Geomechanics and Engineering, *Vol. 7, No. 4 (2014) 335-353*
DOI: http://dx.doi.org/10.12989/gae.2014.7.4.335

A softening hyperelastic model and simulation of the failure of granular materials

Jiangfang Chang, Xihua Chu* and Yuanjie Xu

Department of engineering mechanics, Wuhan University, Wuhan 430072, China

(Received September 11, 2013, Revised June 02, 2014, Accepted June 07, 2014)

Abstract. The softening hyperelastic model based on the strain energy limitation is of clear concepts and simple forms to describe the failure of materials. In this study, a linear and a nonlinear softening hyperelastic model are proposed to characterize the deformation and the failure in granular materials by introducing a softening function into the shear part of the strain energy. A method to determine material parameters introduced in the models is suggested. Based on the proposed models the numerical examples focus on bearing capacity and strain localization of granular materials. Compared with Volokh softening hyperelasticity and classical Mohr-Coulomb plasticity, our proposed models are able to capture the typical characters of granular materials such as the strain softening and the critical state. In addition, the issue of mesh dependency of the proposed models is investigated.

Keywords: granular materials; hyperelastic model; strain energy; strain softening; critical state

1. Introduction

Granular materials, though extensively existing in nature and widely used in engineering, are not always well understood. There are many problems remain unresolved, such as the static stress distribution (De Gennes 1996) and the nonlinear stress-strain relationship (Duncan and Chang 1970). Granular materials under small strain and static conditions are generally considered as elasticity or isotropic linear elasticity in engineering fields. However, the stress-strain relationship of granular materials is nonlinear on macro scale according to the Hertz contact theory (Johnson 1985), and it is also proved by numerous experiments. But, it is difficult to find an appropriate theoretical expression to describe their nonlinear behaviors. An earlier attempt on adopting nonlinear stress-strain relationship or stress-dependent elastic modulus was made by Boussinesq, who thought the elastic modulus was related to the square root of the trace of the small strain tensor. Duffy and Mindlin (1957) extended the Hertz model by considering the tangential forces, while it could not well be applied to describe the elastic behavior of granular materials due to its path dependence. Zytynski *et al.* (1978) first pointed out the theoretical problems existing in the former nonlinear models, as the elastic modulus was not derived from an appropriate free energy, the elastic response was not always conservative. In order to avoid such problems, several

*Corresponding author, Associate Professor, E-mail: Chuxh@whu.edu.cn
[a] Ph.D. Student, E-mail: cjf881024@163.com
[b] Professor, E-mail: xyj9781@163.com

http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

nonlinear hyperelastic models for granular materials were proposed in recent years, for example, the granular elasticity model (GE model) (Jiang and Liu 2003). It was established by multiplying the elastic strain energy with Δ^a (where a is the power of volumetric strain) based on the Helmholtz free energy. Such a model will give elastic modulus that is power law function of the strains. Jiang and Liu considered that GE model is a special case of granular solid hydrodynamics (GSH) (Jiang and Liu 2009, Gudehus *et al*. 2011) under static condition, and they examined the validity of the GE model on the description of mechanics behavior for granular media in a series of papers (Jiang and Liu 2007, Jiang *et al*. 2012). In addition, there are several analogous models, such as the EP model (Einav and Puzrin 2004) based on the Gibbs free energy and the HAR model (Houlsby *et al*. 2005). Comparison between the recent nonlinear hyperelastic models and experimental data was made by Humrickhouse *et al*. (2010), and the advantages and the limitations of them were expounded respectively. Studies have shown that these nonlinear hyperelastic models can well describe some characteristics of granular materials, such as the inability to take tension, the loss of stability and the Reynolds dilatancy.

However, the traditional hyperelastic strain energy will increase to infinity with the strain increase, this makes such models cannot well describe the failure behavior of real materials. To account for materials failure a softening hyperelastic model targeted to brittle materials and soft hyperelastic materials was proposed by Volokh (2007). The softening is controlled by a constant Φ, the critical failure energy, which can also be interpreted as the material “toughness” similar to the critical energy release rate in the classical fracture mechanics. Volokh model can simulate the softening failure behavior of brittle materials effectively. And it will not produce physically meaningless results due to the inexistence of the surface energy and the length-dependence compared with the classical fracture mechanics.

Considering that the failure behavior of granular materials is mainly caused by shear, classical elastic-plastic theory, such as the Mohr-Coulomb and the Drucker-Prager yield criteria, mainly focus on the shear yielding. Even though the compression can also result in granular material failure, it remains relatively difficult to occur. It means that shear deformation plays a more significant role in the failure progress of granular materials. However, the softening hyperelastic model proposed by Volokh brings the softening mechanism in the whole elastic strain energy, resulting in that the material softening is controlled by only one constant Φ, which indicates that the shear part and the volume part of the strain energy on materials failure are equally important in Volokh model. Based on the failure characters of granular materials, a model with two different energy limiters introduced in the shear part and the volume part of the strain energy respectively may be more appropriate. In the present work, for simplicity, we temporarily only consider the shear failure. A linear and a nonlinear hyperelastic model are modified by introducing a softening function into the shear part of the strain energy based on the softening mode proposed by Volokh. This means that there is a limitation of the shear strain energy, however the volumetric strain energy can increase unlimitedly. Two additional parameters are introduced in the proposed models, namely a shearing strain energy limit Φ and a proportionality constant α, which can be regarded as parameters controlling materials softening and the critical state. The method to determine these parameters is suggested. The proposed models are compared with the classical Mohr-Coulomb model and the Volokh model. In addition, the issue of mesh dependency of the proposed models is investigated.

2. Linear hyperelastic models

2.1 Linear hyperelastic model for ideal materials

The strain energy for linear isotropic hyperelastic model (denoted as LH-0 in the paper) can be written as

$$F = \frac{1}{2} K\Delta^2 + G u_s^2 \tag{1}$$

where K is the bulk elastic modulus, G is the shear elastic modulus, Δ is the volumetric strain, and u_s is the equivalent strain.

$$\Delta = -\varepsilon_{ii}; \tag{2a}$$

$$u_s^2 = e_{ij} e_{ij}; \tag{2b}$$

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\Delta\delta_{ij} \tag{2c}$$

where $\Delta = -\varepsilon_{ii}$ is positive in compression. The elastic modulus tensor is given as

$$D_{ijkl} = \frac{\partial^2 F}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}} = K\delta_{ij}\delta_{kl} + 2G\left(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \tag{3}$$

The stress invariants are expressed as

$$p = \frac{\partial F}{\partial \Delta} = K\Delta \tag{4}$$

$$q = \frac{\partial F}{\partial u_s} = 2Gu_s \tag{5}$$

2.2 The softening hyperelastic model proposed by Volokh

A softening hyperelastic model applied to many kinds of isotropic linear materials was proposed by Volokh, we call it as LH-Volokh in the present paper.

