Horizontal pullout capacity of a group of two vertical plate anchors in clay

Paramita Bhattacharya[*1] and Jyant Kumar[2a]

[1] *Civil Engineering Dept., Indian Institute of Technology Kharagpur, INDIA 721302*
[2] *Civil Engineering Dept., Indian Institute of Science Bangalore, INDIA - 560012*

Abstract. The horizontal pullout capacity of a group of two vertical strip plate anchors, placed along the same vertical plane, in a fully cohesive soil has been computed by using the lower bound finite element limit analysis. The effect of spacing between the plate anchors on the magnitude of total group failure load (P_{uT}) has been evaluated. An increase of soil cohesion with depth has also been incorporated in the analysis. For a weightless medium, the total pullout resistance of the group becomes maximum corresponding to a certain optimum spacing between the anchor plates which has been found to vary generally between $0.5B$ and B; where B is the width of the anchor plate. As compared to a single plate anchor, the increase in the pullout resistance for a group of two anchors becomes greater at a higher embedment ratio. The effect of soil unit weight has also been analyzed. It is noted that the interference effect on the pullout resistance increases further with an increase in the unit weight of soil mass.

Keywords: anchors; clays; failure; finite elements; limit analysis; pullout load

1. Introduction

Vertical plate anchors are often used to generate lateral support for retaining walls, sheet piles and bulkheads. A number of investigations have been reported by different researchers to predict the pullout resistance of a single vertical plate anchor by means of conventional 1-g model tests (Meyerhof 1973, Das *et al.* 1985), centrifuge model tests (Dickin and Leung 1983), elastoplastic 2-D finite element analysis (Rowe and Davis 1982), 3-D large deformation finite element analysis (Yu *et al.* 2009), the lower and upper bound limit analyses (Merifield *et al.* 2001, 2003, Merifield and Sloan 2006) and small scale model tests in reinforced soils (Ei Sawwaf and Nazir 2006). However, except the recent upper bound solutions of Sahoo and Kumar (2012), on the basis of finite elements and linear optimization, no exclusive research investigation seems to have been reported in literature to examine the horizontal pullout capacity for a group of vertical plate anchors embedded in undrained clay by using the lower bound limit analysis. Such a situation often arises when a deep sheet pile wall needs to be constructed and an additional horizontal support is required in the form of plate anchors. The lower and upper bounds solutions are often used to bracket the true collapse load in a bound form. The lower bound solution provides the safe

---

∗Corresponding author, Assistant Professor, E-mail: paramitabh@gmail.com
[a] Professor, E-mail: jkumar@civil.iisc.ernet.in

estimates of the design load for a material following an associated flow rule. In the present research, an attempt has made to perform a lower bound finite element limit analysis for computing the horizontal pullout resistance for a group of two vertical strip anchors embedded in a fully cohesive soil. The effect of spacing between the plate anchors on the magnitude of the group failure load has been examined in detail. An increase of soil cohesion with depth has also been incorporated in the analysis for a weightless medium; it needs to be pointed out that the cohesion for saturated normally consolidated and lightly over consolidated clays increases almost linearly with depth (Bishop 1966). The computational results have been obtained for both smooth and rough plate anchors. Failure patterns have also been drawn for a number of cases.

2. Problem formulation

2.1 Problem definition

Two strip plate anchors, each having width B, are embedded vertically along the same vertical plane in a fully cohesive soil ($\phi = 0$) at a vertical clear spacing S; where ϕ is internal friction angle of soil mass. The thickness of the anchor plates is assumed to be negligible, and H is the depth of the bottom edge of the lower anchor plate from ground surface as indicated in Fig. 1. The soil mass is assumed to follow the Mohr-Coulomb failure criterion and an associated flow rule in order that the lower bound theorem of the limit analysis remains applicable. The cohesion of soil mass is assumed to increase linearly with depth (h) and is given by following expression

$$c = c_0 + mc_0\, h/B \qquad (1)$$

where c_0 and c are the values of cohesion at ground surface and at a depth h, respectively, and m is a non-dimensional factor which accounts for the rate at which cohesion increases linearly with depth.

In the present investigation, two separate cases have been analyzed. In the first case, the soil mass is assumed to be weightless ($\gamma = 0$), and the increase of soil cohesion with depth has been incorporated. In the second case, the effect of unit weight of soil mass has been included but only by using a constant value of cohesion throughout the depth, that is, by taking $m = 0$.

