Geomechanics and Engineering, Vol. 4, No. 3 (2012) 209-218
DOI: http://dx.doi.org/10.12989/gae.2012.4.3.209

An analytical expression for the dynamic active thrust from c-ϕ soil backfill on retaining walls with wall friction and adhesion

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(Received November 16, 2011, Revised July 10, 2012; Accepted July 13, 2012)

Abstract. This paper presents the derivation of an analytical expression for the dynamic active thrust from c-ϕ (c = cohesion, ϕ = angle of shearing resistance) soil backfill on rigid retaining walls with wall friction and adhesion. The derivation uses the pseudo-static approach considering tension cracks in the backfill, a uniform surcharge on the backfill, and horizontal and vertical seismic loadings. The development of an explicit analytical expression for the critical inclination of the failure plane within the soil backfill is described. It is shown that the analytical expression gives the same results for simpler special cases previously reported in the literature.

Keywords: c-ϕ soil backfill; dynamic active thrust; retaining wall; seismic loads; surcharge; tension cracks; wall friction and adhesion.

1. Introduction

For cohesionless soil backfills (ϕ soil backfills), the Mononobe-Okabe (M-O) expression is widely used to calculate the total load acting against the back of a rigid retaining wall due to the combined effect of static and seismic-induced inertial loads (Mononobe 1924, Okabe 1924, Mononobe and Matsuo 1929, Seed and Whitman 1970, Zarrabi 1979, Bowles 1996, Kramer 1996, Das and Ramana 2011). This load is called the dynamic active thrust (or total dynamic active pressure) in the current study consistent with earlier related papers by the first author. Analytical expressions for the dynamic active thrust from cohesive soil backfills (c-ϕ soil backfills) have also been reported (Okabe 1924, Saran and Prakash 1968, Richards and Shi 1994, Das and Puri 1996, Saran and Gupta 2003, Shukla *et al.* 2009, Greco 2010, Shukla and Zahid 2010, Shukla 2011). However, no analytical expression is currently available in explicit form for the dynamic active thrust from c-ϕ

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soil backfill that considers tension cracks in the backfill, a uniform surcharge at the backfill surface, both horizontal and vertical seismic loadings, and wall friction and adhesion. This combination of conditions is possible for conventional rigid retaining walls in the field. Therefore, this paper presents the derivation of an analytical expression for the dynamic active thrust, considering all these factors. Additionally, the development of an analytical expression in explicit form for the critical inclination of the failure plane within the c-ϕ soil backfill for the active state is also presented.

2. Analytical derivation

Fig. 1 shows a trial failure wedge $A_1A_2A_3$ consisting of c-ϕ soil backfill of weight W behind the vertical back face A_1A_2 of a retaining wall of height H. The parameters related to the wall geometry, soil backfill and applied loads are labelled in the figure. All the forces shown in the figure are expressed per unit running length of the wall. It is assumed that the failure occurs along a plane A_2A_3 that propagates from the bottom of the wall at an inclination α to the horizontal. The tension crack zone extends to depth z_c below the top surface of the backfill. The horizontal and vertical seismic inertial forces, k_hW and k_vW are also applied to the sliding wedge; the outward direction of the horizontal inertia vector is the critical case for the dynamic active thrust. The critical case for non-zero values of the vertical inertial force will vary depending on the magnitude of the horizontal inertial value; therefore both vertically downward and upward cases have been taken by considering positive and negative signs, respectively. Here k_h and k_v are the horizontal and vertical seismic coefficients, respectively. A uniform surcharge placed at the top of the slope A_1A_3 (= B) applies a

