Geomechanics and Engineering, *Vol. 23, No. 3 (2020) 217-225*
DOI: https://doi.org/10.12989/gae.2020.23.3.217

Fractional order GL model on thermoelastic interaction in porous media due to pulse heat flux

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(Received April 16, 2020, Revised September 26, 2020, Accepted October 14, 2020)

Abstract. In this paper, the thermoelastic interactions in a two-dimension porous body are studied. This problem is solved by using the Green and Lindsay (GL) generalized thermoelasticity model under fractional time derivative. The derived approaches are estimated. with numeral results which are applied to the porous mediums in simplifying geometrical. The bounding plane surface of the present half-space continuum is subjected to a pulse heat flux. We use the Laplace-Fourier transforms methods with the eigenvalues approach to solve the problem. The numerical solutions for the field functions are obtained numerically using the numerical Laplace inversion technique. The effects of the fractional parameter and the thermal relaxation times on the temperature field, the displacement field, the change in volume fraction field of voids distribution and the stress fields have been calculated and displayed graphically and the obtained results are discussed.

Keywords: Laplace-Fourier transforms; Green and Lindsay model; porous medium; eigenvalues approach; fractional derivative

1. Introduction

Biot (1956) presented the coupled thermoelastic model to overcome the first lack of classical thermoelastic model, that it presented two conflicting phenomenons with the physical observation. (Lord and Shulman 1967) have presented the generalized thermo-elastic theory with one relaxation time for the particular state of an isotropic media. In this model an improved law of thermal conduction that includes both the heat flux and its temporary derivatives replaced the conventional Fourier law. (Green and Lindsay 1972) obtained the second theory of generalized of thermoelasticity with two relaxation time. The thermal equation associated with this model is hyperbolic and, therefore, removes the paradox of infinite propagation velocities inherent in thermoelastic coupled and uncoupled theories. (Green and Naghdi 1991, Green and Naghdi 1993) introduced a new generalized thermoelasticity theory by depend on the displacement-heating gradient among the separate constitutive variables. (Lata and Kaur 2019) studied the deformation in the transversely isotropic thermoelastic medium using new modified couple stress theory in the frequency domain.

The generalization of the concepts of derivatives and integrals to a non-integer order has been the subject of several methods and various alternative definitions of fractional derivatives have emerged. In the context of generalized thermoelastic theories, (Youssef 2010, Youssef and Al-Lehaibi 2010) have presented the generalized fractional-order thermoelastic of low and high thermal conductivities. Based on a Taylor expansion of time-fractional order, (Ezzat and El Karamany 2011) have given other model for a fractional order generalized thermo-elasticity. A new model is presented by using the heat conduction law as in (Sherief *et al.* 2010). As an important branch of mechanical properties of solid, the literature (Abbas 2006, Youssef 2012, El-Naggar, Kishka *et al.* 2013, Othman *et al.* 2013, Sarkar and Lahiri 2013, Abbas and Kumar 2014, Deswal and Kalkal 2014, Kakar and Kakar 2014, Sur and Kanoria 2014, Hussein 2015, Wang *et al.* 2015, Abbas and Alzahrani 2016, Alzahrani and Abbas 2016, Lata and Kaur 2019, Othman and Abd-Elaziz 2019, Lata and Singh 2020, Othman *et al.* 2020, Othman and Sur 2020, Sarkar and Mondal 2020, Sur 2020a, b, c, Sur *et al.* 2020a, b) have considered different problems by numerical and analytical methods. (Ezzat *et al.* 2016) used the memory-dependent derivative model in generalized thermoelasticity theory. (Sarkar 2017) studied the effects of fractional-order two-temperatures model on waves propagations in magneto-thermoelastic solid half-space. (Hobiny and Abbas 2020) studied the fractional order thermoelastic wave assessment in a two-dimension medium with voids. (Aouadi *et al.* 2017) have presented a porous thermoelastic diffusion theory of types II and III. (Singh 2007) have investigated the thermoelastic interactions of wave propagations in media containing voids. (Bachher *et al.* 2014) studied the impacts of fractional-order derivative heat transfer in infinite thermoelastic mediums containing voids subjected to instant thermal sources. (Ezzat 1994) has studied the state-space approach to unsteady two-dimension

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ISSN: 2005-307X (Print), 2092-6219 (Online)

free convections flow through a porous medium. (Lata and Zakhmi 2019) presented the fractional-order generalized thermoelastic study in the orthotropic medium of type GN-III. Several authors (Marin *et al.* 2017, Milani Shirvan *et al.* 2017a, b Ellahi *et al.* 2019, Monda *et al.* 2019, Sheikholeslami *et al.* 2019, Zeeshan *et al.* 2019) have given the solutions of others problems for porous media under various models.

This article investigates the effects of fractional time derivative and thermal relaxation times in a two-dimension porous material based on Green and Lindsay model. By using the eigenvalue approach and Fourier-Laplace transformations, the governing equations are processed by the analytical-numerical method. The effects of the fractional parameter and the thermal relaxation times on the temperature field, the displacement field, the change in volume fraction field of voids distribution and the stress fields have been calculated and displayed graphically and the obtained results are discussed.

2. Basic equations

We consider an isotropic, homogeneous and porous thermoelastic half-space. The governing equations under (Green and Lindsay 1972) model, according to (Singh 2007) in absences of heat sources and body force can be presented by:

$$(\lambda+\mu)u_{j,ij}+\mu u_{i,jj}+b\varphi_{,i}-\gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta_{,i}=\rho\frac{\partial^2 u_i}{\partial t^2}, \quad (1)$$

$$\alpha\varphi_{,jj}-bu_{j,j}-\zeta_1\varphi-\omega_o\frac{\partial\varphi}{\partial t}+m\Theta=\rho\psi\frac{\partial^2\varphi}{\partial t^2}, \quad (2)$$

$$K\Theta_{,jj}=\rho c_e\left(1+\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\frac{\partial\Theta}{\partial t}+\gamma_t T_o\left(1+n\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\left(mT_o\frac{\partial\varphi}{\partial t}+\frac{\partial u_{j,j}}{\partial t}\right),\qquad 0<\beta\le 1 \quad (3)$$

$$\sigma_{ij}=\mu\left(u_{i,j}+u_{j,i}\right)+\left(\lambda u_{k,k}+b\varphi-\gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta\right)\delta_{ij} \quad (4)$$

By taking into consideration the above definition can be expressed by

$$\frac{\partial^\beta g(\boldsymbol{r},t)}{\partial t^\beta}=\begin{cases} g(\boldsymbol{r},t)-g(\boldsymbol{r},0), & \beta\to 0,\\ I^{\alpha-1}\dfrac{\partial g(\boldsymbol{r},t)}{\partial t}, & 0<\beta<1,\\ \dfrac{\partial g(\boldsymbol{r},t)}{\partial t}, & \beta=1,\end{cases} \quad (5)$$

where I^β is the Riemann-Liouville integral fraction introduced as a natural generalization of the well-known integral $I^\beta g(\boldsymbol{r},t)$ that can be written as a convolution type.