2.2 Problem domain, boundary conditions and finite element mesh

A rectangular problem domain EFHG, as shown in Fig. 1, is chosen. In this domain, the two plate anchors are positioned along the vertical lines OP and MN, respectively. The back and front vertical boundaries (EF and GH) and the horizontal boundary (FH) of the domain are kept at sufficient distances away from the anchor plates. The horizontal distances: (i) L_F between the front face of the anchor plate and the vertical boundary GH, and (ii) L_B between the back face of the anchor plate and the vertical boundary EF are kept equal to $50B$ and $20B$, respectively. The vertical extent of the domain (D) below the bottom edge of the lower anchor plate (MN) is taken between $40B$ and $50B$ depending upon the value of H/B. The values of L_F, L_B and D are chosen in a way such that (i) the yielded elements do not touch any of the chosen domain boundaries (EF, GH and FH), and (ii) an increment in the size of the domain does not bring any change in the magnitude of the collapse load.
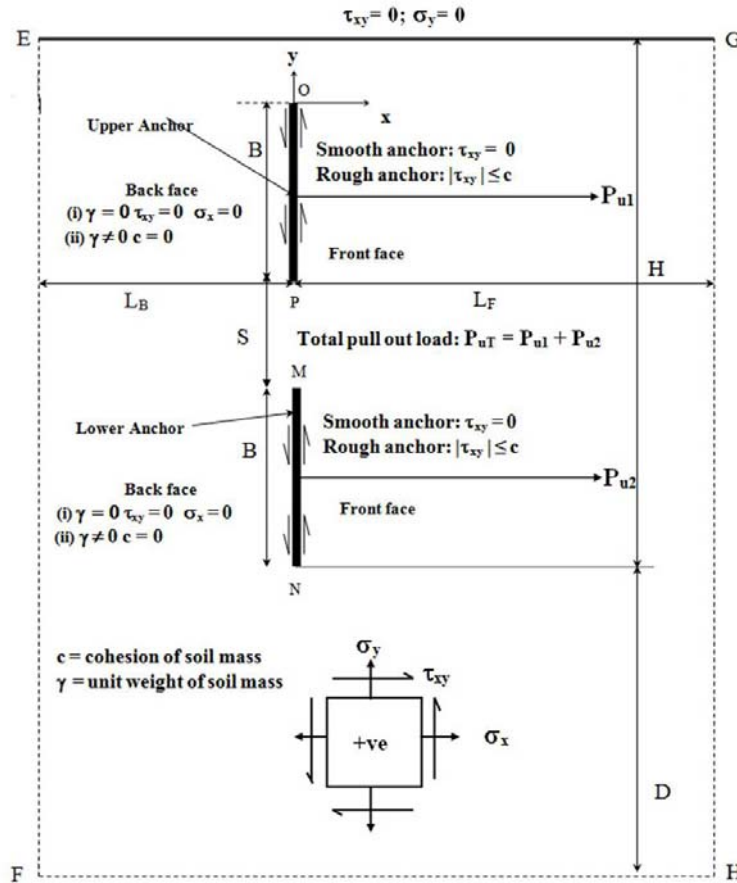
Fig. 1 Chosen domain, boundary conditions and pullout load

The stress boundary conditions along the different boundaries of the domain are presented in Fig. 1. Along the ground surface (EG), the normal stress (σ_y) and shear stress (τ_{xy}) are specified both equal to zero. No stress boundary conditions are imposed on the chosen boundary lines EF, FH and GH. For a weightless medium, it has been assumed that a separation occurs between the back surface of the anchor plate and the adjoining soil mass. Therefore, it is specified that the values of σ_x and τ_{xy} become equal to 0 along the back surface for both the plates for $\gamma = 0$. On the other hand, for $\gamma > 0$, it is specified that $c = 0$ along the back surface of both the plate anchors in order to avoid the development of the shear resistance at the interface between the back surface of the plates and surrounding soil mass; however, no separation is considered between the back surface of the anchor plate and surrounding soil mass for this condition. These two different conditions have been imposed considering the fact that for a frictionless medium with $c = 0$ but for $\gamma > 0$, for instance water, it would not be possible to avoid the generation of the normal stresses along the front as well as along the back sides of the anchor plates. Along the interface between the front face of the anchor plate and adjoining soil mass, the following stress boundary conditions are specified

$$|\tau_{xy}| \leq c \quad \text{for rough anchor front face} \tag{2a}$$

$$\tau_{xy} = 0 \quad \text{for smooth anchor front face} \tag{2b}$$

One can choose different values of c_m / c in order to obtain the solution for different roughness values; where c_m is the mobilized cohesion of soil.

The problem domain has been discretized into a number of three noded triangular elements in a manner such that the sizes of the elements reduce continuously towards top and bottom edges of each anchor plate. Typical finite element meshes, with embedment ratio (H/B) equal to 7 for $S/B = 0.5$ and 1.0, are shown in Figs. 2(a) and (b); where N, E and D_c refer to total number of nodes, elements and stress discontinuities, respectively. It should be mentioned that adequate numbers of nodes, elements and stress discontinuities were taken for solving the problem. It was checked that a further increase in the numbers of nodes, elements and stress discontinuities does not affect significantly the solution.
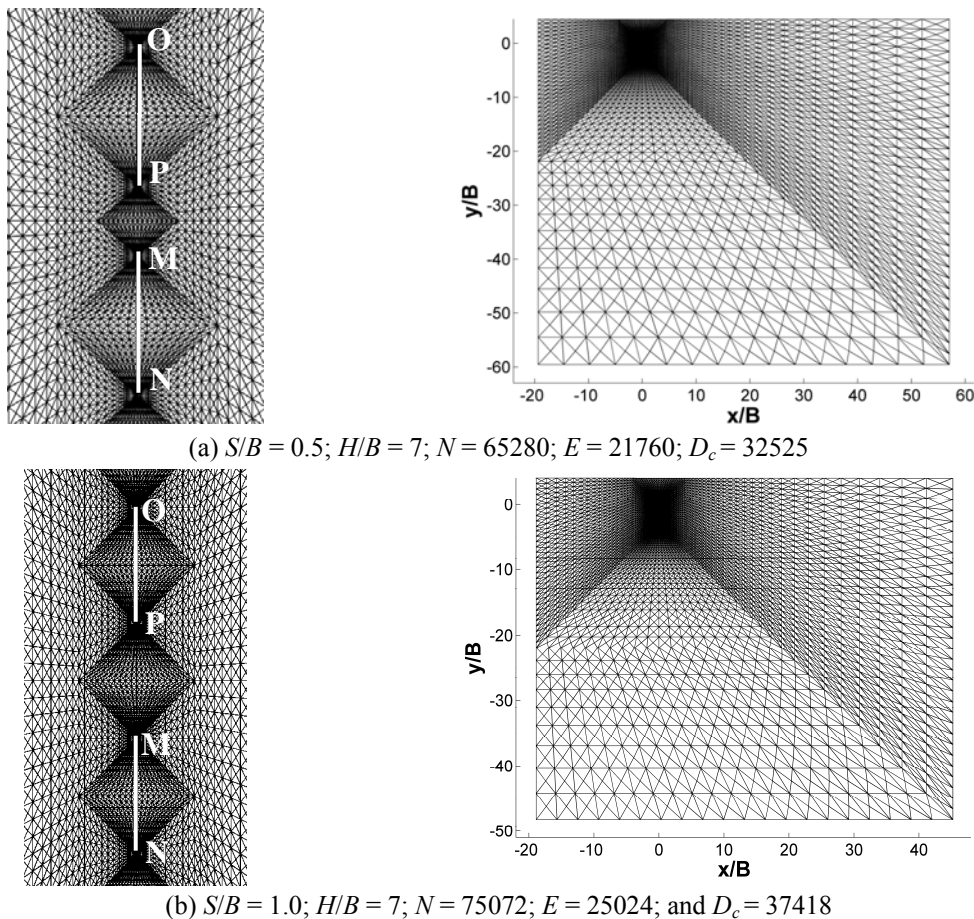

