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A simple quasi-3D HDST for dynamic behavior of advanced composite plates with the effect of variables elastic foundations

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Abstract. In this study, dynamics responses of advanced composite plates resting variable elastic foundations via a quasi-3D theory are developed using an analytical approach. This higher shear deformation theory (HSDT) is included the shear deformation theory and effect stretching that has five unknowns, which is even inferior to normal deformation theories found literature and other theories. The quasi-three-dimensional (quasi-3D) theory accounts for a parabolic distribution of the transverse shear deformation and satisfies the zero traction boundary conditions on the surfaces of the advanced composite plate without needing shear correction factors. The plates assumed to be rest on two-parameter elastic foundations, the Winkler parameter is supposed to be constant but the Pasternak parameter varies along the long side of the plate with three distributions (linear, parabolic and sinusoidal). The material properties of the advanced composite plates gradually vary through the thickness according to two distribution models (power law and Mori-Tanaka). Governing differential equations and associated boundary conditions for dynamics responses of the advanced composite plates are derived using the Hamilton principle and are solved by using an analytical solution of Navier's technique. The present results and validations of our modal with literature are presented that permitted to demonstrate the accuracy of the present quasi-3D theory to predict the effect of variables elastic foundation on dynamics responses of advanced composite plates.

Keywords: dynamics responses; advanced composite plates; quasi-3D theory; variables elastic foundations

1. Introduction

For the first time, the concept of functionally graded materials (FGM) was introduced by scientists in fields materials in the Sendai region of Japan in 1984 (Koizumi 1997, Yaghoobi *et al.* 2015, Hadji *et al.* 2015, Avcar 2019, Ahmed *et al.* 2019, Balubaid *et al.* 2019, Addou *et al.* 2019, Boutaleb *et al.* 2019, Zarga *et al.* 2019, Karami *et al.* 2019ab, Hellal *et al.* 2019, Salah *et al.* 2019, Menasria *et al.* 2020, Zine *et al.* 2020, Khiloun *et al.* 2020, Matouk *et al.* 2020, Hussain *et al.* 2020, Rahmani *et al.* 2020, Kaddari *et al.* 2020, Boussoula *et al.* 2020, Rachedi *et al.* 2020). Functional graded materials are a new class of advanced composites that are progressively changing in the constitution of materials from one surface to another. The main feature of FGM is to eliminate the stress concentration of conventional laminated composites (Lee *et al.* 2015).

Functionally graded materials and structures attract many scientists in the various fields of research to develop their concept through theoretical and experimental research. Reddy (2000) analyzed functionally graded plates subjected thermo-mechanical loads using third-order theory and employed analytical approached and finite element method.

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Matsunaga (2008) in studied the dynamics behaviors and buckling functionally graded materials by taking into account the effects of transverse shear and normal deformations and rotatory inertia. Grover *et al.* (2013) proposed a new inverse hyperbolic shear deformation theory of laminated composite and sandwich plates for the static and buckling responses. Jha *et al.* (2013) studied the static and free vibration analyses of functionally graded (FG) elastic, rectangular, and simply (diaphragm) supported plates by using a higher order shear and normal deformations plate theory. Thai and Kim (2013) developed a simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates. Nguyen *et al.* (2014) presented the static, buckling and free vibration analyses of isotropic and functionally graded (FG) sandwich plates using an inverse trigonometric shear deformation theory. Pandey and Pradyumna (2015) developed a layerwise finite element formulation for dynamic analysis of two types of functionally graded material (FGM) sandwich plates with nonlinear temperature variation along the thickness and the FGM having temperature dependent material properties. The studies of dynamics and mechanical behavior of composite structures are developed in many research using higher order shear deformation theory (Katariya and Panda 2016, Sahoo *et al.* 2016a, b, Singh *et al.* 2016 and 2018, Panda and Kolahchi 2018, Faleh *et al.* 2018, Hirwani *et al.* 2017a, b and 2019,

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Hussain and Naeem 2019, Belbachir *et al.* 2019, Sahla *et al.* 2019, Abualnour *et al.* 2019, Belbachir *et al.* 2020, Katariya *et al.* 2017and 2020, Sahu *et al.* 2020). A comparison between a three-dimensional (3D) exact solution and several two-dimensional (2D) numerical solutions using numerical methods include classical 2D finite elements (FEs), and classical and refined 2D generalized differential quadrature (GDQ) solutions are presented by (Brischetto *et al.* 2016). Recently, Tran and Kim (2018) studied static and free vibration of multilayered plates based on isogeometric analysis (IGA) and higher-order shear and normal deformation theory. Sayyad and Ghugal (2018) investigated the bending, buckling, and vibration behavior of shear deformable laminated composite and sandwich beams using trigonometric shear and normal deformation theory.