$$I^\beta\beta(\boldsymbol{r},t)=\int_0^t\frac{(t-s)^\beta}{\Gamma(\beta)}g(\boldsymbol{r},s)ds\,,\beta>0, \quad (6)$$

where $g(\boldsymbol{r},t)$ is a Lebesgue's integral function and $\Gamma(\beta)$ is the Gamma function. In the case where $g(\boldsymbol{r},t)$ is definitely continuous, then it is possible to write

$$\lim_{\beta\to 1}\frac{\partial^\beta g(\boldsymbol{r},t)}{\partial t^\beta}=\frac{\partial g(\boldsymbol{r},t)}{\partial t} \quad (7)$$

where the different values of fractional parameter $0<\beta\le 1$ cover two types of conductivity, $\beta=1$ for normal conductivity and $0<\beta<1$ for low conductivity, u_i are the components of displacement, σ_{ij} are the components of stress, c_e is the specific heat at constant strain, ρ is the density of material, $\omega_o,\alpha,b,m,\psi,\zeta_1$ are the voids material constants, τ_o, τ_1 are the thermal relaxation times, λ,μ are the Lame's constants $\Theta=T-T_o$, T_o is the reference temperature, K is the coefficient of thermal conductivity,t is the time, $\gamma_t=(3\lambda+2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion and $i,j,k=1,2,3$. Taking into account a two-dimension porous material, the components of variables can be expressed as

$$\boldsymbol{u}=(u,v,0),v=v(x,y,t),u=u(x,y,t),\varphi=\varphi(x,y,t),\Theta=\Theta(x,y,t) \quad (8)$$

Therefore, the Eqs. (1)- (4) are expressed as:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial y^2}+b\frac{\partial\varphi}{\partial x}-\gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\Theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2} \quad (9)$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 v}{\partial x^2}+b\frac{\partial\varphi}{\partial y}-\gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\Theta}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2} \quad (10)$$

$$\alpha\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)-b\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\zeta_1\varphi-\omega_o\frac{\partial\varphi}{\partial t}+m\Theta=\rho\psi\frac{\partial^2\varphi}{\partial t^2}, \quad (11)$$

$$K\left(\frac{\partial^2\Theta}{\partial x^2}+\frac{\partial^2\Theta}{\partial y^2}\right)=\rho c_e\left(1+\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\frac{\partial\Theta}{\partial t}+\left(\frac{\partial}{\partial t}+n\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\left(mT_o\varphi+\gamma_t T_o\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right), \quad (12)$$

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y}+b\varphi-\gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta,\sigma_{xy}=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right). \quad (13)$$

3. Applications

The initial conditions of the problem can be defined as

$$u(x,y,0)=\frac{\partial u(x,y,0)}{\partial t}=0,v(x,y,0)=\frac{\partial v(x,y,0)}{\partial t}=0,\Theta(x,y,0)=\frac{\partial\Theta(x,y,0)}{\partial t}=0,\varphi(x,y,0)=\frac{\partial\varphi(x,y,0)}{\partial t}=0. \quad (14)$$

The boundary conditions of half-space $(x = 0)$ are due to the gradient temperature exponentially decaying pulse with heat flux by

$$-K\frac{\partial\Theta(x,y,t)}{\partial x} = q_o\frac{t^2 e^{-\frac{t}{\tau_p}}}{16\tau_p^2}H(a-|y|), \tag{15}$$

where H is the Heaviside unit step function, q_o is a constant and τ_p is the pulse heat flux characteristic time. We assume that, on the boundary $x = 0$, the change in volume fraction field of voids distribution φ and the displacement u of the body does not depend on x and the material is subjected to a rough and rigid foundation enough to prevent the displacement v and hence one can be obtains

$$v(x,y,t) = 0, \frac{\partial u(x,y,t)}{\partial x} = \frac{\partial\varphi(x,y,t)}{\partial x} = 0. \tag{16}$$

For conveniences, the non-dimensional varibles can be defined as

$$(t^*,\tau_o^*,\tau_1^*,\tau_p^*) = \eta c^2(t,\tau_o,\tau_1,\tau_p),(x^*,y^*,u^*,v^*) = \eta c(x,y,u,v),$$
$$\varphi' = \psi\eta^2c^2\varphi, \Theta^* = \frac{\Theta}{T_o}, (\sigma_{xx}^*,\sigma_{xy}^*) = \frac{(\sigma_{xx},\sigma_{xy})}{(\lambda+2\mu)}. \tag{17}$$

where $c = \sqrt{\frac{\lambda+2\mu}{\rho}}$ and $\eta = \frac{\rho c_e}{k}$. In these non-dimensional terms of the variables in Eq. (17), the basic equations with initial and boundary conditions are presented as the following forms (the star has been deleted for conveniences)

$$\frac{\partial^2 u}{\partial x^2} + (1-s_1)\frac{\partial^2 v}{\partial x\partial y} + s_1\frac{\partial^2 u}{\partial y^2} + s_2\frac{\partial\varphi}{\partial x} - s_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\Theta}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{18}$$

$$\frac{\partial^2 v}{\partial y^2} + (1-s_1)\frac{\partial^2 u}{\partial x\partial y} + s_1\frac{\partial^2 v}{\partial x^2} + s_2\frac{\partial\varphi}{\partial y} - s_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\Theta}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{19}$$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - s_4\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - s_5\varphi - s_6\frac{\partial\varphi}{\partial t} + s_7\Theta = s_8\frac{\partial^2\varphi}{\partial t^2}, \tag{20}$$

$$\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2} = \left(1+\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\frac{\partial\Theta}{\partial t} + \left(1+n\frac{\tau_o^\beta}{\Gamma(\beta+1)}\frac{\partial^\beta}{\partial t^\beta}\right)\left(s_9\frac{\partial\varphi}{\partial t} + s_{10}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right), \tag{21}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} + (1-2s_1)\frac{\partial v}{\partial y} + s_2\varphi - s_3\left(1+\tau_1\frac{\partial}{\partial t}\right)\Theta, \sigma_{xy} = s_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{22}$$

with initial and boundary conditions as

$$u(x,y,0) = \frac{\partial u(x,y,0)}{\partial t} = 0, v(x,y,0) = \frac{\partial v(x,y,0)}{\partial t} = 0, \Theta(x,y,0) = \frac{\partial\Theta(x,y,0)}{\partial t} = 0, \varphi(x,y,0) = \frac{\partial\varphi(x,y,0)}{\partial t} = 0, \tag{23}$$

