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Generalized Schmertmann Equation for settlement estimation of shallow footings in saturated and unsaturated sands

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Abstract. Simple relationships are proposed in this paper by modifying the Schmertmann's equation for settlement estimations of footings (i.e., $B/L \approx 1$) carrying vertical loads in saturated and unsaturated sandy soils. The modified method is developed using model plate load tests (PLTs) and cone penetration tests (CPTs) results conducted in saturated and unsaturated sand in a controlled laboratory environment. Seven in-situ large-scale footings tested under both saturated and unsaturated conditions in sands were used to validate the proposed technique. The results of the study are encouraging as they provide reliable estimates of the settlement of shallow footings in both saturated and unsaturated sands using the conventional CPT results.

Keywords: shallow footings; unsaturated sand; matric suction; settlement; PLT; CPT

1. Introduction

Shallow foundations such as isolated footings are preferred for light and low-rise buildings in comparison to deep foundations where relatively favorable soils are available. The main function of shallow foundations is to transfer the loads from the superstructure to the supporting soil with an adequate factor of safety against bearing capacity failure. In addition, the footings settlement should be lower than an allowable value, which is typically recommended as 25 mm for sands (Terzaghi and Peck 1967). In most cases of shallow foundations design, it is the settlement that is the governing parameter rather than the bearing capacity, particularly for sands. The settlements of footings in sands are typically estimated from in-situ tests such as plate load tests (PLTs), standard penetration tests (SPTs), or cone penetration tests (CPTs) without considering the influence of capillary stresses or matric suction (i.e., negative pore-water pressure with respect to atmospheric pressure) in the soil. The empirical correlations using the CPT results are more commonly used as a tool in the estimation of settlement of shallow footings in sands in engineering practice (Robertson 2009). A number of empirical equations are available in the geotechnical literature that can be used for the estimation of the settlement of footings in sands using the CPTs results (for example, Meyerhof 1956, Schmertmann *et al*. 1978, Mayne and Illingworth 2010). Schmertmann *et al*. (1978) method used the same correlation factor to relate the cone resistance, q_c to the

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modulus of elasticity, E_s of soils. The settlement 1994 prediction session held in Texas clearly demonstrated the deficiencies in the present settlement prediction or estimation methods (Das and Sivakugan 2007, and Swamy *et al*. 2011).

The conventional methods are particularly conservative for unsaturated sands as they do not take account of the influence of matric suction while estimating settlements of shallow footings. Several researchers have used calibration chambers with flexible walls to conduct CPTs in either dry or saturated soils (Schmertmann 1976, Bellotti *et al*. 1982, Houlsby and Hitchman 1988, Iwasaki *et al*. 1988, Parkin 1988, Schnaid and Houlsby 1991). A calibration chamber with rigid walls has also been used by Salgado *et al*. (1998) for CPTs in saturated sand. Hryciw and Dowding (1987) conducted a series of CPTs in partially saturated sands and found that the capillary tensions (i.e., matric suction) significantly increase the penetration resistance of saturated or dry condition. Miller *et al*. (2002) and Pournaghiazar *et al*. (2012) used a new calibration chamber to perform CPTs in unsaturated soils where matric suction values are controlled and demonstrated that the measured penetration resistances were influenced by the capillary stresses. They also suggested that the measured cone penetration values obtained in the chamber can be related to the equivalent field values to be used in practice. In addition, several researchers carried out investigations on the bearing capacity of unsaturated soils (Broms 1963, Steensen-Bach *et al*. 1987, Oloo *et al*. 1997, Costa *et al*. 2003, and Mohamed and Vanapalli 2006). All these studies have shown a significant contribution of matric suction to increase the bearing capacity of unsaturated soils. Recent studies also show that matric suction within the stress bulb region of a footing substantially influences the settlement behaviour of footings that are located above the groundwater level (Vanapalli 2009, Oh and Vanapalli 2011).

The focus of the present study is to provide experimental evidence to demonstrate the influence of the matric suction on the settlement behaviour of footings in unsaturated sands. Simple relationships in a form of correlation factors are also proposed to be used for settlement estimations of footings in both saturated and unsaturated sands.

2. Tested material

The basic properties of the sand used in the present study are summarized in Table 1. The tested soil is classified according to the Unified Soil Classification System (*USCS*) as poorly graded sand (fine sand).

Table 1 Summary of properties of the tested soil

Parameter or soil properties	Value
Average dry unit weight, γ_d , kN/m³	16.02
Min. dry unit weight, $\gamma_{d\,(\mathrm{min})}$, kN/m³	14.23
Max. dry unit weight, $\gamma_{d\,(\mathrm{max})}$, kN/m³	17.25
Optimum water content, *o.w.c*, % (*Standard Proctor Test*)	14.6
Void ratio, *e* (*after compaction*)	0.62 – 0.64
Effective cohesion, *c'*, kN/m^2 (*Drained condition*)	0.6
Effective peak internal friction angle, ϕ'(°) (*Drained condition*)	35.3

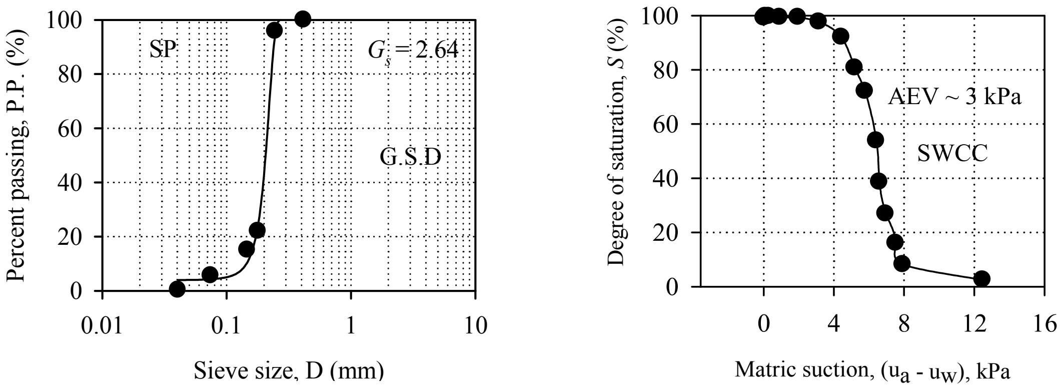

Fig. 1 The grain-size distribution (G.S.D) and the Soil-Water Characteristic Curve of the tested soil (SWCC)

3. Test equipment

3.1 Setup and Test Tank (UOBCE - 2006)

Model footing tests were conducted by Mohamed and Vanapalli (2006) determine the stress versus settlement response using a specially designed equipment (UOBCE-2006; the University of Ottawa Bearing Capacity Equipment) (see Fig. 2).

