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A new analytical model to determine dynamic displacement of foundations adjacent to slope

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Abstract. Estimating seismic displacements has a great importance for foundations on or adjacent to slope surfaces. However, dynamic solution of the problem has received little attention by previous researchers. This paper presents a new analytical model to determine seismic displacements of the shallow foundations adjacent to slopes. For this purpose, a dynamic equilibrium equation is written for the foundation with failure wedge. Stiffness and damping at the sliding surface are considered variable and a simple method is proposed for its estimation. Finally, for different failure surfaces, the calculated dynamic displacement and the surfaces with maximum strain are selected as the critical failure surface. Analysis results are presented as curves for different slope angles and different foundation distances from edge of the slope and are then compared with the experimental studies and software results. The comparison shows that the proposed model is capable of estimating seismic displacement of the shallow foundations adjacent to slopes. Also, the results demonstrate that, with increased slope angle and decreased foundation distances from the slope edge, seismic displacement increases in a non-linear trend. With increasing the slope angle and failure wedge angle, maximum strain of failure wedge increases. In addition, effect of slope on foundation settlement could be neglected for the foundation distances over 3B to 5B.

Keywords: seismic settlement; analytical method; foundation; adjacent to slope; soil stiffness; failure surface

1. Introduction

Industrial development of towns on the one hand and lack of relatively flat and suitable land in mountainous areas on the other hand have caused growth in optimal utilization of sloped grounds. Seismic response of the shallow foundations adjacent to slope is different from their dynamic response on the flat half-space. Accordingly, slope effects must be considered for estimating dynamic bearing capacity of the foundations adjacent to slope. In many cases, footings of structures such as retaining walls, power transmission towers, retaining walls along the bridge and foundations of bridges are located next to the slope or on a slope surface. According to the discussed points, nowadays, there is great demand for research about dynamic response of the shallow foundations adjacent to slope.

Seismic bearing capacity and displacement of shallow foundations have been estimated by

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researchers in the second half of the twentieth century. Lysmer and Richart (1966) studied dynamic response of shallow foundations and Novak and Bereduge (1972) investigated dynamic response of embedded foundations regardless of the existence of slope. Studies about calculating stiffness and damping coefficients for different foundations placed on a horizontal ground surface have been done in the following years (Woods *et al*. 1974, Dobry and Gazetas 1986, Prakash and Puri 1988).

Previous studies about bearing capacity of the shallow foundations adjacent to slopes have been classified into three analytical, experimental and numerical categories. In the category of analytical studies, Ghosh and Kumar (2005) determined bearing capacity of the shallow foundations adjacent to slope by limit analysis method and effects of seismic forces were considered to be pseudo-static. Choudhury and Subba Rao (2006) investigated bearing capacity of the shallow strip footings embedded in slope by using limit equilibrium method and considering horizontal and vertical seismic accelerations to be pseudo-static. Based on the theory of viscoelasticity and fractional calculus, Zhu *et al*. (2012) proposed a fractional Kelvin-Voigt model to account for time-dependent behavior of soil foundation under vertical line load. Ghanbari *et al*. (2013) applied limit equilibrium and horizontal slices method to propose a new formulation for estimating seismic displacements of a reinforced slope under earthquake records.

In the analytical studies, dynamic load has been applied as equivalent pseudo-static loading and dynamic analysis has not been completely assessed yet. Also, force of inertia, stiffness and damping of the soil wedge beneath the foundation has not been considered in the analysis. Moreover, the calculations limited to finding bearing capacity and seismic displacement of foundations have not been investigated.

In the category of laboratory studies, Alamshahi and Hataf (2009) studied bearing capacity of the shallow foundations near reinforced sand slopes with geogrid and grid-anchor both experimentally and numerically. These researchers proposed different curves for impact of reinforcement on bearing capacity of the shallow foundations near the slopes. Sawwaf and Nazir (2011) studied effect of shallow foundations on reinforced sand slopes with geosynthetics and unreinforced sand slopes under permanent and cyclic vertical loads. They presented the curves which showed influence of slope reinforcement on settlement and bearing capacity. In addition, influence of external load frequency and foundation distance from the slope edge was investigated.

Islam and Gnanendran (2012, 2013) studied changes in ultimate bearing capacity and permanent deformation behavior of a strip footing near the crest of a slope at varying slope angles due to cyclic loading. Moreover, they presented some curves to show influence of slope angles, frequency of the external load and number of loading cycles on permanent displacements and bearing capacity. Other experimental studies have been performed on dynamic behavior of the foundations adjacent to slopes, which generally indicate significant impact of slopes on dynamic response of foundations.

Kourkoulis *et al*. (2010) numerically studied the interaction of shallow foundations and soil slopes under earthquake loads and presented curves for determining the foundation's safe distances from edge of the slope and specifying displacements considering the interaction between the system components. Arabshahi *et al*. (2010) determined bearing capacity of shallow foundations adjacent to slopes under horizontal seismic acceleration using discrete element method. Salloum *et al*. (2011) used Plaxis software and pseudo-static method to investigate stability of the shallow foundations adjacent to slopes. These researchers presented curves for reliability index and introduced some design charts for estimating safety factor of the soil-footing system. Lakshmanan *et al*. (2009) studied dynamic analysis of the framed foundations supporting high-speed machines by applying spectral finite element method. Other studies have been conducted by numerical methods on this issue which show that the resulting values obtained by numerical analysis are lower than the analytically obtained ones.