(a) $S/B = 0.5$; $H/B = 7$; $N = 65280$; $E = 21760$; $D_c = 32525$

(b) $S/B = 1.0$; $H/B = 7$; $N = 75072$; $E = 25024$; and $D_c = 37418$

Fig. 2 Finite element meshes for a group of two vertical anchors with $H/B = 7$ for (a) $S/B = 0.5$; and (b) $S/B = 1.0$

3. Solution procedure

3.1 Analysis

The analysis was carried out by using the lower bound finite element limit analysis in combination with linear programming. The method is described well in detail originally by Sloan (1988) and later by Kumar and Khatri (2008). The computational procedure is, therefore, not repeated herein. The nodal stresses (σ_x, σ_y and σ_{xy}) are considered as basic unknown stress variables in the analysis. The element equilibrium conditions need to be satisfied throughout the problem domain. Statically admissible stress discontinuities are permitted along the common edges shared by any two adjacent elements, that is, the values of shear and normal stresses are specified to be continuous along any stress discontinuity line. In addition, it is also ensured that the yield condition is not violated anywhere in the domain. For solving the problem, the linear optimization procedure has been followed. The Mohr-Coulomb yield criterion is a nonlinear function of basic nodal stress variables as shown in following expression.

$$F = \left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2 - \left(2c\,\cos\varphi - \left(\sigma_x + \sigma_y\right)\sin\varphi\right)^2 = 0 \tag{3}$$

For the purpose of linearization, this nonlinear yield function, which becomes a circle in X and Y co-ordinates system, is replaced by a regular polygon of sides "p" inscribed to the parent yield circle; where $X = 2\tau_{xy}$ and $Y = (\sigma_x - \sigma_y)$. Following Bottero *et al.* (1980), each side of the yield polygon becomes a linear function of nodal stress variables as shown below

$$A_k\sigma_x + B_k\sigma_y + C_k\tau_{xy} \leq D; \qquad k = 1,2\ldots p \tag{4}$$

where $A_k = \cos(2\pi k / p) + \sin\varphi\,\cos(\pi/p)$; $B_k = \sin\varphi\,\cos(\pi/p) - \cos(2\pi k / p)$

$$C_k = 2\sin\left(2\pi k/p\right) ; \quad D = 2c\,\cos\varphi\,\cos\left(\pi/p\right)$$

The value of p has been taken equal to 21 in the present analysis. The basic expression of the total collapse load P_{uT} is derived by integrating the normal stresses along the front surfaces of two anchors plates by using the following expression

$$P_{u1} = P_{u1} + P_{u2} \tag{5}$$

where P_{u1} and P_{u2} are the pullout load per unit length of the upper and the lower anchor plates, respectively. The expressions for P_{u1} and P_{u2} are given below.

$$P_{u1} = \int_{\text{Front face}} (-\sigma_x dy) \tag{6a}$$

$$P_{u2} = \int_{\text{Front face}} (-\sigma_x dy) \tag{6b}$$

The magnitude of the collapse load (P_{uT}) is maximized subjected to a set of equality and inequality linear constraints. The linear optimization problem is then stated in the following

canonical form

$$\text{Maximize the objective function} \quad -\{c\}^T\{\sigma\} \tag{7a}$$

$$\text{Subjected to (i) quality constraints}: \quad -\{A_e\}\{\sigma\} = \{b_e\} \tag{7b}$$

$$\text{(ii) inequality constraints}: \quad -\{A_{in}\}\{\sigma\} \le \{b_{in}\} \tag{7c}$$

$$-\infty \le \{\sigma\} \le +\infty \tag{7d}$$

It should be mentioned that the nodal stresses have real values either tensile or compressive in nature. This condition is mathematically represented by inequality condition Eq. (7d). The LINPROG function available in MATLAB is used to perform the necessary linear optimization.

3.2 Definition of pullout capacity factor F_c and $F_{c\gamma}$ for a single plate anchor

The horizontal pullout capacity of a single isolated vertical strip plate anchor is obtained in terms of a non-dimensional factor, (i) F_c for $\gamma = 0$ and (ii) $F_{c\gamma}$ for $\gamma > 0$, as defined herein

$$F_{c\gamma} \text{ and } F_c = \frac{P_u}{c_0 B} \tag{8}$$

where P_u is the magnitude of the horizontal collapse load per unit length of the single strip anchor plate.

3.3 Definition of the group efficiency factor η_c

The total horizontal pullout load (P_{uT}) of the group of two vertical strip anchors, placed at a clear spacing S, is normalized with respect to the horizontal pullout load (P_u) of a single isolated vertical plate anchor for the same values of B and H, in terms of a non-dimensional factor, namely, group efficiency factor, η_c for $\gamma = 0$ and $\eta_{c\gamma}$ for $\gamma > 0$, as defined below

$$\eta_{c\gamma} \text{ and } \eta_c = \frac{P_{uT}}{P_u} \tag{9}$$

Note that while computing P_{uT} and P_u, the positions of (i) the lower anchor plate for a group of two anchors and (ii) the single isolated anchor plate have been kept exactly at the same level.

