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Deflection and buckling of buried flexible pipe-soil system in a spatially variable soil profile

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Abstract. Response of buried flexible pipe-soil system is studied, through numerical analysis, with respect to deflection and buckling in a spatially varying soil media. In numerical modeling procedure, soil parameters are modeled as two-dimensional non-Gaussian homogeneous random field using *Cholesky decomposition* technique. Numerical analysis is performed using random field theory combined with finite difference numerical code FLAC 5.0 (2D). Monte Carlo simulations are performed to obtain the statistics, i.e., mean and variance of deflection and circumferential (buckling) stresses of buried flexible pipe-soil system in a spatially varying soil media. Results are compared and discussed in the light of available analytical solutions as well as conventional numerical procedures in which soil parameters are considered as uniformly constant. The statistical information obtained from Monte Carlo simulations is further utilized for the reliability analysis of buried flexible pipe-soil system with respect to deflection and buckling. The results of the reliability analysis clearly demonstrate the influence of extent of variation and spatial correlation structure of soil parameters on the performance assessment of buried flexible pipe-soil systems, which is not well captured in conventional procedures.

Keywords: buried pipes; flexible; deflection; Monte Carlo; numerical; random; spatial.

1. Introduction

The pioneering work on the buried flexible pipe-soil system to determine its horizontal and vertical deflections, its bending moments, and its tangential thrusts is done by Marston and Anderson (1913), Marston (1930) and Spangler (1941). Full-scale experiments on flexible culverts were conducted and the design formulas, developed from the load hypothesis, were verified by Spangler (1941). The hypothesis assumed the parabolic distribution of passive horizontal pressures on the sides of a pipe. In due course of time many of the design standards all over the world adopted the Marston-Spangler theory. In most of the situations, the failure of a pipe is characterized by (i) excessive deflection, and (ii) buckling. Closed form solutions as well as results of the finite element analysis for these modes of failures are available in the literature (Watkins and Spangler 1958, Burns and Richards 1964, Moser 1990, AASHTO 2004). In general, design of buried flexible pipe-soil system is controlled by deflection. Carrier (2005) indicated that there are following

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situations in which buckling may control the design, viz., (i) shallow cover with an internal vacuum pressure and (ii) shallow cover, submerged in deep water, with atmospheric internal pressure. Carrier (2005) further stated that in these cases, the standard short-term deflection criterion of 3% should be reduced sufficiently, in order to ensure that deflection controls the design.

The conventional design procedures require that the geometric details of the buried flexible pipes should meet the following requirements for successful performance (also termed as performance limits), i.e. (i) estimated deflection should be within tolerable limits, and (ii) total stress on the circumference of the pipe should not exceed the allowable buckling pressure.

In conventional approach, the above-mentioned criteria are satisfied with a certain margin of safety expressed in terms of *factor of safety*. It is implicitly assumed that the adopted factor of safety will take care of (a) all sources of uncertainties involved in the estimation of design parameters, as well as, (b) in the derivation of analytical formulation based on simplified assumptions. In this approach, the soil is treated as a material with uniformly constant soil properties defining its strength and stiffness characteristics.

A question that often arises, in practice, is to know "how safe is safe?" or to what extent the factors of safety that are routinely used to address the question of safety and economy are adequate. These factors of safety represent the combined influence of total variability and deviations from analytical formulations based on simplified assumptions. To take care of different sources of uncertainties involved in the estimation of input strength and stiffness parameters, a selection of appropriate value of factor of safety comes from past experiences and good engineering judgments. It is well understood that the approach is simple and straightforward but does take into account the variability in an appropriate manner (Schweiger *et al*. 2001).

Soil, being a natural material, the stochastic analysis of soil spatial variability and use of probabilistic models is deemed necessary for the performance assessment of buried flexible pipe-soil system. The effect of spatial variation of soil properties on the response of various geotechnical structures received considerable attention in recent years. The approach combines the numerical analysis based upon finite element or finite difference schemes and random field theory proposed by Vanmarcke (1983).

In spatial variability modelling, random variables vary continuously over space and are referred to as random fields. In a random field, the variable exhibits auto-correlation, which means the values of the variable at one point are correlated to the values at nearby points. To characterize a random field, in addition to the mean and standard deviation (or variance), quantification of the correlation structure is also required. A classic paper introducing spatial correlation concepts to the geotechnical profession was published by Vanmarcke (1977a). Some recent papers further summarizing the concept include those by Degroot (1996), Fenton (1996), Lacasse and Nadim (1996), Phoon and Kulhawy (1996), Uzielli *et al*. (2007).

Griffiths and Fenton (1993, 2001, 2004), Fenton and Griffiths (1995, 2003) studied various geotechnical problems using random field finite element method (RFEM) and highlighted several advantages of RFEM. Fenton and Griffiths (2003), in solving bearing capacity problem of spatially random cohesive-frictional soil, mentioned that the random finite element method (RFEM) actually models the physical locations of weak and strong zones in the domain. When the soil block is compressed, progressive failure occurs, and the failure mechanism 'seeks out' the weakest path through the soil.

Considering slope stability problem, Griffiths and Fenton (2004) indicated that RFEM offers many advantages over traditional probabilistic techniques, i.e. (i) it enables slope failure to develop naturally by seeking out the most critical failure mechanism, and (ii) it also overcomes the drawback of traditional probabilistic analysis in which spatial variability is ignored by assuming perfect correlation, which may lead to unconservative estimates of the probability of failure. Griffiths and Fenton (2007) highlighted that the RFEM is able to properly account for the important influence of spatial correlation and local averaging in geotechnical problems involving highly variable soils.

2. Objectives of the present study

The objective of the present study is (i) to study the response of buried flexible pipe-soil system with respect to deflection and buckling in a spatially varying soil media considering a typical case in which 1.4 m diameter steel pipe is buried at a depth of 4.8 m and there is a uniform surcharge of 50 kPa at the ground surface to stimulate the traffic load; (ii) to discuss results in the light of available analytical solutions and conventional numerical procedures in which geotechnical properties are considered as uniformly constant; (iii) to perform probabilistic analysis of buried flexible pipe-soil system with respect to deflection and buckling through Monte Carlo simulations. A parametric study is performed to investigate the effect of extent of variation and auto-correlation structure of input soil parameters on the reliability of buried flexible pipe-soil system.