In the previous studies, dynamic forces have been considered to be pseudo-static. Also, forces of inertia, stiffness and damping of the failure wedge have been neglected. With regard to economic gain in designs and reduced damage due to strong earthquakes, a more appropriate model should be presented for seismic performance of the shallow foundations near slopes. This performance can increase efficiency while these foundations are used after earthquakes for machineries and piers of bridges or other such structures.

In this paper, dynamic equilibrium equation of the shallow foundations adjacent to slope and the beneath failure wedge is investigated. Using a simple method, stiffness and damping are obtained for different parts of the failure surface. External load is considered cyclic in the vertical direction and, with resisting inertia forces, stiffness and damping of soil are inserted in the system of equations. After solving dynamic equilibrium equation for the system, dynamic response is calculated as a cyclic displacement. This analysis is done for different angles of failure surface and each of the failure surfaces which have the highest strain is selected as the critical failure surface. Finally, seismic bearing capacity and displacement of the shallow foundations adjacent to slopes are calculated.

2. Proposed model

Lysmer and Richart (1966) indicated that vertical vibration of a rigid circular foundation on an elastic half-space could be modeled by a system of mass spring dashpot. For the shallow foundations adjacent to slopes, because of the existence of incomplete elastic half-space, stiffness and damping coefficients could not be used for the foundation placed on elastic half-space.

In the present study as illustrated in Fig. 1, failure surface of the soil below the foundation is assumed to be planar. Also, stiffness and damping at different points of the failure surface are considered to be variable. By applying dynamic equilibrium equations for both foundations and its failure wedge, displacement values of different angles of the failure surface could be obtained and the surface with the highest strain range is selected as the critical failure surface.

The considered assumptions are as follows:

(1) The failure surface is planar which starts from the bottom of the foundation and intersects the slope face.
(2) The foundation located on slope is of strip footing type and a plane strain behavior is assumed.
(3) The soil below the foundation is granular, dry, homogeneous and isotropic and has a linear elastic behavior.
(4) The foundation is shallow and the load applied to the foundation is cyclic and time-dependent.
(5) Failure wedge under the foundation is rigid.
(6) As illustrated in Fig. 1, because of applying dynamic force, the triangular wedge below the foundation starts to slip and displacement occurs rigidly. To obtain seismic displacement of the system, the dynamic equilibrium equation in the direction of failure surface and perpendicular direction to failure surface is written as the following equations

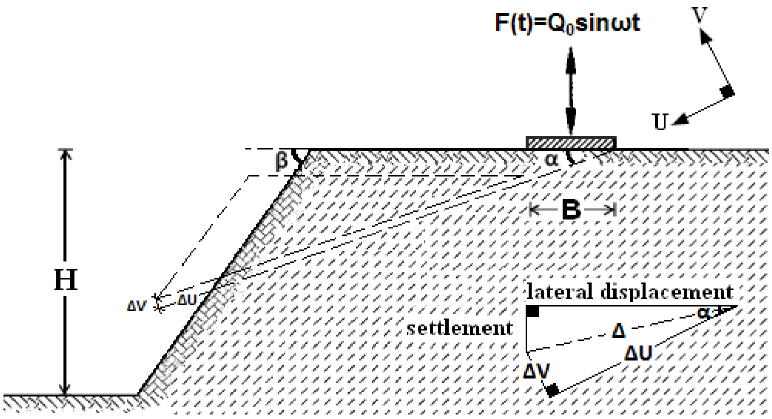

Fig. 1 Planar failure surface and displacement occur in the failed wedge due to the vertical cyclic loading applied to the foundation

$$\sum F_U = 0 \rightarrow m\Delta\ddot{U} + \sum_{i=1}^{n} C_{Ui}\Delta\dot{U} + \sum_{i=1}^{n} K_{Ui}\Delta U = F(t).\sin\ \alpha\alpha \tag{1}$$

$$\sum F_V = 0 \rightarrow m\Delta\ddot{V} + \sum_{i=1}^{n} C_{Vi}\Delta\dot{V} + \sum_{i=1}^{n} K_{Vi}\Delta V = F(t).\cos\ \alpha \tag{2}$$

In Eqs. (1) and (2), $F(t)$ is external cyclic dynamic load with amplitude of Q_o and angular velocity of $\omega = 2\pi f$ which is applied vertically to the foundation. ΔU and ΔV are displacements in the direction of failure surface and perpendicular to failure surface, respectively. K_{Ui} and K_{Vi} are coefficients for stiffness in the direction of failure surface and perpendicular to the failure surface for different points of the failure surface, respectively. Also, C_{Ui} and C_{vi} are coefficients of damping in the direction of failure surface and perpendicular to the failure surface at different points of the failure surface, respectively. System displacement equation could be considered as Eq. (3) in the failure surface direction and Eq. (4) perpendicular to the failure surface

$$\Delta U = A_1 \sin\varpi t\ +\ A_2 \cos\omega\ t \tag{3}$$

$$\Delta V = B_1 \sin\varpi\,\mathrm{t} + B_2 \cos\omega t \tag{4}$$

In order to obtain stiffness at different points of the failure surface, trend of changes in vertical and horizontal stiffness of the embankment slope is considered the product of two hyperbolic functions, as illustrated in Fig. 2. Equations of these functions are as shown in Eqs. (5) and (6).