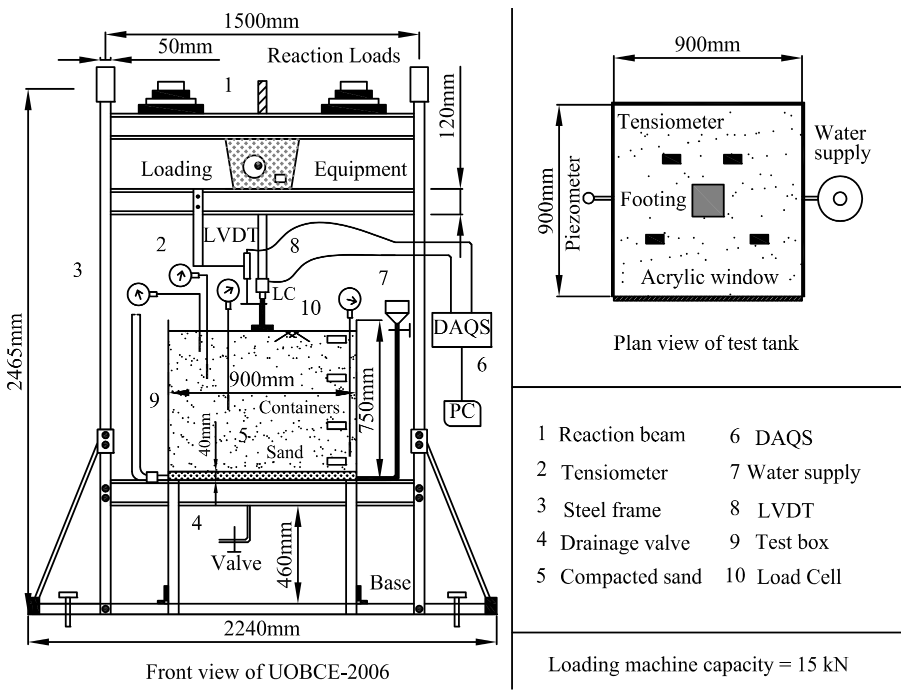

Fig. 2 Schematic to illustrate the details of the UOBCE-2006

The model footings of sizes 100 mm × 100 mm and 150 mm × 150 mm were placed on the surface the compacted sand in the test tank and tested. The test tank has a system for saturating (matric suction = 0 kPa) and desaturating the sand (matric suction = 1 to 6 kPa). The equipment consists of a rigid-steel tank of 900 mm (length) × 900 mm (width) × 750 mm (depth). The test tank can hold up to 1.2 tons of a compacted sandy soil. The sand in the test tank was compacted manually using a 5-kg flat steel-plate. The optimum water content value of 14.6 % determined from Standard Proctor test was simulated in the test tank to achieve an average dry density of 16.02 kN/m^3 approximately. A Linear Variable Displacement Transducer (LVDT) was attached to the vertical loading arm and the tip of the LVDT was placed directly on the surface of the model footing to measure the settlement, δ as the load is being applied. A load cell (LC) capable of measuring 15 kN was mounted on the loading arm to measure the load being applied. Both the LVDT and the LC were connected to a data acquisition system (DAQS) to monitor and record data during testing (i.e., the DAQS is compatible with LabView software) and a computer. The model footing tests were carried out under different average matric suction values in the stress bulb region (i.e., 1.5B) of each model footing (i.e., 0 kPa, 2 kPa, 4 kPa and 6 kPa where the water table was located at 150 mm, 200 mm, 350 mm and 600 mm respectively.

3.2 Setup and Test Tank (UOBCE - 2011)

This equipment (i.e., UOBCE-2011) has twice the loading capacity of the UOBCE-2006 described in section 3.1. The schematic of the equipment is shown in Fig. 3. This equipment consists of a rigid-steel frame made of rectangular sections with a thickness of 6 mm and a test tank of 1500 mm (length) × 1200 mm (width) × 1060 mm (depth). The test tank can hold up to 3 tons of sand. The sand in the test tank was compacted to achieve similar compaction properties as in the UOBCE-2006 test tank (i.e., optimum water content ~14.6 % and an average dry density

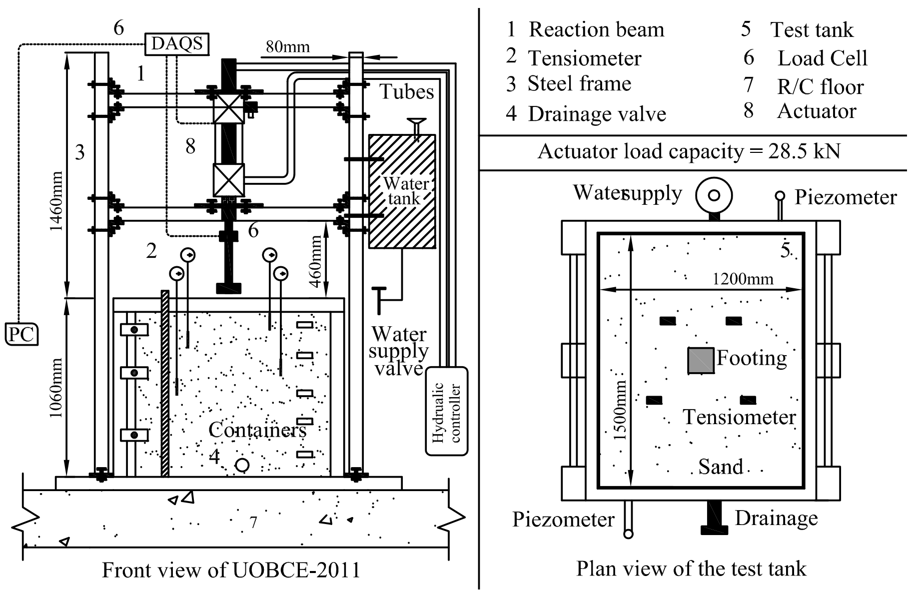

Fig. 3 Schematic to illustrate the details of the UOBCE-2011

of 16.02 kN/m^3). The capacity of the loading machine (i.e., Model 244-Hydraulic Actuator with a maximum stroke length of 270 mm connected to MTS controller) is 28.5 kN. The built-in (LVDT) and the (LC) were (DAQS) and a computer.

4. Laboratory Model Footing Tests (PLTs)

4.1 Surface and embedded PLTs under fully saturated soil condition

The water table was slowly raised from the base of the test tank (UOBCE-2006 and UOBCE-2011) through a 50 mm aggregate layer that was covered by a thin layer of a geotextile. The objective was to prevent the sand from being washed out through the aggregate during desaturation process (see Fig. 4). This technique facilitated escape of air from bottom to the surface layers of the soil in the test tank gradually to ensure saturated condition (i.e., $(u_a - u_w) = 0$ kPa). The adjustments of the water level in the test tank were inspected periodically using the piezometers (i.e., transparent plastic tubes attached to the test tank as in Figs. 2 and 3). The supply valve was closed once the water level reached the soil surface in the test tank. The applied stress and the settlement of the saturated soil were measured (for 100 mm × 100 mm and 150 mm × 150 mm PLTs) during the loading process of the model footings at a constant rate of 1.2 mm/min assuming drained condition. All Tensiometers in the test tank (as shown in Figs. 2 and 3) indicated zero matric suction values after saturation and during the testing period.

