Geomechanics and Engineering, *Vol. 5, No. 5 (2013) 485-497*
DOI: http://dx.doi.org/10.12989/gae.2013.5.5.485

Factor of safety in limit analysis of slopes

Antoni Florkiewicz [a] and Albert Kubzdela*

Department of Civil and Environmental Engineering, Poznan University of Technology, Piotrowo 5, 60-965 Poznan, Poland

(Received November 29, 2012, Revised April 28, 2013, Accepted May 08, 2013)

Abstract. The factor of safety is the most common measure of the safety margin for slopes. When the traditionally defined factor is used in kinematic approach of limit analysis, calculations can become elaborate, and iterative methods have to be used. To avoid this inconvenience, the safety factor was defined in terms of the work rates that are part of the work balance equation used in limit analysis. It was demonstrated for two simple slopes that the safety factors calculated according to the new definition fall close to those calculated using the traditional definition. Statistical analysis was carried out to find out whether, given normal distribution of the strength parameters, the distribution of the safety factor can be approximated with a well-defined probability density function. Knowing this function would make it convenient to calculate the probability of failure. The results indicated that the normal distribution could be used for low internal friction angle (up to about 16°) and the Johnson distribution could be used for larger angles ϕ. The data limited to two simple slopes, however, does not allow assuming these distributions *a priori* for other slopes.

Keywords: slope stability analysis; factor of safety

1. Introduction

Among different parameters used for assessment of the safety of slopes, such as critical height, limit inclination, or critical acceleration, it is the factor of safety that is most often used by engineers. Historically, the definition of the safety factor was specific to the methods of solution; for instance, one of the earlier proposals was the ratio of the resisting moment to the moment of gravity forces (e.g., Taylor 1948, Lambe and Whitman 1968). This definition was the outcome of the use of circular failure surfaces employed in safety assessment. As the methods of analysis evolved, a more popular definition of the factor of safety used today relates material strength properties of the soil comprising the slope

$$F^* = \frac{c}{c_d} = \frac{\tan\phi}{\tan\phi_d} \tag{1}$$

where c = cohesion and ϕ = internal friction angle, and c_d and ϕ_d are their respective values needed

*Corresponding author, Associate Professor, E-mail: albert.kubzdela@put.poznan.pl
[a] Professor

http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

to maintain limit equilibrium of the slope. The definition in Eq. (1) is the most common definition of the factor of safety for slopes used today.

The use of this definition, however, can be intricate in methods where the internal friction angle affects the solution implicitly, for instance through the kinematics relations that enter the energy considerations in limit analysis. An effective, but mathematically inconvenient method, is one where the true strength properties of the soil are reduced successively to reach the failure state when the respective ratio is equal to the factor of safety. This method was used by Michalowski (1989) in relation to slopes, and it has been termed in the literature the *strength reduction method*.

When results of a stability analysis are reported for a wide range of parameters, recovering the safety factor, as defined in Eq. (1), requires an iterative technique. A way to avoid this inconvenience is to report results as a function of a parameter that is independent of the safety factor. An early proposal of such presentation is due to Bell (1966) in the context of a slice method analysis, and the chosen dimensionless parameter independent of the safety factor was $c/\gamma H \tan\phi$, where γ is the soil unit weight and H is the slope height. An efficient application of this method was shown by Michalowski (2002, 2010) who reported results of limit analysis in charts that are iteration-free for both two-dimensional and three-dimensional failures. A different approach to presenting the safety factor was shown recently by Klar *et al.* (2011). For a slope of given geometry and given unit weight, they constructed a graph with limit state combinations of strength parameters, and the safety factor can be found as a ratio of two collinear vectors (magnitudes) on that graph.

Limit analysis in stability calculations of earth structures becomes increasingly popular, and this note is focused on the development of a measure of safety that is directly related to the energy terms embedded in the kinematic approach of limit analysis. A similar attempt was made earlier by Karal (1977), Izbicki (1983), and Derski *et al.* (1988), but it did not materialize in an accepted measure of safety for slopes.