Researchers have extensively studied the interactions between structures, as like plates, beams and shell, and elastic foundations because of their use in many fields such as civil engineering, mechanics, etc. The Winkler linear model is the first type of elastic foundation in one parameter that consists of a series of separate springs without coupling effects between them (Winkler 1867). The Pasternak elastic foundation is a two-parameter parameter with a shear layer added to the Winkler spring to describe the interactions between them (Pasternak 1954). Huang *et al.* (2008) presented exact solutions for static behavior of functionally graded thick plates using the three-dimensional theory of elasticity. Malekzadeh (2009) studied the three-dimensional free vibration analyses of functionally graded plates are limited to plates with simply supported boundary conditions and with elastic foundations. Kumar *et al.* (2011) presented the hygro-thermal effects on the free vibration of laminated composite plates resting on elastic foundations with random system properties using micromechanical model via finite element method. Han *et al.* (2016) presented a quasi-three-dimensional theory for dynamic responses of power law and sigmoid Functionally Graded Material plates. The first order shear theory are employed to predict free vibration and static responses of homogeneous and functionally graded structures resting on elastic foundations by (Mantari and Granados 2016, Park and Choi 2017). The mechanical responses and free vibration of functionally graded structures resting elastic foundation are widely studied using higher shear deformation theory (Said *et al.* 2014, Xiang *et al.* 2014, Lee *et al.* 2015, Shahsavari *et al.* 2018, Majeed and Sadiq 2018, Rezaiee-Pajand *et al.* 2018, Batou *et al.* 2019, Salah *et al.* 2019, Nebab *et al.* 2019, Chaabane *et al.* 2019, Tounsi *et al.* 2020, Rabhi *et al.* 2020, Chikr *et al.*2020, Refrafi *et al.* 2020). Recently, Lin and Shi (2018) investigated three-dimensional formulation for the free vibrations of thick rectangular plates with general boundary conditions and resting on elastic foundations by using the Rayleigh-Ritz method. Zhang *et al.* (2018) established a thin orthotropic rectangular fluid-structure coupled system resting on varying elastic Winkler and Pasternak foundations. The main objective of this study is the investigation of the effect of variables elastic foundations on dynamic responses of advanced composite plates using a new quasi-three-dimensional theory. This present quasi-3D theory has only four variables; the numbers of variables are reduced by using undefined integral. In order to check the accuracy of the present theory. The Hamilton principle is utilized to establish the equation of motion, where solved by analytical solution series. The results are compared with the exiting results found in the literature. The effects of the power law index, two-parameter elastic foundation, aspect side-to-length ratios and side-to-thickness ratios on advanced composite plates responses on the Pasternak elastic foundation were examined.

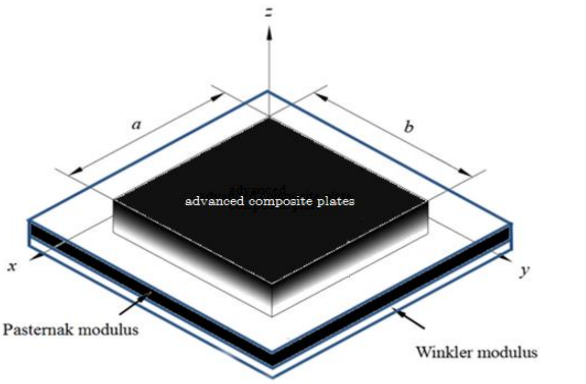

Fig. 1 Typical advanced composite plates with Cartesian coordinates

2. Problem formulation

2.1 Geometrical configuration

Let's consider an isotropic advanced composite plate resting on two-parameters elastic foundation of length (*a*), width (*b*), with uniform constant thickness (*h*), as shown in Fig. 1, the plate has simply supported edge in four sides. The Cartesian coordinate system (*x,y,z*) is assumed to the basis extract mathematical formulations when x and y-axis are located in mid-plane of the plate.