4.2 Surface and embedded PLTs under unsaturated soil conditions

The soil in the tank was first saturated as detailed in the previous section. The water table was then lowered down slowly (using drainage valves shown in Fig. 4) to different levels of depth from

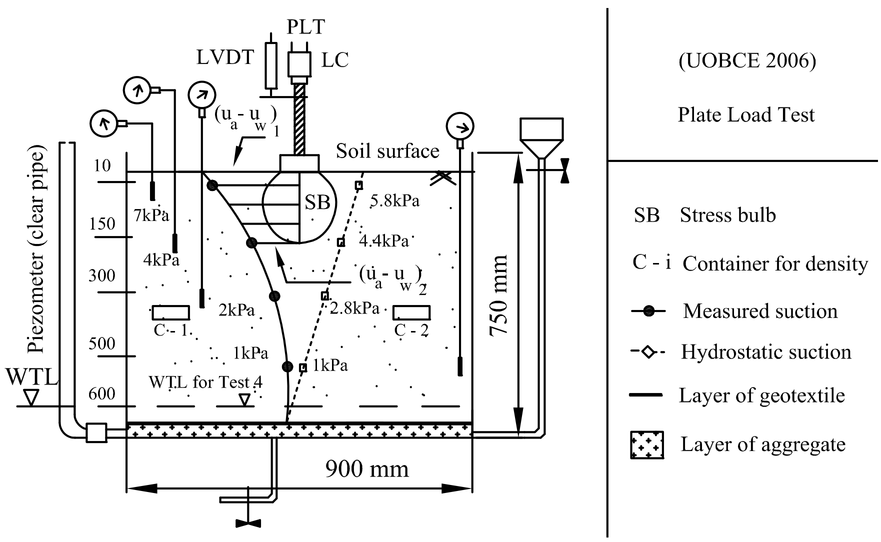

Fig. 4 Sectional view of the test tank of the UOBCE-2006 illustrating the variation of average suction of 6 kPa in the stress bulb zone of a surface PLTs

Table 2 Typical data from the test tank for an average matric suction of 6 kPa in the stress bulb zone (i.e., 1.5B) (100 mm × 100 mm) surface model footing using UOBCE-2006

Parameter or property					
[1]D (mm)	[2]γ_t (kN/m^3)	[3]γ_d (kN/m^3)	[4]w (%)	[5]S (%)	[6]$(u_a - u_w)_{AVR}$ (kPa)
10	18.17	15.94	14.0	58	6.0
150	18.75	15.85	18.3	76	4.0
300	19.27	16.07	20.0	86	2.0
500	19.40	15.77	23.0	94	1.0
600	19.74	15.95	23.8	100	0

[1]Depth of a Tensiometer from the soil surface; [2]total unit weight; [3]dry unit weight; [4]water content; [5]degree of saturation, [6]average matric suction in the stress bulb zone

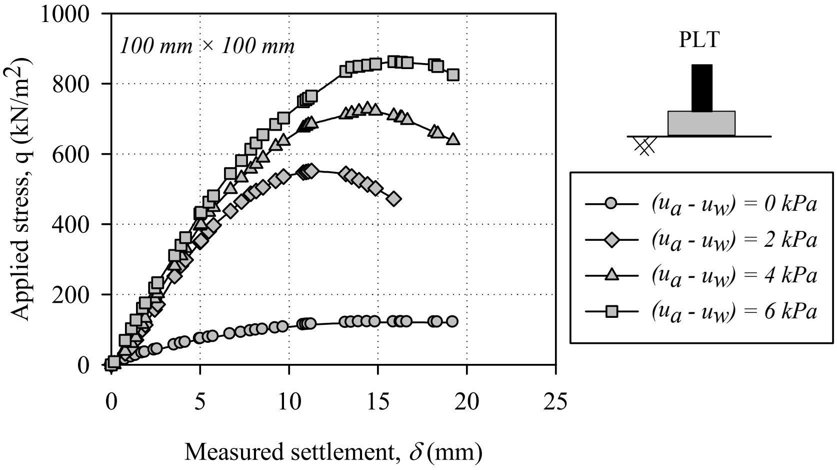

Fig. 5 Applied stress versus settlement behaviour of surface model footing tests (PLTs) of 100 mm × 100 mm

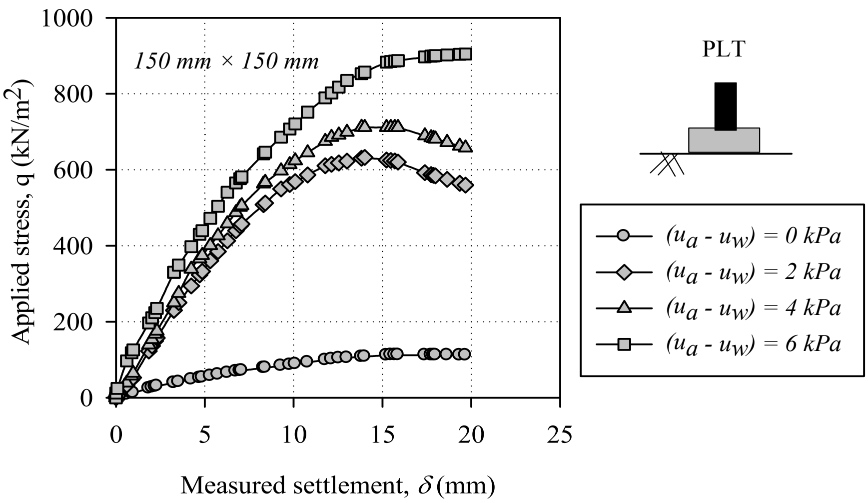

Fig. 6 Applied stress versus settlement behaviour of surface model footing tests (PLTs) of 150 mm × 150 mm

the soil surface to achieve different desired matric suction values. Equilibrium conditions with respect to matric suction value in the stress bulb zone (i.e., depth of 1.5*B*) were typically achieved in a time period of 24 to 48 hours. As reported by Poulos and Davis (1974), Steensen-Bach (1987), and Agarwal and Rana (1987) the zone of depth in which the stresses due to loading are predominant is 1.5*B*. The applied stress and the settlement were measured during the tests under different average matric suction values (i.e., 2 kPa, 4 kPa and 6 kPa in the 1.5*B* region). While the matric suction values were measured using the Tensiometers, the gravimetric water contents at equilibrium conditions were determined approximately at the same levels where soil specimens were collected in small aluminum cups with perforations that were embedded inside the compacted sand prior to conducting the tests. These cups were placed within the proximity of the model footing (PLT). Fig. 4 shows the cross-section of the test tank (i.e., UOBCE-2006) in a schematic form and provides the details of the placement of Tensiometers and the aluminum cups of 15 mm (height) × 40 mm (diameter) at different depths (labeled as C-1, C-2…etc). This figure also shows the variation of matric suction (i.e., measured and hydrostatic matric suction profiles) with respect to depth in the test tank. Table 2 summarizes the data set of results in which the average matric suction value in the stress bulb zone was 6 kPa (achieved by placing the water table at a depth of 600 mm). The results of the experimental work conducted using two different model footings (i.e., surface PLTs) of 100 mm × 100 mm and 150 mm × 150 mm in the laboratory are plotted in Figs. 5 and 6 respectively.

The stress versus settlement relationships obtained using the two model footings in the laboratory show dramatic increase in the applied stress, q and a significant decrease of the measured settlement, δ with an increase in the matric suction value below the footing. It should be noted that the matric suction values (i.e., 2 kPa, 4 kPa and 6 kPa) for each test represents an average value of the capillary stress with the stress bulb depth which is equal to 1.5 times the width, B of each model footing (i.e., $(u_a - u_w)_{\mathrm{AVR}} = [(u_a - u_w)_1 + (u_a - u_w)_2]/2$) (see Fig. 4).