$$\frac{\partial\Theta(x,y,t)}{\partial x} = -q_o\frac{t^2 e^{-\frac{t}{\tau_p}}}{16\tau_p^2}H(a-|y|), v(x,y,t) = 0, \frac{\partial u(x,y,t)}{\partial x} = 0, \frac{\partial\varphi(x,y,t)}{\partial x} = 0, \tag{24}$$

where

$$s_1 = \frac{\mu}{\rho c^2}, \quad s_2 = \frac{b}{\rho\psi\eta^2c^4}, \quad s_3 = \frac{\gamma_t T_o}{\rho c^2}, \ s_4 = \frac{b\psi}{\alpha}, \ s_5 = \frac{\zeta_1}{\alpha\eta^2c^2}, \ s_6 = \frac{\omega_o}{\alpha\eta}, \ s_7 = \frac{m\psi T_o}{\alpha}, \ s_8 = \frac{\rho c^2\psi}{\alpha}, \ s_9 = \frac{m}{\psi\eta^3c^2k}, \ s_{10} = \frac{\gamma_t}{\rho c_e}.$$

4. Laplace-Fourier transforms

Now, the Laplace transform for every function $f(x,y,t)$, are given as

$$\bar{f}(x,y,s) = \int_0^\infty f(x,y,t)e^{-st}dt, s > 0, \tag{25}$$

while, the Fourier transforms for every function $\bar{f}(x,y,s)$ are defined as (Debnath and Bhatta 2014)

$$\bar{f}^*(x,q,s) = \int_{-\infty}^\infty \bar{f}(x,y,s)e^{-iqy}dy. \tag{26}$$

Thus, the basic Eqs. (18)-(22) under initial conditions (23) with the problem boundary conditions (24) are presented to obtain the ordinary differential system as

$$\frac{d^2\bar{u}^*}{dx^2} = (s^2+s_1q^2)\bar{u}^* - iq(1-s_1)\frac{d\bar{v}^*}{dx} - s_2\frac{d\bar{\varphi}^*}{dx} + s_3(1+s\tau_1)\frac{d\bar{\Theta}^*}{dx} \tag{27}$$

$$\frac{d^2\bar{v}^*}{dx^2} = \frac{(s^2+q^2)}{s_1}\bar{v}^* - \frac{s_2iq}{s_1}\bar{\varphi}^* + \frac{s_3iq}{s_1}(1+s\tau_1)\bar{\Theta}^* - \frac{iq(1-s_1)}{s_1}\frac{d\bar{u}^*}{dx}, \tag{28}$$

$$\frac{d^2\bar{\varphi}^*}{dx^2} = s_4iq\bar{v}^* + (s^2s_8+q^2+s_5+s_6s)\bar{\varphi}^* - s_7\bar{\Theta}^* + s_4\frac{d\bar{u}^*}{dx}, \tag{29}$$

$$\frac{d^2\bar{\Theta}^*}{dx^2} = s_{10}iqs(1+nd)\bar{v}^* + ss_9(1+nd)\bar{\varphi}^* + \left(q^2 + s(1+d)\right)\bar{\Theta}^* + s(1+nd)s_{10}\frac{d\bar{u}^*}{dx}, \tag{30}$$

$$\bar{\sigma}_{xx}^* = \frac{d\bar{u}^*}{dx} + iq(1-2f_1)\bar{v}^* + s_2\bar{\varphi}^* - s_3(1+s\tau_1)\bar{\Theta}^*, \ \bar{\sigma}_{xy}^* = s_1\left(iq\bar{u}^* + \frac{d\bar{v}^*}{dx}\right), \tag{31}$$

$$\frac{d\bar{u}^*}{dx} = 0, \bar{v}^* = 0, \frac{d\bar{\varphi}^*}{dx} = 0, \frac{d\bar{\Theta}^*}{dx} = -\frac{q_o t_p}{8(st_p+1)^3}\sqrt{\frac{2}{\pi}}\frac{\sin(qa)}{q}, \tag{32}$$

where $d = \frac{s^\beta\tau_o^\beta}{\Gamma(\beta+1)}$. The matrix-vector differential relations of Eqs. (27)-(30) are given by

$$\frac{dV}{dx} = AV \tag{33}$$

where A and V can be defined as in appendix A. By using the eigenvalue method which presented by (Das *et al.* 1997, Abbas 2015), the analytical solutions of Eq. (33) can be presented. Then, the characteristic equation of the matrix A can be written as:

$$\xi^8 - m_1\xi^6 + m_2\xi^4 + m_3\xi^2 + m_4 = 0 \tag{34}$$

where m_1, m_2, m_3 and m_4 can be defined as in appendix B. To get the solutions of Eq. (32), the eigenvalues and its eigenvectors of matrix A are calculated. In the cases $\xi_1, -\xi_1, \xi_2, -\xi_2, \xi_3, -\xi_3, \xi_4$ and $-\xi_4$ are the eigenvalues, the conforming eigenvector of eigenvalues ξ are presented as in appendix C. Hence, the analytical solutions of Eq. (33) can be expressed as:

$$V(x,q,s)=\sum_{i=1}^{4}B_iY_ie^{-\xi_ix} \tag{35}$$

Therefore, the general solutions of the physical quantities are considered for q,x and t as:

$$\bar{\Theta}^*(x,q,s)=\sum_{i=1}^{4}B_iT_ie^{-\xi_ix}, \tag{36}$$

$$\bar{u}^*(x,q,s)=\sum_{i=1}^{4}B_iu_ie^{-\xi_ix}, \tag{37}$$

$$\bar{v}^*(x,q,s)=\sum_{i=1}^{4}B_iv_ie^{-\xi_ix}, \tag{38}$$

$$\bar{\varphi}^*(x,q,s)=\sum_{i=1}^{4}B_i\varphi_ie^{-\xi_ix}, \tag{39}$$

$$\bar{\sigma}_{xx}^*(x,q,s)=\sum_{i=1}^{4}B_i(-\xi_iu_i+iq(1-2s_1)v_i+s_2N_i-s_3(1+s\tau_1)T_i)e^{-\xi_ix}, \tag{40}$$

$$\bar{\sigma}_{xy}^*(x,q,s)=\sum_{i=1}^{4}B_is_1(-\xi_iv_i+iqu_i)e^{-\xi_ix}, \tag{41}$$

where B_1,B_2,B_3 and B_4 are constants which are determined by using boundary conditions of the problem such that the terms containing exponentials of rising nature in the spatial variable x have been discarded due to the regularity condition of the solution at infinity, while u_i,v_i,T_i and φ_i are the corresponding eigenvector components. Now, the inverse of Fourier transforms for $\bar{f}^*(x,q,s)$ can be defined by

$$\bar{f}(x,y,s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\bar{f}^*(x,q,s)e^{iqy}dq, \tag{42}$$

Finally, to obtain the general solutions of the change in volume fraction field of voids distribution, the displacement components, the increment of temperature and the components of stress versus x distance and y distance for any time t, the numerical inversion (Stehfest 1970) scheme was used. In this scheme, the inverse of Laplace transforms for $\bar{f}(x,y,s)$ are considered by

$$f(x,y,t)=\frac{ln(2)}{t}\sum_{n=1}^{N}V_n\bar{f}\left(x,y,n\frac{ln(2)}{t}\right), \tag{43}$$

where

$$V_n=(-1)^{\left(\frac{N}{2}+1\right)}\sum_{p=\frac{n+1}{2}}^{min\left(n,\frac{N}{2}\right)}\frac{(2p)!p^{\left(\frac{N}{2}+1\right)}}{p!(n-p)!\left(\frac{N}{2}-p\right)!(2n-1)!}, \tag{44}$$

where N is the term number.