$$\xi_1\left(\frac{b}{B},\beta\right) = \tanh\left\{a_1\left(\frac{b+\frac{B}{2}}{B}.\frac{1}{\beta}\right)+b_1\right\} \tag{5}$$

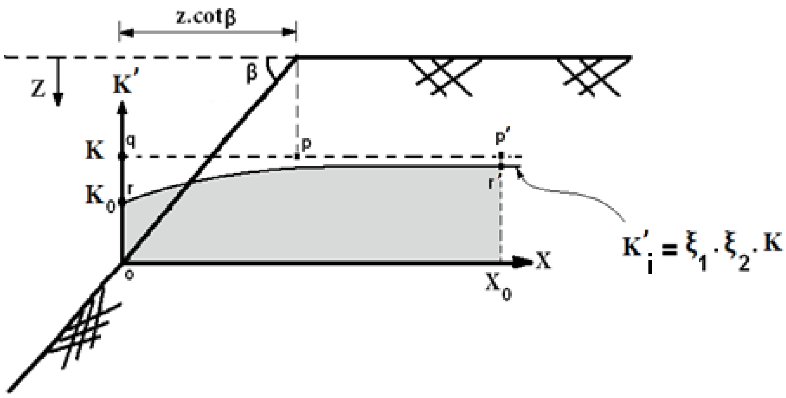

Fig. 2 The considered trend for variations of vertical and horizontal stiffness at slope adjacent point

$$\xi_2(S) = \tanh[a_2.(S) + b_2] = \frac{e^{[a_2.(S)+b_2]} - e^{-[a_2.(S)+b_2]}}{e^{[a_2.(S)+b_2]} + e^{-[a_2.(S)+b_2]}}, \quad S = \frac{x}{z} \tag{6}$$

So, vertical and horizontal stiffness along the assumed failure surface can be calculated by Eqs. (7) and (8), respectively.

$$K'_z = \sum \xi_1\left(\frac{b}{B}, \beta\right).\xi_{2z}(S).\frac{K_z}{L} \tag{7}$$

$$K'_z = \sum \xi_1\left(\frac{b}{B}, \beta\right).\xi_{2x}(S).\frac{K_z}{L} \tag{8}$$

Where K'_i is stiffness of a point adjacent to slope. Also, ξ_1 and ξ_2 are non-dimensional coefficients which show reduction in stiffness due to slope existence and are obtained from Eqs. (5) and (6), respectively. Also, a_1, b_1, a_2 and b_2 are non-dimensional coefficients which will be calculated. In the above equations, L as length of failure surface and K_z and K_x obtained by Eqs. (9) and (10), respectively, were presented by Mylonakis *et al*. (2006) for the strip foundations placed on flat ground with limited depth.

$$K_z = \frac{0.73G}{1-\mu}\left(1 + 3.5\frac{B}{H}\right) \tag{9}$$

$$K_x = \frac{2G}{2-\mu}\left(1 + 2\frac{B}{H}\right) \tag{10}$$

For the points on slope surface of the soil stiffness represented by K_o ($K_o = K_{zo}, K_{xo}$), value of K_o can be calculated as the model illustrated in Fig. 3. According to Sawwaf and Nazir's (2011) results for the small-scale strip footing, it is assumed that, if the foundation distance from edge of the slope exceeds 3B, slope effect will be negligible on the footing displacement. Moreover, variation of $K0$ is linear and reaches zero in the slope end.

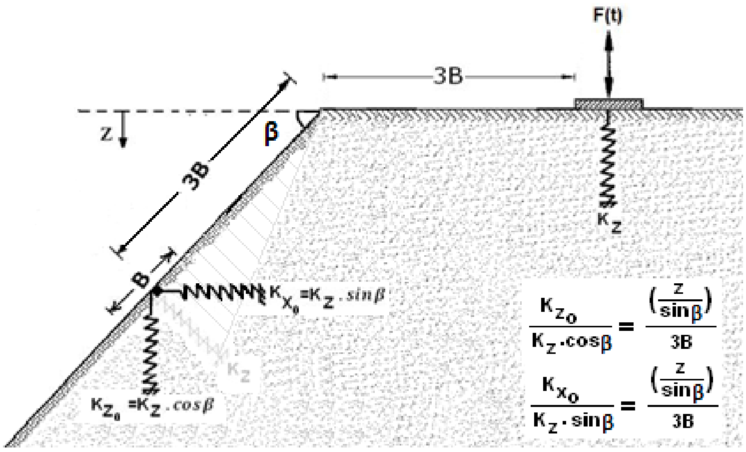

Fig. 3 Variations of horizontal and vertical stiffness at the points on slope surface

Also, according to the studies by Sawwaf and Nazir (2011), for the function of ξ_1 which depends on geometric properties of the system and is equal for vertical and horizontal stiffness, two following conditions are considered:

When foundation distance from edge of the slope is three to four times of width of the foundation and the slope angle is 35°, horizontal and vertical stiffness are reduced to 90 and 95 percent of stiffness of the foundation when placed on the flat ground, respectively.

$$\left(\frac{b}{B} = 4,\ \beta = 35^\circ \Rightarrow \xi_1 = 0.95\right),\ \left(\frac{b}{B} = 3,\ \beta = 35^\circ \Rightarrow \xi_1 = 0.9\right) \tag{11}$$