Embedded model footing tests (PLTs) of 150 mm × 150 mm were also conducted by placing the model footing at a depth of 150 mm below the soil surface in the bearing capacity equipment, which is referred to as UOBCE-2011. Table 3 summarizes the data set of results in which the average matric suction value in the stress bulb zone was 6 kPa achieved by placing the water table at a depth of 800 mm. These tests were conducted to investigate the influence of matric suction on

Table 3 Typical data from the test tank for an average matric suction of 6 kPa in the stress bulb zone (i.e., 1.5*B*) (150 mm × 150 mm) embedded model footing using UOBCE-2011

Parameter or property					
[1]D (mm)	[2]γ_t (kN/m^3)	[3]γ_d (kN/m^3)	[4]w (%)	[5]S (%)	[6]$(u_a - u_w)_{\mathrm{AVR}}$ (kPa)
12	18.16	16.20	12.1	53	8.0
150	19.00	16.24	17.0	75	7.0
355	19.20	16.13	19.0	82	5.0
500	19.50	16.12	21.0	91	2.0
700	19.74	16.03	23.1	98	1.0
800	19.75	15.95	23.8	100	0

[1]Depth of a Tensiometer from the soil surface; [2]total unit weight; [3]dry unit weight; [4]water content; [5]degree of saturation, [6]average matric suction in the stress bulb zone

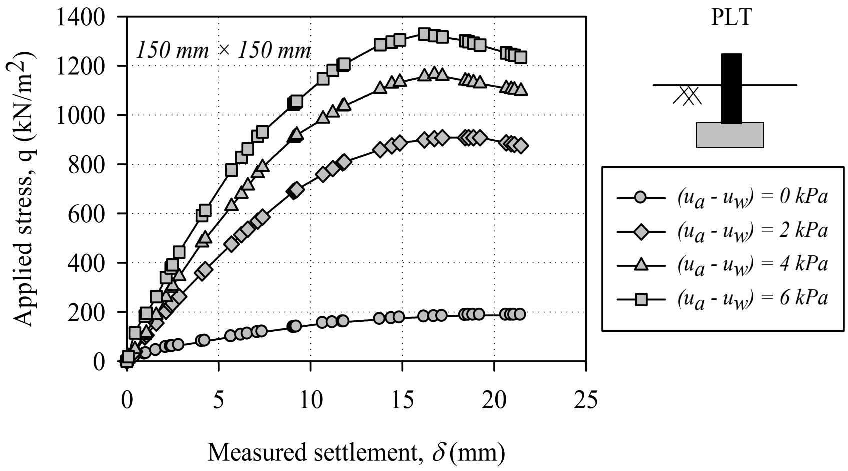

Fig. 7 Applied stress versus settlement behaviour embedded model footing tests (PLTs) of 150 mm × 150 mm

the applied stress and settlement behaviour of footings in sands. Fig. 7 presents the applied stress versus settlement for different tests using 150 mm × 150 mm embedded model footing. The matric suction values (i.e., 0 kPa, 2 kPa, 4 kPa and 6 kPa) for each test represent an average value of capillary stress in the proximity of the stress bulb (i.e., 1.5*B*) of the model footing in the UOBCE-2011.

The applied stress versus settlement relationships in Figs. 5, 6, and 7 show that the maximum applied stress (i.e., bearing capacity) increases with an increase in matric suction in the range of 0 to 6 kPa of the tested compacted sand. As the matric suction increases, it contributes to the component of the apparent cohesion in the shear strength of the unsaturated sand (Vanapalli *et al.* 1996).

5. Laboratory Cone Penetration Tests (CPTs)

In addition to the PLTs, cone penetration tests (CPTs) were also conducted in the same sand that was previously used for PLTs. The UOBCE-2006 was modified and a cone penetrometer was specially designed for conducting this research program. The cone penetrometer has a diameter of 40 mm and apex angle of the cone was 60°. The projected area of the cone was 1256 mm^2. A smooth shaft of 600 mm was connected to the cone along with the loading machine. Commercial manufacturers typically use cone diameter, d_c = 35.7 mm for penetrometers. The cone resistance, q_c results (form cone resistance, q_c versus penetration depth, d profile as shown in Fig. 8) can be used for the estimation of settlements of different footing sizes (e.g., ~ 150 mm × 150 mm to ~3000 mm × 3000 mm). The ASTMD5778 standard suggests using cone penetrometer sizes in the range of 35.7 mm to 43.7 mm. This guideline has encouraged the researchers of this study using 40 mm diameter cone. Lunne *et al.* (1997) stated that “cone penetrometers with diameters differing from the standard 35.7 mm are quite frequently used”. Therefore, a cone penetrometer of 40 mm in diameter was used since the test tank was large enough for conducting the CPTs with negligible size effects (width of test tank/diameter of cone ≈ 23).

A long stroke Linear Displacement Transducer (LDT) which can penetrate the sand up to 500 mm was connected with the cone penetrometer to measure the penetration depth. A series of CPTs conducted on the compacted sand (i.e., the same sand used for the PLTs with a relative density, D_r of 65 %) in the UOBCE-2006 under saturated and unsaturated sand conditions as shown in Fig. 8.

The cone penetrometer was pushed through the compacted sand at a constant rate of 1.2 mm/min. The saturated coefficient of permeability of sand is relatively high and the penetration rate used (1.2 mm/min) is relatively slow enough to assure a negligible pore-water pressure (assuming $q_c = q_t$). Iwasaki *et al.* (1988) performed CPT at 2 mm/sec to assure fully drained condition of Toyoura sand. Robertson and Cabal (2010) suggest that q_c may be assumed to be equal to q_t for sandy soils.

The sand in the test tank was compacted prior to testing as discussed earlier in section 3. The first series of tests (i.e., CPTs) was carried out under saturated condition (i.e., water level in the

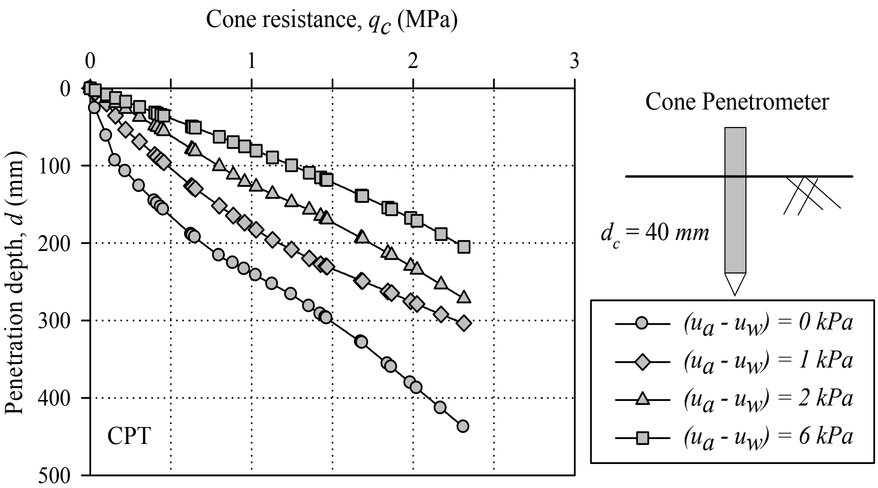

Fig. 8 Variation of cone resistance from CPTs with penetration depth in sand under saturated and unsaturated conditions

Table 4 Typical data from the test tank for an average matric suction of 6 kPa in the influence zone (i.e., 1.5B; B = footing width) of the cone penetrometer

Parameter or property					
[1]D (mm)	[2]γ_t (kN/m^3)	[3]γ_d (kN/m^3)	[4]w (%)	[5]S (%)	[6]$(u_a - u_w)_{AVR}$ (kPa)
10	18.17	15.94	14.0	59	6.0
150	18.76	15.85	18.3	76	4.0
300	19.20	16.07	19.5	83	2.0
500	19.30	15.77	22.5	93	1.0
600	19.74	15.95	23.8	100	0

[1]Depth of a Tensiometer from the soil surface; [2]total unit weight; [3]dry unit weight; [4]water content; [5]degree of saturation, [6]average matric suction in the stress bulb zone

Table 5 Data collected from PLTs and CPTs conducted in the laboratory using UOBCE-2006

Parameter or property						
Soil	B (m)	[1]d (m)	Water level (m)	[2]$(u_a - u_w)$ (kPa)	[3]D_r (%)	[4]$q_{c\ \mathrm{AVR}}$ (kN/m^2)
Surface model PLT from this research						
SP	0.1	0	0	0	65	118
SP	0.1	0	0.2	2.0	65	565
SP	0.1	0	0.6	6.0	65	805