5. Numerical outcomes and discussion

For numeral example, the magnesium material was chosen for objective of numerical estimations. The magnesium (Mg) parameters values as porous media can be talking from (Othman and Marin 2017)

$$\omega_o=0.0787\times10^{-3}(N)(m^{-2})(s^{-1}),\rho=1740(kg)(m^{-3}),\alpha=3.688\times10^{-5}(N),$$

$$\zeta_1=1.475\times10^{10}(N)(m^{-2}),\lambda=2.17\times10^{10}(N)(m^{-2}),\mu=3.278\times10^{10}(N)(m^{-2}),$$

$$\psi=1.753\times10^{-15}(m^2),\alpha_t=1.98\times10^{-6}(k^{-1}),T_o=298(k),K=1.7\ (W)(m^{-1})(k^{-1}),$$

$$\beta=2.68\times10^{6}(N)(m^{-2})(k^{-1}),c_e=1040\ (J)(kg^{-1})(k^{-1}),\ t=0.3,a=0.25,$$

$$b=1.13840\times10^{10}(N)(m^{-2}),m=2\times10^{6}(N)(m^{-2})(k^{-1}).$$

On the basis of the above dataset, Figs. 1-12 show the considering variables are calculated numerically with respect to differences values of x and y. The above data have been applied to study the different among the fractional Green and Lindsay (FGL) model, the Green and Lindsay (GL) model and the classical dynamical

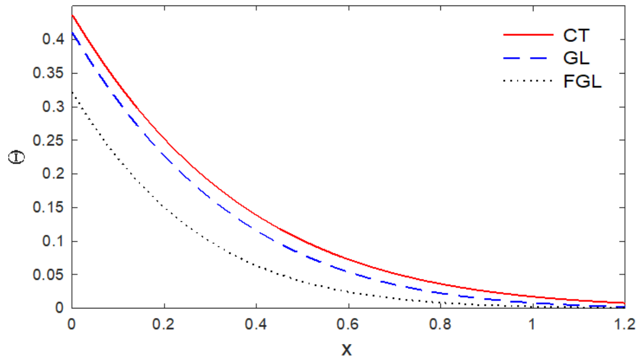

Fig. 1 The variations of temperature Θ with respect to x for different theories when y= 0.5

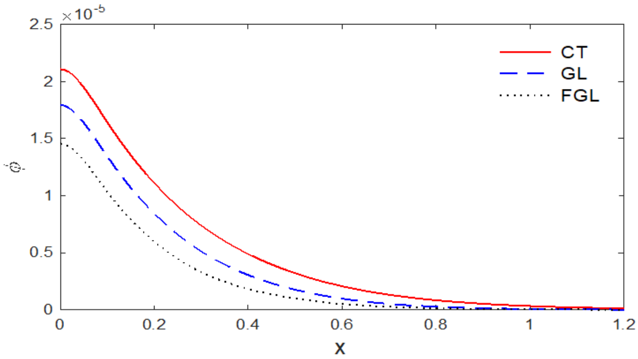

Fig. 2 The changes in volume fraction field of voids distributions φ with respect to x for different theories when y= 0.5

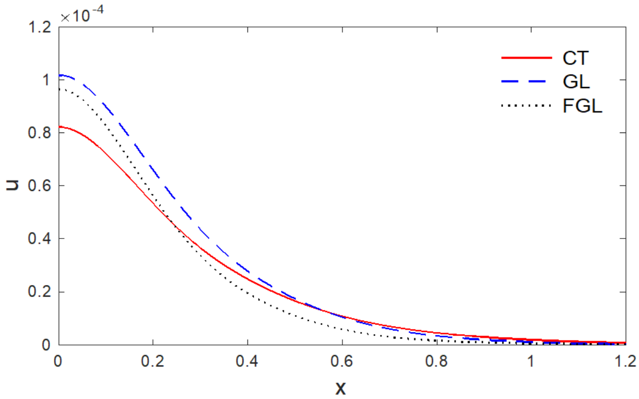

Fig. 3 The variations of horizontal displacement u with respect to x for different theories when y= 0.5

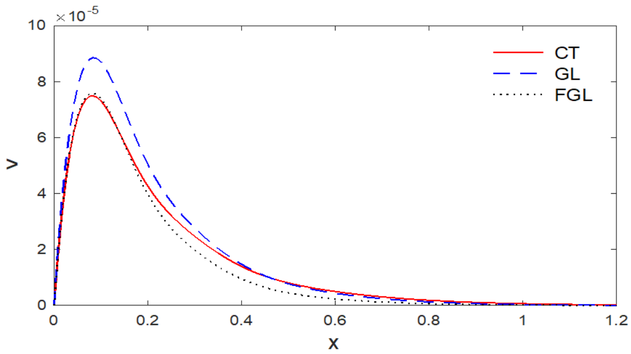

Fig. 4 The variations of vertical displacement v with respect to x for different theories when y= 0.5

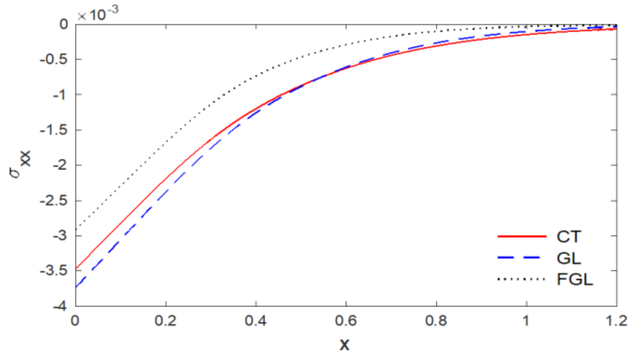

Fig. 5 The variations of stress σ_{xx} with respect to x for different theories when y= 0.5