2. Factor of safety in terms of work rate

Limit analysis was developed in structural engineering primarily for estimates of limit loads, but it is equally applicable for calculations of bounds on other critical parameters, such as the critical height, or the safety factor. Earlier attempts to extract the factor of safety from the kinematical approach of limit analysis (Karal 1977, Izbicki 1983, Derski *et al.* 1988) proposed the factor of safety to be calculated as a ratio of the rate of work dissipation to the rate of work of external forces. However, such a safety factor becomes zero if the material does not dissipate energy when subjected to plastic deformation. First, we will arrive at this factor (F_1) directly from the work rate balance equation written for a stable slope

$$\int_V \sigma_{ij}\dot{\varepsilon}_{ij}\,dV = F_1\left[\gamma\int_V n_i v_i\,dV + \int_S p_i v_i\,dS\right] \tag{2}$$

The left-hand side of Eq. (2) represents the rate of work dissipation, and the two terms on the right-hand side denote the work rate of the soil weight (with unit weight γ) and the work rate of the given distributed load p_i on boundary S [see Michalowski (2010) for definition of other symbols in Eq. (2)]. Because the slope is stable, the available dissipation is larger than the work of the forces driving the collapse, and factor F_1 is the measure of the safety margin, interpreted as the factor of safety. Consequently, the definition of F_1 can be written as the ratio of the rate of plastic work D (dissipation) to the rate of the external work W_γ and W_p (gravity and boundary loads), all corresponding to the respective integrals in Eq. (2)

$$F_1 = \frac{D}{W_\gamma + W_p} \tag{3}$$

This definition works best for undrained soil ($\phi = 0$), or the short-term analysis (Karal 1977), and it is equivalent to the definition in Eq. (1) for undrained analysis. For frictional soils, however, it yields factors different than those from Eq. (1), and it fails when $c = 0$, since it implies $D = 0$ for soils governed by the Mohr-Coulomb yield condition and the associative flow rule.

It is proposed that the work rate balance in Eq. (2) be modified to reflect that part of the soil weight (in volume V^-) may be doing negative work during collapse (resisting failure), $\gamma \int_{V^-} n_i v_i dV^-$, and part of the boundary load (given on boundary S^-) can do negative work, $\int_{S^-} p_i v_i dS^-$. Including these terms on the left-hand side of the work rate balance, one can write

$$\int_V \sigma_{ij} \dot{\varepsilon}_{ij} dV - \gamma \int_{V^-} n_i v_i dV^- - \int_{S^-} p_i v_i dS^- = F_2 \left[\gamma \int_{V^-} n_i v_i dV^+ - \int_{S^+} p_i v_i dS^+ \right] \tag{4}$$

with factor F_2 being the new measure of the margin of safety. Replacing now the respective terms in Eq. (4) with W_γ^-, W_p^- and W_γ^+, W_p^+ the following definition of safety factor F_2 results

$$F_2 = \frac{D - W_\gamma^- - W_p^-}{W_\gamma^+ + W_p^+} \tag{5}$$

The safety factor modified in this manner is applicable to cases of $c = 0$, and the formula in Eq. (5) defines the safety factor essentially as the ratio of the resisting work to the work causing the collapse. Note that the non-dissipative terms in the numerator of Eq. (5) are negative, thus they are algebraically added to the dissipation rate D.

It needs to be emphasized that the concept behind the definition in Eq. (5) is conceptually very different from that based on the mobilized strength in Eq. (1). The new definition has more in common with the very traditional definitions based on separation of moments (or forces) causing failure and those resisting collapse (see, e.g., Lambe and Whitman 1968), here, it is the respective work rates of moments (or forces).

The new definition, while convenient in calculations, has two drawbacks. First, it cannot be applied to slopes when $c = 0$. This is because the critical mechanism in such cases tends to a single block (with vanishing volume) moving down the slope, leading to a singularity in Eq. (5). The second drawback is its applicability in limit analysis only. This is because this safety factor is defined in terms of the work rates, and more traditional methods do not call for calculation of work rates in the analysis.

As the definition in Eq. (5) is new, there may be some reluctance using it in practice until more experience is gained regarding its performance, i.e., the ability to depict the true margin of safety. However, it has a computational advantage over the definition in Eq. (1). For a given mechanism, this definition does not require iterative procedures to calculate F_2 as the work rate terms on the right-hand-side are all independent of F_2. Calculating the factor of safety according to the definition in Eq. (1), on the other hand, requires iterative procedures (unless $\phi = 0$). This is because the geometry of the collapse mechanism is an implicit function of the internal friction angle (no matter whether the mechanism is constructed analytically or numerically). This is a result of the kinematic constraints that follow from the flow rule associated with the Mohr-Coulomb yield condition. Consequently, some iterative method (such as the strength reduction procedure) is needed to calculate the safety factor according to definition in Eq. (1). This is independent of whether a semi-analytical solution technique is used or the numerical technique based on finite elements. The definition in Eq. (5) does not call for iterative procedure, but it does require separation of the rate of work of external forces into positive and negative components. When semi-analytical rigid-block analysis is used, this separation is straightforward. It is equally straightforward in numerical methods once the nodal velocities are determined. However, both definitions of the factor of safety require iterative trials in the optimization procedure needed to identify the most critical collapse mechanism, but this is independent of the definition of the factor of safety.