$$\psi = \Phi - \Phi\exp(-F/\Phi) \tag{6}$$

where Φ is the critical failure energy which indicates the maximum strain energy that the infinitesimal material volume can sustain without failure and it controls material softening. F is the strain energy as shown in Eq. (1). The elastic modulus tensor can be expressed as

$$\begin{aligned} D_{ijkl} = \frac{\partial^2\psi}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}} &= \exp(-F/\Phi)\left(K\delta_{ij}\delta_{kl} + 2G(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{ij}\delta_{kl})\right) \\ &- \frac{1}{\Phi}\exp(-F/\Phi)(K\Delta\delta_{kl} + 2Ge_{kl})(K\Delta\delta_{ij} + 2Ge_{ij}) \end{aligned} \tag{7}$$

Similarly, the stress invariants are given by

$$p = \exp(-F/\Phi)K\Delta \tag{8}$$

$$q = \exp(-F/\Phi)2Gu_s \tag{9}$$

2.3 The modified model

In this section, we present a modified softening hyperelastic model based on the softening mode proposed by Volokh.

2.3.1 The modified strain energy

The failure and softening behavior of granular materials are mainly caused by the shearing deformation, just as the classical Drucker-Prager and Mohr-Coulomb yield criterions described. Therefore we try to introduce a softening function into the shear part of the strain energy

$$\psi = \frac{1}{2}K\Delta^2 + \Phi\left(1 - \exp\left(-\frac{Gu_s^2}{\Phi}\right)\right) \tag{10}$$

This model is denoted by LH-M. Similar to the previous, the elastic modulus tensor and the stress invariants can be written as

$$\begin{aligned} D_{ijkl} = \frac{\partial^2\psi}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}} &= K\delta_{ij}\delta_{kl} + \exp\left(-\frac{Gu_s^2}{\Phi}\right)\left(-\frac{1}{\Phi}\right)2Ge_{kl}2Ge_{ij} \\ &+ \exp\left(-\frac{Gu_s^2}{\Phi}\right)2G\left(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \end{aligned} \tag{11}$$

$$p = K\Delta \tag{12}$$

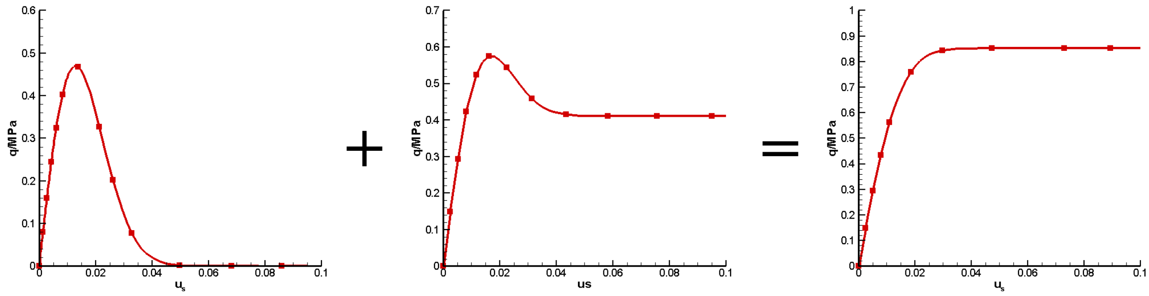
$$q = \exp\left(-\frac{Gu_s^2}{\Phi}\right)2Gu_s \tag{13}$$

where Eq. (13) is the total form of q, the resulting curve of $q \sim u_s$ is shown as Fig. 1(a), it can represent the softening failure of brittle materials just as what Volokh held. However, numerous researches have shown that granular materials have the critical state and the residual strength (Schofield and Worth 1968).

We find that a $q \sim u_s$ curve like Fig. 1(b) can be obtained from the modified incremental form of q as Eq. (14b) shown, which is modified by cutting out the rightmost term of incremental form of Eq. (14a).

$$\delta q = \exp\left(-\frac{Gu_s^2}{\Phi}\right)2G\delta u_s\left(1 - \frac{1}{\Phi}2Gu_s^2\right) \tag{14a}$$

$$\delta q = \exp\left(-\frac{Gu_s^2}{\Phi}\right)2G\delta u_s \tag{14b}$$



(a) total form q_T (b) modified incremental form q_I (c) superposed form q

Fig. 1 The method of curve superposition for $q \sim u_s$

Considering the mechanical properties of real materials, we suggest taking a superposition like Eq. (15) of the total form q_T shown in Eq. (13) and the modified incremental form q_I shown in Eq. (14b), and a curve of $q \sim u_s$ like Fig. 1(c) is obtained (here $\alpha = 0.5$). Thus the modified model can simulate brittle materials with $\alpha = 1.0$, materials like loose sand with $\alpha = 0$ and materials like dense sand with $\alpha = 0 \sim 1$.

$$q = \alpha q_T + (1-\alpha) q_I \tag{15}$$

2.3.2 Determination of Φ and α

According to Eq. (15) and Figs. 1(a)-(c), the residual strength of q is supposed to be $(1-\alpha)(q_I)_{\max}$, strictly speaking the peak value of q_I does not exist known from Eq. (14b), here we consider the peak value in Fig. 1(b), or say the critical value in Fig. 1(c), can be obtained when the following condition is satisfied.

$$(G)_{eff} = \frac{1}{2}\frac{\delta q_I}{\delta u_s} = \exp\left(-\frac{G u_s^2}{\Phi}\right) G = 10^{-8} G \tag{16}$$

Then parameter Φ can be calculated as the following

$$\Phi = \frac{G (u_s)_{q_{cr}}^2}{8 \ln 10} \tag{17}$$

where $(u_s)_{g_{cr}}$ can be obtained from the experimental stress–strain responses as shown in Fig. 2.