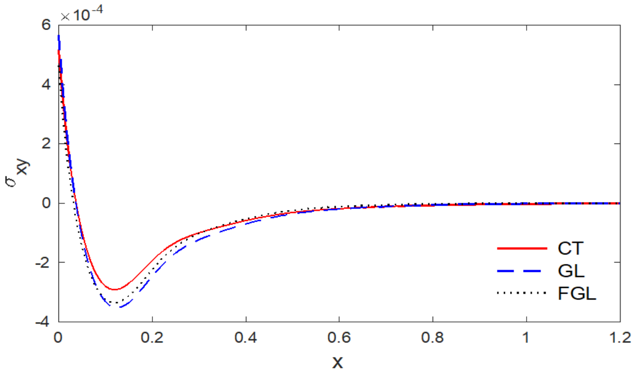

Fig. 6 The variations of stress σ_{xy} with respect to x for different theories when y= 0.5

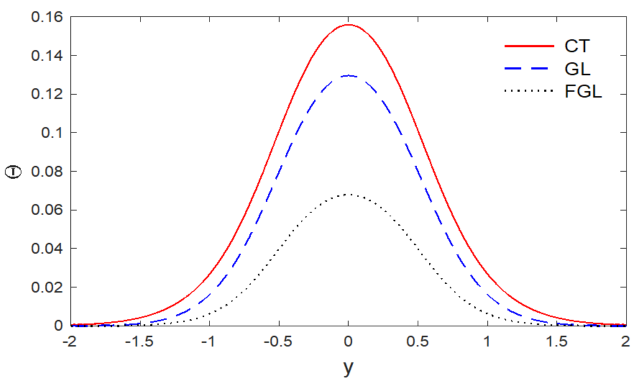

Fig. 7 The variation of temperature Θ with respect to y when x= 0.5 for different theories

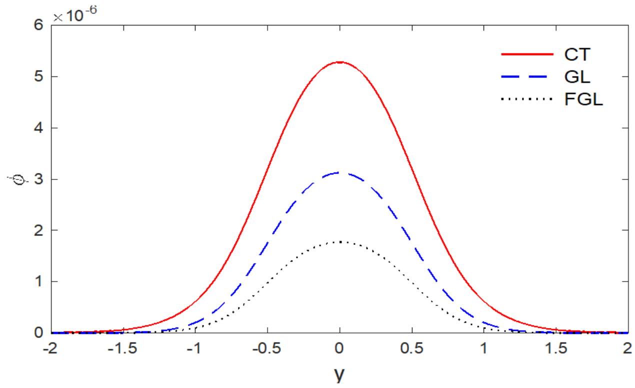

Fig. 8 The changes in volume fraction field of voids distribution φ with respect to y when x = 0.5 for different theories

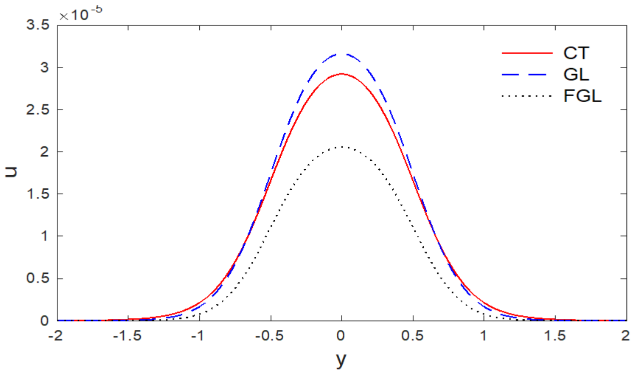

Fig. 9 The variations of horizontal displacement u with respect to y when x= 0.5 for different theories

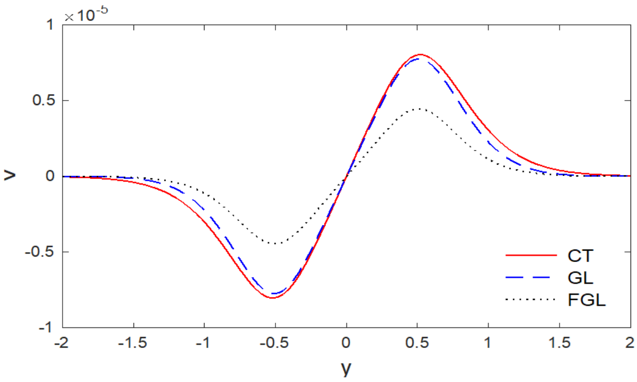

Fig. 10 The variations of vertical displacement v with respect to y when x= 0.5 for different theories

coupled (CT) model in the variations of temperature Θ, the displacement components u, v, the components of stress σ_{xx}, σ_{xy} and the change in volume fraction field of voids distributions φ. The material is considered to be an isotropic two-dimensional porous medium.

Fig. 1 displays the variation of temperature with respect to the distance x. It is noticed that it starts from maximum value according to the problem boundary condition and reduces with the increasing x to closed to zero. Fig. 2 shows the change in volume fraction field of voids distribution φ along to the distance x. It is observed that it

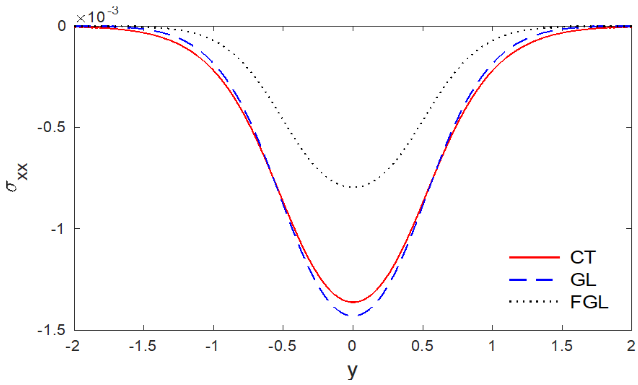

Fig. 11 The variations of stress σ_{xx} with respect to y when x= 0.5 for different theories