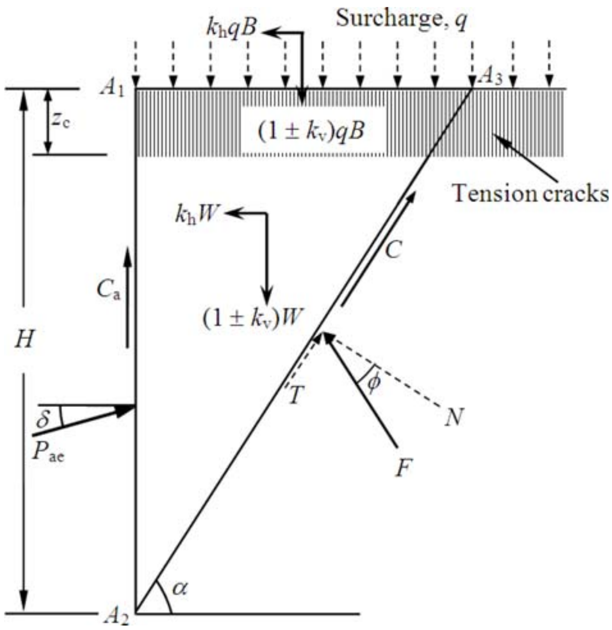

Fig. 1 Forces acting on a trial failure wedge consisting of c-ϕ soil backfill with tension cracks behind a rigid retaining wall in active state

vertical pressure q per unit surface area together with horizontal and vertical seismic inertial forces, $k_h qB$ and $k_v qB$, respectively. The force F is the resultant of the frictional component of the shear force T and the normal force N acting on the failure plane. C is the total cohesive force on the failure plane A_2A_3, and C_a is the total adhesive force mobilised along the wall-backfill interface A_1A_2. P_{ae} is the dynamic active thrust inclined at an angle δ to the normal to the back face of the wall.

From the geometry of Fig. 1,

$$\overline{A_1A_3} = B = H \cot \alpha \tag{1}$$

The weight of the soil wedge $A_1A_2A_3$ is

$$W = \frac{1}{2}(\overline{A_1A_2})(\overline{A_1A_3})\gamma = \frac{1}{2}(H)(H \cot \alpha) = \frac{1}{2}\gamma H^2 \cot \alpha \tag{2}$$

The total cohesive force mobilised along the failure plane A_2A_3 is

$$C = \bar{c} \times \overline{A_2A_3} = \bar{c}H \operatorname{cosec}\alpha \tag{3}$$

where $\bar{c}$ is the average cohesion of the backfill defined as

$$\bar{c} = \frac{1}{H}\left\{c(H-z_c) + \left(\frac{c}{2}\right)z_c\right\} = \left(1-\frac{z_c}{2H}\right)c \tag{4}$$

It should be noted that Eq. (4) is based on the assumption that the mobilized cohesive resistance within the tension crack zone varies linearly from c at the bottom of the tension crack to zero at the top of the tension crack (Lambe and Whitman 1979).

The average adhesion $\overline{c_a}$ of the soil backfill behind the wall can also be defined as

$$\overline{c_a} = \frac{1}{H}\left\{c_a(H-z_c) + \left(\frac{c_a}{2}\right)z_c\right\} = \left(1-\frac{z_c}{2H}\right)c_a \tag{5}$$

where c_a is the adhesion between the wall back face and the soil backfill.

From Eqs. (4) and (5)

$$\frac{\overline{c_a}}{c} = \frac{c_a}{c} = a_f, \text{ say} \tag{6}$$

where a_f is the adhesion factor with a value in the range [0, 1].

The total adhesive force mobilised along the wall-backfill interface A_1A_2 is

$$C_a = \overline{c_a} \times (\overline{A_1A_2}) = \overline{c_a}H \tag{7a}$$

Using Eq. (6), Eq. (7a) becomes

$$C_a = a_f \bar{c}H \tag{7b}$$

In Fig. 1, it is important to note that the interaction between the soil backfill and the back face of the retaining wall occurs through friction and adhesion. The former is considered by the inclination of the dynamic active thrust denoted by angle δ to the horizontal in the free-body diagram of the backfill. The latter is considered by the adhesive force C_a given in Eq. (7b). Actual values of factor a_f can be calculated using both c_a and c from laboratory shear tests performed on project-specific materials. However, for simplicity in routine design practice, one can assume that $c_a \approx c$. For the case of cohesionless soil backfills when $c_a \to 0$ and $c \to 0$, then $a_f \to 1$ from Eq. (6), resulting in $C_a = 0$ from Eq. (7b). This is the typical case for static and dynamic active thrust analyses for cohesionless soil backfills.