Parameter α can be determined from the extreme condition by taking derivative to Eq. (12)

$$\alpha \delta q_T + (1-\alpha) \delta q_I = 0 \tag{18}$$

where δq_T and δq_I are given respectively as Eqs. (14a) and (14b), substituting them in Eq. (18) results in

$$1 - \alpha \frac{1}{\Phi} 2G (u_s)_{q_{\max}}^2 = 0 \tag{19}$$

Thus, for a given G, substituting $(u_s)_{g_{\max}}$ and Eq. (17) in Eq. (19), where $(u_s)_{g_{\max}}$ can be obtained

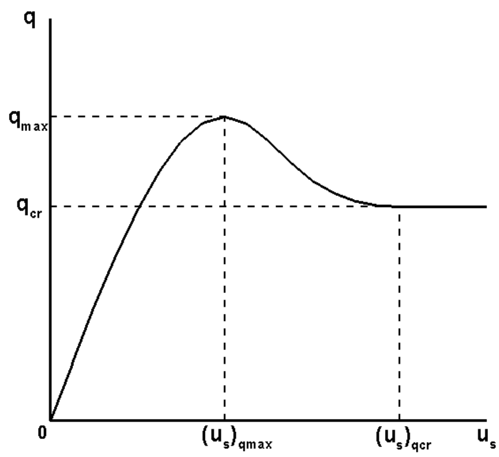


Fig. 2 The sketch of $q \sim u_s$ curve

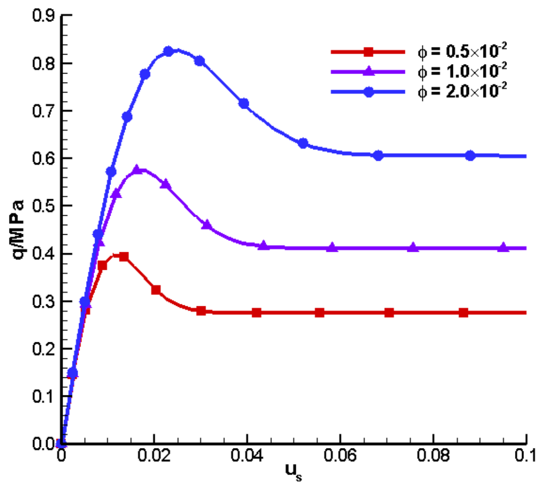


Fig. 3 Comparison of $q \sim u_s$ curves with different Φ ($\alpha = 0.5$)

from experimental data as shown in Fig. 2, we can get the value of α as follows

$$\alpha = \frac{\Phi}{2G(u_s)^2_{q_{\max}}} \tag{20}$$

As an example, the experimental data of true triaxial tests on prismatic specimens of dense Santa Monica Beach sand with a confining pressure σ_3 of 49 KPa and b-Values of 0.20 $\left(b = \dfrac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}\right)$ are adopted here (Wang and Lade 2001). The two parameters can be calculated according Eqs. (17) and (20) respectively as: $\Phi = 0.692238 \times 10^{-2}$ MJ/m^3 and $\alpha = 0.07832$.

2.3.3 Parametric study

In this section, the influence of Φ and α on materials constitutive behavior will be investigated.

As mentioned, Φ is the ultimate value of the shear strain energy density that represents the strength of granular materials. Fig. 3 shows the $q \sim u_s$ curves corresponding to $\alpha = 0.5$ and $\Phi = 0.5 \times 10^{-2}$ MJ/m^3, 1.0×10^{-1} MJ/m^3 and 2.0×10^{-2} MJ/m^3. It can be seen that parameter Φ has a significant effect on the peak stress and the residual stress and it controls the materials softening.

Comparison of $q \sim u_s$ curves with different α is shown in Fig. 4. Parameters adopted are $\Phi = 1.0 \times 10^{-2}$ MJ/m^3 and $\alpha = 0.3$, 0.5 and 0.7. We can know that α and $1-\alpha$ respectively represent the proportion of q_T and q_I in Eq. (15), the peak stress and the residual stress will decrease as the parameter α increases. Hence, α can be regarded as another strength parameter.

2.3.4 Remarks

In Section 2.3, a modified hyperelastic model intended to simulate the mechanical behavior of granular materials is presented. A softening function is introduced in the shear part of the strain energy, and a superposition of total form and modified incremental form for stress is adopted. Thus, there are 4 parameters for the proposed model, namely, K, G, Φ, α, the latter two can be regarded as parameters which control materials strength and softening, and an approach to determine them according to the experimental data is suggested. Based on the investigation on constitutive behavior of the proposed model, it can be seen that the proposed models can well reflect some typical characteristics of granular materials, such as the strain softening and the critical state.

3. Nonlinear hyperelastic models

3.1 GE-C model

The following nonlinear hyperelastic strain energy has been proposed by Jiang and Liu (2003) for granular materials.

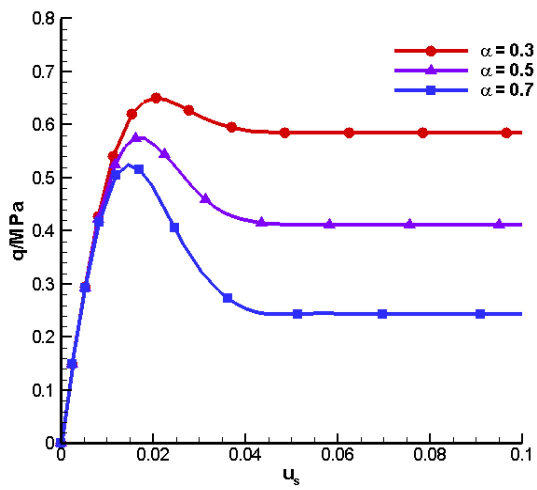


Fig. 4 Comparison of $q \sim u_s$ curves with different α ($\Phi = 1.0 \times 10^{-2}$ MJ/m^3)

$$F = \kappa\Delta^a\left(\frac{2}{5}\xi\Delta^2 + u_s^2\right) \tag{21}$$

Here ξ, a are material constants, κ is dependent on material density. Jiang and Liu take $a = 1/2$ (consistent with Hertz contact and K, $G \sim p^{1/3}$), and call it as "granular elastic" or GE. Humrickhous *et al*. (2010) suggest $a = 1$ as a large body of experimental data has shown, which is referred to as GE-cubic or GE-C. Some other researchers also have explored the issue that what value a should be taken (Goddard 1990, De Gennes 1996). In this paper we consider the GE-C model and denoted it as NLH-GEC. The elastic modulus of NLH-GEC is given as

$$\begin{aligned} D_{ijkl} = \frac{\partial^2 F}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}} &= \frac{2(a+2)(a+1)}{5}\kappa\xi\Delta^a\delta_{ij}\delta_{kl} + \kappa a(a-1)\Delta^{a-2}u_s^2\delta_{ij}\delta_{kl} \\ &+ 2\kappa a\Delta^{a-1}\delta_{ij}e_{kl} + 2\kappa a\Delta^{a-1}e_{ij}\delta_{kl} + 2\kappa\Delta^a\left(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{ij}\delta_{kl}\right) \end{aligned} \tag{22}$$

The stress invariants are written as

$$p = \frac{2}{5}\kappa\xi(a+2)\Delta^{a+1} + a\kappa\Delta^{a-1}u_s^2 \tag{23}$$

$$q = 2\kappa\Delta^a u_s \tag{24}$$

3.2 The modified model

A similar softening mode used in the linear modified model is introduced in the nonlinear model.