4. Results and comparisons

4.1 For a weightless medium

The results were obtained for: (i) different values of H/B between 1 and 7 for a single vertical plate anchor, (ii) two different values of H/B, namely, 5 and 7 for a group of two plate anchors, (iii) four different values of m, namely, 0, 0.5, 1 and 2, and (iv) both smooth and rough anchor-soil interface conditions. For a group of two plate anchors, the clear spacing between the anchors was

varied from 0 to the maximum possible value, that is, (*H-2B*). All the results are presented herein.

4.1.1 The variation of F_c

For a single isolated plate anchor, the variation of F_c with H/B for different values of m is shown in Fig. 3(a) for both smooth and rough anchors. Note that the magnitude of F_c for $m > 0$ increases quite considerably with increases in the values of both H/B and m; for $m = 0$ the increase in F_c with H/B has been found to be, however, only marginal. The values of F_c for rough plate anchors become only marginally greater than that for smooth plate anchors especially for smaller values of H/B.

The values of F_c computed from the present analysis for a single plate anchor were compared with the numerical lower and upper bound solutions of Merifield *et al*. (2001) for rough plate anchors. The comparison of all these results has been presented in Fig. 3(b) for two different values of *m*, namely, 0 and 1. It can be noted that the present values of F_c are found to be only a little lower than the upper bound solutions given by Merifield *et al*. (2001). On the other hand, the lower bound solution of Merifield *et al*. (2001) compares very well with the present results. The comparison for a single anchor plate, therefore, validates the present computational procedure.

4.1.2 The variation of η_c

The variation of the anchor group efficiency factor η_c with changes in S/B, corresponding to different combinations of H/B values and m, has been presented in Fig. 4. For different combinations of H/B and m, Table 1 provides the maximum values of η_c, which is referred to as η_{c-max}, and the associated optimum values of S/B. Following observations are drawn from (i) Fig. 4, and (ii) Table 1.
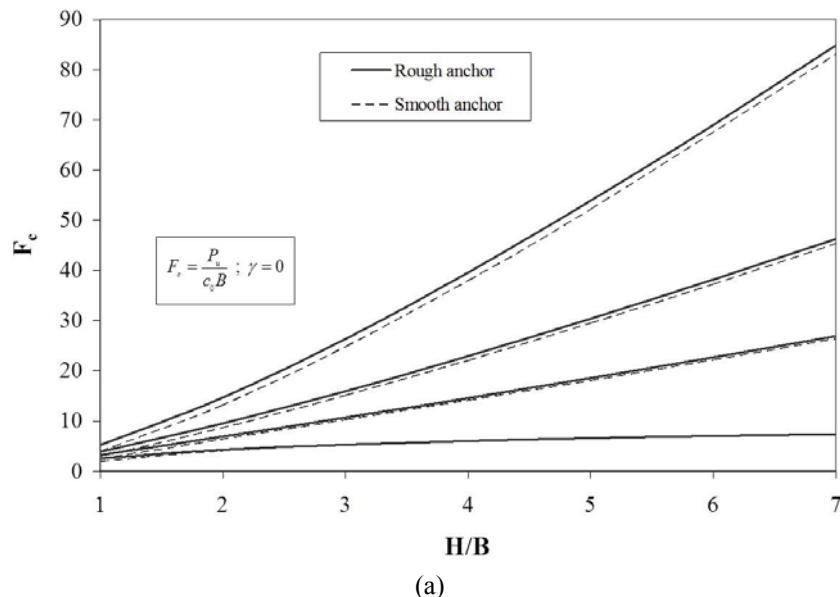

(a)

Fig. 3 (a) The variation of F_c with H/B for a single vertical plate anchor for different values
of *m*; and (b) a comparison of F_c for a single rough vertical plate anchor with the
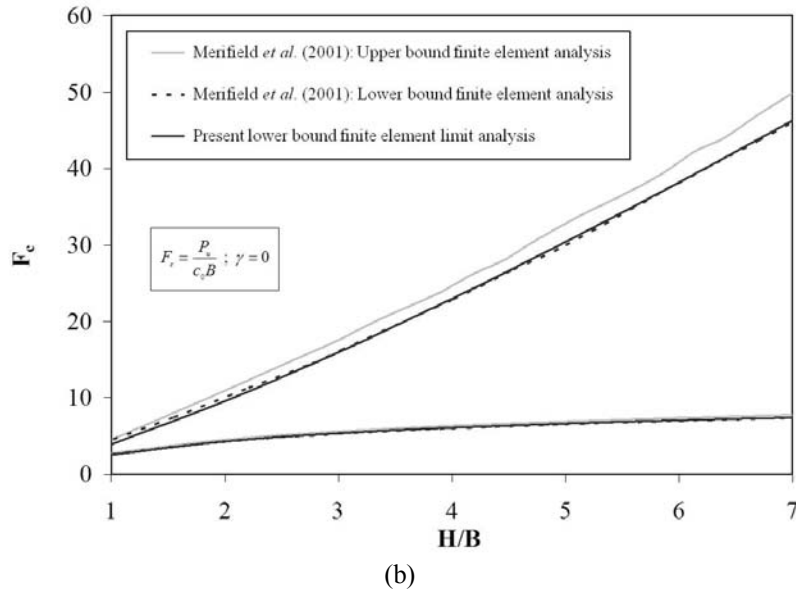results of Merifield *et al*. (2001)

(b)