The maximum depth of tension crack z_c can be selected based on field observation/experience or it may be computed using the following expression based on Rankine theory (Taylor 1948, Lambe and Whitman 1979, Terzaghi *et al.* 1996, Das 2008).

$$z_c = \frac{2c}{\gamma}\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right) \tag{8}$$

Considering equilibrium of forces [↓+ and ↑−] in the vertical direction

$$-P_{ae}\sin\delta + (1 \pm k_v)W - F\cos(\alpha-\phi) + (1 \pm k_v)qB - C\sin\alpha - C_a = 0 \tag{9}$$

Substituting C and C_a from Eqs. (3) and (7b), respectively, Eq. (9) becomes

$$-P_{ae}\sin\delta + (1 \pm k_v)(W+qB) - F\cos(\alpha-\phi) - \bar{c}H - a_f\bar{c}H = 0 \tag{10}$$

Considering equilibrium of forces [→+ and ← −] in the horizontal direction

$$P_{ae}\cos\delta - k_hW - F\sin(\alpha-\phi) - k_hqB + C\cos\alpha = 0 \tag{11}$$

Substituting C from Eq. (3), Eq. (11) becomes

$$P_{ae}\cos\delta - k_h(W+qB) - F\sin(\alpha-\phi) + \bar{c}H\cot\alpha = 0 \tag{12}$$

Eliminating F from Eqs. (10) and (12),

$$P_{ae} = (1 \pm k_v)(W+qB)\frac{\tan\theta + \tan(\alpha-\phi)}{\cos\delta + \sin\delta\tan(\alpha-\phi)} - \bar{c}H\frac{(a_f+1)\tan(\alpha-\phi)+\cot\alpha}{\cos\delta + \sin\delta\tan(\alpha-\phi)} \tag{13}$$

where

$$\theta = \tan^{-1}\left(\frac{k_h}{1 \pm k_v}\right) \tag{14}$$

is the seismic inertia angle.

Eq. (13) is simplified as

$$P_{ae}\left(\frac{1 \pm k_v}{\cos\theta}\right)(W+qB)\frac{\sin(\theta-\phi+\alpha)}{\cos(\delta+\phi-\alpha)} - \bar{c}H\frac{a_f\sin(\alpha-\phi) + \cos\phi\,\mathrm{cosec}\,\alpha}{\cos(\delta+\phi-\alpha)} \tag{15}$$

Substituting B and W from Eqs. (1) and (2), respectively, Eq. (15) reduces to

$$P_{ae} = \left(\frac{1 \pm k_v}{\cos\theta}\right)\left(q + \frac{1}{2}\gamma H\right)H\frac{\cos\alpha\sin(\theta - \phi + \alpha)}{\sin\alpha\cos(\delta + \phi - \alpha)} - a_f\bar{c}H\frac{\sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)} - \bar{c}H\frac{\cos\phi}{\sin\alpha\cos(\delta + \phi - \alpha)} \quad (16)$$

Using Eq. (4), Eq. (16) is expressed as

$$P_{ae} = \left(\frac{1 \pm k_v}{\cos\theta}\right)\left(q + \frac{1}{2}\gamma H\right)H\frac{\cos\alpha\sin(\theta - \phi + \alpha)}{\sin\alpha\cos(\delta + \phi - \alpha)} - a_f cH\left(1 - \frac{z_c}{2H}\right)\frac{\sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)}$$

$$-cH\left(1 - \frac{z_c}{2H}\right)\frac{\cos\phi}{\sin\alpha\cos(\delta + \phi - \alpha)} \quad (17)$$