3.2.1 The modified strain energy

We suggest the following softening model and call it as NLH-M

$$\psi = \frac{2}{5}\kappa\xi\Delta^{2+a} + \Phi\left(1 - \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\right) \tag{25}$$

The elastic modulus is given as follows

$$\begin{aligned} D_{ijkl} = \frac{\partial^2\psi}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}} &= \frac{2(2+a)(a+1)}{5}\kappa\xi\Delta^a\delta_{ij}\delta_{kl} \\ &+ \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\left[-\frac{1}{\Phi}\left(\kappa a\Delta^{a-1}\delta_{kl}u_s^2 + 2\kappa\Delta^a e_{kl}\right)\right]\left(\kappa a\Delta^{a-1}\delta_{ij}u_s^2 + 2\kappa\Delta^a e_{ij}\right) \\ &+ \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\left[-\kappa a(a-1)\Delta^{a-2}\delta_{kl}\delta_{ij}u_s^2 + 2\kappa a\Delta^{a-1}\delta_{ij}e_{kl}\right. \\ &\left.+ 2\kappa a\Delta^{a-1}\delta_{kl}e_{ij} + 2\kappa\Delta^a\left(\delta_{ik}\delta_{jl} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)\right] \end{aligned} \tag{26}$$

The stress invariants are given by

$$p = \frac{2}{5}\kappa\xi(a+2)\Delta^{a+1} + \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\kappa a\Delta^{a-1}u_s^2 \tag{27}$$

$$q = \frac{\partial\psi}{\partial u_s} = \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\kappa\Delta^a 2u_s \tag{28}$$

Where Eq. (28) is the total form of q, the modified incremental form is shown as Eq. (29) similar to Eq.(14), and the calculation of q is the same as Eq. (15) in Section 2.3.1.

$$\delta q = 2\kappa\Delta^a \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\delta u_s \tag{29}$$

3.2.2 Determination of Φ and α

The determination of Φ and α of the NLH-M model is similar to that of the LH-M model in Section 2.3.2, and the only difference is the existence of the coupling term Δ^a. A same assumption used in the LH-M model is adopted here, viz, It is assumed that the peak value of q_I can be obtained when Eq. (30) is satisfied

$$(G)_{eff} = \frac{1}{2}\frac{\delta q_I}{\delta u_s} = \exp\left(-\frac{\kappa\Delta^a u_s^2}{\Phi}\right)\kappa\Delta^a = 10^{-8}\kappa\Delta^a \tag{30}$$

Then according to the values for $\Delta_{g_{cr}}$ and $(u_s)_{g_{cr}}$ at the critical point of $q \sim u_s$ curve as Figs. 5(a)-(b) shown, the parameter Φ can be calculated

$$\Phi = \frac{\kappa\Delta_{q_{cr}}^a (u_s)_{q_{cr}}^2}{8\ln 10} \tag{31}$$

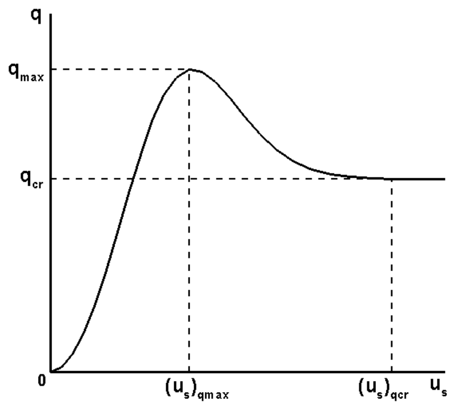
The parameter α can be derived based on the condition similar to Eq. (18) in Section 2.3.3, we can get Eq. (32)

$$1 - \alpha\frac{1}{\Phi}\kappa(\Delta)_{q_{\max}}^a (u_s)_{q_{\max}}^2 = 0 \tag{32}$$

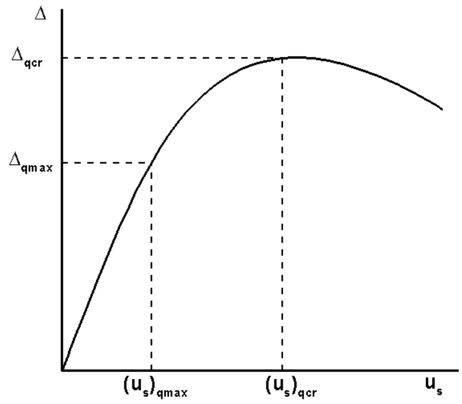
Substituting Φ, $\Delta_{q_{\max}}$ and $(u_s)_{q_{\max}}$ in Eq. (32), where $\Delta_{q_{\max}}$ and $(u_s)_{q_{\max}}$ can be determined from experimental stress–strain responses as shown in Fig. 5, we obtain

$$\alpha = \frac{\Phi}{\kappa(\Delta)_{q_{\max}}^a (u_s)_{q_{\max}}^2} \tag{33}$$

Noticing that $a = 1/2$ in the NLH-M model, which requires $\Delta \geq 0$. This implies that current version of NLH-M only can capture the shear contraction characteristic of some geomaterials. As an example, experimental data obtained from the drained triaxial tests on cylinder specimens (98 mm in diameter) of stiff fissured clays are employed to calculate Φ and α (Marsland 1972). With a confining pressure of 67.66 KPa the values of the two parameters are: $\Phi = 5.46623 \times 10^{-4}$ mJ/m^3, $\alpha = 0.1975$.



(a) $q \sim u_s$



(b) $\Delta \sim u_s$

Fig. 5 The sketches of $q \sim u_s$ and $\Delta \sim u_s$

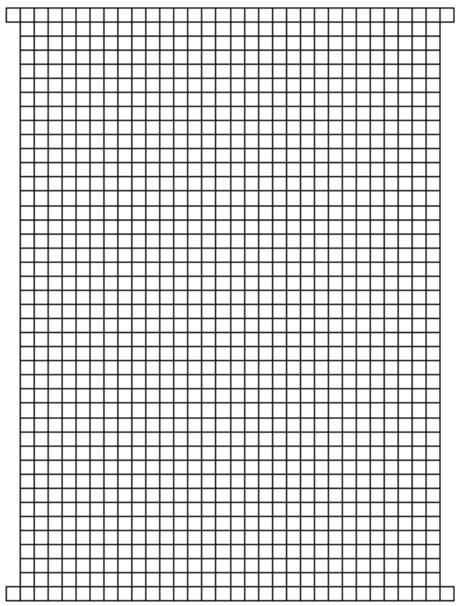


Fig. 6 The plate model

4. Numerical examples

The former sections have made an analysis to behaviors of linear and nonlinear hyperelastic models for granular materials, and in this section we will compare their performance of modeling boundary value problem. In addition, the mesh dependence of the modified models will be investigated.