3. Available analytical solutions

3.1 Deflection

Deflection is quantified in terms of the ratio of the horizontal increase in diameter (ΔX) (or vertical decrease in diameter, Δ_y) to the pipe diameter (D). Spangler (1941) developed the following semi-empirical equation based on the modified Lowa formula for calculating the deflection of buried flexible pipe-soil systems under earth load

$$\Delta X = \frac{D_L K_b W_c R_p^3}{E_p I + 0.061 E' R_p^4} \tag{1}$$

where, ΔX = horizontal deflection or change in diameter (inch); D_L = deflection lag factor; K_b = bedding constant; W_c = Marston's load per unit length of pipe (lb/inch); R_p = mean radius of pipe (inch); E_p = Modulus of elasticity of pipe material (lb/in^2), I = moment of inertia of pipe wall per unit length (inch^3) = $t^3/12$ (for thin walled pipe); t = thickness of pipe wall (inch); E' = soil modulus of reaction (lb/in^2). It should be noted that the Spangler equation needs semi-empirical properties such as K_b, D_L, and E', which are cumbersome to determine (Kang *et al.* 2008).

Burns and Richards (1964) provided the following theoretical solution for deflection, i.e. vertical decrease in diameter, Δ_y, expressed in percentage (%) with respect to pipe diameter, D

For a full bonded interface

$$\frac{\Delta_y}{D}(\%) = \frac{q}{4M_s}[UF(1-a_o^*)+VF(1-a_2^*-2b_2^*)]\times 100 \tag{2}$$

For a free-slip surface

$$\frac{\Delta_y}{D}(\%) = \frac{q}{4M_s}\left[UF(1-a_o^*)+\frac{2}{3}VF(1+a_2^{**}-4b_2^{**})\right]\times 100 \tag{3}$$

where

$$UF = \frac{2B_k M_s R_p}{E_p A_p} \tag{4}$$

$$VF = \frac{C_k M_s R_p^3}{3E_p I} \tag{5}$$

$$M_s = \frac{E_s(1-\nu)}{(1+\nu)(1-2\nu)} \tag{6}$$

$$a_o^* = \frac{UF-1}{UF+(B_k/C_k)} \tag{7}$$

$$a_2^* = \frac{C_k(1-UF)VF-UF(C_k/B_k)+2B_k}{(1+B_k)VF+C_k(VF+1/B_k)+2(1+C_k)} \tag{8}$$

$$a_2^{**} = \frac{2VF-1+(1/B_k)}{2VF-1+(3/B_k)} \tag{9}$$

$$b_2^* = \frac{(B_k+C_k\times UF)VF-2B_k}{(1+B_k)VF+C_k(VF+1/B_k)UF+2(1+C_k)} \tag{10}$$

$$b_2^{**} = \frac{2VF-1}{2VF-1+(3/B_k)} \tag{11}$$

In the above expressions, Δ_y = vertical decrease in diameter; q = vertical stress (surface stress) on the crown of the pipe; M_s = one–dimensional constrained soil modulus; A_p = area of pipe wall per unit length; UF = extensional flexibility ratio; VF = bending flexibility ratio; B_k = non-dimensional parameter = $(1 + K_s)/2$; C_k = non-dimensional parameter; K_s = lateral stress ratio = $\nu/(1-\nu)$; and ν = Poisson's ratio of soil.

3.2 Buckling

Buckling is a premature failure in which the structure becomes unstable at a stress level that is well below the yield strength of the structural material. The allowable buckling pressure (f_a) can be calculated using the following expression (Luscher 1965)

$$f_a = \frac{\left\{\left[8(n^2-1)\frac{E_pI}{D^3}\right]+\frac{BE_s}{(n^2-1)}\right\}}{FS_b}; \quad (n=2,3,4) \text{ and } p \le f_a \tag{12a}$$

where, p = total stress on the circumference of the pipe, including internal vacuum, if any (kPa); B' = coefficient of elastic support; FS_b = factor of safety against buckling (usually 2.5 to 4); n is the number of nodes in the circumference of the buckled pipe (≥ 2).

The above formulation (Eq. 12(a)) may also include the Poisson's ratio of the pipe material and the ovality (or ellipticity) of the pipe (Baikie and Meyerhof 1982). Carrier (2005) stated that these additional corrections have only minor influence and are, to a certain extent, compensating. Thus, in most geotechnical applications, these terms are neglected, subsumed into the factor of safety.

In extreme limits, E_s = 0 (no soil support and pipe is surrounded by a fluid) and n = 2, Eq. (12a) reduces to

$$p = \frac{24E_pI}{D^3FS_b} \tag{12b}$$

If E_s is sufficiently high (for a soil supported buried pipe), then $n \geq 3$ and Eq. (12a) is approximated as

$$p = \frac{[32BE_sE_pI]^{1/2}}{D^3}\frac{1}{FS_b}; \quad (n \geq 3) \tag{12c}$$

The algebraic value of n is given by the following expression

$$n = \left[\frac{BE_sD^3}{8E_pI}\right]^{1/4}; \quad (n \geq 3) \tag{13}$$

AWWA (1983, 1989, and 1996) modified Eq. (12c) by introducing various parameters

$$f_a = \left[\frac{32R_wB'E'E_pI}{D^3}\right]^{1/2}\frac{1}{FS_b} \tag{14}$$

where, R_w = water buoyancy factor. The expressions for R_w and B' are given below

$$R_w = 1 - 0.33(h_w/H); \quad 0 \leq h_w \leq H \tag{15}$$

$$B' = \frac{1}{[1+(4e^{-0.213H})]} \tag{16}$$

where, h_w = height of water surface above top of pipe (m); and H = height of fill above pipe (m). The allowable buckling pressure (f_a) evaluated from Eq. (14) must be greater than the actual stress calculated in the circumference of pipe wall (q_b) using Eq. (17)

$$q_b = 0.00981[(R_wH\rho_s)+(\rho_wh_w)]+\frac{1000W_L}{OD} \tag{17}$$

where, ρ_s = density of backfill soil (kg/m^3); ρ_w = water density (1000 kg/m^3); W_L = live load (kPa); OD = outside diameter of pipe (mm).

4. Uncertainties in buried flexible pipe design

The design methodology discussed in previous section incorporates input parameters, viz. soil density (ρ_s), elastic modulus of pipe (E_p) and one-dimensional constrained soil modulus (M_s). To overcome variability in the estimation of these input parameters conventional methods incorporate *factors of safety* for deflection and buckling pressure. Considering the fact that most flexible pipes (steel, PVC and HDPE) can tolerate deflections from 2% to 5% of the diameter of the pipe without developing any structural problem (Moser 1990); the allowable deflection limit is taken as 2% as per the recommendations provided in Stephenson (1976). Safety factors of 2.0 is used for buckling (AASHTO 2004).