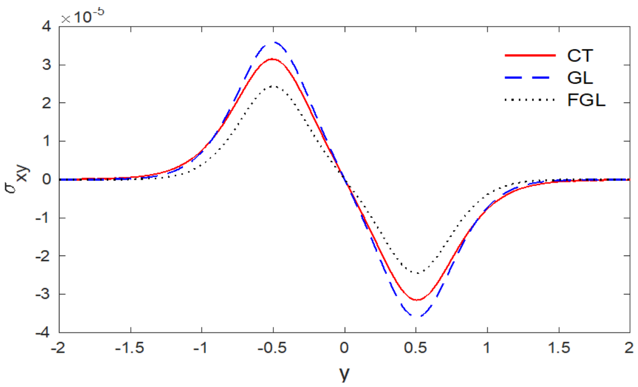

Fig. 12 The variations of stress σ_{xy} with respect to y when x= 0.5 for different theories

reduces with the rising x till reach zeros. Fig. 3 depicts the variation of horizontal displacement u along x. It is clear that it starts from maximum value after that reduces with the rising x to come to zeros. Fig. 4 shows the variation of vertical displacement versus x. It is noticed that the vertical displacement starts from zero which satisfied the problem boundary condition after that it rising up to maximum values at a particular location proximately nearby the surface after that the vertical displacement decreases to close to zeros. Figs. 5 and 6 display the stress components variations σ_{xx} and σ_{xy} with respect to the distance x. It is clear that the stress magnitudes, always started from the maximum values and then decreases with the increasing the distance x to reach to zeros. Figs. 7 and 8 show the temperature variations Θ and the change in volume fraction field of voids distribution φ along the distance y and they indicate that the variations of temperature and the change in volume fraction field of voids have maximum values at the length of heating surface $\left((|y| \leq 0.5)\right)$ and they start to reduce completely close to the edge $\left((|y| \leq 0.5)\right)$ where they reduce smoothly and ultimately come to the zero value. Figs. 9 and 11 display the horizontal displacement variation u and the stress component σ_{xx} with respect to the distance y and it point that they have maximum values at the length of the thermal surface $(|y| \leq 0.5)$, and they start to reduce completely close to the edge $(y = \pm 0.5)$, then decreases to zero values. Figs. 10 and 12 display the vertical displacement variation v and the stress component σ_{xy} with respect to y. It is noticed that they begins increasing at the starting and ending of the thermal surface $(|y| \leq 0.5)$, and have small values at the middle of the thermal surface, then they start increasing and come to maximum value just near the edges $(y = \pm 0.5)$, then it decreases to reach to zero. As expected, it can be found that the fractional parameter has the great effects on the values of all the physical quantities. According to the numerical results, this new fractional Green and Lindsay (FGL) model of thermoelasticity offers finite speed of the thermal wave and mechanical wave propagation.

6. Conclusions

The results of this paper discuss the fractional-order generalized thermo-elastic model as a new improvement in the field of thermoelasticity. According to this model, we have to construct a new classification for all the materials according to its fractional parameter, where this parameter becomes a new indicator of its ability to conduct the thermal energy. Accordingly, we can consider the generalized thermo-elastic model with the normal conductivity and the low conductivity as an advancement in the investigate of elastic porous medium. The non-dimensional resulting has been solved employing the Laplace and Fourier transformations techniques and have been solved by the eigenvalue approach. The great effect of the fractional derivative parameter are discussed for all physical quantities.

Acknowledgments

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, under grant No (DF-481-130-1441). The authors, therefore, gratefully acknowledge DSR technical and financial support.

References

Abbas, I. (2006), "Natural frequencies of a poroelastic hollow cylinder", *Acta Mechanica*. **186**(1-4), 229-237. https://doi.org/10.1007/s00707-006-0314-y.

Abbas, I.A. (2015), "The effects of relaxation times and a moving heat source on a two-temperature generalized thermoelastic thin slim strip", *Can. J. Phys*., **93**(5), 585-590. https://doi.org/10.1139/cjp-2014-0387.

Abbas, I.A. and Alzahrani, F.S. (2016), "Analytical solution of a two-dimensional thermoelastic problem subjected to laser pulse", *Steel Compos. Struct*., **21**(4), 791-803. https://doi.org/10.12989/scs.2016.21.4.791.

Abbas, I.A. and Kumar, R. (2014), "Deformation due to thermal source in micropolar generalized thermoelastic half-space by finite element method", *J. Comput. Theor. Nanosci*., **11**(1), 185-190. https://doi.org/10.1166/jctn.2013.3193.

Alzahrani, F.S. and Abbas, I.A. (2016), "The effect of magnetic field on a thermoelastic fiber-reinforced material under GN-III theory", *Steel Compos. Struct.*, **22**(2), 369-386. https://doi.org/10.12989/scs.2016.22.2.369.

Aouadi, M., Ciarletta, M. and Iovane, G. (2017), "A porous thermoelastic diffusion theory of types II and III", *Acta Mechanica*, **228**(3), 931-949. https://doi.org/10.1007/s00707-016-1749-4.

Bachher, M., Sarkar, N. and Lahiri, A. (2014), "Generalized thermoelastic infinite medium with voids subjected to a instantaneous heat sources with fractional derivative heat transfer", *Int. J. Mech. Sci.*, **89**, 84-91. https://doi.org/10.1016/j.ijmecsci.2014.08.029.

Biot, M.A. (1956), "Thermoelasticity and irreversible thermodynamics", *J. Appl. Phys.*, **27**(3), 240-253. https://doi.org/10.1063/1.1722351.

Das, N.C., Lahiri, A. and Giri, R.R. (1997), "Eigenvalue approach to generalized thermoelasticity", *Indian J. Pure Appl. Math.*, **28**(12), 1573-1594.

Debnath, L. and Bhatta, D. (2014), *Integral Transforms and Their Applications*, Chapman and Hall/CRC.

Deswal, S. and Kalkal, K.K. (2014), "Plane waves in a fractional order micropolar magneto-thermoelastic half-space", *Wave Motion*, **51**(1), 100-113. https://doi.org/10.1016/j.wavemoti.2013.06.009.

El-Naggar, A., Kishka, Z., Abd-Alla, A., Abbas, I., Abo-Dahab, S. and Elsagheer, M. (2013), "On the initial stress, magnetic field, voids and rotation effects on plane waves in generalized thermoelasticity", *J. Comput. Theor. Nanosci.*, **10**(6), 1408-1417. https://doi.org/10.1166/jctn.2013.2862.

Ellahi, R., Sait, S.M., Shehzad, N. and Ayaz, Z. (2019), "A hybrid investigation on numerical and analytical solutions of electro-magnetohydrodynamics flow of nanofluid through porous media with entropy generation", *Int. J. Numer. Meth. Heat Fluid Flow*. https://doi.org/10.1108/HFF-06-2019-0506.

Ezzat, M., El-Karamany, A. and El-Bary, A. (2016), "Modeling of memory-dependent derivative in generalized thermoelasticity", *Eur. Phys. J. Plus*, **131**(10), 372. https://doi.org/10.1140/epjp/i2016-16372-3.

Ezzat, M.A. (1994), "State space approach to unsteady two-dimensional free convection flow through a porous medium", *Can. J. Phys.*, **72**(5-6), 311-317. https://doi.org/10.1139/p94-045.

Ezzat, M.A. and El Karamany, A.S. (2011), "Theory of fractional order in electro-thermoelasticity", *Eur. J. Mech. A/Solids*, **30**(4), 491-500. https://doi.org/10.1016/j.euromechsol.2011.02.004.

Green, A. and Lindsay, K. (1972), "Thermoelasticity", *J. Elasticity*, **2**(1), 1-7. https://doi.org/10.1007/BF00045689.