We take the plain strain model with a scale of $0.6 \times 0.8\ m^2$ as shown in Fig. 6. The upper and bottom are rigid. The DOF in x direction of the upper and bottom boundary is fixed, DOF in y

direction of the bottom boundary is fixed too, and the left and right boundary is free. The plate is under pressure via a vertical displacement in the middle node of the top rigid plate.

4.1 Comparison of the linear hyperelastic models

The $q \sim u_s$ curves and the load-displacement curves of the three linear models proposed in Section 2 are shown in Figs. 7 and 8 respectively with parameters $K = 60$ MPa, $G = 30$ MPa, $\Phi = 1.0 \times 10^{-2}$ MJ/m^3, $\alpha = 0.5$, and loading displacement $u = -0.03$ m. Where the $q \sim u_s$ curves are drawn from the integration point located in the top left corner of the panel and can represent the material response in the shear band. It can be seen that curves of model LH-0 keep straight lines in the whole loading process, which correspond to the ideal materials. The equilibrium iteration of

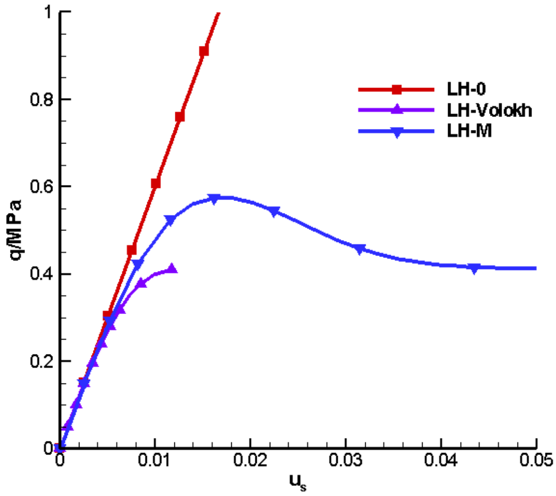


Fig. 7 Comparison of the $q \sim u_s$ curves for the three linear models

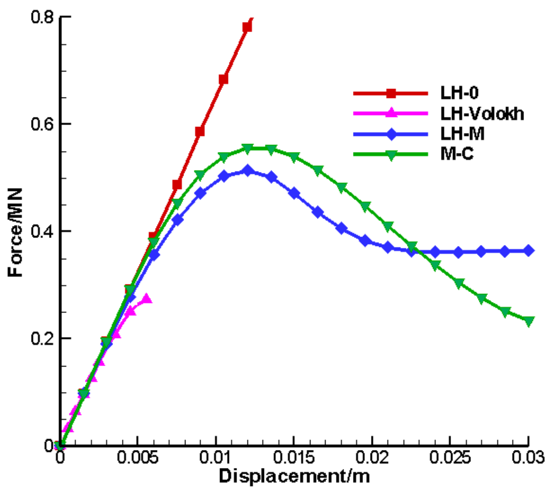


Fig. 8 Comparison of the load-displacement curves for the three linear models and the M-C model

the softening model proposed by Volokh cannot converge when loading displacement increase to – 0.006 m, which can be observed both from Figs. 7 and 8, so its performance is somewhat weak compared with other models under the same condition. Although the curves of the M-C model and the proposed model are very close before softening, the classical M-C model cannot well simulate the critical state of granular materials.

Noting that the proposed models have introduced two new parameters Φ and α, we consider they may have some potential links with the parameters of c and φ (the cohesion and the angle of internal friction) in the M-C model. Further study is needed to carry out this possible relation.

4.2 Comparison of the nonlinear hyperelastic models

The $q \sim u_s$ curves and the load-displacement curves of the two nonlinear models are shown in Figs. 9 and 10 respectively. Parameters used in the models are κ = 30 MPa, ξ = 5/2, a = 0.5, Φ = 0.5×10^{-5} MJ/m^3 and α = 0.5. Similar to Section 4.1, the modified nonlinear softening model can well simulate some characteristics of granular materials.

Compared with the LH-M model which is relatively simple without considering the nonlinear characteristics of the elastic stage and can capture both of shear contraction and dilatancy, the NLH-M model can effectively reflect the nonlinear feature of the stress-strain relation even in small strain due to the coupling of Δ^a in the strain energy. It must be noted that a = 1/2 in the NLH-M model, which requires $\Delta \geq 0$, this means current version of the NLH-M model is only applicable to describe the shear contraction behavior of granular materials. However, it is well known that the dilatancy behavior of granular materials is very complex, and that shear contraction or dilatation appears depends on the initial state and confining pressure, and has important role on capacity and deformation of granular materials (Chu *et al*. 2012).

4.3 Investigation of the mesh dependence

According to the classical elastic-plastic theory, the phenomenon of strain softening is usually

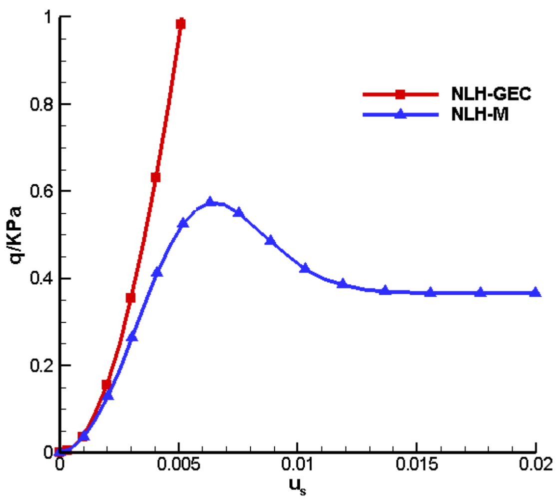


Fig. 9 Comparison of the $q \sim u_s$ curves for the two nonlinear models

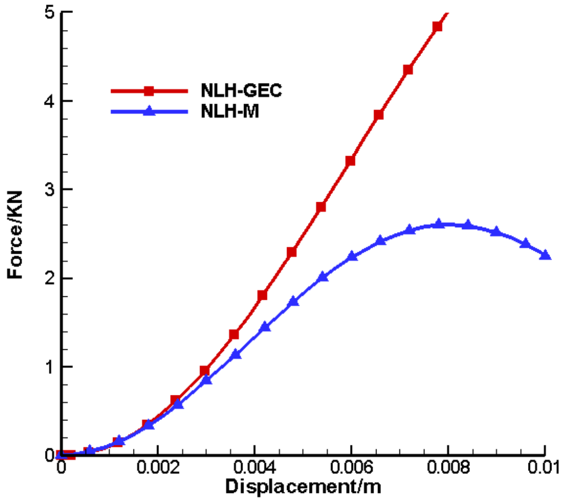


Fig. 10 Comparison of the load-displacement curves for the two nonlinear models

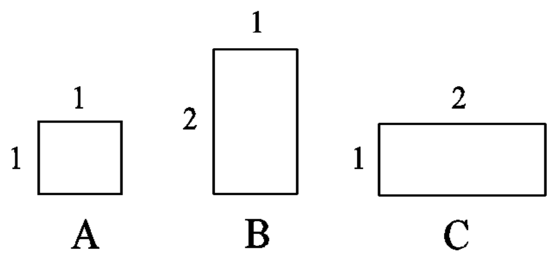


Fig. 11 Meshes used for investigation of mesh alignment

accompanied with strain localization, and the subsequent loss of ellipticity or hyperbolicity of the governing equations with softening will lead to the pathological mesh-dependency problem. Volokh (2007) implied that the mesh dependency may exist in the softening hyperelastic model, and he gave some approaches to solve this problem in the discussion part. In this section, the mesh dependency of the proposed models is investigated. Three types of mesh alignment are considered.

