim}4)\, d_w$ is used with a mean value of 2.0.

Smear effects can significantly reduce the permeability and the coefficient of consolidation. The effect on the coefficient of permeability is generally considered as the reduction ratio with respect to the coefficient of horizontal permeability, k_h / k_s. Researchers suggested using a value of the reduction ratio in the range of 2 to 6 (Hansbo 1981, Onoue 1992, Indraratna and Redna 1998, Hird and Mosely 2000). Hansbo (1997) proposed to use the coefficient of permeability for the smear zone, k_s as same as the coefficient of vertical permeability, k_v. In this study, the range of the minimum and maximum permeability reduction ratio $k_h / k_s = (1.0{\sim}6.0)$ is used with a mean value of 3.0. In the absence of a more reliable laboratory or field data, representative ratios k_h / k_v for soft clays are in the range from about 1.0 to 5.0 (Rixner *et al.* 1986) depending on the layering and consistency of the soil, in which an average value of 2.0 is used in this study.

Consolidation and permeability characteristics are important to quantify stress-strain relations such as settlement, and the time-dependent behavior of very soft cohesive soils (Abu-Hejleh and Znidarčić 1995) such as degree of consolidation. Variability of these soil properties is a major contributor to the uncertainty in optimum design of PVD spacing and preloading height and their associated costs, as well as the reliability of the estimated settlement and degree of consolidation. The coefficient of variation (COV) has been commonly used to describe the inherent variation of many geotechnical soil properties and insitu test parameters, which represents a relative and dimensionless measure of dispersion and is expressed as

$$COV(\%) = \frac{\sigma_w}{\mu_x}(100) \tag{5}$$

where μ_x = mean and σ_x = standard deviation. Table 1 presents tabulated COV data of some relevant soil consolidation parameters such as compression index (C_c), coefficient of permeability (k), coefficient of consolidation (C_v), void ratio (e), and unit weight (γ).

The lognormal distribution is used as the type of statistical description for most geotechnical parameters such as soil for three reasons. First, it results if many individual random variables are multiplied together. Hence, any process that is the product of individual random variables will tend to be described by a lognormal distribution. Second, the lognormal distribution models variables cannot be less than zero. Since many engineering properties, such as strength and stiffness, are nonnegative, the lognormal distribution is a reasonable model. Finally, the lognormal distribution is convenient for modeling quantities that vary over several orders of magnitude, such as hydraulic conductivity (Griffiths and Fenton 2007).

2.3 Stochastic cost optimization of ground improvement with PVD and preloading

With PVD and surcharge preloading method, the total cost of ground improvement (G) can be approximated as a function of the PVD spacing (s) and height of surcharge preloading (h) as stochastic parameters as derived in the following.

Given the total area (A) to be that requires ground improvement with PVD and preloading, the total number of PVD installations (N) for both triangular and square patterns (Fig. 1(b)) can be approximated as given by Eq. (6)

$$N = \frac{A}{s^2} \tag{6}$$

where the PVD spacing is given as (Fig. 1(b))

$$s = \frac{D_e}{1.05}, \quad \text{for triangular pattern} \tag{7}$$

$$s = \frac{D_e}{1.128}, \quad \text{for square pattern} \tag{8}$$

in which the equivalent diameter (D_e) of the soil cylinder dewatered by the PVD is related to the consolidation parameters as given by Eqs. (1) to (3).

The total length (L) of PVD required for the whole ground improvement area (A) is then calculated by multiplying Eq. (6) with the average PVD length (L') per installation

$$L = N \cdot L' \tag{9}$$

When additional preloading is required in combination with PVD to satisfy design criteria in terms of target settlement or degree of consolidation at a given construction period, the total required volume (V) of preloading embankment fill can be approximated for any required preloading height (h) as

$$V = A \cdot h \cdot \alpha \tag{10}$$

where α = volume factor to account for preloading embankment side slope (A.h = volume of fill with vertical sides), in which the side slope can be approximated as equal to the angle of repose (θ) of the fill. For rectangular preloading embankment fill having base dimensions B_P = width, L_P = length, and θ = side slope, the volume factor (α) is a function of height h and θ which can be approximated as