[1]Depth of plate base from soil surface; [2]Measured matric suction in the test box; [3]Relative density; [4]Average cone penetration resistance (in kN/m^2) from laboratory CPTs

Table 6 Data collected from PLTs and CPTs conducted in the laboratory using UOBCE-2006

Parameter or property						
Soil	B (m)	[1]d (m)	Water level (m)	[2]$(u_a - u_w)$ (kPa)	[3]D_r (%)	[4]$q_{c\ \mathrm{AVR}}$ (kN/m^2)
Surface model PLT from this research						
SP	0.15	0	0	0	65	270
SP	0.15	0	0.2	2.0	65	900
SP	0.15	0	0.6	6.0	65	1235

[1]Depth of plate base from soil surface; [2]Measured matric suction in the test box; [3]Relative density; [4]Average cone penetration resistance (in kN/m^2) from laboratory CPTs

Table 7 Data collected from PLTs and CPTs conducted in the laboratory using UOBCE-2011

Parameter or property						
Soil	B (m)	[1]d (m)	Water level (m)	[2]$(u_a - u_w)$ (kPa)	[3]D_r (%)	[4]$q_{c\ \mathrm{AVR}}$ (kN/m^2)
Surface model PLT from this research						
SP	0.15	0.15	0	0	65	550
SP	0.15	0.15	0.45	2.0	65	1200
SP	0.15	0.15	0.8	6.0	65	1600

[1]Depth of plate base from soil surface; [2]Measured matric suction in the test box; [3]Relative density; [4]Average cone penetration resistance (in kN/m^2) from laboratory CPTs

test tank was at the soil surface and the matric suction = 0 kPa). The second series of tests was conducted under unsaturated conditions. Poulos and Davis (1974), Steensen-Bach *et al*. (1987), and Agarwal and Rana (1987) reported that the zone of depth in which the stresses due to loading are predominant is 1.5B. A representative value of cone resistance, q_c of each of the series of the CPTs results is taken as the average of the cone resistance in a depth of 1.5B. Typical set of results of the CPT tests with an average matric suction of 6 kPa is summarized in Table 4. The experimental results of the variation of the cone resistance, q_c with penetration depth are plotted in Fig. 8. From the measured CPTs results, the cone resistance, q_c under unsaturated conditions (average matric suction values of 1 kPa, 2 kPa and 6 kPa) found to be two to three times higher than the cone resistance for saturated condition. The cone resistance increased as the soil condition changed from saturated (0 kPa) condition to unsaturated (1 kPa, 2 kPa and 6 kPa) conditions in the capillary zone. The increase in the CPT values can be attributed to the contribution of the matric suction to the shear strength of the tested sand. These results were also consistent with the observations of Russell and Khalili (2006) and Pournaghiazar *et al*. (2012).

A summary of the sand properties along with the model plate footing tests (i.e., PLTs of 100 mm × 100 mm and 150 mm × 150 mm) and the cone penetration tests (CPTs) used in the development of the proposed technique (i.e., relationships) is presented in Tables 5, 6 and 7.

6. In-situ footing load tests and cone penetration tests (PLTs and CPTs)

6.1 *In-situ footing load tests* (*FLTs*)

Five large-scale footing load tests (FLTs) of 1.0 m × 1.0 m, 1.5 m × 1.5 m, 2.5 m × 2.5 m, 3.0 m × 3.0 m (north) and 3.0 m × 3.0 m (south) were conducted in-situ (in a sandy soil with some silt) by Giddens and Briaud (1994). These FLTs were used to validate the proposed technique for estimating the settlement of footings in the present study. The footings were loaded in sand at the Texas A&M University National Geotechnical Experimentation site (see data in Table 8). The in-situ results of the CPT conducted nearby the footing 3.0 m × 3.0 m (south) provided a measured q_c which is much lower compared to the q_c values of the other CPTs conducted close to each of the

Table 8 FLTs and CPTs data summarized from the literature and used to validate the proposed technique (Large-scale footings from Giddens and Briaud 1994)

Parameter or property						
Soil	B (m)	[1]d (m)	Water level (m)	[2]$(u_a - u_w)$ (kPa)	[3]D_r (%)	[4]$q_{c\ \mathrm{AVR}}$ (kN/m^2)
SP	1	0.5-1.5	4.9	~10	48	5400
SP	1.5	0.5-1.5	4.9	~10	46	6000
SP	2.5	0.5-1.5	4.9	~10	53	6500
SP	3 (north)	0.5-1.5	4.9	~10	57	7500

[1]Depth of footing from soil surface; [2] Estimated matric suction in-situ; [3]Relative density estimated from the CPT data; [4] Average measured cone penetration resistance (in kN/m^2) from in-situ CPTs

Table 9 FLTs and CPTs data summarized from the literature and used to validate the proposed technique (Large-scale footings from Bergdahl *et al*. 1985)

Parameter or property						
Soil	B (m)	[1]d (m)	Water level (m)	[2]$(u_a - u_w)$ (kPa)	[3]D_r (%)	[4]$q_{c\ \mathrm{AVR}}$ (kN/m^2)
S	0.55	0.4-1.1	1.5	~7	30	2300
S	1.6	0.4-1.1	1.5	~7	30	3000
S	2.3	0.4-1.1	1.5	~7	30	3300
S	0.55	0.4-1.1	1.5	~7	30	2300

[1]Depth of footing from soil surface; [2] Estimated matric suction in-situ; [3]Relative density estimated from the CPT data; [4] Average measured cone penetration resistance (in kN/m^2) from in-situ CPTs

other footings. Lee and Salgado (2002) also commented that the significant load under prediction resulting from the application of Schmertmann's method to the 3-m footing (south side) was due to the very low cone resistance at a depth equal to about 3 m observed in the CPT test used in the analysis. The researchers also speculated that these results may not be reflective of the true soil condition underneath the footing. Therefore, the 3.0 m × 3.0 m footing (south) and the CPT-07 in Giddens and Briaud (1994) results were not considered for the analysis in the presented study. In addition to these tests, four footing load tests (0.55 m × 0.65 m, 1.1 m × 1.3 m, 1.60 m × 1.80 m, and 2.30 m × 2.50 m) results (in a sandy soil) conducted in Fittja site in Sweden by Bergdahl *et al*. (1985) were investigated in the present research (see data in Table 9). Nevertheless, the footing 1.10 m × 1.80 m was omitted as their results showed a very low stress and large settlement because of the existence of clay lens underneath it, as the focus of the present research is directed towards testing only sandy soils. The sands at both sites (i.e., National Geotechnical Experimentation site in Texas and Fittja site) are considered to be in unsaturated conditions as the groundwater table levels were at 4.9 m and 1.5 m deep respectively (from the ground surface). The average matric suction value $(u_a - u_w)_{AVR}$ for the sand at the Texas site was assumed to be uniform as the water content values (reported by Giddens and Briaud 1994) above the groundwater table were also uniform and equal to 5%. The Soil-Water Characteristic Curve (SWCC) of Sollerod sand (tested by Steensen-Bach *et al*. 1987) which has similar grain-size distribution of sand in Texas site was used to back calculate the matric suction value of the Texas site. A matric suction value of 10 kPa that corresponds to 5% water content from the SWCC was used in the present study. The groundwater table at the Fittja site was at a shallow depth; therefore, a hydrostatic distribution was assumed and the average matric suction in the stress bulb zone was taken as 7 kPa.