Phoon and Kulhawy (1999a, 1999b) indicated that the major sources of uncertainties in the estimation of geotechnical parameters include factors such as: (a) natural heterogeneity or inherent variability (the physical phenomenon contributing to the variability), (b) measurement error (due to equipment, procedural-operator, and random testing errors), and (c) model transformation uncertainty (due to approximation present in empirical, semi-empirical or theoretical models to relate measured quantities to design parameters). Quantitative assessment of soil uncertainty modeling requires the use of statistics as well as probabilistic modeling, which relies on sets of measured data. The uncertainty in the measured data (say m no. of measured data sets) is expressed in terms of sample mean (μ) and variance (σ^2), which is evaluated from the following expression

Sample Mean

$$\mu = \frac{1}{m}\sum_{i=1}^{m} x_i \tag{23}$$

Variance (σ^2) : It is a measure of dispersion of data about the mean value. The square root of variance is defined as standard deviation (σ).

$$\sigma^2 = \frac{1}{(m-1)}\sum_{i=1}^{m}(x_i-\mu)^2 \tag{24}$$

The coefficient of variation (CoV%), which is obtained by dividing the sample standard deviation (σ) by the sample mean (μ), is commonly used in quantifying the geotechnical uncertainty. Several studies in the past (Lacasse and Nadim 1996, Duncan 2000, Uzielli *et al*. (2007) provided the generic range of coefficient of variation (CoV%) in the geotechnical parameters. Table 1 summarizes the generic range of coefficients of variation of the relevant geotechnical parameters used in the present analysis.

The uncertainties in the input soil parameters and their impact on the performance of a geotechnical system are studied using the reliability-based design procedures. Reliability analysis

Table 1 Coefficient of variation (CoV%) for selected geotechnical parameters

Property	CoV% range
Dry unit weight (γ_d)	2-13
Undrained shear strength (c_u)	6-80
Effective friction angle (ϕ')	7-20
Elastic modulus (E_s)	15-70

focuses on the most important aspect, i.e. probability of failure (p_f). In the reliability analysis, the input soil parameters are modeled as continuous random variables defined by their probability density functions (*pdf*) and the parameters of distributions. Normally, in geotechnical practice, the input soil parameters are modeled either as normally distributed or log-normally distributed continuous random variables (Baecher and Christian 2003). The parameters of the normal and log-normal probability distribution functions (*pdf*) are directly related to the unbiased estimates of statistical moments, i.e. sample mean (μ) and variance (σ^2) of the measured data.

5. Spatial variation and random field modeling

It is well understood that the second moment statistics, i.e. mean (μ) and variance (σ^2), alone are insufficient to describe the spatial variation of soil properties, which vary in the 2- or 3-dimensional space. The spatial variation of *in situ* soil represented by the mean (μ), coefficient of variation (CoV%), and auto-correlation distance (δ_z) is well represented by the random field theory developed by Vanmarcke (1983). For the spatial variability modeling, a parameter, i.e. an auto-correlation distance (δ_z) is defined as "the distance within which the soil property exhibits relatively strong correlation". It is noted that for low values of δ_z the domain is similar to erratic field and as δ_z increases the field becomes more homogeneous.

For representing a log-normally distributed continuous random variable input soil property (say, c), represented by parameters such as mean (μ_c), standard deviation (σ_c), auto-correlation distance (δ_z), the following equation is utilized

$$c(\tilde{x}_i) = \exp\{\mu_{\ln c}(\tilde{x}) + \sigma_{\ln c}(\tilde{x}) \cdot G_i(\tilde{x})\} \tag{25}$$

where $\tilde{x}$ is the spatial position at which c is desired. $G(\tilde{x})$ is a normally distributed random field with zero mean and unit variance. The values of $\mu_{\ln c}$ and $\sigma_{\ln c}$ are determined using log-normal distribution transformations given by the following expressions

$$\mu_{\ln c} = \ln\mu_c - \frac{1}{2}\sigma_{\ln c}^2 \tag{26}$$

$$\sigma_{\ln c}^2 = \ln\left(1 + \frac{\sigma_c^2}{\mu_c^2}\right) = \ln(1 + COV_c^2) \tag{27}$$

Fenton and Griffiths (2003) indicated that the correlation coefficient, $\rho_c(\tau)$, between log-normally distributed soil properties at two points, viz., $\ln c(x)$ and $\ln c(x+\tau)$, separated by τ, follows a Gauss-Markov model, which is an exponentially decaying function of separation distance. Hence, in this study, correlation function is considered to be exponentially decaying as given by the following equation

$$\rho_c(\tau) = \exp\left(-\frac{2\tau}{\delta_v}\right) \tag{28}$$

where, $\tau = |\tilde{x}_1 - \tilde{x}_2|$ is the absolute distance between the two points. The auto-correlation matrix is decomposed into the product of a lower triangular matrix and its transpose by *Cholesky decomposition* (Press *et al*. 2002)

$$L \cdot L^T = \rho_c \tag{29}$$

Given the matrix L, the correlated standard normal random field is obtained as follows

$$G_i = \sum_{j=1}^{i} L_{ij} Z_j, \qquad i = 1,2,3 \ldots n \tag{30}$$

where, Z_j is the sequence of independent standard normal random variables. In the present study, the spatial correlation distance in the vertical and horizontal directions are assumed to be equal and that represents the isotropic correlation structure, which is sufficient to establish the basic stochastic behavior as discussed by Griffiths *et al*. (2002). Different correlation distances in the two directions represent anisotropic correlation structure, which is not a scope of the present paper.

6. Reliability analysis

The limit state, $g(X)$, is a function of random variables (soil and pipe properties) that define the failure or safe state at the design point; $g(X) < 0$ represents the failure state, $g(X) > 0$ indicates the safe state and $g(X) = 0$ represents the limit state boundary, which separates the safety and failure domain. The probabilistic assessment of the safety of the system is made in terms of reliability index.

Defining the limit state function as $g(X) = R - S$; where R is the resistance or capacity and D is the demand, the reliability index (β) values can be obtained from the following equations (Baecher and Christian 2003)

For uncorrelated normally distributed R and S

$$\beta = \frac{R - S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{31}$$

For uncorrelated log-normally distributed R and S

$$\beta = \frac{\ln\left[\frac{\mu_R}{\mu_S}\sqrt{\frac{(1 + CoV_S^2)}{(1 + CoV_R^2)}}\right]}{\sqrt{\ln[(1 + CoV_R^2)(1 + CoV_S^2)]}} \tag{32}$$

where, μ, σ and CoV are mean value, standard deviation and coefficient of variation, respectively. R and S in the subscripts stand for resistance and demand, respectively. The statistical information about R and S, i.e. mean (μ_R, μ_S) and standard deviation (σ_R, σ_S) are obtained from Monte Carlo simulation runs.