As shown in Figs. 2 and 3, for the function ξ_2 which depends on coordinates of the specified point relative to ground surface and the distance from edge of the slope, three boundary conditions are considered as follows:

(I) At $x0$ distance from the slope edge, stiffness value is 90 percent of the foundation stiffness placed on the flat ground.

$$\left(x = x_0 \Rightarrow \xi_{2x}(S) = 0.9\right),\ \left(x = x_0 \Rightarrow \xi_{2z}(S) = 0.9\right) \tag{12}$$

(ii) Decreased stiffness up to x_0 distance from the slope edge is equal to the area of o.p.q triangle, as illustrated in Fig. 2. So

$$\left(\int_0^{x_0} \left[K_z - K_z'\right] dx = \frac{1}{2} K_z . z.\ \cot\beta\right),\ \left(\int_0^{x_0} \left[K_x - K_x'\right] dx = \frac{1}{2} K_x . z . \cot\beta\right) \tag{13}$$

As shown in Fig. 3, stiffness of the points placed on the slope surface is as follows

$$\left(x = 0 \Rightarrow K_z' = K_{z_0} = \frac{z.K_z . \cot\beta}{3B}\right),\ \left(x = 0 \Rightarrow K_x' = K_{x_0} = \frac{z.K_z . K_x}{3B}\right) \tag{14}$$

Using these conditions, parameters of hyperbolic Eqs. (5) and (6) such as a_1, b_1, a_2 and b_2 can be obtained as follows

$$a_{1z} = a_{1x} = 0.21947, \quad b_{1z} = b_{1x} = 0.42793 \tag{15}$$

$$a_{2z} = \frac{4b_z^2 - 14.611b_z + 13.083}{\cot\beta}, \quad a_{2x} = \frac{4b_x^2 - 14.611b_x + 13.083}{\cot\beta} \tag{16}$$

$$b_{2z} = \frac{1}{2}\ln\left(\frac{z\cot\beta + 3B}{3B - z\cot\beta}\right), \quad b_{2x} = \frac{1}{2}\ln\left(\frac{z + 3B}{3B - z}\right).\left(\frac{K_z}{K_x}\right) \tag{17}$$

Total damping is the sum of geometric and internal damping. It is assumed that trend of reduction in geometric damping is similar to that of stiffness reduction. Therefore, damping values for the sloped ground could be obtained by Eqs. (18) to (20). Values of C_z and C_x in Eq. (20) can be extracted from Mylonakis *et al*. (2006). Note that, in this study, total damping is considered to be just internal damping with value of 5%.

$$C_{zi} = (C_{zi} + \widetilde{C}_{zi}), \quad C_{xi} = (C_{xi} + \widetilde{C}_{xi}) \tag{18}$$

$$C_{zi} = D.\left(2\sqrt{K_{zi}m}\right), \quad C_{xi} = D.\left(2\sqrt{K_{xi}m}\right), \quad D = \frac{C}{C_c} = \frac{C}{2\sqrt{K.m}} \tag{19}$$

$$\widetilde{C}_{zi} = \rho.V_{La}(B.L)(\widetilde{c}_z), \quad \widetilde{C}_{xi} = \rho.V_{La}(B.L)(\widetilde{c}_z) \tag{20}$$

Therefore, horizontal and vertical stiffness and damping of each point of the slope can be obtained. It must be noted that stiffness at any point can be obtained by dividing the achieved stiffness by failure length. As shown in Fig. 4, stiffness and damping values in failure direction and perpendicular to failure direction could be achieved by projecting horizontal and vertical

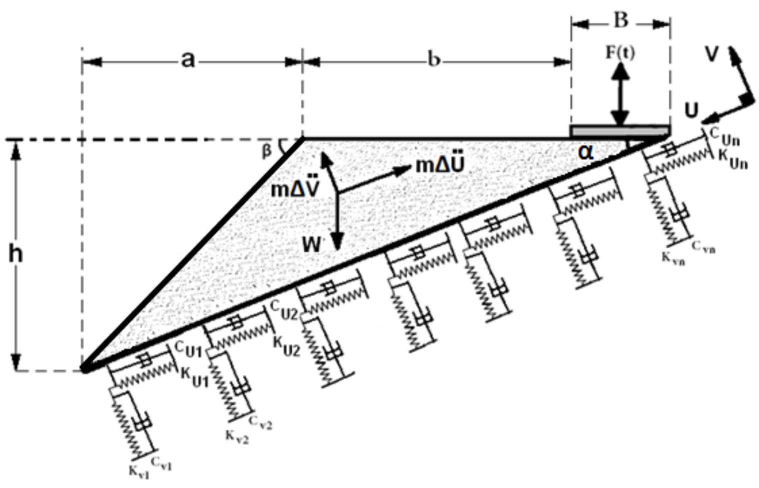

Fig. 4 Free body diagram of the failed wedge by the applied forces

stiffness and damping on special so-called directions. Finally, total mass and mass of failure wedge profile can be obtained by Eqs. (21) and (22), respectively.

$$m = m_s + m_f \tag{21}$$

$$m_s = \frac{1}{2}(B+b)^2\left(1+\frac{\tan\alpha}{\tan\beta-\tan\alpha}\right).\rho_s \tag{22}$$