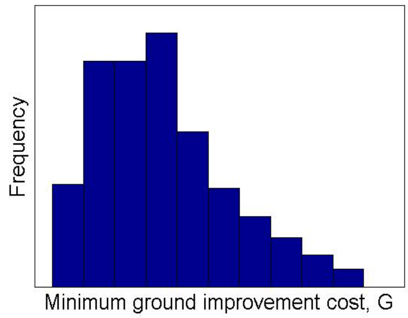
$$\alpha = \frac{0.5(A + A')}{A} = \frac{0.50\left[B_P L_P + \left(B_P - \frac{2h}{\tan\theta}\right)\left(L_P - \frac{2h}{\tan\theta}\right)\right]}{B_P L_P} \tag{11}$$

where A = area of base of preloading embankment, and A' = area of top of preloading embankment at height h and side slope θ.

Combining Eqs. (9) and (10) while multiplying with their respective unit rates, the total cost of ground improvement (G) with PVD and preloading can be approximated as

$$G = L \cdot C_D + V \cdot C_P \tag{12}$$

where C_D = unit cost of PVD per linear meter (\$/m), and C_P = unit cost of preloading (\$/m^3) per cubic meter. The unit cost of PVD (C_D) and preloading (C_P) should be given as to include the respective sum of all the direct and indirect costs such as costs of materials, labor, and equipment. Referring to Eqs. (6) and Eqs. (10)-(11) it can be seen that G (Eq. (12)) is a function of the PVD spacing (s) and preloading height (h).



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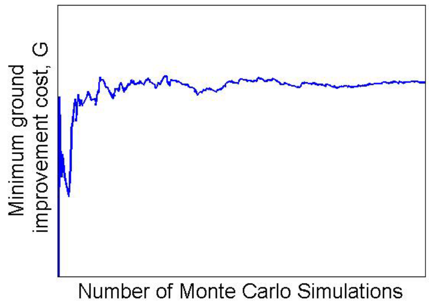

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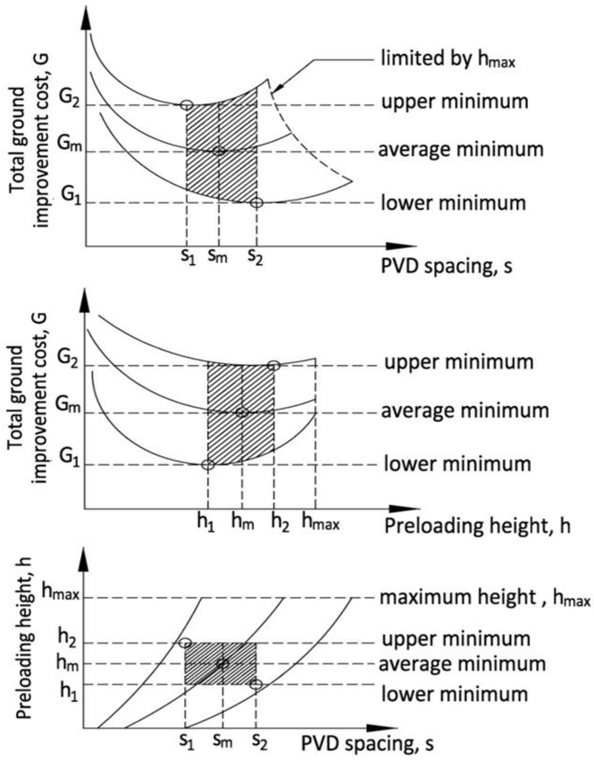


(c)

Fig. 2 (a) Frequency distribution of minimum ground improvement cost, G; (b) average minimum ground improvement cost vs. number of MC simulation cycles; (c) plot of G vs. s; (d) plot of G vs. h; and (e) plot of optimized s vs. h

For a range of minimum and maximum PVD spacing ($s_{\min}$ and $s_{\max}$), and a range of preloading height from zero (no preloading) to $h_{\max}$ to prevent stability problems, an optimization method is implement in Matlab (Mathworks 2010) by direct Monte Carlo (MC) simulation with importance sampling (IS) techniques considering the criteria of minimum consolidation time ($t_{\min}$), and minimum degree of consolidation $U_{\min}$ (assuming $U \approx U_h$). MC simulation combined with IS techniques tends to increase the efficiency of the simulation by reducing the number of iteration steps and solution time for analysis, in which samples within the range of the minimum and maximum value of the soil parameter from the original randomly distributed data are only selected for calculation.