USACE (1997) made specific recommendation on target probability of failure (p_f) and reliability indices (β) in geotechnical and infrastructure projects. The suggested guidelines are given in Fig. 1 that indicates that a reliability index (β) value of at least 4.0 is required for a good performance of the system while a value of at least 3.0 is needed for an above-average performance (Phoon 2004).

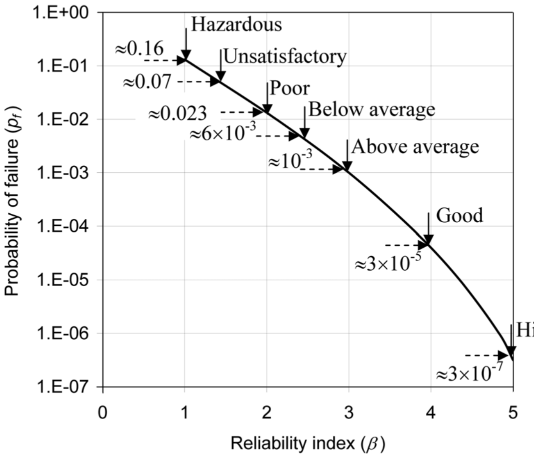

Fig. 1 USACE (1997) guidelines for reliability index (β) and corresponding probability of failure (p_f) (adapted from Table B-1) (Phoon 2004)

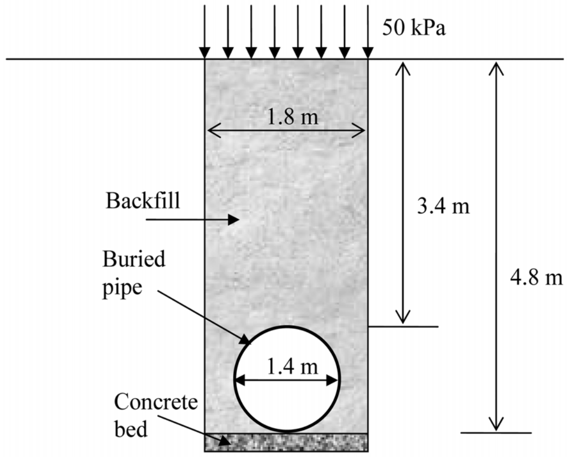

Fig. 2 Geometrical details of the buried flexible pipe

Table 2 Properties of soil and pipe material (Sivakumar Babu and Rajaparthy 2005, Sivakumar Babu *et al*. 2006)

Material properties	Pipe	Native soil	Backfill
Material behavior	Elastic	Mohr-Coulomb	Mohr-Coulomb
Unit weight (γ), kN/m^3	78.5	20	16
Elastic modulus (Pa)	2.15×10^{11}	6.770×10^6	2.39×10^6
Poison's ratio (ν)	0.30	0.34	0.21
Cohesion (c'), kPa	0	5.0	0
Friction angle (ϕ')	0	30°	26°

7. Results of the analysis and discussion

7.1 Analytical solutions

For a typical geometrical configuration of buried flexible pipe-soil system as shown in Fig. 2 and using the properties of pipe material, native soil and backfill material provided in Table 2 and other parameters given in Table 3, the results of the calculations for deflection and buckling are provided in Table 4. It can be noted that the calculated values of deflection and circumferential stress (buckling stress) are well below their respective allowable limits providing enough margin of safety expressed in terms of factor of safety. The buried flexible pipe-soil configuration as shown in Fig. 2 should be considered as a stable structural system. For the mean value of material properties and other parameters provided in Table 2 and 3, the deflection and circumferential stress values are obtained as 24.70 mm and 88.68 kPa, respectively. Though these values, which are based on the mean values of parameters, are less than the corresponding permissible limits, it is imperative that the reliability of the system should also be ensured in the context of spatial variation of geotechnical parameters.

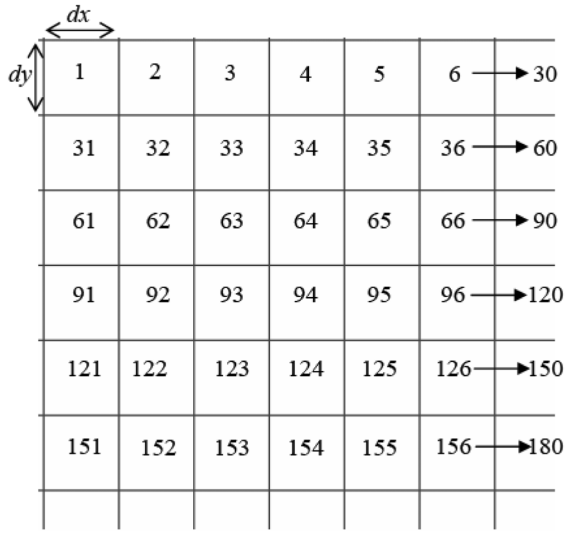

Fig. 3 Discretization of finite difference grid in the random filed modeling

Table 3 Other properties and parameters

Symbol description	Value
Thickness of pipe (t), m	0.008
Buoyancy factor (R_w)	1.00
Trench width (B_d), m	1.80
Height of the backfill (H), m	3.40
Pipe diameter ($D = 2R$), m	1.40
Moment of inertia of the pipe section (I), m^4/m	4.27×10^{-8}
Soil constrained modulus, (M_s) kPa	2690
Overburden stress (kPa)	50
Vertical stress on the crown of pipe (q), kPa	104.4
x-sectional area of pipe wall per unit length (A_p), m^2/m	0.008
x-sectional area of pipe section (A), m^2	0.0354

Table 4 Results of deflection and buckling pressure values obtained for given case of buried flexible pipe-soil system

Deflection	
For the mean value of the soil properties	24.70 mm
Allowable deflection (2.0% of pipe diameter)	28.00 mm (ok)
Buckling pressure	
Height of the water table from the pipe invert (H_w), m	0
Critical buckling pressure (f_{cr}), kPa	225.94
Actual buckling pressure (q_a), kPa	88.68
Available factor of safety (it should be at least 2.0)	2.55 (ok)