Types A consist of elements with equal height and width, type B consists of rectangular elements with double the height(stretched vertically) and type C consists rectangular elements with double the width(stretched horizontally). In addition, for each mesh alignment, three types of mesh density are also considered, they are 24 × 32, 30 × 40, 60 × 80 for alignment A, 48 × 32, 60 × 40, 120 × 80 for alignment B, and 24 × 64, 30 × 80, 60 × 160 for alignment C. Fig. 11 shows the example of mesh types used in the analysis.

Figs. 12 and 13 show the equivalent strain distribution of different meshes from the linear and nonlinear modified models respectively. We set the color of the equivalent strain (u_s as shown in Eq. (2b)) above 0.1 as red and below 0 as blue in the legend for the linear modified model, and set it above 0.02 as red and below 0 as blue for the nonlinear modified model. It can be observed that the distribution pattern of the equivalent strain is about the same either for different mesh alignment or for different mesh size. The width of shear band remains nearly the same. However,

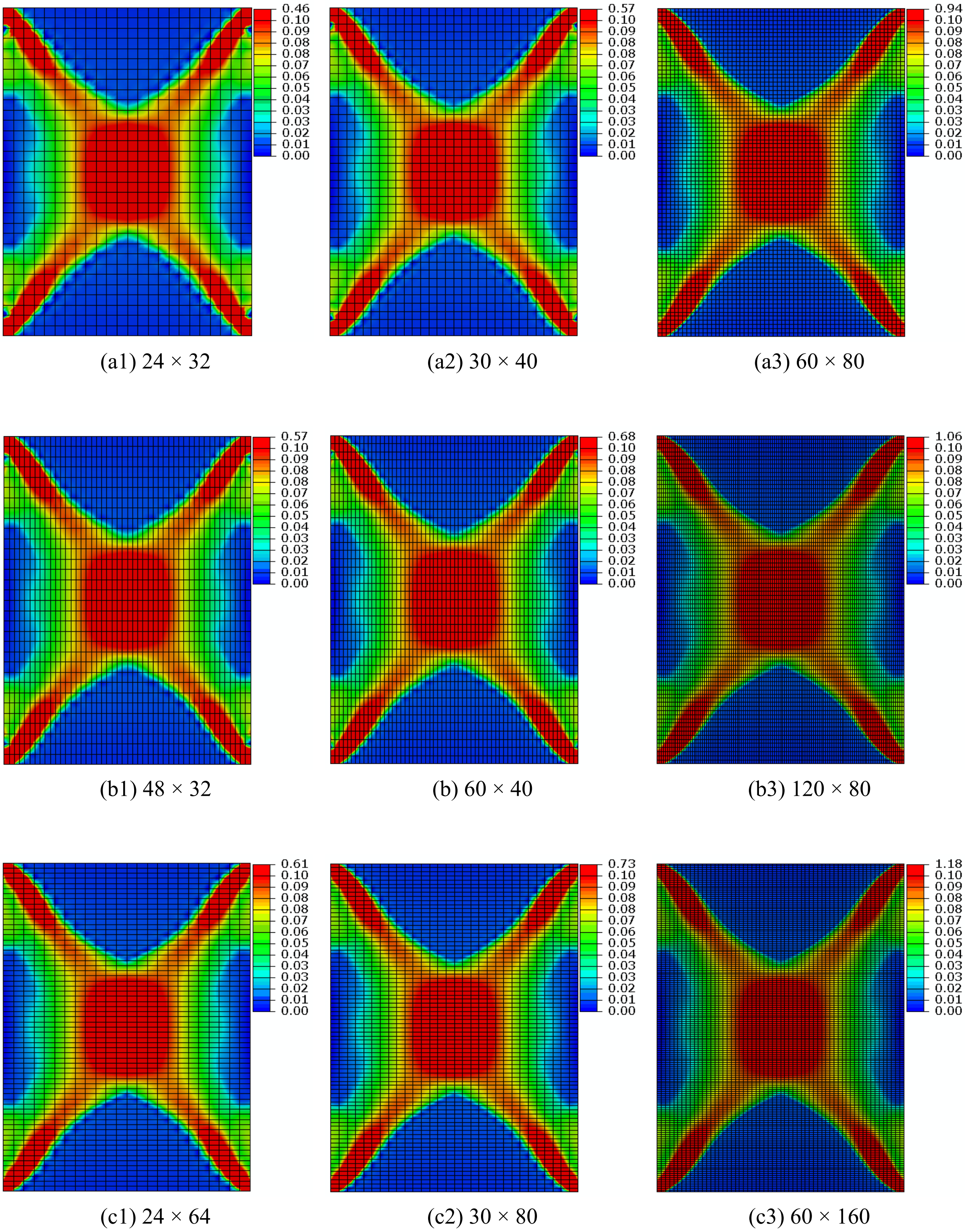
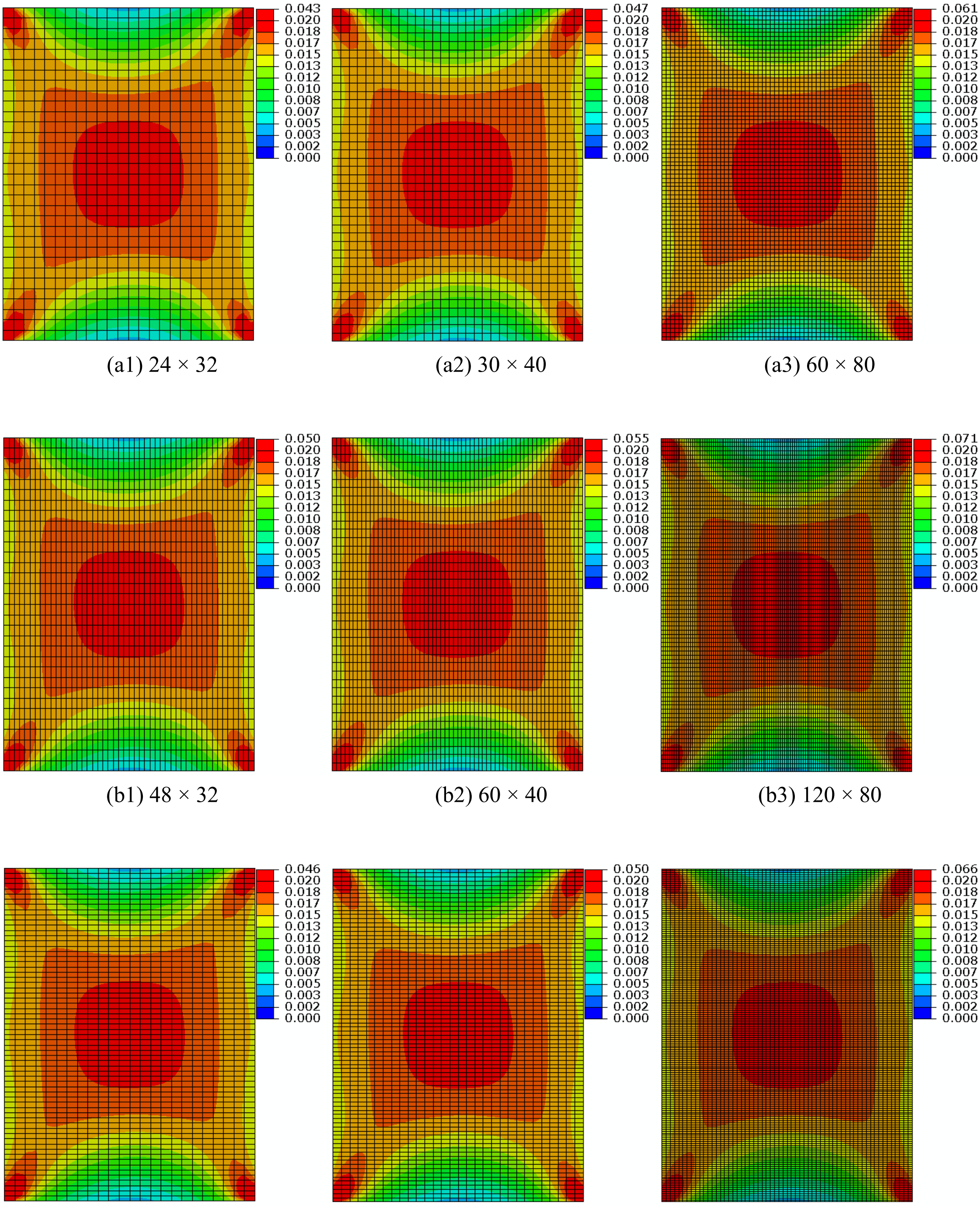