Fig. 3 Continued

Table 1 The values of S_{opt} and $\eta_{c\text{-max}}$ for different values of m with $H/B = 5$ and 7 with $\gamma = 0$

m	Anchor-soil interface	$H/B = 5$		$H/B = 7$	
		S_{opt}/B	$\eta_{c\text{-max}}$	S_{opt}/B	$\eta_{c\text{-max}}$
0.0	Smooth	0.7	1.55	0.8-0.9	1.65
	Rough	0.7	1.57	1.0	1.64
0.5	Smooth	0.6	1.40	0.7	1.48
	Rough	0.6	1.40	0.7	1.49
1.0	Smooth	0.5 - 0.6	1.36	0.6 - 0.7	1.46
	Rough	0.5 - 0.6	1.36	0.6 - 0.7	1.46
2.0	Smooth	0.5 - 0.6	1.33	0.6	1.44
	Rough	0.6 - 0.7	1.34	0.6	1.44

The value of η_c in all the cases becomes maximum corresponding to a certain optimum spacing between the plate anchors. For given values of H/B and S/B, the magnitude of η_c becomes continuously lower with an increase in m. The maximum value of η_c varies (i) between 1.33 and 1.55 for $H/B = 5$ and (ii) between 1.44 and 1.65 for $H/B = 7$. It indirectly reveals that the group effect of two plate anchors becomes more advantageous for greater values of H/B. The value of S_{opt}/B varies between (i) 0.5 and 0.7 for $H/B = 5$; and (ii) 0.6 and 1.0 for $H/B = 7$; it implies that the value of the optimum spacing becomes higher for greater values of H/B. The value of S_{opt}/B decreases continuously with an increase in the value of m. The peak values of η_c (namely, $\eta_{c\text{-max}}$) have also been marked in Fig. 4. The values of S_{opt}/B and η_c do not show much variation with changes in anchor-soil interface rough condition. Note that at $S = 0$, the total horizontal pullout
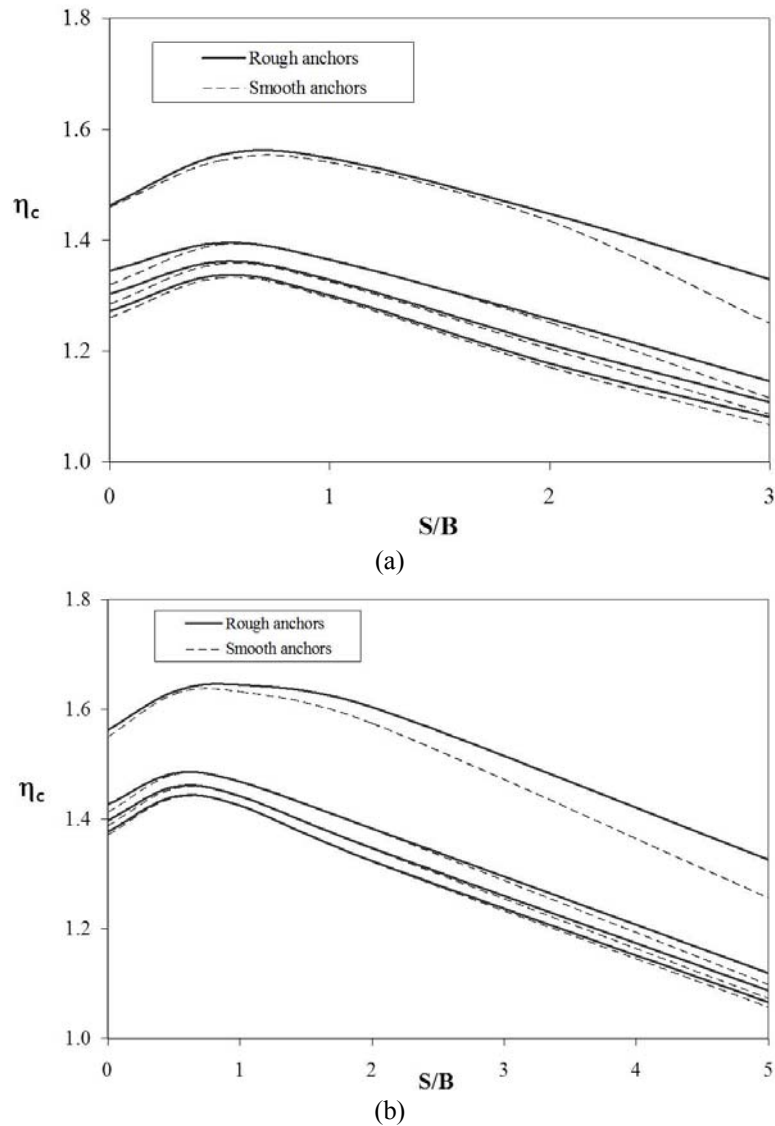
Fig. 4 The variation of η_c with S/B for (a) $H/B = 5$; and (b) $H/B = 7$

capacity of two vertical anchor plates becomes exactly the same as that of a single plate anchor with width $2B$ and embedment ratio $H/2B$; the results, therefore, for this case can be simply obtained from the Section 4.1.1 and Figs. 3(a) and (b).