7.2 Implementation of random field

Following the procedure explained in previous section (5) the auto-correlation matrix is first generated using Eq. (28). The value of lag distance (τ) is considered to be the center distance of the constitutive grid. Fig. 3 explains the evaluation of auto-correlation matrix after considering the discretization of finite difference grid. For example, if the center to center distance between grids 1 and 2 is *dx*, the auto-correlation between these two grids can be calculated by putting the value $\tau = dx$ in Eq. (28). Similarly, auto-correlation of grid 1 with 3, 4, 5 can be established by placing $\tau = 2 \times dx$, $3 \times dx$ and $4 \times dx$, respectively and of grid 1 with 31, 32, 33 can be given by *dy*, and $\sqrt{dx^2 + dy^2}$, $\sqrt{(2dx)^2 + dy^2}$ respectively and so on. Therefore, the values in the first row of the auto-correlation matrix are the auto-correlation coefficients between grid 1 and other grids, and this leads to 900 values in a row (if suppose the size of grids is 30 × 30). Hence, considering all the grids, the size of the auto-correlation matrix is 900 × 900. Once the auto-correlation matrix is established it is decomposed into lower and upper triangular matrices using *Cholesky decomposition technique*. The correlated standard normal random field is obtained by generating a sequence of independent standard normal random variables (with zero mean and unit standard deviation) and decomposed auto-correlation matrix as obtained by Eq. (30). The realization of log-normally distributed cohesion value at each grid location is obtained by transformation presented in Eq. (25) for a specified mean and standard deviation of cohesion parameter *c*.

The implementation of the above calculation procedure in the numerical code is done by developing a subroutine in '*FISH*' code in *FLAC*. *FISH* is a programming language, which is used to define new variables and functions and is embedded in *FLAC*. For example, new variables may be plotted or printed, special grid generators may be implemented, servo-control may be applied to a numerical test, non-normal distributions of properties may be specified and parameter studies may be automated. For a detailed discussion on the topic the reader may refer to the *FLAC* reference manual (Itasca 2007).

The buried flexible pipe-soil system (as shown in Fig. 2) is numerically modeled using FLAC code. The side boundaries are fixed in horizontal direction and bottom boundary is fixed in both directions. The grid size is taken as 0.1 m × 0.1 m. Elastic perfectly plastic Mohr-coulomb model with non-associated flow rule is used for backfill material and native soil. The pipe material is considered as elastic material and the analysis is performed assuming full bonded interface. The extent of boundaries is selected in such a way that it does not influence the results.

Fig. 4 illustrates the typical realization of spatial variation of angle of internal friction (ϕ') of backfill material and native soil for a given mean and *CoV*% (Table 5) and auto-correlation distance (δ_z) = 0.0 m. The values of mean and variance in the input parameters (or *CoV*%) have been obtained from the published work by Sivakumar Babu and Rajaparthy (2005) and Sivakumar Babu *et al*. (2006).

7.3 Numerical analysis results-deterministic vs. stochastic

In the deterministic numerical analysis the mean values of pipe and soil properties are taken from Table 5 and the deflection and circumferential stress values are computed as 14.62 mm and 32.56 kPa, respectively. It can be observed that these values are well below the corresponding values obtained from the available analytical solutions. It should also be noted that the analytical solutions have idealized assumptions and, therefore, provide a conservative estimate of deflection and circum-

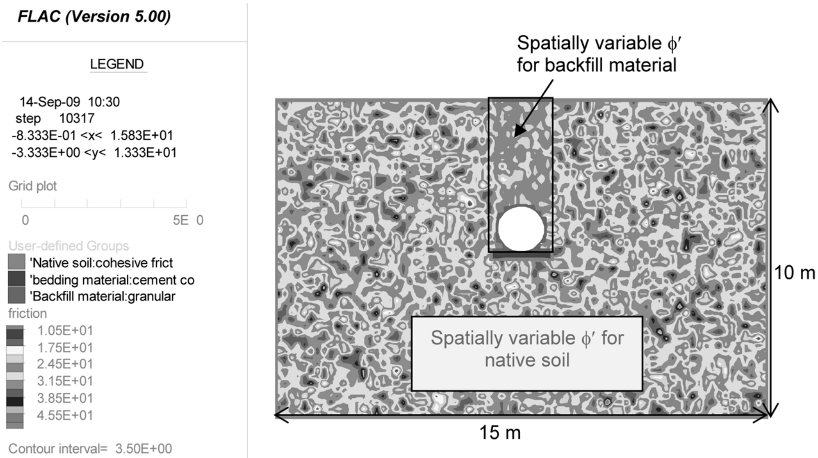

Fig. 4 Typical realization of spatially variable frictional angle for the backfill material and native soil (mean and *CoV*% as per Table 5)

Table 5 Statistical properties (mean value and coefficient of variation) of the material adopted from (Sivakumar Babu and Rajaparthy 2005, Sivakumar Babu *et al*. 2006)

Material properties	Mean (μ)	*CoV*(%)	Standard deviation (σ)
Elastic modulus of pipe, E_p	2.15×10^{11}	–	–
Elastic modulus of backfill material, E_s (Pa)	2.39×10^6	30%	0.717×10^6
Friction angle for backfill material, ϕ'	26°	18%	4.68°
Unit weight of backfill soil, γ_s (kN/m^3)	16	15%	2.40
Elastic modulus of native soil, E_n (Pa)	6.77×10^6	30%	2.031×10^6
Unit weight of native soil, γ_n (kN/m^3)	20	15%	3.0
Cohesion parameter for native soil, c_n, kPa	5.0	25%	1.25
Friction angle for native soil, ϕ_n'	30°	18%	5.4°

ferential stresses, which is highlighted from the comparison of the results. As mentioned earlier, in the conventional numerical analysis soil parameters are considered as uniformly constant. Fig. 6 shows the shear strain increment (*ssi*) contours, which indicates equal displacement on both sides of the pipes due to consideration of uniformly constant soil properties.

Since the objective of the present work is to study the response of buried flexible pipe-soil system in a spatially varying soil media, variation in the properties of pipe material and geometric uncertainties of the buried pipe-soil system are neglected. In stochastic numerical analysis, modeling spatial variation of geotechnical parameters, i.e. cohesion (c), angle of internal friction (ϕ), unit weight (γ) and elastic modulus (E_s) of backfill material and native soil required statistical information from Table 5 and correlation distance value of 0 m is taken. It should be noted that in each run the '*FISH*' program (subroutine) assigns different realizations of random field in finite difference grids and, therefore, the response of the buried flexible pipe-soil system in each run would be different. Hence, Monte Carlo Simulations (*MCS*) are performed to obtain the statistical information, i.e. mean (μ) and variance (σ^2) of deflection and circumferential (buckling) stress.