Shown in Table 1 is the coefficient of variation (COV) of inherent soil variability for consolidation and permeability parameters, which represents the relative and dimensionless measure of dispersion. Rather than using a deterministic description of the geotechnical property in terms of a single value using the mean or average (μ_x) as typically used in a deterministic method of analysis, the relevant geotechnical input parameters are statistically defined in a stochastic method of analysis. The relevant geotechnical input property is therefore defined by the following range of values from minimum ($x_{\min}$) to maximum ($x_{\max}$) with total number of samples (n), having the statistical property of mean (μ_x), COV, standard deviation (σ_x), and statistical distribution (ex. Lognormal). Applying the various sampling and combinations for the range from minimum to maximum values for each of the relevant geotechnical properties in Eqs. (1) to (11) will produce lower bound and upper bound results for the calculated total ground improvement cost (G) in Eq. (12) as shown in Fig. 2.

Based on the above and from the results of the stochastic analysis generated for all n-samples, total ground improvement cost (G) as calculated by Eq. (12) is also statistically described such as a plot of its frequency distribution and mean as typically shown in Figs. 2(a)-(b), respectively. The optimized PVD spacing (s_m) and preloading height (h_m) is then determined using the mean total ground improvement cost (G_m) as typically shown in Figs. 2(c) and (d), respectively. At the slope of the curve $d(G) = 0$, in which the ground improvement cost is minimum, represents the optimum combination of PVD spacing and preloading height (s_m, h_m). A relationship between other combinations of s and h meeting the required minimum criteria for consolidation can be plotted as typically shown in Fig. 2(d).

3. Case study and analysis

A reclamation work for a development project was reported by Chen (2004) on a coastal area at Pulau Indah, Klang, Malaysia on a mangrove swamp site approximately 200 m wide and 650 m long, in which the average ground level within the area was about + 5 m. The development required to have a designed surface level of + 7.2 m, in which an average of 2.2 m fill was required. The subsoil at site mainly consists of very soft and highly compressible silty clay, with a range and mean compressibility properties obtained from laboratory tests as shown in Fig. 3 and summarized Table 2. The statistical description of the soil consolidation parameters that is used in this study were based on typical information available from the literature as described in Section 2.2 and summarized in Table 1. The preconsolidation pressures obtained from the tests show that the soft clay can be treated as normally consolidated clay with an average effective unit weight of about 5 kN/m^3. It was decided to improve the soil with PVD and preload so that the anticipated long term and large settlement can be eliminated or significantly reduced. A targeted resting period for minimum degree of consolidation $U_{\min}$ = 95% with preloading was 4 months ($t_{\min}$). A band shaped PV drain was used with an equivalent diameter (d_w) of 50 mm and average discharge capacity (q_w) of 1890 m^3/year. For the purpose of this investigative study, and while it is considered that unit prices and price increase may vary from country to country depending on several factors such as location of the project site, distance from material source to site, etc.; a typical unit cost of \$2.5/m for PVD and \$10.0/m^3 for preloading was used in this example in accordance with typical unit rate price ranges as suggested by Townsend and Anderson (2004), USACE (1999), and based on local

Table 1 COV of inherent soil variability for consolidation and permeability parameters (Jones *et al*. 2002)

Property	Soil type	COV (%)	Reference
C_c	Sandy clay	26	Harr (1987)
	Clay	30	
	*	37	Kulhawy (1992)
k	*	$240^{(a)}$	Harr (1987)
	*	$90^{(b)}$	
C_v	*	33 - 68	Duncan (2000)
e	All soil types	7 - 30	Lacasse and Nadim (1996)
	All soil types	7 - 9	Kulhawy (1992), Phoon and Kulhawy (1999)

* Not reported: (a) 80% saturation; (b) 100% saturation

and global project experiences gained by the authors in the region while working in the Engineering and Construction industry. The suggested unit rate presented in the paper is then for illustration purpose only while emphasizing the importance of accounting for the variability and uncertainties of geotechnical parameters in the ground improvement analysis, design, construction schedule, and costs.