3. Application example

To illustrate the application of this definition, two simple examples in Figs. 1(a) and (b) are considered. The hodograph for describing the magnitudes of blocs and their relative velocities is presented in Fig. 1(c). Collapse mechanisms are also illustrated in Figs. 1(a) and (b). These are relatively simple failure mechanisms, but the purpose of this note is only to illustrate the application of the new definition of the safety factor. Applying the definition of the factor of safety in Eq. (3), the following expression results after substituting the specific terms representing the work dissipation rate and the work rate of the external forces

$$F_1 = \frac{c \cdot \cos\phi \cdot (v_1 l_{AB} + v_2 l_{BC} + v_3 l_{CD} + v_{12} l_{BF} + v_{23} l_{CE})}{\gamma \cdot (V_1 \cdot v_{1y} + V_2 \cdot v_{2y} + V_3 \cdot v_{3y})} \tag{6}$$

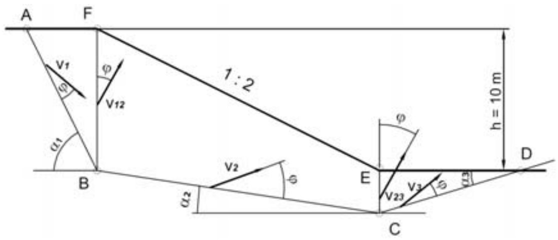
where v_i = velocity magnitude of block i, v_{XX} = velocity jump magnitude on discontinuity XX, l_{XX} = length of discontinuity XX, and V_i = volume of block i.

When the safety factor for the given example is taken as in Eq. (5), the calculations can be carried out according to the following expression

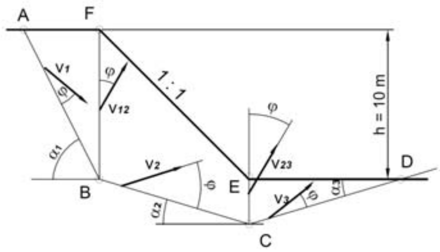
$$F_2 = \frac{c \cdot \cos\phi \cdot (v_1 l_{AB} + v_2 l_{BC} + v_3 l_{CD} + v_{12} l_{BF} + v_{23} l_{CE}) - \gamma \cdot \min(0, V_2 \cdot v_{2y}) - \gamma \cdot \min(0, V_3 \cdot v_{3y})}{\gamma \cdot (V_1 \cdot v_{1y} + \max(0, V_2 \cdot v_{2y}) + \max(0, V_3 \cdot v_{3y}))} \tag{7}$$

where symbols min() and max() denote the algebraic minimum or maximum of zero and the dot product in the parentheses; this assures that the negative terms are added to the numerator and the positive ones are embedded in the denominator.

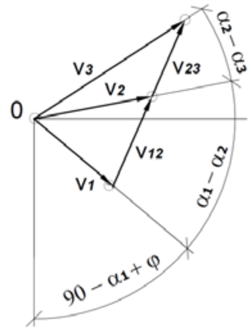
As this approach yields an upper bound to the factor of safety, we search for the minimum and F_2, with the lengths of discontinuities AB, BC, CD, BF, and CE being varied. Deterministic results for the two examples of slopes in Fig. 1 are presented in Fig. 2 as functions of the internal



(a)



(b)



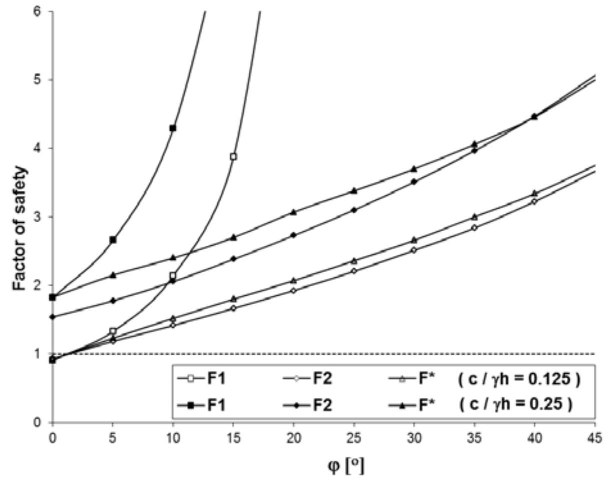
(c)

Fig. 1 Example mechanisms of collapse: (a) slope A; (b) slope B; and (c) hodograph

