Nonlinear consolidation of soft clays subjected to cyclic loading - Part II: Verification and application

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Abstract. In the companion paper, the nonlinear consolidation of soft clays subjected to cyclic loading was analytically investigated. This paper reports the results of an experimental program conducted to verify some critical assumptions made in the analytical study. It, also, includes a numerical study carried out to examine the capability of the proposed theory to determine the consolidation characteristics of soft clays subjected to cyclic loading. Results show that the permeability of the soft clays does not significantly change during the cyclic loading. It is also shown that, compared to the Terzaghi's solution for a linear clay, the inherit nonlinearity of the clay tends to decrease the degree of consolidation due to the smaller rate of dissipation in the excess pore water pressure.

Keywords: nonlinear; consolidation; cyclic loading; finite difference method; soft clays.

1. Introduction

In the companion paper, two nonlinear partial differential equations were derived to predict the consolidation characteristics of normally consolidated (NC) and over consolidated (OC) soft clays subjected to cyclic loading (Yazdani 2008, Yazdani *et al.* 2010). An analytical discussion was provided in which it was assumed that the slopes of the $e-\log k$ response in the NC and OC states are the same. This paper reports the results of an experimental program conducted to verify the assumption. It also reports the results of a numerical study carried out to investigate the influence of the nonlinear behavior of soft clay on its consolidation settlement.

2. Experimental study

The critical assumption made in the companion paper, in which it was assumed that the slopes of $e-\log k$ in the NC and OC states are the same, is experimentally examined. Rearranging the modulus of volume compressibility (Terzaghi *et al.* 1996), we have

$$k = C_v \gamma_w m_v \tag{1}$$

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Defining PR, Permeability Ratio, as

$$PR = \frac{k_{NC}}{k_{OC}} \tag{2}$$

and substituting Eq. (2) in Eq. (1) results in

$$PR = \frac{CCR}{CR} \tag{3}$$

A series of consolidation tests was performed on soft clay to investigate the variation of $e-\log k$ slope in the NC and OC states. Each consolidation test was carried out in four stages. In the first stage, the initial static pressure, σ_i , was applied in order to obtain the consolidation and compressibility coefficients of the clay in the NC state. Holding the initial static pressure, the second stage was carried out by applying a cyclic loading with the intensity of Δp and the period of T_c . The third stage included unloading the clay when the specimen reached a steady state in which more loading cycles had no incremental effect on its settlements. Finally, a static pressure of Δp was applied to determine the coefficients in the OC state. Specifications of 20 tests are shown in Table 1. Last column in Table 1 shows that the assumption made in the companion paper, ($N_P = O_P$ or PR = 1), is effectively reasonable.

No.	σ _i (kPa)	Δp (kPa)	T_c (min)	$m_{vNC} (10^4 \text{ kPa})^{-1}$	$C_{\nu NC}$ (cm ² /min)	CCR	CR	PR
1	200	50	30	3.3557	0.0014	0.070	0.100	0.70
2	200	50	60	3.5321	0.0014	0.100	0.100	1.00
3	275	25	60	1.2712	0.0012	0.095	0.090	1.06
4	300	50	60	2.4872	0.0008	0.105	0.100	1.05
5	300	75	40	2.7053	0.0014	0.105	0.110	0.95
6	300	75	20	2.8233	0.0014	0.100	0.110	0.91
7	400	50	60	0.7755	0.0029	0.095	0.095	1.00
8	400	50	30	0.7502	0.0029	0.080	0.095	0.84
9	400	100	60	1.1763	0.0021	0.100	0.110	0.91
10	450	50	20	2.0408	0.0005	0.100	0.100	1.00
11	500	62.5	20	0.7773	0.0015	0.065	0.100	0.65
12	500	62.5	40	0.7926	0.0015	0.080	0.100	0.80
13	600	60	30	0.5946	0.0011	0.093	0.110	0.85
14	600	60	60	0.6006	0.0011	0.065	0.110	0.59
15	650	100	60	0.7923	0.0021	0.090	0.090	1.00
16	700	100	60	0.8076	0.0015	0.100	0.090	1.11
17	1400	200	30	0.2586	0.0096	0.090	0.090	1.00
18	1400	200	20	0.2568	0.0096	0.090	0.090	1.00
19	1600	100	30	0.208	0.0026	0.090	0.090	1.00
20	1600	100	60	0.2065	0.0026	0.090	0.090	1.00

Table 1 Consolidation tests on 20 samples under cyclic loading

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3. Numerical study

The piece-wise linearization is a commonly used method to deal with nonlinear problems. This method, in which both spatial and time coordinates are divided into a finite number of small intervals, is utilized to maintain the validity of the assumptions supporting the Terzaghi's consolidation theory (Terzaghi *et al.* 1996). During a short period of time, a thin layer of soil can be considered homogenous, $\partial k/\partial z = 0$, and the gradients of effective stress and pore-water pressure can be deemed the same, $\partial^2 u/\partial z^2 = -\partial^2 \sigma'/\partial z^2$. Therefore the Terzaghi's consolidation theory is applicable in the analysis of nonlinear consolidation of soft clays under cyclic loading. Analysis begins with an initial permeability and compressibility. Then the permeability and compressibility are updated for every time step so that it can be reasonably assumed that *k* and *m_v* are constant for every small time increment. A finite difference method is used in order to numerically study the consolidation of a clay layer.

Consider the consolidation of a saturated soft clay layer subjected to a rectangular cyclic loading. The scheme of the clay layer, drained at the top and bottom, is shown in Fig. 1. The layer is overburdened by a sand layer and considered to be normally consolidated before the cyclic loading, Fig. 2, is applied. The groundwater is at the top of the clay layer.