6.2 *In-situ cone penetration tests* (*CPTs*)

Five cone penetration tests (CPTs) were conducted by Giddens and Briaud (1994) at Texas A&M University National Geotechnical Experimentation site near the locations of the spread footing load tests (FLTs) described earlier. The values of average cone resistance, q_c for the site was between 5400 kPa to 7500 kPa.

7. Settlement estimations using the available CPT-based methods

One of the most commonly used CPT-based methods in practice for estimating the settlement of shallow footings in sand is the Schmertmann *et al*. (1978) method. The modulus of elasticity, E_s typically increases with depth, and the stresses induced by the applied load decrease with depth. The stresses are typically negligible for depths greater than 1.5B (refer to Fig. 4 in Section 4). Schmertmann *et al*. (1978) suggested an equation (i.e., Eq. (1)) for the calculation of footing settlement in sands using average cone resistance over a depth of 2B from the base of the footing. Meyerhof (1974) also suggested an empirical equation (i.e., Eq. (2)) for estimating settlements of footings on sandy soils using the CPTs results. More recently, Mayne and Illingworth (2010) analyzed a large database which consisted of large-scale footings and proposed an empirical equation (i.e., Eq. (3)) for settlement estimation of large-scale footings in the range of 0.5 m $\leq B \leq$ 6.0 m on different sands. The key parameter required for the estimation of settlement of shallow footings is the modulus of elasticity (see Eq. (1)). Stress history, natural cementation, apparent or total cohesion due to matric suction and over consolidation are other significant factors that influence the modulus of elasticity of cohesionless soils (e.g., sands). The measurement of the modulus of elasticity from field tests is complicated and also expensive. The measurement of modulus of elasticity using laboratory tests is not only time consuming but also difficult due to the problems associated with sampling disturbance for sands. Due to these reasons, the CPT has been widely used as a tool to estimate the modulus of elasticity from empirical correlations (Meyerhof 1974, Schmertmann *et al.* 1978, and Robertson and Campanella 1986). The modulus of elasticity of sands can be estimated from the CPT results with a low degree of uncertainty in comparison to the SPT and PLT results. In practice, the soil modulus of elasticity is usually determined by multiplying the average cone resistance, q_c by a correlation factor such as f_i. Three CPTs-based equations reported in the literature for settlement estimations are listed below.

Schmertmann *et al.* (1978): $$\delta = C_1 C_2 (q_a - \sigma'_{z,d}) \sum_0^{2B} [(I_{zi} \Delta_{zi})/(E_s)] \tag{1}$$

Meyerhof (1974): $$\delta = (q_a)[B/(2q_c)] \tag{2}$$

Mayne and Illingworth (2010): $$\delta = (q_a)^2 [5B/(3q_c)] \tag{3}$$

where: C_1 = depth factor (i.e., $C_1 = 1 - 0.5[\sigma'_{z,\mathrm{d}} / (q_a - \sigma'_{z,d})$, C_2 = time factor (i.e., $C_2 = 1 - 0.21 \log [t/0.1]$), δ = settlement, (mm), q_a = footing pressure, (kN/m^2), $\sigma'_{z,d}$ = vertical effective stress at footing base level, (kN/m^2), E_s = elastic modulus of soil (i.e., $E_s = f_i \times q_{ci}$), (kN/m^2), I_{zi} = influence factor, B = footing width, (mm), q_{ci} = resistance of each layer, (kN/m^2), f_i = correlation factor, t = time, (year), and Δ_{zi} = thickness of the soil layer, (mm), q_c = average cone resistance, (kN/m^2) over an influence depth in the range of B to $2B$ below the footing base.

8. Proposed correlations between cone resistance and settlement of footings in saturated and unsaturated sands

Bowles (1996) suggested that the modulus of elasticity, E_s can be determined from CPT results using the general form of $E_s = C_3 + C_4 (q_c)$; where, $C_3 = 0$ and $C_4 = 2.5$–3.0 for normally consolidated sand. Vesic (1970) suggested that E_s varies with relative density according to the relation $C_4 = 2 \times (1 + D_r^2)$ and used it to correlate q_c to E_s. The proposed relationships of f_1 and f_2 as functions of the relative density are to be used to correlate the cone tip resistance, q_c with the modulus of elasticity, E_s. Eq. (4) is suggested to estimate the modulus of elasticity for saturated sands (i.e., $(u_a - u_w) = 0$ kPa) as in Eq. (4).

$$E_{s(sat)} = f_1 \times q_{c\,(sat)} \tag{4}$$

where: $E_{s\ (sat)}$ = modulus of elasticity for saturated sand, $f_1 = 1.5 \times ((D_r/100)^2 + 3)$ (i.e., f_1 is a correlation factor and D_r is the relative density in %), $q_{c\ (sat)}$ = average cone resistance under saturated sands condition within an influence zone, I_Z equal to $1.5B$ from the footing base level, and B = footing width.

Eq. (5) is suggested to estimate the modulus of elasticity for unsaturated sands (i.e., $(u_a - u_w) >$ 0 kPa) as in Eq. (5)

$$E_{s(unsat)} = f_2 \times q_{c\,(unsat)} \qquad (5)$$

where: $E_{s\,(unsat)}$ = modulus of elasticity for unsaturated sands, $q_{c\,(unsat)}$ = average cone resistance under unsaturated sands conditions within influence zone, I_Z equal to 1.5B.

The two correlation factors, f_1 and f_2 were proposed using the database of the experimental results of both PLTs and CPTs presented in this study. The form which was proposed by Vesic (1970) was re-examined using the laboratory investigation results. The constant numbers (i.e., 2 and 1 which are referred to herein as X_1 and X_2) were back calculated to be used in the correlation relationships. The correlation factors, f_1 and f_2 were developed as presented in Tables 10 and 11 for sands in saturated and unsaturated conditions, respectively. The general form of the proposed

Table 10 Database used for proposing the correlation factor, f_1

Analysis for saturated condition, $(u_s - u_w)$ = 0 kPa							
B (mm)	$E_{s(m)}$ (MPa) *measured from PLTs*	q_c (MPa) *measured from CPTs*	D_r (%) *determined based on density from the test tank*	X_1	X_2	$E_{s(e)}$ (MPa) *Estimated using f_1 in Eq. (4)*	$E_{s(e)}/E_{s(m)}$
[1]100	1.0	0.118	65	1.5	3.0	0.61	0.61
[1]150	1.45	0.27	65	1.5	3.0	1.39	0.96
[2]150	2.9	0.55	65	1.5	3.0	2.82	0.97
						AVR	0.85

[1] Surface square footing (*matric suction* = 0 kPa)
[2] Embedded square footing (*matric suction* = 0 kPa)
Thus, $f_1 = 1.5 \times ((D_r/100)^2 + 3.0)$ for saturated sands.