Green, A. and Naghdi, P. (1991), "A re-examination of the basic postulates of thermomechanics", *Proc. Royal Soc. London Ser. A Math. Phys. Sci.*, **432**(1885), 171-194. https://doi.org/10.1098/rspa.1991.0012.

Green, A.E. and Naghdi, P.M. (1993), "Thermoelasticity without energy dissipation", *J. Elasticity*, **31**(3), 189-208. https://doi.org/10.1007/BF00044969.

Hobiny, A.D. and Abbas, I.A. (2020), "Fractional order thermoelastic wave assessment in a two-dimension medium with voids", *Geomech. Eng.*, **21**(1), 85-93. http://doi.org/10.12989/gae.2020.21.1.085.

Hussein, E.M. (2015), "Fractional order thermoelastic problem for an infinitely long solid circular cylinder", *J. Therm. Stresses*, **38**(2), 133-145. https://doi.org/10.1080/01495739.2014.936253.

Kakar, R. and Kakar, S. (2014), "Electro-magneto-thermoelastic surface waves in non-homogeneous orthotropic granular half space", *Geomech. Eng.*, **7**(1), 1-36. http://doi.org/10.12989/gae.2014.7.1.001.

Lata, P. and Kaur, H. (2019), "Deformation in transversely isotropic thermoelastic medium using new modified couple stress theory in frequency domain", *Geomech. Eng.*, **19**(5), 369-381. http://doi.org/10.12989/gae.2019.19.5.369.

Lata, P. and Kaur, I. (2019), "Effect of time harmonic sources on transversely isotropic thermoelastic thin circular plate", *Geomech. Eng.*, **19**(1), 29-36. http://doi.org/10.12989/gae.2019.19.1.029.

Lata, P. and Singh, S. (2020), "Deformation in a nonlocal magneto-thermoelastic solid with hall current due to normal force", *Geomech. Eng.*, **22**(2), 109-117. http://doi.org/10.12989/gae.2020.22.2.109.

Lata, P. and Zakhmi, H. (2019), "Fractional order generalized thermoelastic study in orthotropic medium of type GN-III", *Geomech. Eng.*, **19**(4), 295-305. http://doi.org/10.12989/gae.2019.19.4.295.

Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids*, **15**(5), 299-309. https://doi.org/10.1016/0022-5096(67)90024-5.

Marin, M., Baleanu, D. and Vlase, S. (2017), "Effect of microtemperatures for micropolar thermoelastic bodies", *Struct. Eng. Mech.*, **61**(3), 381-387. http://doi.org/10.12989/sem.2017.61.3.381.

Milani Shirvan, K., Mamourian, M. and Ellahi, R. (2017a), "Numerical investigation and optimization of mixed convection in ventilated square cavity filled with nanofluid of different inlet and outlet port", *Int. J. Numer. Meth. Heat Fluid Flow*, **27**(9), 2053-2069. https://doi.org/10.1108/HFF-08-2016-0317.

Milani Shirvan, K., Mamourian, M., Mirzakhanlari, S., Rahimi, A. and Ellahi, R. (2017b), "Numerical study of surface radiation and combined natural convection heat transfer in a solar cavity receiver", *Int. J. Numer. Meth. Heat Fluid Flow*, **27**(10), 2385-2399. https://doi.org/10.1108/HFF-10-2016-0419.

Mondal, S., Sahu, S. and Pankaj, K. (2019), "Transference of Love-type waves in a bedded structure containing a functionally graded material and a porous piezoelectric medium", *Appl. Math. Mech.*, **40**(8), 1083-1096. https://doi.org/10.1007/s10483-019-2505-6

Othman, M., Sarkar, N. and Atwa, S.Y. (2013), "Effect of fractional parameter on plane waves of generalized magneto-thermoelastic diffusion with reference temperature-dependent elastic medium", *Comput. Math. Appl.*, **65**(7), 1103-1118. https://doi.org/10.1016/j.camwa.2013.01.047.

Othman, M.I. and Abd-Elaziz, E.M. (2019), "Influence of gravity and micro-temperatures on the thermoelastic porous medium under three theories", *Int. J. Numer. Meth. Heat Fluid Flow*. https://doi.org/10.1108/HFF-12-2018-0763.

Othman, M.I. and Marin, M. (2017), "Effect of thermal loading due to laser pulse on thermoelastic porous medium under GN theory", *Results Phys.*, **7**, 3863-3872. https://doi.org/10.1016/j.rinp.2017.10.012.

Othman, M.I. and Sur, A. (2020), "Transient response in an elasto-thermo-diffusive medium in the context of memory-dependent heat transfer", *Waves Random Complex Media*, 1-24. https://doi.org/10.1080/17455030.2020.1737758.

Othman, M.I., Alharbi, A.M. and Al-Autabi, A.A.M.K. (2020), "Micropolar thermoelastic medium with voids under the effect of rotation concerned with 3PHL model", *Geomech. Eng.*, **21**(5), 447-459. http://doi.org/10.12989/gae.2020.21.5.447.

Sarkar, N. (2017), "Wave propagation in an initially stressed elastic half-space solids under time-fractional order two-temperature magneto-thermoelasticity", *Eur. Phys. J. Plus*, **132**(4), 154. https://doi.org/10.1140/epjp/i2017-11426-8.

Sarkar, N. and Lahiri, A. (2013), "The effect of fractional parameter on a perfect conducting elastic half-space in generalized magneto-thermoelasticity", *Meccanica*, **48**(1), 231-245. https://doi.org/10.1007/s11012-012-9597-3.

Sarkar, N. and Mondal, S. (2020), "Thermoelastic plane waves under the modified Green-Lindsay model with two-temperature formulation", *ZAMM J. Appl. Math. Mech.*, e201900267.

https://doi.org/10.1002/zamm.201900267.

Sheikholeslami, M., Ellahi, R., Shafee, A. and Li, Z. (2019), "Numerical investigation for second law analysis of ferrofluid inside a porous semi annulus: An application of entropy generation and exergy loss", *Int. J. Numer. Meth. Heat Fluid Flow*, **29**(3), 1079-1102. https://doi.org/10.1108/HFF-10-2018-0606.

Sherief, H.H., El-Sayed, A.M.A. and Abd El-Latief, A.M. (2010), "Fractional order theory of thermoelasticity", *Int. J. Solids Struct.*, **47**(2), 269-275. https://doi.org/10.1016/j.ijsolstr.2009.09.034.

Singh, B. (2007), "Wave propagation in a generalized thermoelastic material with voids", *Appl. Math. Comput.*, **189**(1), 698-709. https://doi.org/10.1016/j.amc.2006.11.123.

Stehfest, H. (1970), "Algorithm 368: Numerical inversion of Laplace transforms [D5]", *Commun. ACM*, **13**(1), 47-49. https://doi.org/10.1145/361953.361969.