4.2 Effect of soil unit weight

4.2.1 For a single plate anchor

In present study, the effect of soil unit weight has been examined only for $m = 0$ with a rough plate anchor. The variation of $F_{c\gamma}$ with changes in $\gamma H/c_0$ has been presented in Fig. 5(a) for H/B

equal to 3, 5 and 7. It is noted that the value of $F_{c\gamma}$ increases linearly with increasing value of $\gamma H/c_0$ up to a certain limiting value beyond which the plate anchor behaves like a deep anchor irrespective of the embedment depth. The limiting value of $F_{c\gamma}$ has been found to vary between 11.1 and 11.2 with an increase in H/B from to 3 to 7. This limiting value of $F_{c\gamma}$ has been found to match well with the numerical results reported by Merifield *et al.* (2001); it should be mentioned that Merifield *et al.* (2001) have found the limiting value of the breakout factor between 11.16 and 11.86.
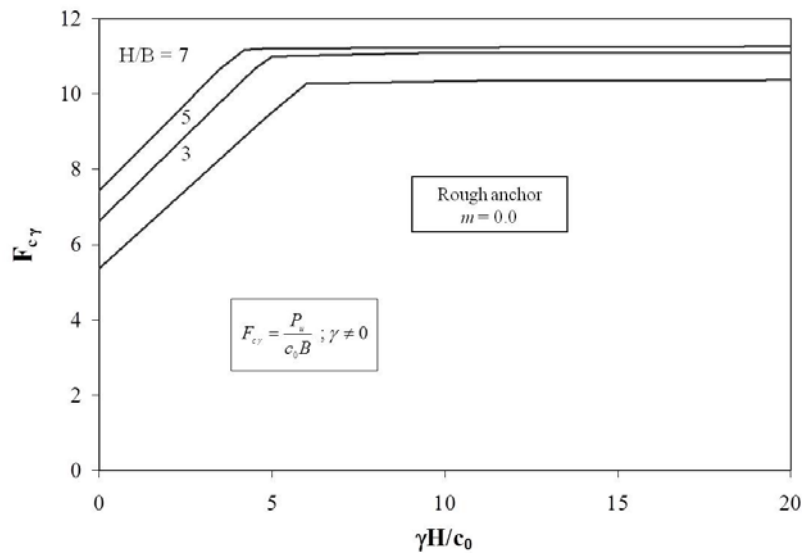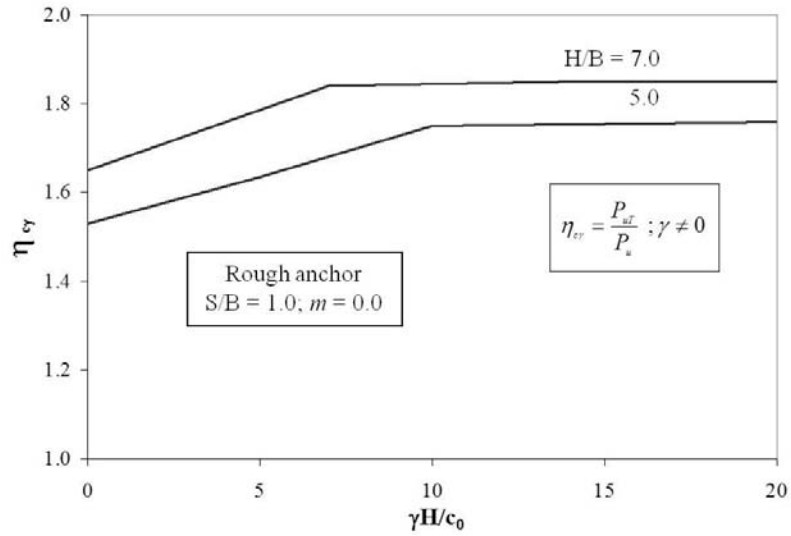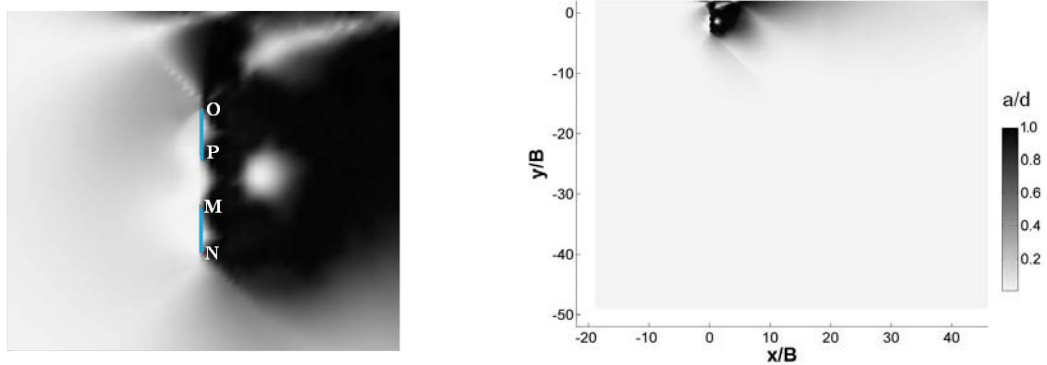

(a)
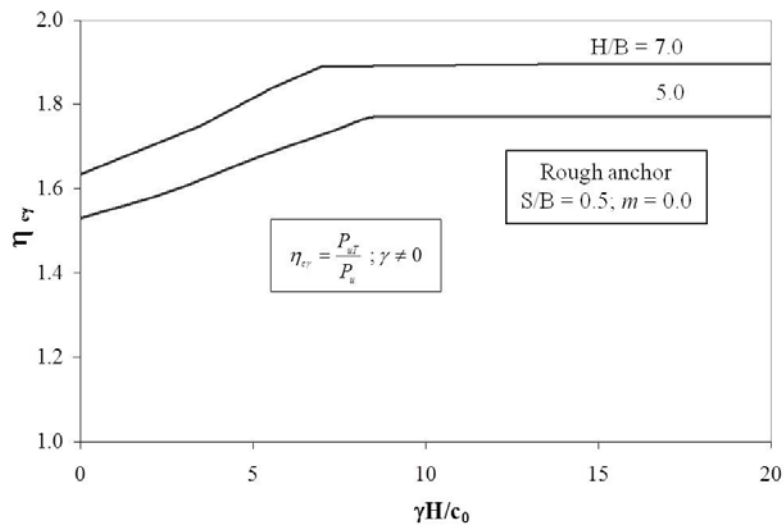
(b)