Using the ground improvement optimization method shown in Fig. 2, the stochastic MC consolidation analysis was implemented in Matlab (Mathworks 2010) from a generated 10,000 samples of random soil properties. With importance sampling method, that is, by limiting the sampled data within the range of the minimum and maximum value of the soil parameter, only about 380 samples were used in the calculations to derive the results. Figs. 4(a)-(b) show the average total ground improvement costs (G_m) versus number of MC simulation cycles with a mean and frequent value of about \$5.1 M. Considering the variability and uncertainties of the soil

Table 2 Statistical description of soil consolidation parameters

Parameter	Range	Mean, μ	COV
Compression ratio, $CR = C_c / (1 + e_0)$	0.15-0.30	0.25	0.30
Vertical coefficient of consolidation, C_v (m^2/year)	1.0-3.0	2.0	0.30
Permeability ratio, k_h / k_v	1.0-3.0	2.0	0.50
Diameter ratio of smear zone, d_s / d_w	1.5-4	2.0	0.25
Permeability ratio of smear zone, k_h / k_s	1.0-6.0	3.0	0.50
Eff. unit weight of compressible soil layer, γ' (kN/m^3)	4.0-7.0	5.0	0.08
Unit weight of fill and preloading soil, γ_s (kN/m^3)	15.0-19.0	18.0	0.08
Effective permanent load due to total thickness of fill accounting for settlements, Δp (kPa)	50.0-56.0	52.5	0.08
Equivalent diameter of PVD, d_w (m)		0.05	
Discharge capacity of PVD, q_w (m^3/year)		1890	
Range of PVD spacing, s (m)	0.5-2.5		
Range of surcharge preloading height, h (m)	0.0-4.0		

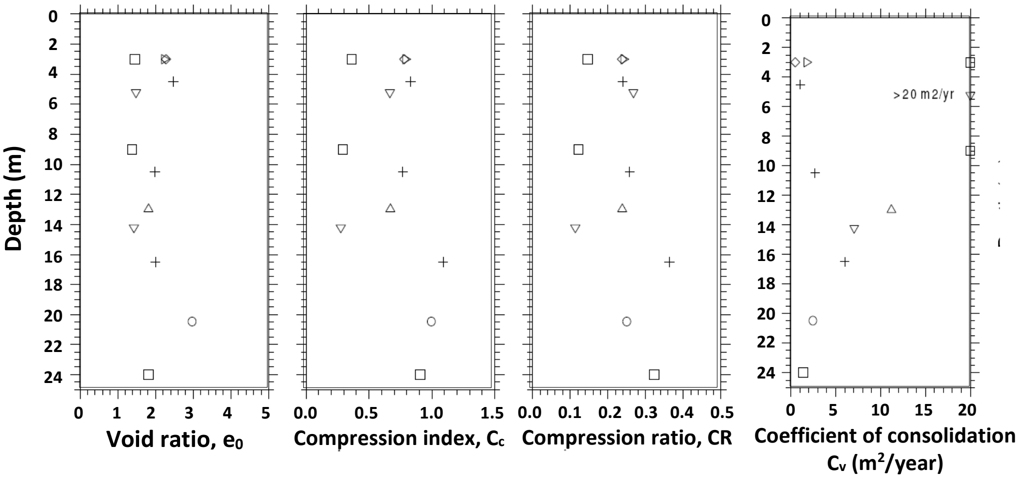
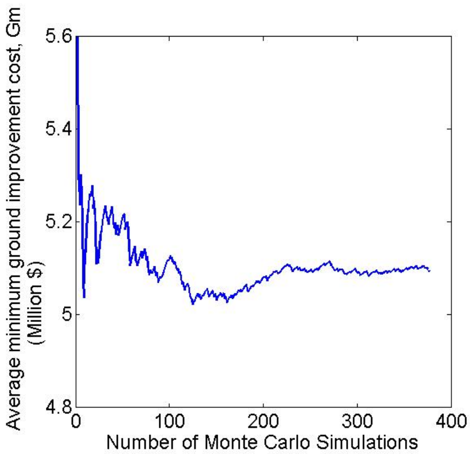