Sur, A. (2020a), "Wave propagation analysis of porous asphalts on account of memory responses", *Mech. Based Des. Struct. Machines*, 1-19. https://doi.org/10.1080/15397734.2020.1712553.

Sur, A. (2020b), "Memory responses in a three-dimensional thermo-viscoelastic medium", *Waves Random Complex Media*, 1-18. https://doi.org/10.1080/17455030.2020.1766726.

Sur, A. (2020c), "A memory response on the elasto-thermodiffusive interaction subjected to rectangular thermal pulse and chemical shock", *Mechanics Based Des. Struct. Machines*, 1-22. https://doi.org/10.1080/15397734.2020.1772086.

Sur, A. and Kanoria, M. (2014), "Fractional order generalized thermoelastic functionally graded solid with variable material properties", *J. Solid Mech.*, **6**(1), 54-69.

Sur, A., Mondal, S. and Kanoria, M. (2020a), "Effect of nonlocality and memory responses in the thermoelastic problem with a Mode I crack", *Waves Random Complex Media*, 1-26. https://doi.org/10.1080/17455030.2020.1800860.

Sur, A., Mondal, S. and Kanoria, M. (2020b), "Effect of hydrostatic pressure and memory effect on magneto-thermoelastic materials with two-temperatures", *Waves Random Complex Media*, 1-30. https://doi.org/10.1080/17455030.2020.1805524.

Wang, Y., Liu, D. and Wang, Q. (2015), "Effect of fractional order parameter on thermoelastic behaviors in infinite elastic medium with a cylindrical cavity", *Acta Mechanica Solida Sinica*, **28**(3), 285-293. https://doi.org/10.1016/S0894-9166(15)30015-X.

Youssef, H.M. (2010), "Theory of fractional order generalized thermoelasticity", *J. Heat Transfer*, **132**(6), 1-7. https://doi.org/10.1115/1.4000705.

Youssef, H.M. (2012), "Two-dimensional thermal shock problem of fractional order generalized thermoelasticity", *Acta Mech.*, **223**(6), 1219-1231. https://doi.org/10.1007/s00707-012-0627-y.

Youssef, H.M. and Al-Lehaibi, E.A. (2010), "Variational principle of fractional order generalized thermoelasticity", *Appl. Math. Lett.*, **23**(10), 1183-1187. https://doi.org/10.1016/j.aml.2010.05.008.

Zeeshan, A., Ellahi, R., Mabood, F. and Hussain, F. (2019), "Numerical study on bi-phase coupled stress fluid in the presence of Hafnium and metallic nanoparticles over an inclined plane", *Int. J. Numer. Meth. Heat Fluid Flow*, https://doi.org/10.1108/HFF-11-2018-0677.

Appendix A

The matrix $A = [a_{ij}] = 0, i, j = 1 \dots 8$ except for $a_{48} = 1, a_{37} = 1, a_{26} = 1, a_{15} = 1,$

$a_{51} = s^2 + s_1 q^2, a_{56} = -iq(1 - s_1), a_{57} = -s_2, a_{58} = s_3(1 + s\tau_1), a_{62} = \frac{(s^2+q^2)}{s_1},$

$a_{63} = -\frac{s_2 iq}{s_1}, a_{64} = \frac{s_3 iq}{s_1}(1 + s\tau_1), a_{65} = -\frac{iq(1-s_1)}{s_1}, a_{72} = s_4 iq, a_{73} = s^2 s_8 + q^2 + s_5 + s_6 s,,$

$a_{74} = -s_7, a_{75} = s_4, a_{82} = s_{10} iq(s + nsd), a_{83} = s_9(s + nsd), a_{84} = q^2 + (s + sd),$

$a_{85} = (s + nsd)s_{10}$ and V = $\begin{bmatrix} \bar{u}^* & \bar{v}^* & \bar{\varphi}^* & \bar{\Theta}^* & \frac{d\bar{u}^*}{dx} & \frac{d\bar{v}^*}{dx} & \frac{d\bar{\varphi}^*}{dx} & \frac{d\bar{\Theta}^*}{dx} \end{bmatrix}^T.$

Appendix B

$m_1 = a_{73} + a_{57}a_{75} + a_{84} + a_{62} + a_{56}a_{65} + a_{58}a_{85} + a_{51}.$

$m_2 = a_{51}a_{73} - a_{64}a_{82} - a_{58}a_{65}a_{82} - a_{74}a_{83} - a_{58}a_{75}a_{83} + a_{58}a_{51}a_{62} - a_{63}a_{72} - a_{57}a_{65}a_{72} + a_{62}a_{85} - a_{56}a_{64}a_{85} + a_{58}a_{73}a_{85} - a_{57}a_{74}a_{85} + a_{51}a_{84} + a_{62}a_{84} + a_{56}a_{65}a_{84} + a_{73}a_{84} + a_{57}a_{75}a_{84} + a_{62}a_{73} + a_{56}a_{65}a_{73} + a_{57}a_{62}a_{75} - a_{56}a_{63}a_{75},$

$m_3 = -a_{62}a_{73}a_{84} + a_{58}a_{62}a_{75}a_{83} - a_{57}a_{64}a_{72}a_{85} - a_{58}a_{62}a_{73}a_{85} - a_{58}a_{65}a_{72}a_{83} + a_{51}a_{64}a_{82} + a_{56}a_{64}a_{73}a_{85} + a_{58}a_{63}a_{72}a_{85} + a_{57}a_{62}a_{74}a_{85} - a_{56}a_{63}a_{74}a_{85} - a_{56}a_{65}a_{73}a_{84} - a_{57}a_{62}a_{75}a_{84} + a_{51}a_{63}a_{72} - a_{51}a_{73}a_{84} - a_{56}a_{64}a_{75}a_{83} - a_{51}a_{62}a_{84} - a_{58}a_{63}a_{75}a_{82} + a_{57}a_{64}a_{75}a_{82} - a_{64}a_{72}a_{83} + a_{58}a_{65}a_{73}a_{82} + a_{64}a_{73}a_{82} + a_{56}a_{63}a_{75}a_{84} + a_{51}a_{74}a_{83} + a_{62}a_{74}a_{83} + a_{57}a_{65}a_{72}a_{84} + a_{56}a_{65}a_{74}a_{83} - a_{51}a_{62}a_{73} - a_{63}a_{74}a_{82} + a_{63}a_{72}a_{84} - a_{57}a_{65}a_{74}a_{82},$

$m_4 = a_{51}a_{63}a_{74}a_{82} + a_{51}a_{62}a_{73}a_{84} - a_{51}a_{62}a_{74}a_{83} + a_{51}a_{64}a_{72}a_{83} - a_{51}a_{64}a_{73}a_{82} - a_{51}a_{63}a_{72}a_{84}.$