Fig. 5 The variation of (a) $F_{c\gamma}$ with $\gamma H/c_0$ for a single rough plate anchor; (b) $\eta_{c\gamma}$ with $\gamma H/c_0$ for rough plate anchors at $S/B = 0.5$ and $m = 0$; and (c) $\eta_{c\gamma}$ with $\gamma H/c_0$ for rough plate anchors at $S/B = 1.0$ and $m = 0.0$
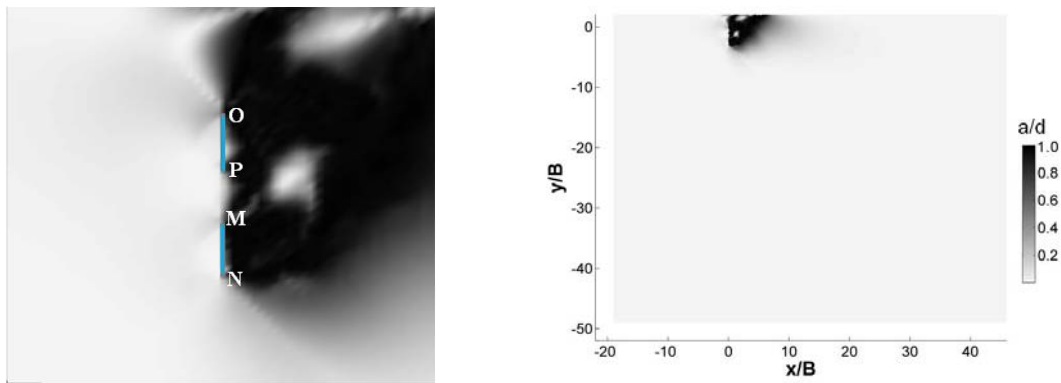
(c)

Fig. 5 Continued

(a) Rough anchors with $H/B = 5$; $S/B = 1$; and $m = 0$

(b) Rough anchors with $H/B = 5$; $S/B = 1$; and $m = 2$

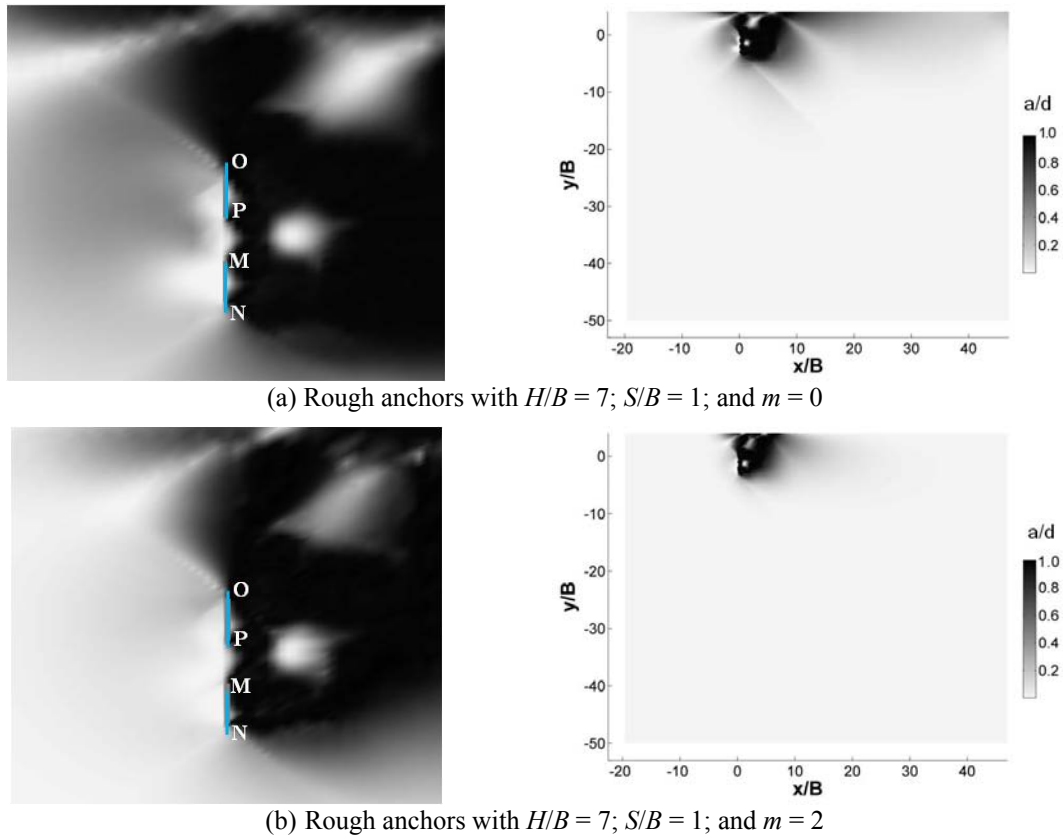Fig. 6 Failure patterns for rough anchors with $S/B = 1$ and $H/B = 5$ at (a) $m = 0$; and (b) $m = 2$

(a) Rough anchors with *H/B* = 7; *S/B* = 1; and *m* = 0

(b) Rough anchors with *H/B* = 7; *S/B* = 1; and *m* = 2

Fig. 7 Failure patterns for rough anchors with *S/B* = 1 and *H/B* = 7 at (a) *m* = 0; and (b) *m* = 2

4.2.2 For a group of two plate anchors

As it was noted earlier from Fig. 4 that for $\gamma = 0$, the group efficiency factor (η_c) attains a maximum value at a spacing S_{opt} lying between $0.5B$ and B. Therefore, the effect of $\gamma H/c_0$ on the group efficiency factors was analyzed for two different values of spacing, namely $S = 0.5B$ and B. The results were obtained for two different values of H/B, namely, 5 and 7 but only with $m = 0$ and for rough anchors. It was observed that in all the cases the group efficiency factor ($\eta_{c\gamma}$) increases linearly with an increase in $\gamma H/c_0$ up to a certain limiting value. Note that for lower values of H/B, the group efficiency factor attains a limiting value at a little higher values of $\gamma H/c_0$. For $\gamma H/c_0 = 0$, the value of $\eta_{c\gamma}$ becomes simply equal to η_c. It can, therefore, be concluded that the effect of the interference of the two plate anchors will become invariably greater for anchors with higher values of $\gamma H/c_0$.