Fig. 12 Comparison of the equivalent strain distribution for the linear modified model with different mesh alignment and density (Alignment A (a); Alignment B (b); Alignment C (c))



(a1) 24 × 32 (a2) 30 × 40 (a3) 60 × 80

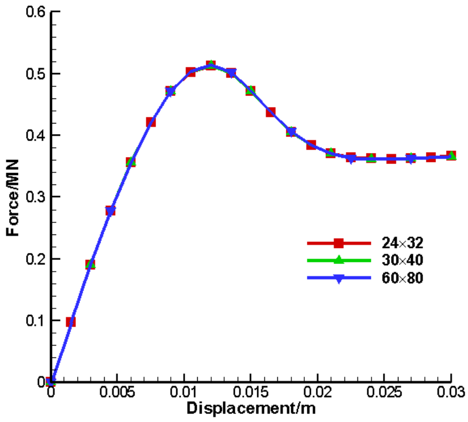
(b1) 48 × 32 (b2) 60 × 40 (b3) 120 × 80

(c1) 24 × 64 (c2) 30 × 80 (c3) 60 × 160

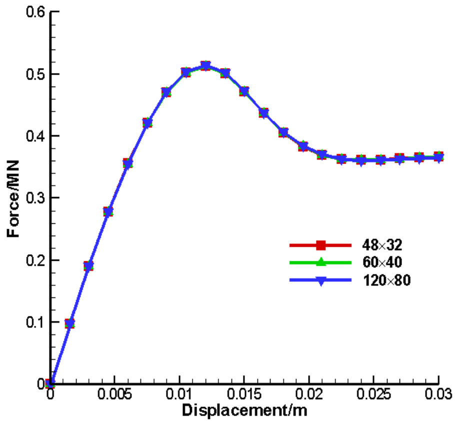
Fig. 13 Comparison of the equivalent strain distribution for the nonlinear modified model with different mesh alignment and density ((a) Alignment A; (b) Alignment B; (c) Alignment C) there is an increase of the maximum equivalent strain with the increase of mesh density both for linear and nonlinear models.

The load-displacement curves for the linear modified model with different meshes are presented in Fig. 14. It is remarkable that the load-displacement curves almost overlap on each other, the bearing capacity is not sensitive to both of the mesh alignment and the mesh size. The load-displacement curves for the nonlinear modified model are shown in Fig. 15. We can see that the bearing capacity of the nonlinear modified model has a slight increase as the mesh density increases, but it is not sensitive to the mesh alignment.

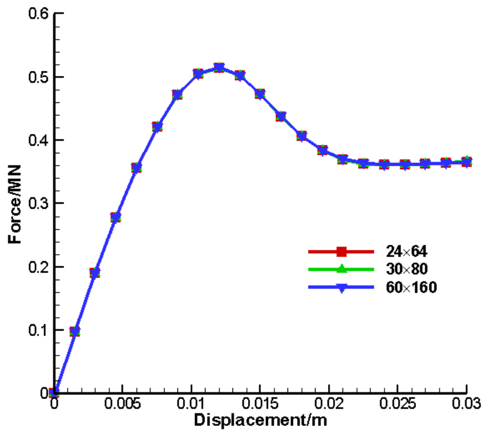
On the whole the proposed constitutive models are not sensitive to the alignment and the size of the rectangular mesh. The possible reason is supposed to the introduction of the potential function in the strain energy, and the stress component is obtained by taking derivation to the strain energy, which is similar to the hyperelasticity to some extent and is different from the classical elastic-plastic approaches.