Table 11 Database used for proposing the correlation factor, f_2

Analysis for unsaturated conditions, $(u_a - u_w)$ > 0 kPa							
B (mm)	$E_{s(m)}$ (MPa) *measured from PLTs*	q_c (MPa) *measured from CPTs*	D_r (%) *determined based on density from the test tank*	X_1	X_2	$E_{s(e)}$ (MPa) *Estimated using f_2 in Eq. (5)*	$E_{s(e)}/E_{s(m)}$
[1]100	5.5	0.565	65	1.7	3.75	4.01	0.73
[2]100	6.75	0.805	65	1.7	3.75	5.71	0.85
[3]150	7.75	0.9	65	1.7	3.75	6.38	0.82
[4]150	11.1	1.235	65	1.7	3.75	8.76	0.97
[5]150	10.5	1.2	65	1.7	3.75	8.51	0.81
						AVR	0.85

[1] Surface square footing (*matric suction* = 2 kPa)
[1,3] Surface square footing (*matric suction* = 2 kPa)
[2,4] Surface square footing (*matric suction* = 6 kPa)
[5] Embedded square footing (*matric suction* = 2 kPa)
[6] Embedded square footing (*matric suction* = 6 kPa)
Thus, $f_2 = 1.7 \times ((D_r/100)^2 + 3.75)$ *for unsaturated sands with* $D_r \geq 50\%$, *and*
f_2 *was reduced as* $f_2 = 1.2 \times ((D_r/100)^2 + 3.75)$ *for unsaturated sands with* $D_r < 50\%$

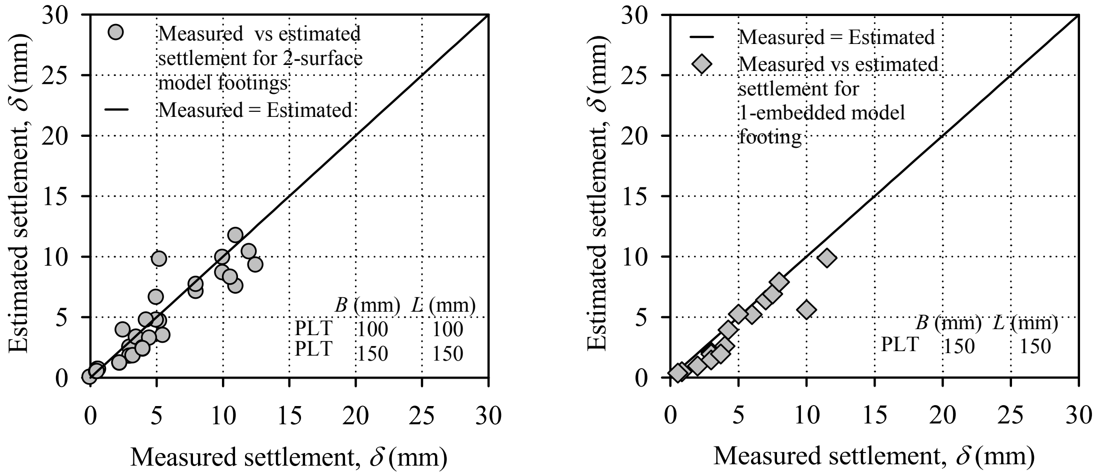

Fig. 9 Effect of loading intensity on settlement for different depths for 150 mm square plate

correlation factors can be as $f_1 = X_1 \times ((D_r/100)^2 + X_2)$; where: D_r in (%); X_1 and X_2 are constants computed by iteration process as the measured E_s values from the PLTs were known.

The relative density, D_r for the case of unsaturated sand (i.e., to calculate f_2) categorised into two groups as follows: $D_r < 50$ % and $D_r \geq 50$ %. The D_r values of sands studied in this research typically varies from 30% to 65%.

Robertson and Cabal (2010) concluded that the reliability of estimating D_r from CPT is high to moderate; therefore, the measured q_c can be used with a greater degree of confidence to estimate the D_r which is required in the proposed relationships in this study. From Tables 10 and 11, it can be seen that the ratio between the estimated $E_{s(\mathrm{e})}$ from the proposed procedure and measured $E_{s(\mathrm{m})}$ from PLTs was in the range of 80 % to 85 % (underestimated) to account for any possible

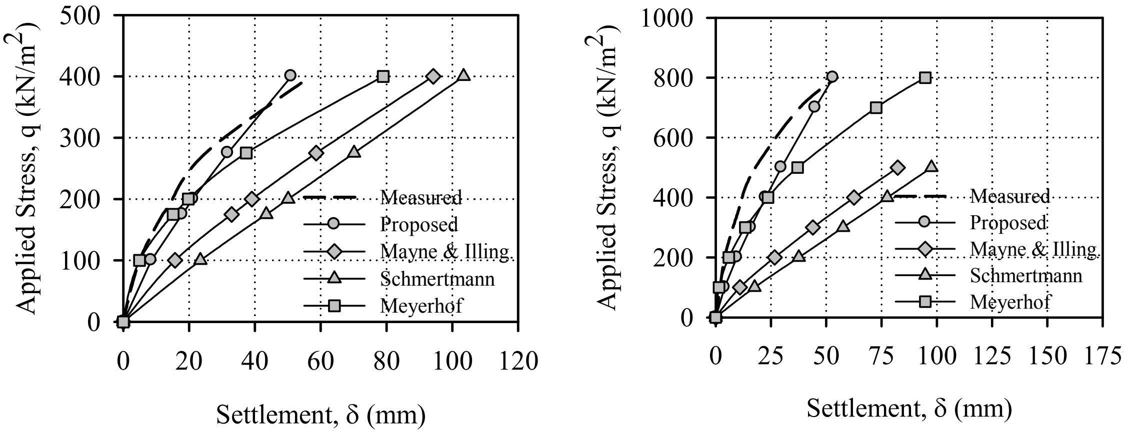

Fig. 10 Comparison between the estimated and measured settlements of two large-scale footings of 1.50 m × 1.50 m and 3.0 m × 3.0 m from Giddens and Briaud (1994)

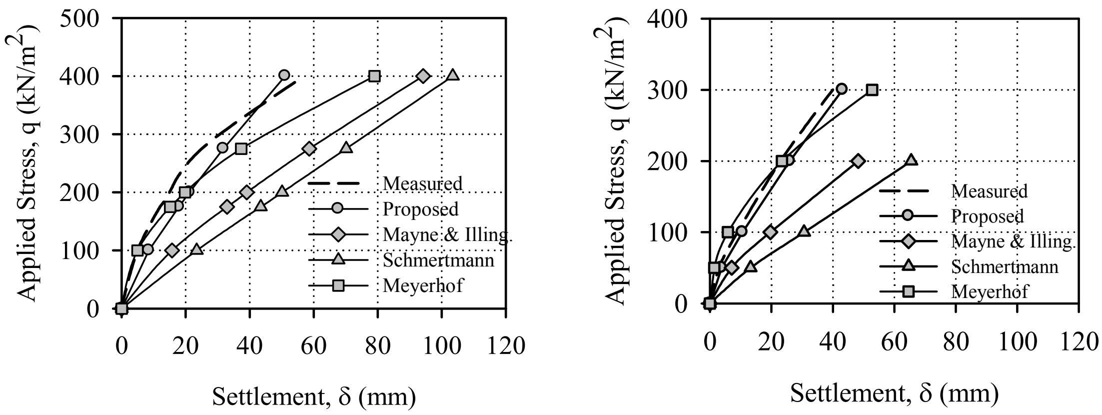

Fig. 11 Comparison between the estimated and measured settlements of two large-scale footing tests of 1.60 m × 1.80 m and 2.30 m × 2.80 m from Bergdahl *et al*. 1985)