Fig. 3 Variation of soil consolidation parameters with depth (Chen 2004)



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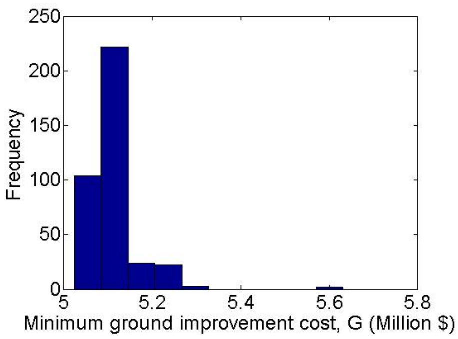

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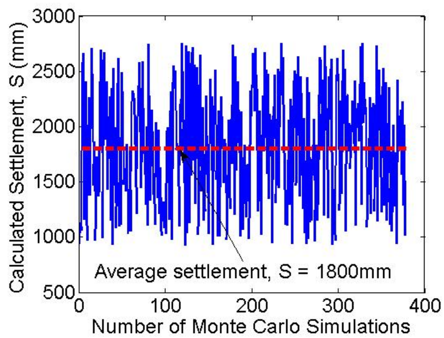
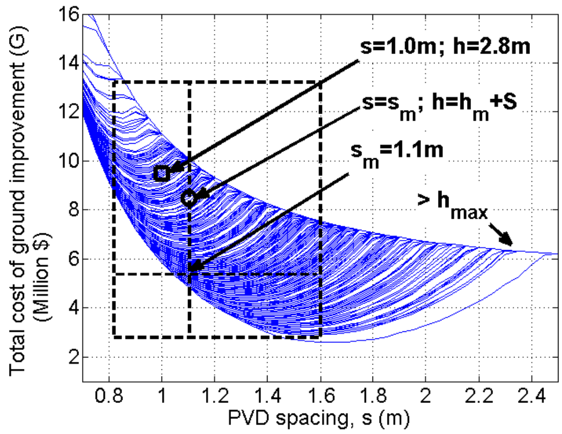


(c)

Fig. 4 (a) Plot of average minimum ground improvement (G) cost vs. number of MC simulations; (b) frequency distribution of minimum ground improvement cost (G); and (c) variation of calculated settlement due to soil variability and uncertainties



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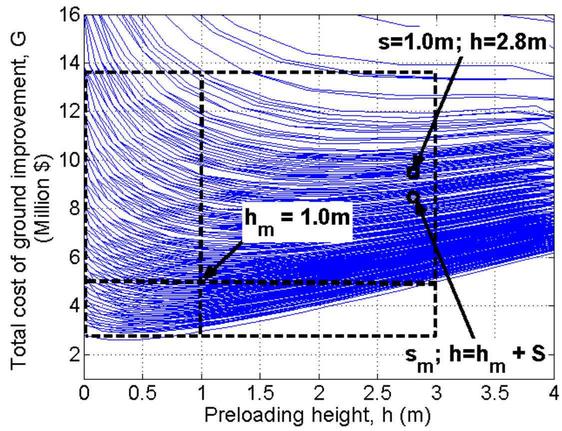