or

$$P_{ae} = \frac{1}{2}\left[\frac{m_1\cos\alpha\sin(\theta - \phi + \alpha) - m_2\sin\alpha\sin(\alpha - \phi) - m_3}{\sin\alpha\cos(\delta + \phi - \alpha)}\right]\gamma H^2 \quad (18)$$

where

$$m_1 = \left(\frac{1 \pm k_v}{\cos\theta}\right)\left(\frac{2q}{\gamma H} + 1\right) \quad (19a)$$

$$m_2 = a_f\left(\frac{2c}{\gamma H}\right)\left(1 - \frac{z_c}{2H}\right) \quad (19b)$$

and

$$m_3 = \left(\frac{2c}{\gamma H}\right)\left(1 - \frac{z_c}{2H}\right)\cos\phi \quad (19c)$$

It should be noted that m_1, m_2 and m_3 are dimensionless, and for given wall geometry, soil backfill properties, surcharge, and seismic coefficients, their values are known.

Eq. (18) can be expressed as

$$P_{ae} = \frac{1}{2}\left(\frac{a_1\tan^2\alpha - b_1\tan\alpha + c_1}{a_2\tan^2\alpha - b_2\tan\alpha}\right)\gamma H^2 \quad (20)$$

where

$$a_1 = m_2\cos\phi + m_3 \quad (21a)$$

$$b_1 = m_1\cos(\theta - \phi) + m_2\sin\phi \quad (21b)$$

$$c_1 = m_3 - m_1\sin(\theta - \phi) \quad (21c)$$

$$a_2 = -\sin(\delta + \phi) \quad (21d)$$

and

$$b_2 = \cos(\delta + \phi) \quad (21e)$$

It should also be noted that a_1, b_1, c_1, a_2 and b_2 are dimensionless, and for given wall geometry, soil backfill properties, surcharge, and seismic coefficients, their values are known.

For the maximum value of the dynamic active thrust P_{ae} from Eq. (20)

$$\frac{\partial P_{ae}}{\partial \alpha} = 0$$

or

$$\frac{\partial P_{ae}}{\partial(\tan\alpha)} = 0$$

or

$$(a_2 b_1 - a_1 b_2)\tan^2\alpha - 2a_2 c_1 \tan\alpha + b_2 c_1 = 0 \tag{22}$$

Eq. (22) is quadratic in $\tan\alpha$, which provides the critical value of inclination of the failure plane, $\alpha = \alpha_c$ as

$$\alpha_c = \tan^{-1}\left[\frac{a_2 c_1 \pm \sqrt{(a_2 c_1)^2 - (a_2 b_1 - a_1 b_2)(b_2 c_1)}}{(a_2 b_1 - a_1 b_2)}\right] \tag{23}$$

Since α_c will lie between 0° and 90°, $\tan\alpha_c$ cannot be negative; therefore '+' or '–' should be considered accordingly based on specific values of a_1, b_1, c_1, a_2 and b_2.

For real values of α_c, the expression under the radical sign in Eq. (23) must be positive, that is

$$(a_2 c_1)^2 - (a_2 b_1 - a_1 b_2)(b_2 c_1) \geq 0 \tag{24}$$

Substituting $\alpha = \alpha_c$ into Eq. (20), the dynamic active thrust is obtained as

$$P_{ae} = \frac{1}{2}\left(\frac{a_1 \tan^2\alpha_c - b_1 \tan\alpha_c + c_1}{a_2 \tan^2\alpha_c - b_2 \tan\alpha_c}\right)\gamma H^2 \tag{25}$$

It should be noted that Greco (2010) presented an equation similar to Eq. (25) based on the approach by Shukla *et al.* (2009) but without describing the derivation steps in detail. The Greco (2010) expression does not consider surcharge, vertically upward seismic inertial force and wall adhesion. Shukla (2010) reported key observations with some minor corrections and explanations to clarify details of the Greco solution.