It should be noted that number of Monte Carlo simulations influences the accuracy of results. An increase in the number of simulations although increases the accuracy but at the same time increases the computational efforts. Hence, a compromise between accuracy and computational time is achieved by estimating the variance (δ_f) of the estimated mean of deflection or circumferential stress for several number of simulation cycles (N). In this approach, a number of simulations are carried out for incrementally large number of cycles till there is no significant change in the value of δ_f.

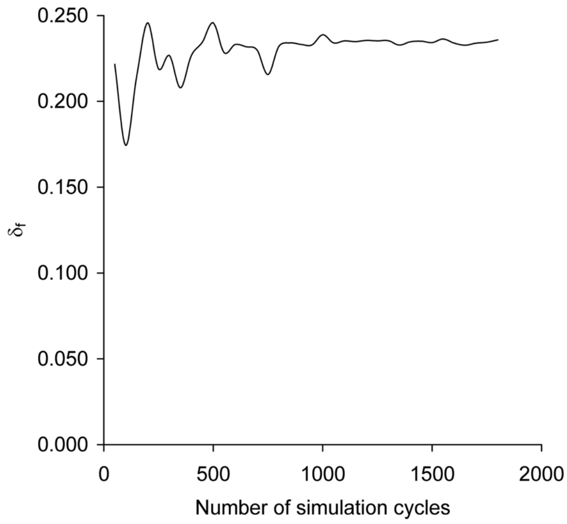

Fig. 5 Variation of δ_f and number of simulation cycles (N)

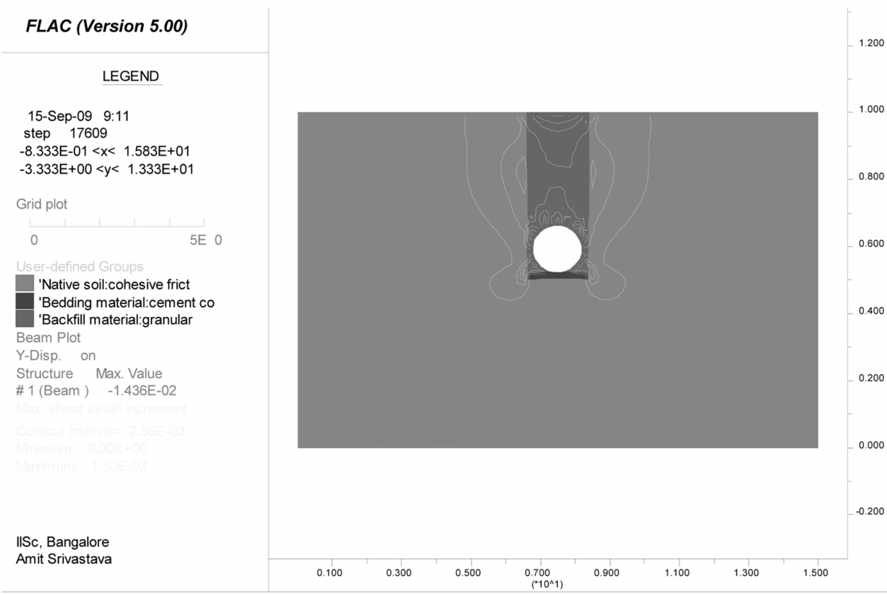

Fig. 6 Shear strain increment (*ssi*) contours indicating uniformity on the both side of the pipes for uniformly constant soil properties (maximum displacement = 14.62 mm)

From Fig. 5 it can be noted that between 1500 and 1800 simulations the variation in the value of δ_f (for deflection) is almost negligible. Hence, it can be expected that a further increase in the number of simulations will not significantly affect the accuracy of the results. Similar results were obtained for buckling pressure criterion for less than 2000 simulation runs. Therefore, in the present analysis, 2000 Monte Carlo simulations are carried out for estimating the statistical parameters, i.e. mean and variance of deflection and circumferential stress for the buried flexible pipe-soil system in which soil properties are spatially variable.

The mean and coefficients of variation for deflection and buckling pressure from 2000 Monte Carlo simulations are evaluated as 19.37 mm (CoV = 13.18%) and 43.14 kPa (CoV = 11.62%), respectively. It can be noted that the mean values of deflection and circumferential stress obtained for spatially varying soil media are relatively higher than the corresponding values obtained from conventional numerical analysis procedure. These values are further utilized for the reliability index (β) calculations using Eqs. (31) and (32). For normally and log-normally distributed R (μ_R = 28 mm, σ_R = 0) and S (μ_S = 19.37 mm, σ_S = 2.55 mm), the reliability index (β) values from deflection criterion are evaluated as 3.38 and 2.88, respectively. On the other hand exceptionally high values of β (36.70 for normally distributed S; and 14.59 for log-normally distributed S) are obtained from buckling criterion (for μ_R = 225.94 kPa, σ_R = 0; and μ_S = 43.87 kPa, σ_S = 4.96 kPa). It may be due to the following two reasons, i.e. (i) huge difference in the allowable limits of buckling pressure (μ_R) and actual values calculated from the numerical analysis (μ_S), and (ii) lower coefficient of variations in the estimated S. In the present case it is understood that deflection is the governing criterion for the design of buried flexible pipe-soil system.

The results of the reliability analysis clearly demonstrate that the calculated values of reliability indices are well below the acceptable limits for good performance (β should be at least 4.0 as per USACE 1997). Hence, the buried flexible pipe–soil system which was acceptable from conventional

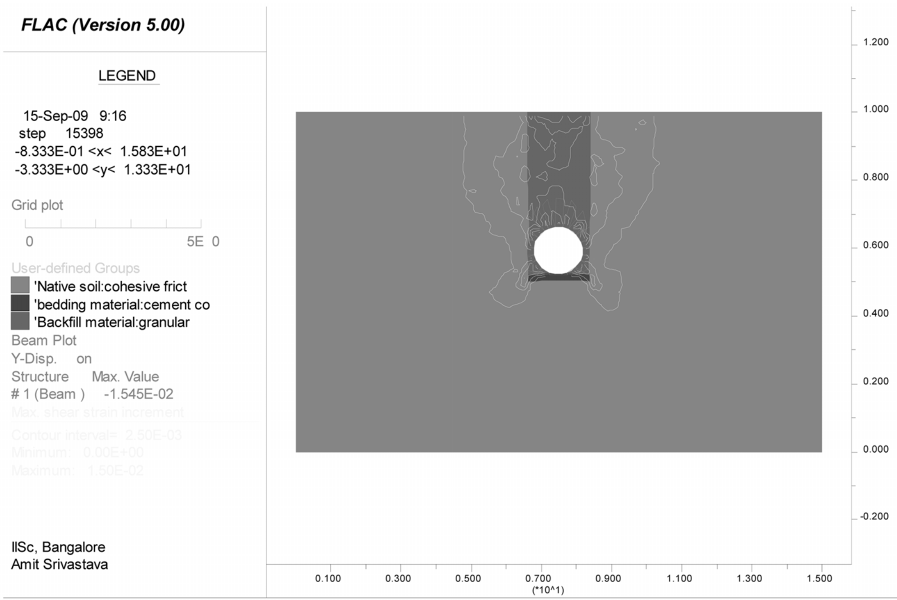

Fig. 7 Shear strain increment (*ssi*) contours indicating non-uniformity for spatially varying soil properties (maximum displacement = 15.45 mm)