In Eq. (21), m_f is foundation mass placed on slope and, in Eq. (22), ρ_s is density of soil beneath the foundation. Thus, by substituting Eq. (3) for Eq. (1), A_1 and A_2 coefficients can be obtained as follows

$$A_1 = \frac{\left(\sum_{i=1}^{n} K_{Ui} - m\omega^2\right)(Q_0.\sin\alpha)}{\left(\sum_{i=1}^{n} K_{Ui} - m\omega^2\right)^2 + \left(\sum_{i=1}^{n} C_{Ui}{}^2\omega^2\right)},\quad A_2 = \frac{-\left(\sum_{i=1}^{n} C_{Ui}\omega\right)(Q_0.\sin\alpha)}{\left(\sum_{i=1}^{n} K_{Ui} - m\omega^2\right)^2 + \left(\sum_{i=1}^{n} C_{Ui}{}^2\omega^2\right)} \tag{23}$$

Also, by substituting Eq. (4) for Eq. (2), values of B_1 and B_2 coefficients can be obtained as follows

$$B_1 = \frac{\left(\sum_{i=1}^{n} K_{Vi} - m\omega^2\right)(Q_0.\sin\alpha)}{\left(\sum_{i=1}^{n} K_{Vi} - m\omega^2\right)^2 + \left(\sum_{i=1}^{n} C_{Vi}{}^2\omega^2\right)},\quad B_2 = \frac{-\left(\sum_{i=1}^{n} C_{Vi}\omega\right)(Q_0.\sin\alpha)}{\left(\sum_{i=1}^{n} K_{Vi} - m\omega^2\right)^2 + \left(\sum_{i=1}^{n} C_{Vi}{}^2\omega^2\right)} \tag{24}$$

Therefore, by substituting Eqs. (23) and (24) for Eqs. (3) and (4), values of displacement in the failure surface direction and perpendicular to the failure surface direction and also the horizontal and vertical settlement can be calculated at any time.

3. Results of the proposed model

Using the obtained formulation for the proposed model in the present study, lateral displacement, foundation settlement and total displacement in seismic conditions can be achieved. In this study, some of the common sloped embankments are analyzed and their seismic settlements are calculated. For this purpose, three types of sandy soil, the properties of which are presented in Table 1, are used. Also, geometry of the investigated models is defined in Table 2. It should be noted that values of frequency and stress are similar to each other in all the comparisons.

Ratio of the foundation stiffness adjacent to slope to foundation stiffness on flat surface without any slope at different distances from the slope is presented in Fig. 5. According to the considered conditions, ratio of stiffness of the foundation adjacent to slope to the one on the flat ground is equal. The results show that, if the foundation moves to distances over 3B to 5B, the slope effect on foundation settlement will be negligible. The distances equal to 3B and 5B are suggested for very mild (20 degrees) and steep (45 degrees) slopes, respectively.

Table 1 Properties of three types of the investigated sandy soil

Type of soil (Current study)	Type of soil (Sawwaf and Nazir 2011)	Specific weight (kN/m^3)	Poisson's Ratio	Shear Modulus (kN/m^2)
Loose sand	un- reinforced loose sand	16	0.3	5000
Medium sand	un- reinforced replaced sand	18	0.3	10000
Dense sand	reinforced replaced sand	20	0.3	20000

Table 2 Geometry characteristics of the model

No		*B* (mm)	*B* (degree)	Foundation height (mm)	Thickness of soil layer, *H* (mm)
1	Proposed method	800	33.69	200	4000
	Sawwaf and Nazir (2011)	80	33.69	20	400
2	Proposed method	800	35.00	200	4000
	Software	800	35.00	200	4000
3	Proposed method	1300	33.69	300	400
	Islam and Gnanendran (2012)	130	33.69	30	400
4	Proposed method	1300	26.56	300	400
	Islam and Gnanendran (2012)	130	26.56	30	400

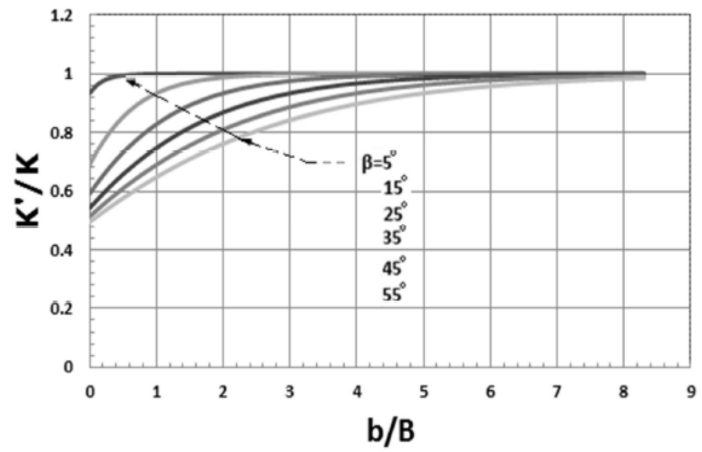

Fig. 5 Variations in ratio of the foundation stiffness adjacent to slope to the one on flat surface for different values of the slope angle and distance from the slope edge