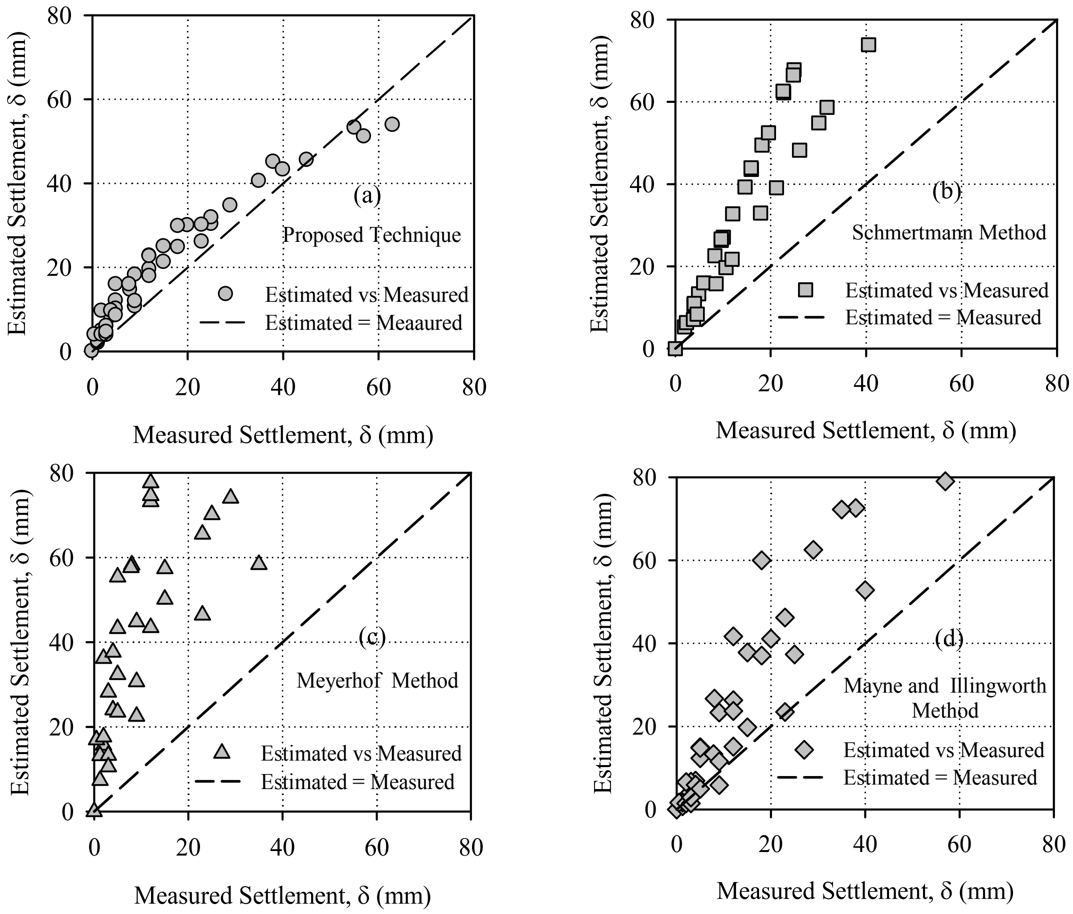

Fig. 12 Comparison between the estimated and measured settlements of seven large-scale footings tested in unsaturated sands corresponding to different applied stress values (Giddens and Briaud 1994, and Bergdahl *et al*. 1985)

experimental errors or boundary effects on the cone penetrometer. The matric suction, (u_a - u_w) was not included in the proposed correlations as its contribution is included in the measured q_c. Typically, the higher the matric suction the higher would be the cone resistance, q_c. The proposed correlation factors provided a good comparison between the estimated and measured settlement values for the two model footings tested in this research (see Fig. 9).

8.1 Validation of the proposed technique

In the present study, an effective penetration depth (i.e., influence zone, I_Z) is chosen to be equal to 1.5*B*. The average cone resistance, $q_{c\ \mathrm{AVR}}$ over the depth of 1.5*B* is used in the analysis of the results.

The influence zone (i.e., a depth of 1.5*B* from the footing base level) provides reasonable correlations between average cone resistance, q_c and the settlement of sand in both saturated and unsaturated conditions using the proposed relationships. Similar influence zone depth, I_Z of 1.5*B* was used by Meyerhof (1956) and Schmertmann *et al*. (1978) to relate the settlement of spread shallow footings to estimate an average cone penetration resistance value. The results summarized in Figs. 10 and 11 show that the settlements estimated using the proposed relationships provide good correlations for both model footings and large-scale in-situ footings. A comparison between the estimated and measured settlement of seven large footings (using the four different methods studied in this research: (a) proposed technique; (b) Schmertmann method; (c) Meyerhof method; (d) Mayne and Illingworth method) is shown in Fig. 12. The proposed relationships in this paper (see Fig. 12(a)) provide better estimations in comparison to the other conventionally used methods from the literature.

9. Results and discussions

The correlation factor, f_1 value for estimating reliable settlement behaviour of shallow footings in sands under saturated condition is typically in the range of 4.5 to 5.0. However, the correlation factor, f_2 value for estimating the settlement of unsaturated sands falls between 4.5 and 7.5 for the sands evaluated. The need for using such a wide range of f_2 values (i.e., 4.5 to 7.5) can be attributed to the influence of matric suction on the cone resistance, q_c values which contributes to reducing the settlement, δ of sands under unsaturated conditions (i.e., (u_a - u_w) > 0 kPa). Several researchers suggested correlation factors between the modulus of elasticity and cone resistance without considering the influence of the relative density or other initial conditions of the sand. Likewise, correlation factors between the modulus of elasticity and cone tip resistance for different sands were suggested by Schmertmann *et al*. (1978) and Robertson and Campanella (1986) as 2.5 - 3.5 for young normally consolidated sand, 3.5 - 6 for aged normally consolidated sands, and 6-10 for over-consolidated sands.

It should be noted that the correlation factors, f_1 and f_2 values for saturated and unsaturated sand conditions respectively are dependent on the relative density, D_r. In both cases, the correlation factor increases proportionally with an increase in the relative density, D_r of the sand. These observations are consistent with the conclusions drawn by Lee and Salgado (2002).

Estimated and measured settlement values of both the model PLTs conducted in the laboratory (e.g., surface and embedded PLTs) and large-scale footings (FLTs) from the two summarized case studies in the geotechnical literature are compared in this research. The results of the research show that Schmertmann *et al*. (1978) method and Meyerhof (1974) method overestimate the measured settlements values by 3 times and 3 to 4 times, respectively (Figs. 12(b) and (c)). Mayne and Illingworth (2010) method provides settlement values of 1.25 to 2 times higher than the measured settlement values (Fig. 12(d)). Comparisons are provided in Fig. 12(a) between the estimated settlements, δ using the proposed technique and the other available methods for the in-situ FLTs showing that the error of the estimated settlement is in the range of ± 15% of the measured settlement values. Comparisons between the estimated and measured elastic settlements are better in the range of 0 to 25 mm both for saturated and unsaturated sands conditions.

10. Conclusions

The Schmertmann *et al*. (1978) method is conventionally used to estimate elastic settlements in sandy soils from the CPTs results using one correlation factor without regardless of the condition of the sand (saturated or unsaturated). Several studies reported in the geotechnical literature have shown that the estimated settlements using this method are typically two to three times higher than the measured settlement values. Two key reasons associated with the discrepancies can be attributed to ignoring the influence of matric suction on settlement behaviour of shallow footings in sandy soils. The experimental investigation performed in this research using model PLTs showed that the settlement of shallow footings located above the groundwater table are less as the sand is in unsaturated condition. Simple relationships are proposed to correlate the cone resistance to modulus of elasticity using the CPT results by modifying the Schmertmann *et al*. (1978) equation. The modified equation using the proposed relationships provides reliable estimates of the settlement in the range of 0 to 25 mm (i.e., allowable settlement) for the large-scale in-situ shallow footings in sands both under saturated and unsaturated conditions. The proposed CPT-based technique is simple, reliable and consistent with methods used for estimation of the settlements of footings in sands by practicing engineers.

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