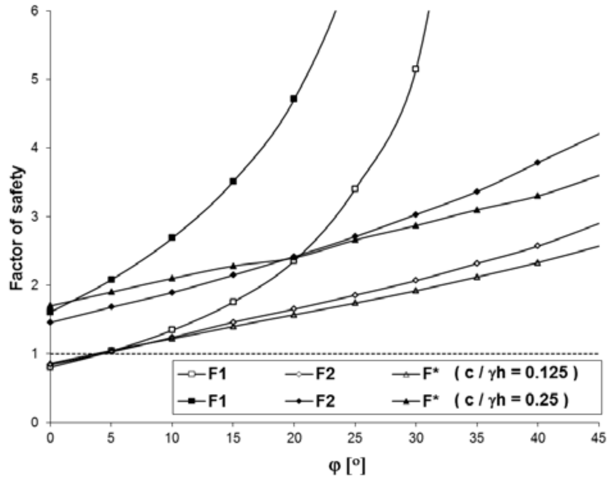
friction angle of the soil in the slopes. Cohesion and the unit weight of the soil are characterized with dimensionless parameter $c/\gamma H$ equal to 0.125 and 0.25. In addition to safety factors calculated according to definitions in Eqs. (3) and (5), the common definition in Eq. (1) was used, and this factor is denoted in the figures as F^*. All of the definitions yield a similar magnitude of the safety factor for undrained analysis ($\phi = 0$), but the safety factor defined in Eq. (3) increases with an increase in the internal friction angle at a much larger rate than the other two. It is rather interesting to notice that for slope A (the gentler of the two) the classical definition in Eq. (1) tends to give slightly higher values of F than that in Eq. (5), whereas this trend is reversed for the steeper slope B. Still, the new definition in Eq. (5) and the classical one in Eq. (1) yield the values of F fairly close to one another. This comparison indicates that the new definition might be an acceptable alternative to that commonly used in practice in Eq. (1), when the limit analysis is used.

4. Probabilistic considerations

The calculations in the previous section were performed for a set of unique material properties of the soil and they led to a well-defined deterministic value of the factor of safety. However, the strength parameters and unit weight of the soil vary in the field, and while deterministic analyses have prevailed in practice, it is natural to ask how the uncertainties in the material properties affect the outcome of the analysis (Christian *et al.* 1994). An approach that considers uncertainties will not result in one well-defined factor of safety; rather, it will lead to a probability of the loss of stability p_f



(a)



(b)

Fig. 2 Comparison of Safety Factors F^*, F_1 and F_2 [Eqs. (1), (3) and (5))]: (a) slope A; and (b) slope B

$$P_F = P[F \leq 1] \tag{8}$$

defined as the probability P that the safety factor F will reach or drop below 1. This probability should not exceed some postulated value, which for slopes with insignificant consequences of failure may be set as large as 10^{-2}, and be orders of magnitude smaller for slopes with severe consequences of failure. If the probability density function f_F of the factor of safety is known, then the loss of stability can be expressed as

$$P_F = P[F \leq 1] = \int_{-\infty}^{1} f_F dF \tag{9}$$

Distribution f_F is not known in slope stability problems, and Monte Carlo simulations can be