After displacement calculation for different wedges, strain can be also calculated for each of the so-called wedges. The wedge which has the highest strain is selected as the critical failure wedge. For the selected critical failure wedge, seismic displacements could be easily extracted at any time. In Fig. 6, variations of maximum cyclic settlement versus load frequency are given for different values of foundation distances from the edge of the slope. The results demonstrate that

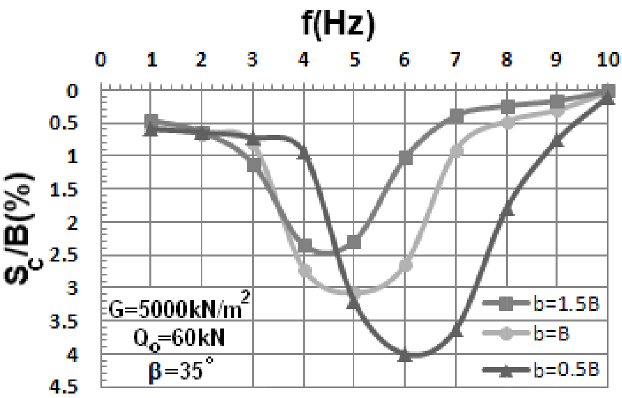

Fig. 6 Variations of maximum cyclic settlement versus load frequency for different values of the foundation distance from the slope edge

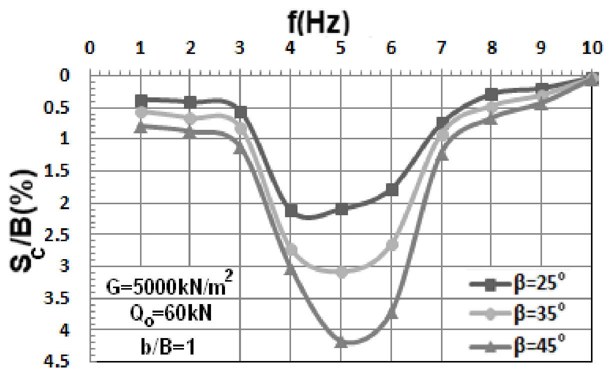

Fig. 7 Variations of maximum cyclic settlement ratio versus load frequency for different slope angles

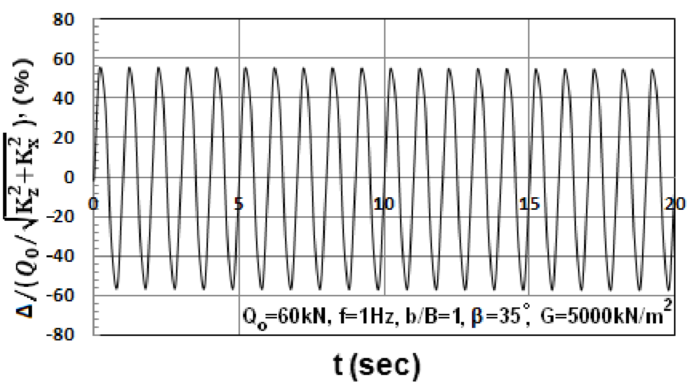

Fig. 8 Total displacement response of a foundation (Δ) due to cyclic vertical loading *C*

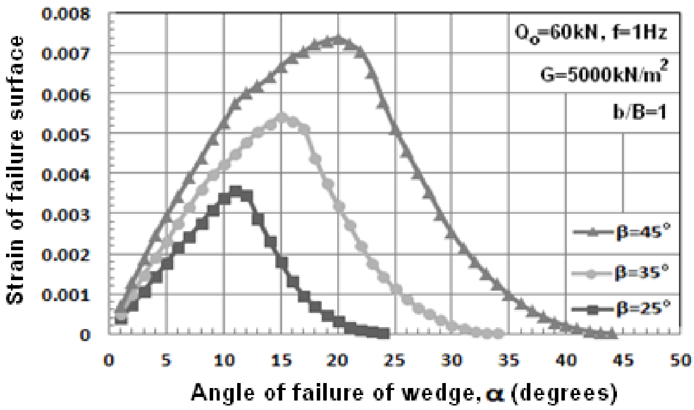

Fig. 9 Variations of strain on the failure surface for different angles of failure wedge

frequency of the external load greatly affects the foundation settlement and the cyclic settlement at natural frequency is much higher than other frequencies. In addition, the cyclic settlement is reduced with the foundation distance increasing from the slope edge.

Variation of cyclic settlement ratio versus load frequency for different slope angles is illustrated in Fig. 7. The results show that the cyclic settlement decreases by decreasing the slope angle.

In Fig. 8, total displacement response of a foundation, due to vertical cyclic load $F(t) = 60\sin(2\pi t)$ for the foundation located adjacent to a slope with the angle of 350, is demonstrated. The so-called response is calculated for different slope angles and different load frequencies. Also, using the proposed model, the strain values can be calculated at a specified point under a foundation for different angles of the failure wedge.

The strain values, when the foundation is located at a distance equal to its width from the slope edge, are represented in Fig. 9 for three different slope angles. The results show that, with increasing the slope angle, failure wedge angle and maximum strain of failure wedge increase. It must be noted that, for the foundation located adjacent to slope, failure mechanism is usually made of sliding along a plane while, for the foundations located on flat ground, logarithmic spiral wedge exists with failure mechanism of the sliding and rotation combination. Accordingly, as shown in Fig. 9, by being the slope steeper, the failure wedge angle increases.