design procedure by proving enough margin of safety showed totally different scenario in probabilistic framework with due consideration of spatial variation of geotechnical parameters.

Fig. 7 shows typical realization of shear strain increment (*ssi*) contours for the spatially varying geotechnical parameters, which indicates non-uniformity in the deformation pattern on the two sides of the buried flexible pipe.

7.4 A parametric study in stochastic numerical analysis

A parametric study is performed to investigate the influence of coefficient of variation (*CoV*s) and auto-correlation distance (δ_z) in input soil parameters on the estimation of deflection and buckling pressure of buried flexible pipe-soil system. A range of values of *CoV*s (10%, 20%, 30%, and 40%) and δ_z (0.0 m, 0.5 m, 1.0 m, 1.5 m, 2.5 m, 5.0 m, and 10 m) for soil parameters are selected.

Fig. 8 compares the results of the analysis for deflection. It can be noted that with increase in the coefficient of variation (*CoV*%) in the input parameters there is an increase in the mean deflection values and there is a decrease in the mean deflection value with increase in the auto-correlation distance. It should be noted that the term mean deflection is used because they are obtained after averaging the calculated deflection values from 2000 Monte Carlo simulations. With an increase in auto-correlation distance the deflection values try to attain the values obtained for uniformly constant soil properties, which is due to the fact that a higher auto-correlation distance indicates homogeneity in the random field domain. Similarly, results were obtained for buckling pressure as shown in Fig. 9 in which there is an increase in the calculated circumferential stress with increase in coefficient of variation or decrease in correlation distance.

Fig. 10 and Fig. 11 show the influence of two important probabilistic characteristics of the soil variability, i.e. coefficient of variation (*CoV*) and auto-correlation distance (δ_z) in the estimation of deflection and circumferential stress, respectively. The results of the numerical analysis with due consideration of spatial variability modeling of soil properties are also compared with the results

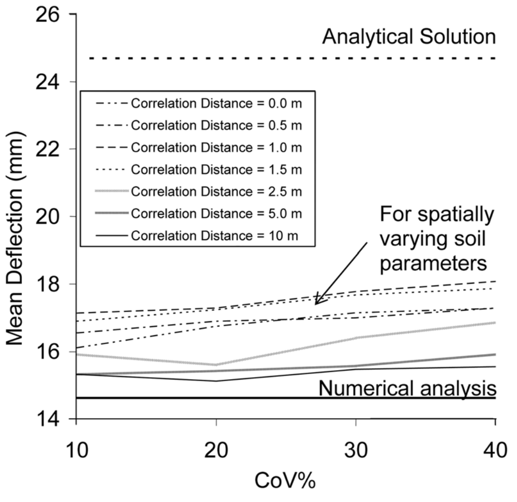

Fig. 8 Comparison of deflection values for different values of *CoV*s and auto-correlation distance (δ_z)

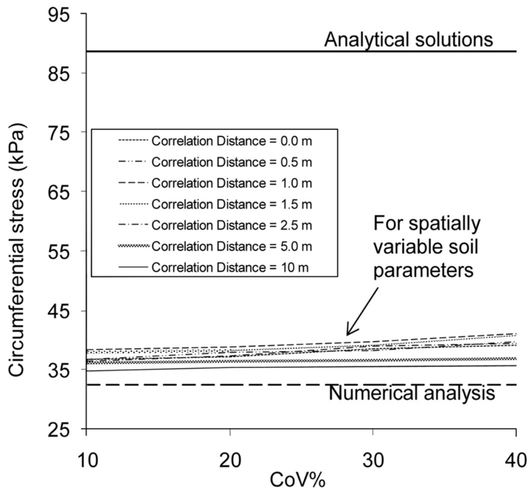

Fig. 9 Comparison of circumferential stress values for different values of *CoV*s and auto-correlation distance (δ_z)