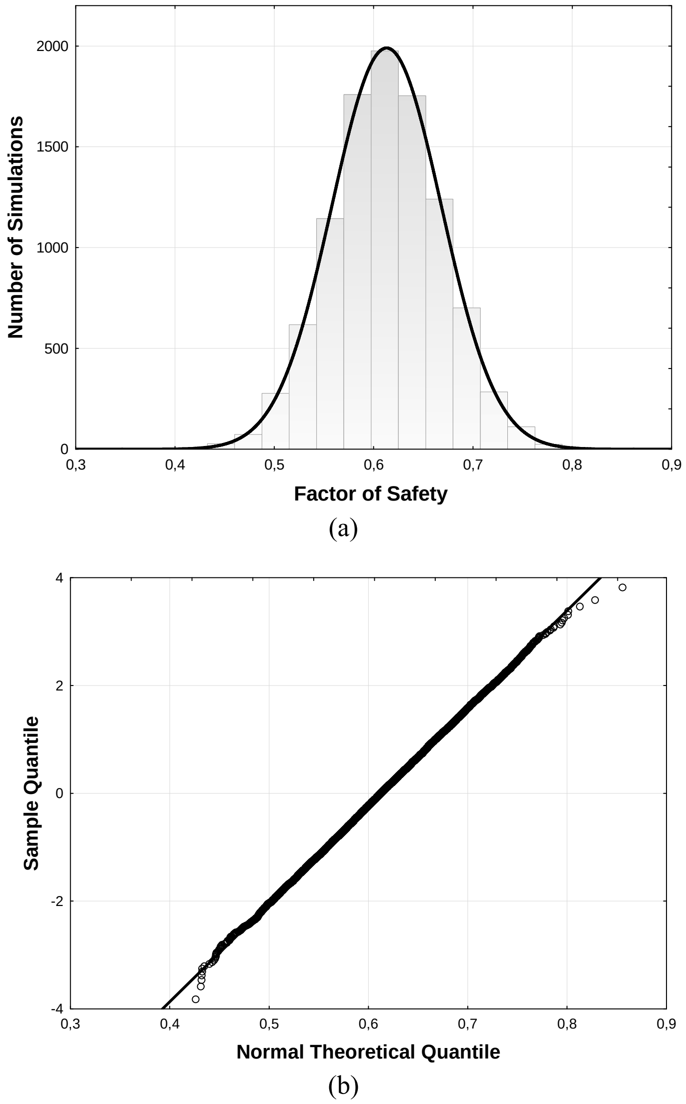
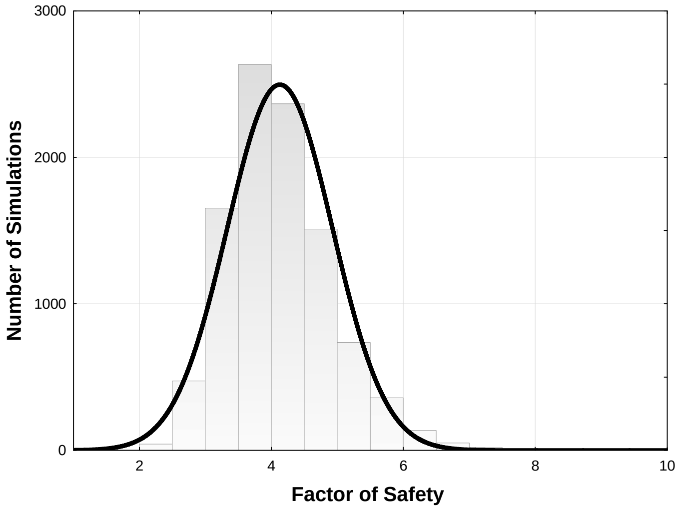
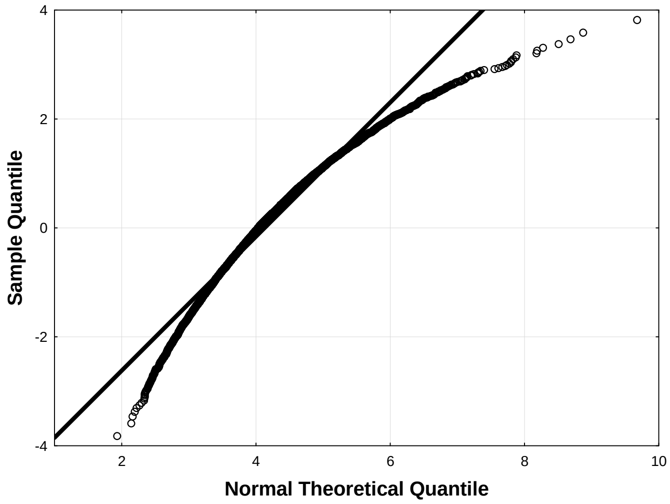


Fig. 3 (a) Histogram of F_2 from Monte Carlo simulations for c = 10 kPa and φ = 0 with the normal distribution fit; and (b) simulation versus normal distribution quantiles indicating good fit

carried out to gain insight into the statistical distribution of F. To shed some light on the statistical behavior of the factors of safety in Eqs. (3) and (5), we repeat the analysis for 110 pairs of strength parameters c and ϕ treated as random variables with normal distributions and coefficient of variation of 0.1 (unit weight γ was assumed not to vary). A Monte Carlo simulation was carried out, where samples of 10,000 safety factors F_1 and F_2 (Eqs. (3) and (5)) were generated for each pair of c and ϕ for slopes in Fig. 1. Acceptable magnitude of probability of failure was set to 10^{-3}, and the results for the slope in Fig. 1(a), in terms of probabilities



(a)



(b)

Fig. 4 (a) Histogram of F_2 from Monte Carlo simulations for $c = 10$ kPa and $\varphi = 36°$ with the normal distribution fit; and (b) simulation versus normal distribution quantiles indicating unacceptable fit