(a) Alignment A

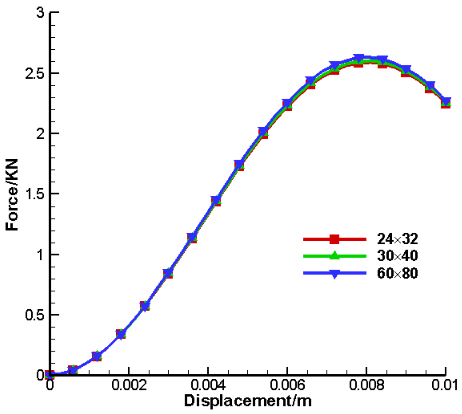


(b) Alignment B

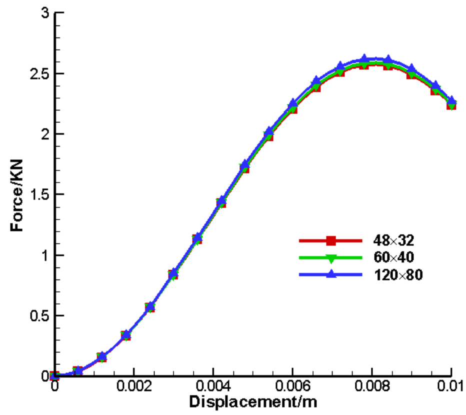


(c) Alignment C

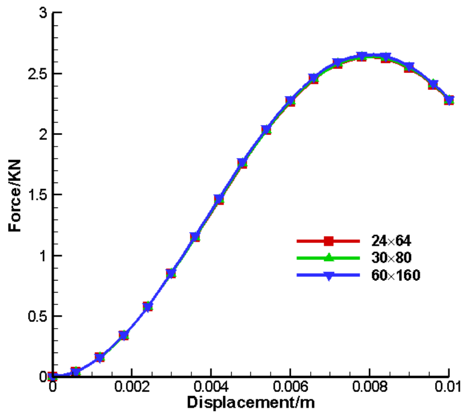
Fig. 14 Comparison of the load-displacement curves for the linear modified model with different mesh alignment and density



(a) Alignment A



(b) Alignment B



(c) Alignment C

Fig. 15 Comparison of the load-displacement curves for the nonlinear modified model with different mesh alignment and density

5. Conclusions

In the present work, a linear hyperelastic model and a nonlinear hyperelastic model are modified to simulate granular materials based on the concept of softening hyperelasticity presented by Volokh. The modification is made by introducing a softening function in the shear part of the strain energy. The materials strength and softening are controlled by parameters Φ and α, and the approach to determine them is suggested. At the same time, comparisons of hyperelastic models with softening and without softening are made both for linear and nonlinear models. Results show that the modified models can well describe some typical characteristics of granular materials, such as the strain softening, the critical state and the strain localization. Investigation on the mesh dependency shows that the proposed models are not sensitive to the alignment and the size of the rectangular mesh. Compared with classical elastic-plastic theory, which describes the material failure through the yield function and the flow rule, the proposed constitutive models do that by introducing a softening function in the strain energy. To some extent, they are two different ways to describe the mechanical behavior of granular materials.

It should be noted that the proposed models are apply to isotropic materials, the anisotropic properties of granular materials is not taken into account. In addition, we mainly focus on the influence of the shearing deformation on the material failure, the volume part has not yet been considered temporarily. So, further studies aimed at these aspects will be the next work.

Acknowledgments

The authors are pleased to acknowledge the support of this work by the National Natural Science Foundation of China through contract/grant number 10802060 and 11172216, and the Natural Key Basic Research and Development Program of China (973 Program) through contract/grant numbers 2010CB731502.

References

Chu, X.H., Yu, C. and Xu, Y.J. (2012), "The dilatancy and numerical simulation of failure behavior of granular materials based on Cosserat model", *Interact. Multisc. Mech.*, **5**(2), 157-168.

De Gennes, P.G. (1996), "Static compression of a granular medium: the "soft shell" model", *Europhys. Lett.*, **35**(2), 145-149.

De Gennes, P.G. (1999), "Granular matter: A tentative view", *J. Rev. Mod. Phys.*, **71**, S374-S382.

Duffy, J. and Mindlin, R.D. (1957), "Stress-strain relations and vibrations of granular media", *J. Appl. Mech.*, **24**, 585-593.

Duncan, J.M. and Chang, C.Y. (1970), "Nonlinear analysis of stress and strain in soils", *J. Soil Mech. Found. Div.*, **96**(5), 1629-1653.

Einav, I. and Puzrin, A.M. (2004), "Pressure-dependent elasticity and energy conservation in elastoplastic models for soils", *J. Geotech. Geoenviron. Eng.*, **130**(1), 81-92.

Gedehus, G., Jiang, Y.M. and Liu, M. (2011), "Seismo- and thermodynamics of granular solids", *Granul. Matter.*, **13**(4), 319-340.

Goddard, J.D. (1990), "Nonlinear elasticity and pressure-dependent wave speeds in granular media", *Proc. R. Soc. Lond.*, **430**(1878), 105-131.

Houlsby, G.T., Amorosi, A. and Rojas, E. (2005), "Elastic moduli of soils dependent on pressure: a hyperelastic formulation", *Geotechnique*, **55**(5), 383-392.

Humrickhouse, P.W., Sharpe, J.P. and Corradini, M.L. (2010), "Comparison of hyperelastic models for granular materials", *Phys. Rev.*, **E81**, 011303.

Jiang, Y.M. and Liu, M. (2003), "Granular elasticity without the coulomb condition", *Phys. Rev. Lett.*, **91**, 144301.

Jiang, Y.M. and Liu, M. (2007), "A brief review of "granular elasticity"", *Eur. Phys. J.*, **22**(3), 255-260.

Jiang, Y.M. and Liu, M. (2009), "Granular solid hydrodynamics", *Granul. Matter.*, **11(**3), 139-156.

Jiang, Y.M., Zheng, H.P., Peng, Z., Fu, L.P., Song, S.X., Sun, Q.C., Mayer, M. and Liu M. (2012), "An expression for the granular elastic energy", *Phys. Rev.*, **85**, 051304.

Johnson, K.L. (1985), *Contact Mechanics*, Cambridge University Press, New York, NY, USA.

Marsland, A. (1971), "The shear strength of stiff fissured clays", *Proceedings of the Roscoe Memorial Symposium Stress-Strain Behavior of Soils*, (Parry, R.H.G., Foulis, G.T. Co. Ltd. Ed.), Cambridge, UK, pp. 59-68.

Schofield, A.N. and Worth, C.P. (1968), Critical State Soil Mechanics, Lecturers in Engineering at Cambridge University.

Volokh, K.Y. (2007), "Hyperelasticity with softening for modeling materials failure", *J. Mech. Phys. Solids.*, **55**(10), 2237-2264.

Wang, Q. and Lade, P.V. (2001), “Shear banding in true triaxial tests and its effect on failure in sand”, *J. Eng. Mech.*, **127**(8), 754-761.
Zytynski, M., Randolph, M.F., Nova, R. and Wroth, C.P. (1978), “On modeling the unloading-reloading behavior of soils”. *Int. J. Numer. Anal. Met.*, **2**, 87-94.

CC