4. Comparison

Results of the proposed model are compared with the experimental results by Sawwaf and Nazir (2011) and Islam and Gnanendran (2012). In addition, the obtained results are compared with the numerical results of finite element software. To compare with the results by Sawwaf and Nazir (2011), three types of sandy soil (as classified in Table 1) are set equivalent to types of soil in their study.

Ratio of cyclic settlement obtained by the experiments done by Sawwaf and Nazir (2011) is compared with results of the proposed model, as illustrated in Fig. 10. The comparison is done for

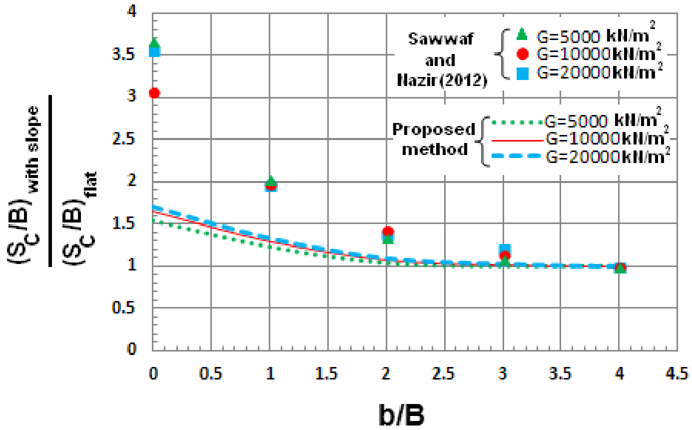

Fig. 10 Comparing results of the proposed model with those of Sawwaf and Nazir's (2011) study at $f = 1$ Hz

different foundation distances at 1 Hz load frequency, which demonstrates that the foundation settlements are in the same reduction trend with increase in the foundation distance. However, results of the experimental studies for settlement are in upper value ranges in comparison to the proposed model.

In Table 3, results of the proposed method are compared with those of an experimental study done by Islam and Gnanendran (2012). Accordingly, results of the proposed method are in higher ranges than those of the experimental work. However, with increasing load intensity and slope angle, results of both methods become closer to each other.

Table 3 Comparing results of the proposed model and those of Islam and Gnanendran's (2012) study

Q_o (kN)	S_c/B (%)			
	$\beta = 33.42°$		$B = 26.56°$	
	Proposed method	Islam and Gnanendran (2012)	Proposed method	Islam and Gnanendran (2012)
5	0.259	0.139	0.202	0.169
10	0.519	0.369	0.404	0.439
15	0.778	0.500	0.606	0.446
20	1.037	0.631	0.808	0.769
25	1.296	0.885	1.010	0.962
30	1.555	1.092	1.212	1.208
35	1.815	1.469	1.414	1.439
40	2.074	2.115	1.616	1.615
45	2.333	---------	1.818	2.039
50	2.592	---------	2.020	2.731

Table 4 Comparing results of the proposed model and software for different foundation distances from the slope edge

b/B	S_c/B (%)					
	G = 5000 (kN/m^2)		G = 10000 (kN/m^2)		G = 20000 (kN/m^2)	
	Proposed method	PLAXIS	Proposed method	PLAXIS	Proposed method	PLAXIS
0	0.699	0.517	0.347	0.254	0.173	0.133
1	0.556	0.501	0.273	0.256	0.135	0.129
2	0.467	0.502	0.227	0.256	0.112	0.129
3	0.450	0.501	0.215	0.255	0.105	0.129
4	0.454	0.496	0.211	0.264	0.102	0.128

In addition to the comparison of the proposed model with two experimental studies, the proposed model is compared using finite element software, named Plaxis 2D, as shown in Table 4. The objective is to evaluate settlement of a strip footing near the slope, specifically to determine distance contribution from edge of slope to the foundation width on the settlement of these footings. For this comparison, parameters of the software model are set the same as analytical model parameters. Also, the foundation and the beneath soil mass are considered in a linear elastic manner in the software. The force acting on the footing is cyclic, similar to the proposed model. The amplitude of loading is considered 60 kN with frequency of 1 Hz. All the dimensions in the numerical analysis are set the same as the analytical analysis.

By trial and error using the upper and lower limits of soil parameters and footing location, the effective area is expanded in the horizontal direction. Beyond these distances, displacement as well as stresses is almost unchanged. A 15-node, triangular element is used in the finite element mesh. A very fine mesh is generated along the footing-soil interfaces and the horizontal surface adjacent to the slope. In the dynamic analysis, absorbing boundaries, except slope edge boundaries, are applied.

The results which are shown in Table 4 demonstrate that both methods predict the same decreasing trend in the settlement with increase in the foundation distance near the slope edge. The results obtained from the displacement contours represent appropriate modeling and suitable coordination assumptions for the failure surface along with displacement manner in the analytical model. However, the proposed model achieves cyclic settlement ratio which was 10 to 20 percent less than the software values.

Therefore, results of the proposed model are in ranges between the results of two experimental studies and are within closer values than the software values.

5. Conclusions

A new analytical model is proposed to estimate seismic displacement of the shallow foundations adjacent to slope. In addition, a simple method is presented for estimating stiffness and damping values on the failure surface. Accordingly, variations of the stiffness and damping values along the failure surface follow a hyperbolic function, the coefficients of which are presented in the paper. The proposed analytical procedure reliably calculates obtaining lateral displacement, settlement and total displacement at each time and also displacement for different frequencies of the external load. Another advantage of this model is in estimating displacement values by basic properties of soil such as shear modulus (G), Poisson's ratio (μ) and specific gravity of soil (γ_s).