It is common practice to present the expression for the static and dynamic active thrusts using earth pressure coefficients (Lambe and Whitman 1979, Terzaghi *et al.* 1994, Kramer 1996, Das 2008, Das and Ramana 2011). In the present general case, three coefficients $K_{ae\gamma}$, K_{aec} and K are introduced in the expression obtained by substituting $\alpha = \alpha_c$ into Eq. (17) as

$$P_{ae} = (1 \pm k_v)\left(q + \frac{1}{2}\gamma H\right)HK_{ae\gamma} - cHK_{aec} + \frac{2Kc^2}{\gamma} \tag{26}$$

where

$$K_{ae\gamma} = \frac{\cos\alpha_c \sin(\theta - \phi + \alpha_c)}{\cos\theta \sin\alpha_c \cos(\delta + \phi - \alpha_c)} \tag{27a}$$

$$K_{aec} = \frac{a_f \sin(\alpha_c - \phi) + \dfrac{\cos\phi}{\sin\alpha_c}}{\cos(\delta + \phi - \alpha_c)} \tag{27b}$$

and

$$K = \left[\frac{a_f \sin(\alpha_c - \phi) + \dfrac{\cos\phi}{\sin\alpha_c}}{2\cos(\delta + \phi - \alpha_c)}\right]\left[\frac{z_c}{\left(\dfrac{2c}{\gamma}\right)}\right] \tag{27c}$$

Eq. (26) provides a general expression for the dynamic active thrust. The factors $K_{ae\gamma}$ and K_{aec} are the active earth pressure coefficients with earthquake/seismic effects associated with unit weight and cohesion, respectively, and K is a tension crack factor.

3. Special cases

Case 1: $c = 0$, $\phi > 0$, $z_c = 0$; $\delta = 0$, $c_a = 0$, $a_f \rightarrow 1$; $k_h = 0$, $k_v = 0$; $q = 0$

Eqs. (14), (19a-c) and (21a-e) give the following:

$\theta = 0$; $m_1 = 1$, $m_2 = 0$, $m_3 = 0$, $a_1 = 0$, $b_1 = \cos\phi$, $c_1 = \sin\phi$, $a_2 = -\sin\phi$, and $b_2 = \cos\phi$. On substitution of these values into Eq. (23), the critical value of inclination of the failure plane is obtained as $\alpha_c = 45^\circ + \frac{\phi}{2}$. For this value of α_c, Eq. (27a) yields

$$K_{ae\gamma} = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45^\circ - \frac{\phi}{2}\right) = K_a \tag{28}$$

where K_a is the Rankine active earth pressure coefficient. Eq. (26) results in

$$P_{ae} = P_a = \frac{1}{2} K_a \gamma H^2 \tag{29}$$

Eq. (29) is the well-known Rankine equation that gives the static active thrust (P_a) from a cohesionless soil backfill.

Case 2: $c = c_u$, $\phi = 0$, $z_c > 0$; $\delta = 0$, $c_a = 0$, $a_f \rightarrow 0$; $k_h = 0$, $k_v = 0$; $q = 0$

Eqs. (14), (19a-c) and (21a-e) give the following:

$\theta = 0$, $m_1 = 1$, $m_2 = 0$, $m_3 = \frac{2c_u}{\gamma H}\left(1 - \frac{z_c}{2H}\right)$, $a_1 = \frac{2c_u}{\gamma H}\left(1 - \frac{z_c}{2H}\right)$, $b_1 = 1$, $c_1 = \frac{2c_u}{\gamma H}\left(1 - \frac{z_c}{2H}\right)$, $a_2 = 0$, and $b_2 = 1$. On substitution of these values into Eq. (23), the critical value of inclination of the failure plane is obtained as $\alpha_c = 45^\circ$. For this value of α_c, Eqs. (27a-c) with Eq. (8) yield $K_{ae\gamma} = 1$, $K_{aec} = 2$ and $K = 1$, respectively. Eq. (26) becomes

$$P_{ae} = P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma} \tag{30}$$

Eq. (30) is reported by Terzaghi *et al.* (1996) and Das (2008) and where γ is the saturated unit weight of soil.