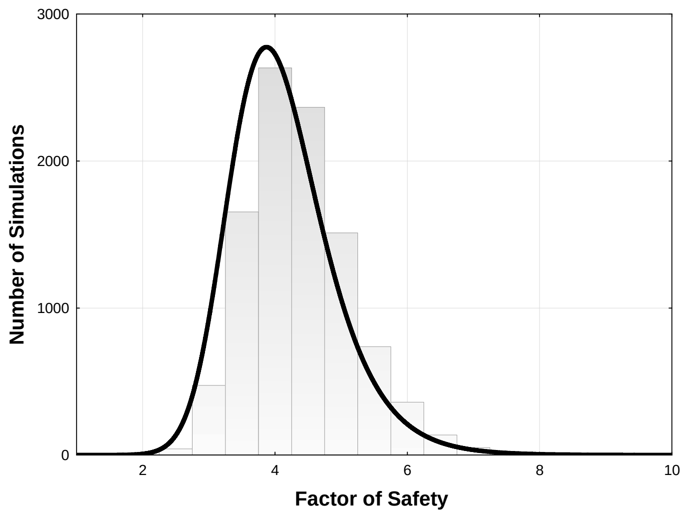
$$P\{F<1\} \approx \frac{1}{N_0}\sum_{i=1}^{N_0} p_i\big(F(c_i,\phi_i)<1\big), \qquad N_0 = 10{,}000$$
$$p_i\big(F(c_i,\phi_i)\big) := \begin{cases} 1 & F(c_i,\phi_i)<1 \\ 0 & F(c_i,\phi_i)\geq 1 \end{cases} \tag{10}$$

are indicated in Table 1 for the stability loss defined as $F_1 \leq 1$, and in Table 2 for $F_2 \leq 1$. Calculated probabilities for both definitions of the safety factor are similar, with values of P ($F_1 \leq 1$) being only slightly larger than P ($F_2 \leq 1$) for the slope in Fig. 1(b).

There is a merit in trying to find probability density functions for factors of safety, as the probability of failure can be calculated easily from Eq. (9) for known distributions, without a need for Monte Carlo simulations. Monte Carlo simulations can be cumbersome if the problem requires sophisticated analysis. Therefore, we will try to assess whether the distribution of the probability density function for the safety factor expressed in Eq. (5) can be predetermined (based on known distributions of c and ϕ), and the probability of stability loss can be calculated directly from Eq. (9), without Monte Carlo simulations.

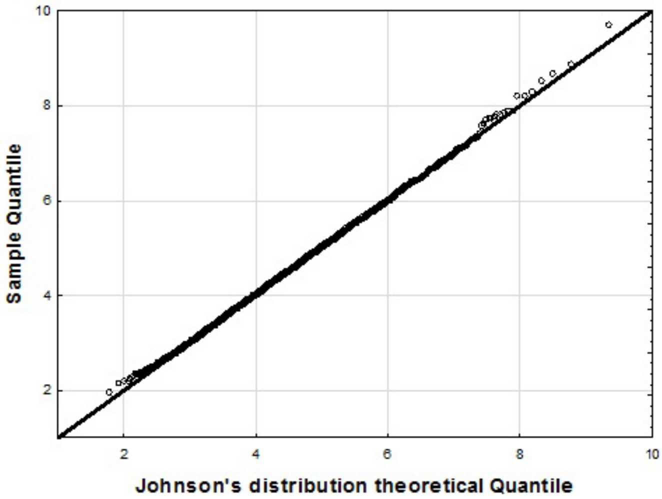
Probabilities of failure generated in Monte Carlo simulation for the first slope (Fig. 1(a)) were subjected to statistical analysis; only the results for the pairs of parameters with average cohesion c = 10 kPa are presented. Based on the Kolmogorow-Smirnow test, the probability distribution of F_1 was found to be approximately normal only for results with ϕ less than about 8° to 10°, depending on the value of c. For larger ϕ, the Johnson distribution was found acceptable up to about ϕ = 16°, but beyond this value, no acceptable fit was found into any standard distribution. Coefficient F_2 conformed to the normal distribution for ϕ up to about 12°. An example of the histogram of the simulated factor of safety (Monte Carlo simulations) and its normal distribution approximation are illustrated in Fig. 3(a), with the quantiles for the two sets of data compared in Fig. 3(b).