4.3 Failure pattern

From the obtained solution, after finding the state of stress at all the nodes, the proximity of the stress state to yield was determined in terms of a ratio, namely, *a/d*, where, $a = \sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2}$; and $d = 2c$. At any point, *a/d* = 1 implies shear failure. On the other

hand, for non-plastic region, the value of a/d remains smaller than unity. The failure patterns are generated in such a way that a very dark color implies a fully plastic region. The failure patterns were drawn only for rough plate anchors with $\gamma = 0$, $S/B = 1$, $H/B = 7$, and $m = 0$ and 2. The corresponding failure patterns are illustrated in Figs. 6(a) and (b) for the value of m equal to 0 and 2, respectively; a zoomed view around the plate anchors has also been presented in these figures. The failure patterns for embedment $H/B = 7$ with $S/B = 1$ have been shown in Figs. 7(a) and (b) corresponding the value of m equal to 0 and 2, respectively. It is noted that in all the cases a plastic shear zone, with a curvilinear boundary, which starts from the bottom edge of the lower anchor plate and extends up to the ground surface, develops on the front side of the anchor plates. A very small region of the plastic zone is also generated on the back side of the plates which starts from the top edge of the upper plate and then extends towards ground surface.

6. Conclusions

The horizontal pullout capacity of a group of two vertical strip plate anchors embedded along the same vertical plane in a fully cohesive soil has been determined by using the lower bound finite element limit analysis. For the group of two plate anchors, the total pullout resistance becomes maximum when the two anchors are placed at a critical spacing (S_{opt}). For the chosen embedment ratios, the value of S_{opt}/B for $\gamma = 0$ has been found to vary between 0.5 and 1.0. The magnitude of the critical spacing (S_{opt}) between the two anchor plates increases (i) with an increase in the embedment ratio, and (ii) with a decrease in the value of m. As compared to a single vertical plate anchor, for $H/B = 7$, the group of two vertical anchors provides a maximum increase in the pullout resistance up to 65%. For greater values of H/B, the group interference effect becomes even more substantial. The group effect on the pullout resistance increases further with an increase in the value of $\gamma H/c_0$.

References

Bishop, A.W. (1966), "The strength of soils as engineering materials", *Géotechnique*, **16**(2), 91-218.
Bottero, A., Negre, R., Pastor, J. and Turgeman, S. (1980), "Finite element method and limit analysis theory for soil mechanics problem", *Comput. Methods. Appl. Mech. Eng.*, **22**(1), 131-149.
Das, B.M., Moreno, R. and Dallo, K.F. (1985), "Ultimate pullout capacity of shallow vertical anchors in clay", *Soils Found.*, **25**(2), 148-152.
Dickin, E.A. and Leung, C.F. (1983), "Centrifugal model tests on vertical anchor plates", *J. Geotech. Eng.*, *ASCE*, **109**(12), 1503-1515.
Ei Sawwaf, M.E. and Nazir, A. (2006), "The effect of soil reinforcement on pullout resistance of an existing vertical anchor plate in sand", *Comput. Geotech.*, **33**(3), 167-176.
Kumar, J. and Khatri, V.N. (2008a), "Effect of footing width on bearing capacity factor N_γ for smooth strip footings", *J. Geotech. Geoenviron. Eng.*, *ASCE*, **134**(9), 1299-1310.
Kumar, J. and Khatri, V.N. (2008b), "Effect of footing width N_γ", *Can. Geotech. J.*, **45**(12), 1673-1684.
Merifield, R.S. and Sloan, S.W. (2006), "The ultimate pullout capacity of anchors in frictional soil", *Can. Geotech. J.*, **43**(8), 852-868.
Merifield, R.S., Lyamin, A.V., Sloan, S.W. and Yu, H.S. (2003), "Three dimensional lower bound solutions for stability of plate anchors in clays", *J. Geotech. Geoenviron. Eng.*, *ASCE*, **129**(3), 243-253.
Merifield, R.S., Sloan, S.W. and Yu, H.S. (2001), "Stability of plate anchors in undrained clays", *Géotechnique*, **51**(2), 141-153.

Meyerhof, G.G. (1973), "Uplift resistance of inclined anchors and piles*", Proeeding of 8th International Conference on Soil Mechanics and Foundation Engineering*, Moscow, **2**, 167-172.

Rowe, R.K. and Davis, E.H. (1982), "The behaviour of anchor plates in clay", *Géotechnique*, **32**(1), 9-23.

Sahoo, J.P. and Kumar, J. (2012), "Horizontal pullout resistance for a group of vertical plate anchors in clays", *Geotech. Geol. Eng.*, **30**(5), 1279-1287.

Sloan, S.W. (1988), "Lower bound limit analysis using finite elements and linear programming", *Int'l J. Num. Anal. Methods Geomech.*, **12**(1), 61-77.

Yu, L., Liu, J., Kong, X.J. and Hu, Y. (2009), "Three-dimensional numerical analysis of the keying of the vertically installed plate anchors in clay", *Comput. Geotech.*, **36**(4), 558-567.

GM