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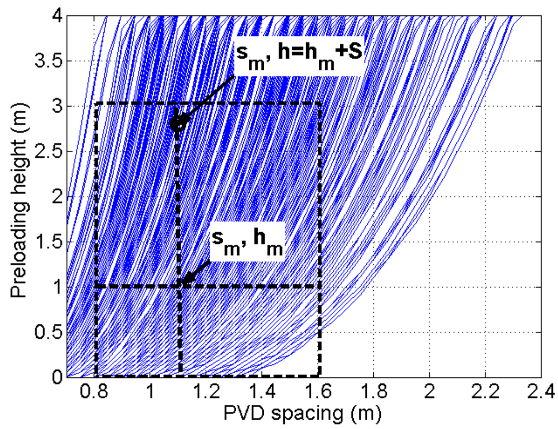

(c)

Fig. 5 MC analysis results for variation of ground improvement cost with: (a) PVD spacing; (b) preloading height; and (c) relationship between PVD spacing and preloading height

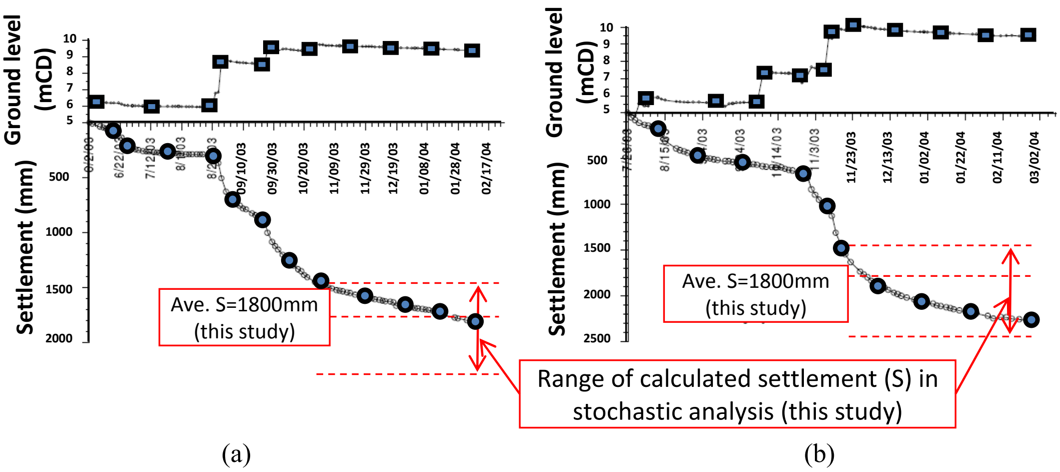

Fig. 6 Comparison of calculated settlements with monitoring results from settlement plates (SP): (a) SP1 (left); and (b) SP5 (right) (after Chen (2004))

consolidation parameters, the calculated settlement varies from about 1.0-2.6 m with an average of about 1.8 m (Fig. 4(c)). The average total ground improvement cost is then plotted in Fig. 5 to determine the optimized PVD spacing s_m = 1.1 m and preloading height h_m = 1.0 m as described in Section 2.3 and Fig. 2. Due to the natural variability of the subsoil properties as well as the limitations of analytical theory, the geotechnical consultant adopted a final design of 1.0 m PVD spacing with surcharge level to elevation + 10 m or about a total 2.8 m preload height including the fill compensation due to settlement, whose recommendations are also in close agreement with the optimized design results from this study as shown in Figs. 5(a)-(b). Results of the stochastic analysis shows a good comparison of calculated settlement with field monitoring results as seen in Fig. 6, in which about 95% degree of consolidation was achieved in about 4 months (Chen 2004).

4. Conclusions

This study presents a ground improvement cost optimization scheme with Prefabricated Vertical Drains (PVD) and preloading by stochastic consolidation analysis with direct Monte Carlo (MC) simulation and importance sampling (IS) technique. In addition to considering the variability and uncertainty of the various soil consolidation parameters in the analysis, advance consolidation theory with PVD is adopted considering the effects of smear and well resistance. Results of the stochastic analysis would provide design guidelines in the selection of the optimum PVD spacing and preloading height at minimum ground improvement cost. The method has been validated with a case study of a PVD improved ground with preloading in which good agreement is obtained with field monitoring data. Results have shown that the minimum ground improvement cost at optimized PVD spacing and preloading height are significantly affected by the variation and uncertainty of the soil consolidation parameters.

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