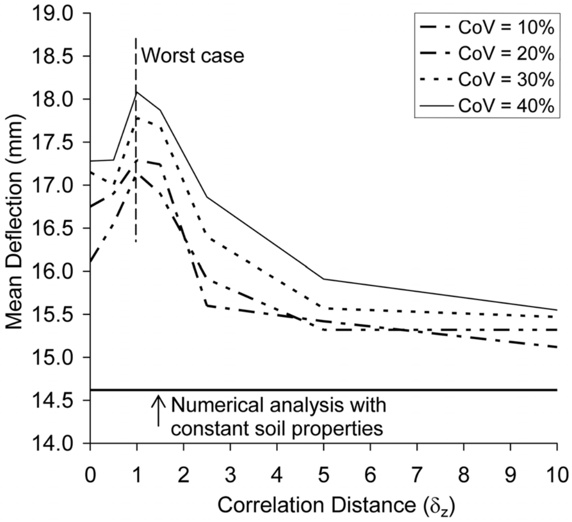

Fig. 10 Mean deflections with variation in *CoV*s and δ_z

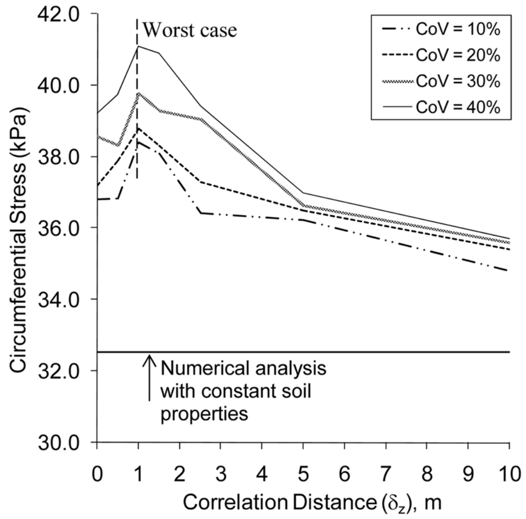

Fig. 11 Mean circumferential stresses with variation in *CoV*s and δ_z

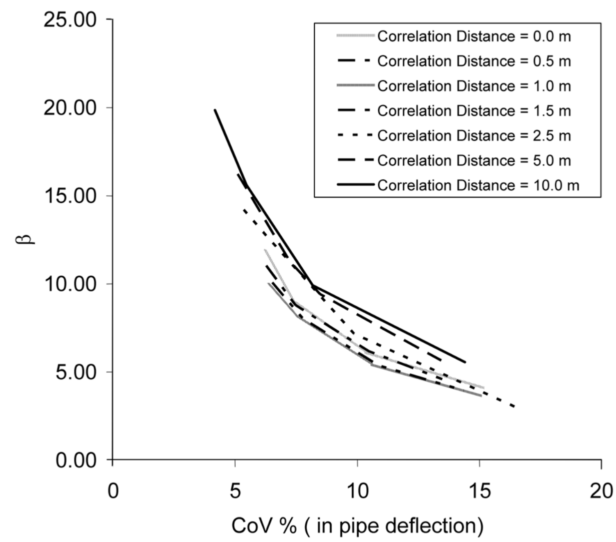

Fig. 12 Effect of *CoV*s and auto-correlation distance (δ_z) on the reliability index (β) values in the buried flexible pipe-soil system from deflection criteria

obtained from analytical solutions as well as numerical analysis for uniformly constant soil properties. It is noted that for a particular value of auto-correlation distance (δ_z = 1.0 m), there is a remarkable increase in the calculated deflection or circumferential (buckling) stress values. As described by Griffiths and Fenton (2007), it can be considered as worst case situation. It should be noted that the worst auto-correlation distance is problem-specific and the value varies within the size of the structure. In the present case, the worst auto-correlation distance is 0.71 times the pipe diameter (D), i.e. $\delta_z^{work} = 0.71D$.

Further, Fig. 12 shows the results of the reliability analysis of the buried flexible pipe-soil system with respect to deflection criterion that are obtained for different values of *CoV*s and auto-correlation distances. It can be noted that as the auto-correlation distance increases from 0 to 1.0 m, the reliability index (β) values decreases and beyond δ_z = 1.0 m, the reliability index (β) values increases. It is due to the fact that the worst auto-correlation distance, in the present case, is evaluated as 1.0 m and the mean values of deflection (Δ) increases till δ_z = 1.0 m, and beyond that point, it starts decreasing. It is also observed that the auto-correlation distance is relevant to the probabilistic interpretation at higher values of *CoV*s. High values of auto-correlation distance (δ_z) are beneficial as they give higher reliability indices.

8. Conclusions

It is noted that available analytical solutions provides too conservative estimates of deflection and buckling pressure compare to the conventional numerical procedure in which soil properties are considered as uniformly constant. The study further highlights the importance of consideration of spatial variation of soil parameters in studying the response of buried flexible pipe-soil system with respect to deflection and buckling. Two important probabilistic characteristics of the soil spatial variability, i.e. coefficient of variation and auto-correlation distance are studied. Monte Carlo simulations combined with numerical analysis is a very useful approach in this regard. Through parametric study, it is noted that there is a significant change in the calculated mean values of deflection and circumferential (buckling) stress values obtained for spatially varying soil media when compared to the corresponding values obtained for uniformly constant soil properties. Reliability analysis is a useful tool in handling uncertainty and soil variability in mathematical framework. It is clearly demonstrated that for the considered case of buried flexible pipe-soil system which indicated enough margin of safety in conventional sense proved to fall well below the acceptable performance level in probabilistic framework.

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Notations

$G(\tilde{x})$	: normally distributed random field with zero mean, unit variance
$\tilde{x}$	: spatial position at which soil property required
$\tau = \|\tilde{x}_1 - \tilde{x}_2\|$	: absolute distance between the two points
$\mu_{\ln c}, \sigma_{\ln c}$	: parameters for log-normal distribution transformations
$a_o^*, a_2^*, a_o^{**}, b_2^*, b_2^{**}$	: non dimensional parameter
B'	: empirical coefficient of elastic support
B_k, C_k	: non dimensional parameters
c'	: cohesion parameter for backfill soil
c_n	: cohesion for native soil
CoV_R	: coefficient of variation of resistance or capacity
CoV_S	: coefficient of variation of load or demand
D	: diameter of the pipe
D_L	: deflection lag factor
E'	: soil modulus of reaction
E_n	: elastic modulus of native soil
E_p	: elastic modulus of pipe material
E_s	: elastic modulus of the backfill material
F	: failure region
f_a	: allowable buckling pressure
f_{cr}	: critical buckling pressure
FS_b	: factor of safety with respect to buckling
$g(X)$	: limit state function
H	: height of the backfill above the crown of the pipe
h_w	: height of water surface above top of the pipe
I	: moment of inertia of pipe wall per unit length
i	: related to the no of input variables (in subscript)
K_b	: bedding constant
K_s	: lateral stress ratio
L_w	: live load distribution width at the crown
m	: total number of observations

M_s	: one-dimensional constrained modulus of backfill material
N	: number of monte carlo simulation runs
n	: number of nodes in the circumference of the buried pipe
p	: total stress on circumference of pipe (including internal vacuum)
pdf	: probability density function
p_f	: probability of failure
q	: vertical stress (surface stress) on the crown of the pipe
q_b	: calculated buckling stress
R	: resistance or capacity
R_p	: mean radius of pipe
R_w	: water buoyancy factor
S	: demand or applied load
t	: thickness of pipe wall
UF	: extensional flexibility ratio
VF	: bending flexibility ratio
W_c	: marston's load per unit length of pipe
W_L	: live load
Z_j	: sequence of independent standard normal random variables
Δ	: calculated deflection
Δ_a	: allowable deflection
ΔX	: horizontal deflection or change in diameter (in)
Δ_y	: vertical decrease in deflection
β	: reliability index
δ_z	: auto-correlation distance
ϕ'	: angle of internal friction for backfill soil
ϕ_n	: angle of internal friction for native soil
γ_s	: unit weight of the backfill soil
μ	: mean
μ_R	: mean value of resistance or capacity
μ_S	: mean value of load or demand
ν	: poisson's ratio of soil
ρ_s	: density of backfill soil
ρ_w	: density of water
σ^2	: variance
σ_R	: standard deviation of resistance or capacity
σ_S	: standard deviation of load or demand