For larger ϕ, leptokurtic and asymmetric properties of the distribution became evident, and an attempt to fit the normal distribution into the Monte Carlo simulation data failed, which is illustrated in Fig. 3. However, a fit of the simulated data with the Johnson distribution was found acceptable for the entire range beyond $\phi = 12°$, this is illustrated in Fig. 3(a), with the comparison of the respective quantiles in Fig. 5(b). Comparison of probabilities of failure from Monte Carlo simulations and those approximated by normal and Johnson distributions, with failure defined by $F_2 \leq 1$, are illustrated in Table 3. Based on this limited attempt, we conclude that the probability density function cannot be assumed *a priori*, and the normal distribution of the strength parameters



(a)

Fig. 5 (a) Histogram of F_2 from Monte Carlo simulations for $c = 10$ kPa and $\varphi = 36°$ with the Johnson distribution fit; and (b) simulation versus Johnson distribution quantiles indicating good fit



(b)

Fig. 5 Continued

does not imply the normal distribution of the safety factor. A similar test was performed on the factors of safety defined in Eq. (1) and obtained from Monte Carlo simulations of slopes with cohesion in the range from 1 to 25 kPa, and internal friction angle from 0 to 45°. The outcome indicated that the factor of safety defined by Eq. (1) can be described with the normal distribution in a wider range of ϕ, i.e., up to about 25°-27°, depending on cohesion. Beyond this limit, Johnson distribution describes well the safety factor distribution.

Table 1 Calculated probabilities for loss of stability of slope A using Monte Carlo simulations and definitions of F in Eqs. (3) and (5)

$P\ (F_1 < 1)$		φ (°)									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
c (kPa)	0	1	1	1	1	0.9785	0.5045	0.0241	0.0008	0.0001	0
	5	1	1	1	0.8453	0.0402	0	0	0	0	0
	10	1	1	0.8536	0.0153	0	0	0	0	0	0
	15	1	0.9873	0.0437	0	0	0	0	0	0	0
	20	0.9998	0.2926	0.0002	0	0	0	0	0	0	0
	25	0.8383	0.0091	0	0	0	0	0	0	0	0
	30	0.2032	0	0	0	0	0	0	0	0	0
	35	0.0174	0	0	0	0	0	0	0	0	0
	40	0.0008	0	0	0	0	0	0	0	0	0
	45	0	0	0	0	0	0	0	0	0	0
	50	0	0	0	0	0	0	0	0	0	0

Table 1 Continued

$P\ (F_2 < 1)$		φ (°)									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
c (kPa)	0	1	1	1	1	0.986	0.5132	0.0311	0.0008	0	0
	5	1	1	1	0.8434	0.0018	0	0	0	0	0
	10	1	1	0.8386	0.0003	0	0	0	0	0	0
	15	1	0.9769	0.0314	0	0	0	0	0	0	0
	20	0.9998	0.275	0	0	0	0	0	0	0	0
	25	0.8416	0.002	0	0	0	0	0	0	0	0
	30	0.1984	0.0001	0	0	0	0	0	0	0	0
	35	0.0144	0	0	0	0	0	0	0	0	0
	40	0.0001	0	0	0	0	0	0	0	0	0
	45	0.0001	0	0	0	0	0	0	0	0	0
	50	0	0	0	0	0	0	0	0	0	0

Table 2 Calculated probabilities for loss of stability of slope B using Monte Carlo simulations and definitions of F in Eqs. (3) and (5)

$P\ (F_1 < 1)$		φ (°)									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
c (kPa)	0	1	1	1	1	1	0.9999	0.9989	0.9086	0.5829	0.473
	5	1	1	1	1	1	0.9042	0.4536	0.0678	0	0
	10	1	1	1	0.9973	0.6487	0.0863	0.0013	0.0001	0	0
	15	1	1	0.9796	0.3472	0.0104	0.0005	0	0	0	0
	20	1	0.9682	0.2665	0.0063	0	0	0	0	0	0
	25	0.9942	0.3528	0.0064	0	0	0	0	0	0	0
	30	0.6451	0.0269	0	0	0	0	0	0	0	0
	35	0.1349	0.0006	0	0	0	0	0	0	0	0
	40	0.0114	0.0001	0	0	0	0	0	0	0	0
	45	0.0015	0	0	0	0	0	0	0	0	0
	50	0	0	0	0	0	0	0	0	0	0
$P\ (F_2 < 1)$		φ (°)									
		0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
c (kPa)	0	1	1	1	1	1	1	0.9985	0.9974	0.8886	0.4236
	5	1	1	1	1	0.9999	0.882	0.4577	0.0001	0	0
	10	1	1	1	0.9957	0.6381	0.0495	0	0	0	0
	15	1	1	0.9768	0.3446	0.0033	0	0	0	0	0
	20	1	0.9649	0.2544	0.0018	0	0	0	0	0	0
	25	0.9912	0.3533	0.0022	0	0	0	0	0	0	0

Table 2 Continued

c (kPa)	30	0.6337	0.0247	0	0	0	0	0	0	0	0
	35	0.1304	0.0004	0	0	0	0	0	0	0	0
	40	0.0116	0	0	0	0	0	0	0	0	0
	45	0.0008	0	0	0	0	0	0	0	0	0
	50	0	0	0	0	0	0	0	0	0	0