Case 3: $c > 0$, $\phi > 0$, $z_c > 0$; $\delta = 0$, $c_a = 0$, $a_f \to 0$; $k_h = 0$, $k_v = 0$; $q = 0$

Eqs. (14), (19a-c) and (21a-e) give the following:

$\theta = 0$, $m_1 = 1$, $m_2 = 0$, $m_3 = \frac{2c}{\gamma H}\left(1 - \frac{z_c}{2H}\right)\cos\phi$, $a_1 = \frac{2c}{\gamma H}\left(1 - \frac{z_c}{2H}\right)\cos\phi$, $b_1 = \cos\phi$, $c_1 = \frac{2c}{\gamma H}\left(1 - \frac{z_c}{2H}\right)\cos\phi$ $+ \sin\phi$, $a_2 = -\sin\phi$, and $b_2 = \cos\phi$. Substitution of these values into Eq. (23) gives the critical value of failure plane inclination $\alpha_c = 45^\circ + \frac{\phi}{2}$, which is the same as for Case 1. For this value of α_c, Eqs. (27a-c) with Eq. (8) yield $K_{ae\gamma} = \frac{1 - \sin\phi}{1 + \sin\phi} = \tan^2\left(45^\circ - \frac{\phi}{2}\right) = K_a$, $K_{aec} = 2\sqrt{K_a}$ and $K = 1$, respectively. Eq. (26) becomes

$$P_{ae} = P_a = \frac{1}{2}K_a \gamma H^2 - 2\sqrt{K_a}cH + \frac{2c^2}{\gamma} \tag{31}$$

Eq. (31) is the well-known Rankine equation for static active thrust (P_a) for the cohesive-frictional soil backfills.

Case 4: $\phi > 0$, $c = 0$, $z_c = 0$; $\delta > 0$, $c_a = 0$, $a_f \to 1$; $k_h > 0$, $k_v > 0$; $q = 0$

This special case results in the M-O equation (Mononobe 1924, Okabe 1924, Mononobe and Matsuo 1929), which is given below using the notation defined in Fig. 1.

$$P_{ae} = \frac{1}{2}(1 \pm k_v)K_{MO}\gamma H^2 \tag{32}$$

where

$$K_{MO} = \frac{\cos^2(\theta - \phi)}{\cos\theta\cos(\theta + \delta)\left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \theta)}{\cos(\delta + \theta)}}\right]^2} \tag{33}$$

It should be noted that Eq. (32) does not consider any surcharge or wall-backfill interface adhesion. A comparison of Eq. (32) with Eq. (26) for this special case shows that $K_{MO} = K_{ae\gamma}$, which is given by Eq. 27(a) in terms of α_c obtained from Eq. (23) using the values of parameters for this special case. From numerical calculations for a common set of parameters, it is found that Eqs. (32) and (26) give the same values of P_{ae}. Additionally, the value of α_c for this special case is also obtained from Eq. (23), which is not reported in the literature in exact form with all practical parameters considered in this paper. Comparison with solutions to α_c reported in the literature for a common set of parameters (Okabe 1924, Zarrabi 1979, Bathurst *et al.* 2012) gives reasonable agreement.

The newly derived generalised expressions [Eqs. (23) and (26)] give the expressions presented earlier by Shukla *et al.* (2009), Shukla and Zahid (2011) and Shukla (2011) for simplified problem conditions.

4. Conclusions

An improved explicit analytical expression [Eq. (26)] is derived in terms of seismic earth pressure coefficients and a tension crack factor for calculating the dynamic active thrust from a *c*-ϕ soil backfill acting at the back of a rigid retaining wall with uniform surcharge and wall-soil friction and adhesion. An explicit expression [Eq. (23)] in exact form for the critical angle of inclination to the horizontal of the failure plane is also presented which is an improvement over other solutions found in the literature. These equations are useful for the calculation of destabilizing earth forces used in the pseudo-static seismic analysis and design of conventional rigid retaining walls with simple geometry, soil properties and boundary conditions.

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