The applied pressure of the rectangular cyclic loading at the N^{th} cycle is

$$\sigma = \begin{cases} \Delta p & (N-1)T_c \le t \le (N-1)T_c + T_c/2\\ 0 & (N-1)T_c + T_c/2 < t < NT_c \end{cases}$$
(4)

Note that a constant load can be defined by setting N = 1 and selecting a long period of loading, $T_c/2$ in Eq. (4). Table 2 includes the characteristics of the clay and the cyclic loading.

In terms of effective stress, the average degree of consolidation is defined as

$$U_{p,av} = \frac{\int_0^H \sigma_z' dz}{(\sigma_i' + \Delta p)H}$$
(5)

where σ_z' is the effective stress at depth z. Eq. (5) is used to calculate the average degree of



Fig. 1 A saturated clay layer subjected to cyclic loading

Fig. 2 The scheme of the applied rectangular cyclic loading

Table 2 The characteristics of the clay layer and the cyclic loading

Parameter	Description	Value
H(m)	Clay layer thickness	5
H'(m)	Sand layer thickness	0.6
γ_{sat} (kN/m ³)	Saturated unit weight of clay	19.81
γ_{sand} (kN/m ³)	Dry unit weight of sand	16.50
$\gamma_w (kN/m^3)$	Unit weight of water	9.81
e_0	Initial void ratio of clay	2.14
C_c	Compression index of clay	0.61
e_{NC}	The void ratio of clay at unit effective stress	2.77
λ	C_c/C_s	5
η	C_c/N_p	0.5
ξ	N_p/O_p	1
μ	e_{Nk}/e_{N0}	2.92
P_{ct}	Preconsolidation pressure at the top of the of the clay layer	100
σ_i' (kPa)	Initial effective stress at the top of the layer	10.7
Δp (kPa)	Maximum loading amplitude	50
T_c (hr)	Period of cyclic loading	1

consolidation for the entire layer. Having calculated the change in void ratio of ith sub-layer, its settlement is determined as

$$\Delta S_i = \frac{\Delta e_i}{1 + e_{0_i}} H_i \tag{6}$$

where H_i , e_{0_i} , and Δe_i are the thickness, initial void ratio, and change in the void ratio of the ith sublayer at a specific time step, respectively. Note that the soil properties are updated in every time step and their initial values in every step are the final values at the end of the previous step. The settlements of *n* sub-layers are added up to calculate the settlement of the entire layer in each step.

Figs. 3 and 4, respectively, show the variation of the average degree of consolidation and settlement under cyclic loading versus real time. The data are shown in every 16 cycles to avoid chaos in the Figs. It can be observed that the soil body reaches the OC state and behaves elastically after 760 cycles (760 hours) and the average degree of consolidation converges to 47% and the settlement converges to 0.44 m. Also, it can be seen that in comparison with the present study, the Terzaghi's solution overestimates the degree of consolidation but underestimates the settlement. The rate of consolidation and difference between the present solutions and those obtained from the Terzaghi's theory depend on the nonlinearity factor (Abbasi *et al.* 2007). It should be stated that the average degree of consolidation can be expressed in terms of settlements. In this case, the rate of variations in the degree of consolidation is essentially the same as that in settlement.

Fig. 5 shows the excess pore water pressure distribution for the present problem in depth. In odd half-cycles, in which the load is applied, the excess pore water pressure is positive while unloading in even half-cycles results in a negative excess pore water pressure in depth. It is seen that after almost 750 cycles (1500 half-cycles) the consolidation reaches the steady state and the value of the water pressure is stabilized around $\Delta p/2$ (here 25 kPa). It confirms the study by Baligh and



Fig. 3 Variations in the average degree of consolidation under cyclic loading



Fig. 4 Variations in the settlement under cyclic loading

Levadoux (1978) that once the steady state is reached, the excess pore water pressure at any depth is $\Delta p/2$ at the end of a loading half-cycles and $-\Delta p/2$ at the end of an unloading half-cycle.

To study the effect of nonlinear behavior of the clay on its average degree of consolidation, all parameters in the existing problem are kept constant except the compression to swell indices ratio, λ . Fig. 6 shows that the average degree of consolidation converges to the steady state more slowly as the soil nonlinearity increases. For an elastic soil, $\lambda = 1$, the average degree of consolidation converges to 50% while the converged average degree of consolidation decreases as the nonlinearity increases (Baligh and Levadoux 1978, Toufigh and Ouria 2009). It can be explained by the variation in the excess pore water pressure; if the soil rebounds along the normal consolidation line ($\lambda = 1$), the available voids would be more compared to that when the soil bounces with a smaller rate ($\lambda > 1$). Therefore, the void ratio for a nonlinear soil is smaller than that for an elastic soil at the end of the unloading half-cycle. Consequently, the excess pore water pressure is dissipated in a slower rate.



Fig. 5 The excess pore water pressure distribution under cyclic loading



Fig. 6 Effect of clay nonlinearity on the average degree of consolidation

4. Conclusions

A series of consolidation tests was performed on soft clay to investigate the variation of $e-\log k$ slope in the NC and OC states. Results showed that the permeability of clays does not considerably change during loading and unloading induced by cyclic loading. A numerical study was carried out to investigate the influence of the nonlinear behavior of soft clay on its consolidation settlement. It was seen that that at the steady state, with Δp being the amplitude of a rectangular cyclic loading, the excess pore water pressure at any depth is $\Delta p/2$ at the end of the loading half-cycles and $-\Delta p/2$ at the end of the unloading half-cycles. It was also shown that as the soil nonlinearity increases, the degree of consolidation decreases at the steady state due to the smaller rate of dissipation in the excess pore water pressure.

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