Table 3 Probability of failure ($F_2 \leq 1$) from simulation and from fitted distributions for c = 1.0 kPa

φ (^{o})	Calculated by Monte Carlo simulation	Estimated using normal distribution	Estimated using Johnson's distribution
10	1	1	1
12	1	1	1
14	1	0.999996	0.999986
16	0.9915	0.993553	0.991062
18	0.8400	0.835516	0.837534
20	0.4546	0.439733	0.455624
22	0.1582	0.158657	0.156857
24	0.0421	0.050883	0.041154
26	0.0075	0.016099	0.008622
28	0.0020	0.006571	0.004581
30	0.0004	0.003402	0.000541
32	0.0001	0.002188	0.000901
34	0	0.001700	0.000246
36	0	0.001908	0.000008
38	0	0.002449	0.000005

5. Conclusions

A successful attempt was made at defining the factor of safety in terms of the work rate components written for an incipient collapse mechanism of a slope. Defining the safety factor as a ratio of all work resisting the collapse to that causing the failure allows one to remove the peculiarities from an earlier definition that took the ratio of the internal work rate (dissipation) to the work rate of all external forces. It was also shown that the safety factor calculated for two slopes using the new definition in terms of the work rate components falls closely to the commonly used definition in terms of soil strength parameters. The new definition was found to slightly underestimate the classical one for a gentle (1:2) slope, and it marginally overestimated the classical value for a steeper (1:1) slope. The new definition, however, has advantages when applied in the kinematic approach of limit analysis, as the solution procedures in limit analysis require explicit formulation of the work rate terms, and these terms are used directly in the new definition.

The factor of safety is a deterministic measure of the safety margin, whereas the properties of the soil are variable throughout the field. To reconcile this inconsistency a probabilistic assessment of the slope stability was attempted, with the cohesion and internal friction angle assumed to have normal distributions with the coefficient of variation of 0.1. Monte Carlo simulations were then carried out to assess whether the probability density function of the factor of safety, calculated according to the proposed definition, can be described with a standard distribution that might be determined *a priori*, without the need for the elaborate Monte Carlo simulations. If such a description was possible then the probability of loss of stability (safety factor reaching or dropping below unity) could be calculated easily through analytical means, by integrating the probability density function in the range of $-\infty$ to 1. For the proposed definition of the safety factor, the Monte Carlo-simulated factor of safety could be approximated with the normal distribution only for rather small internal friction angles (less than 12°), whereas the Johnson distribution could be used for larger angles ϕ. These are very limited results for two slopes, but based on this outcome, it is unlikely that the probability of slope collapse could be determined with acceptable confidence based on known distributions of the strength properties alone.

Acknowledgements

The authors would like to thank Prof. Radoslaw L. Michalowski of the University of Michigan for his help in preparing this manuscript for publication.

References

Bell, J.M. (1966), "Dimensionless parameters for homogeneous earth slopes", *J. Soil Mech. Found. Div., Am. Soc. Civ. Eng.*, **92**(5), 51-65.

Christian, J.T., Ladd, C.C. and Baecher, G. (1994), "Reliability applied to slope stability analysis", *J. Geotech. Eng.*, **120**(12), 2180-2207.

Derski, W., Izbicki, R., Kisiel, I. and Mróz, Z. (1988), *Rock and Soil Mechanics*. PWN, Warszawa.

Izbicki, R.J. (1983), "Application of limit analysis method in stability of slopes", *Archiwum Hydrotechniki*, **29**(3), 275-296. [in Polish]

Karal, K. (1977), "Application of energy method", *J. Geotech. Eng. Div.*, **103**(5), 381-397.

Klar, A., Aharonov, E., Kalderon-Asael, B. and Katz, O. (2011), "Analytical and observational relations between landslide volume and surface area", *J. Geophys. Res.*, **116**(F2), F02001, AGU.

Lambe, T.W. and Whitman, R.V. (1968), *Soil Mechanics*, Wiley, New York.

Michalowski, R.L. (1989), "Three-dimensional analysis of locally loaded slopes", *Géotech.*, **39(**1), 27-38.

Michalowski, R.L. (2002), "Stability charts for uniform slopes", *J. Geotech. Geoenv. Eng.*, **128**(4), 351-355.

Michalowski, R.L. (2010), "Limit analysis and stability charts for 3D slope failures", *J. Geotech. Geoenv. Eng.*, **136**(4), 583-593.

Taylor, D.W. (1948), *Fundamentals of Soil Mechanics*, Wiley, New York.

PL