Comparison of the proposed method with the results of two experiments indicates that trend of settlement is properly predicted. However, results of seismic settlement obtained by the proposed method are in the lower ranges of experimental values in one case and in higher ranges of experimental values in another. Also, in the comparison of the proposed method with the values obtained from the software, tolerances are acceptable. Results of the proposed method are in good agreement with those of finite element software and are in acceptable agreement with mean results of two previous experimental studies.

In addition, results of the proposed model demonstrate that, with increasing the slope angle and decreasing the foundation distance from the slope edge, seismic displacements and strain of failure wedges increase nonlinearly. Also, with increasing slope angle, failure wedge angle and maximum strain of failure wedge increase. In addition, if the foundation is placed at a distance over 3B to 5B, the slope has a negligible effect on the foundation settlement.

References

Alamshahi, S. and Hataf, N. (2009), "Bearing capacity of strip footings on sand slopes reinforced with geogrid and grid-anchor", *Geotext. Geomembr.*, **27**(3), 217-226.

Arabshahi, M., Majidi, A.R. and Mirghasemi, A.A. (2010), "Seismic three dimensional bearing capacity of shallow foundations adjacent to slopes", *Proceedings of the 4th International Conference on Geotechnical Engineering and Soil Mechanics*, Tehran, Iran, November.

Choudhury, D. and Subba Rao, K.S. (2006), "Seismic bearing capacity of shallow strip footings embedded in slope", *Int. J. Geomech.*, *ASCE*, **6**(3), 84-176.

Dobry, R. and Gazetas, G. (1986), "Dynamic response of arbitrary shaped foundations", *J. Soil Mech. Found. Div.*, *ASCE*, **112**(2), 109-135.

Ghanbari, A., Khalilpasha, A., Sabermahani, M. and Heydari, B. (2013), "An analytical technique for estimation of seismic displacements in reinforced slopes based on horizontal slices method (HSM)", *Geomech. Eng.*, *Int. J.*, **5**(2), 143-164.

Ghosh, P. and Kumar, J. (2005), "Seismic bearing capacity of strip footings adjacent to slopes using the upper bound limit analysis", *Electron. J. Geotech. Eng.*, **10**, Bundle C.

Islam, M.A. and Gnanendran, C.T. (2012), "Behaviour of a strip footing near slope crest under vertical cyclic loading", *Proceedings of the GeoCongre*ss, *ASCE*, California, USA, pp. 31-40.

Islam, M.A. and Gnanendran, C.T. (2013), "Slope stability under cyclic foundation loading – Effect of loading frequency", *Proceedings of the GeoCongre*ss, *ASCE*, California, USA, pp. 750-761.

Kourkoulis, R., Anastasopoulos, I., Gelagoti, F. and Gazetas, G. (2010), "Interaction of foundation−structure systems with seismically precarious slopes: Numerical analysis with strain softening constitutive model", *Soil Dyn. Earthq. Eng.*, **30**(12), 1430-1445.

Lakshmanan, N., Gopalakrishnan, N., Rama Rao, G.V. and Sathish kumar, K. (2009), "Dynamic stiffness based computation of response for framed machine foundations", *Geomech. Eng.*, *Int. J.*, **1**(2), 121-142.

Lysmer, J. and Richart, F.E. (1966), "Dynamic response of footings to vertical loading", *J. Soil Mech. Found. Div.*, *ASCE*, **92**(1), 65-91.

Mylonakis, G., Nikolaou, S. and Gazetas, G. (2006), "Footings under seismic loading: Analysis and design issues with emphasis on bridge foundations", *Soil Dyn. Earthq. Eng.*, **26**(9), 824-853.

Novak, M. and Beredugo, Y.O. (1972), "Vertical vibration of embedded footings", *J. Soil Mech. Found. Div.*, *ASCE*, **98**(12), 1291-1310.

Brinkgreve, R.B.J. (2002), Plaxis 2D version 8. A.A. Balkema Publisher, Lisse.

Prakash, S. and Puri, V.K. (1988), *Foundations for Machines*: *Analysis and Design*, John Wiley & Sons, New York, USA.

Salloum, N., Youssef Abdel Massih, D., Soubra, A.H. and Harb, J. (2011), "Deterministic and probabilistic seismic analyses of a slope-footing system", Proceedings of the Georisk 2011, Atlanta, GA, USA , June, pp. 640-647.

Sawwaf, M.A. and Nazir, A.K. (2012), "Cyclic settlement behaviour of strip footings resting on reinforced layered sand slope", *J. Adv. Res.*, **3**(4), 315-324.

Woods, R.D., Barnett, N.E. and Sagesser, R. (1974), "A new tool for dynamics", *J. Geotech. Eng. Div.*, *ASCE*, **100**(GT11), 1231-1247.

Zhu, H.H., Liu, L.C., Pei, H.F. and Shi, B. (2012), "Settlement analysis of viscoelastic foundation under vertical line load using a fractional Kelvin-Voigt model", *Geomech. Eng.*, *Int. J.*, **4**(1), 67-78.

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