2.2 Material properties

Advanced composite materials are functionally graded proprieties where the materials proprieties vary smoothly due to gradually changing the volume fraction of the constituent materials, generally in the thickness direction of plate. The Advanced composite plate is prepared from a mixture of ceramics and metal and the composition varies from the top to the bottom surface. In the case study, we have two types of volume fractional materials to describe its change in thickness, as follows:

2.2.1 Types 1: Metal volume fractions

We supposed the volume fraction in both Mori–Tanaka model and Voigt model follows a simple power law as:

$$V_m = \left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1}$$

Voigt model

Using fraction volume of metal, the effective material properties of functionally graded plates such as Young's modulus E, mass density $\rho$and Poisson's ratio ν are considered to vary gradually through the thickness according to a power law distribution. Note that Eq. (1) also apply to the Voigt model. Which is given in Eq. (2)

$$
\begin{aligned}
E(z) &= E_c + (E_m - E_c)\,V_m \\
\rho(z) &= \rho_c + (\rho_m - \rho_c)\,V_m \\
\nu(z) &= \nu_c + (\nu_m - \nu_c)\,V_m
\end{aligned}
\tag{2}
$$

Mori-tanaka model

According to the Mori-Tanaka scheme, the effective local bulk modulus K_f and the shear modulus G_f are expressed:

$$
K_f = K_c + (K_m - K_c)\cdot\frac{V_m}{1+[1-V_m]\dfrac{3[K_m-K_c]}{3K_c+4G_c}}
\tag{3a}
$$

$$
G_f = G_c + (G_m - G_c)\cdot\frac{V_m}{1+[1-V_m]\dfrac{G_m-G_c}{G_c+f_1}}
\tag{3b}
$$

in which

$$
f_1 = \frac{G_c[9K_c+8G_c]}{6[K_c+2G_c]}
\tag{3}
$$

The effective Young's modulus E_f and Poisson's ratio ν_f can be given by using Eq. (3):

$$
\begin{aligned}
E_f &= \frac{9K_f G_f}{3K_f+G_f} \\
\nu_f &= \frac{3K_f-2G_f}{2(3K_f+G_f)}
\end{aligned}
\tag{4}
$$

2.2.2 Types 2: Ceramic volume fractions

We supposed the volume fraction V_c in Voigt model follows a simple power law as:

$$
V_c = \left(\frac{1}{2}+\frac{z}{h}\right)^p
\tag{5}
$$

Voigt model

Using fraction volume of ceramic, the effective material properties of functionally graded plates such as Young's modulus $E(z)$, mass density $\rho(z)$ and Poisson's ratio $\nu(z)$ are considered to vary gradually through the thickness according to a power law distribution, which is given in Eq (5):

$$
\begin{aligned}
E(z) &= E_m + (E_c - E_m)V_c \\
\rho(z) &= \rho_m + (\rho_c - \rho_m)V_c \\
\nu(z) &= \nu_m + (\nu_c - \nu_m)V_c
\end{aligned}
\tag{6}
$$

2.3 Variables elastic foundations

The advanced composite plates are supposed to be resting on elastic foundation that has two layers. The first layer is from spring of Winkler without considered effect coupled between them and a shear layer of Pasternak that interconnected springs of Winkler. The general formulation described the elastic foundations of Pasternak-Winkler are given as below:

$$
f_e(x) = K_w(x)w - K_p\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)
\tag{7}
$$

where reaction of the elastic foundation is $f_e(x)$, K_p is shear layer is constant, if shear layer is not considered , the elastic foundation becomes Winkler foundation, and $K_w(x)$ is the variable Winkler parameter depend only in x direction, as shown in Fig. 2.

The variation of Winkler elastic foundation is found in previous studies as follow (Pradhan and Murmu 2009, Sobhy 2015, Nebab *et al.* 2019)

$$
K_w(x) = \frac{J_1 D_i}{a^4}\begin{cases} 1+\zeta\dfrac{x}{a} & \text{Linear} \\ 1+\zeta\left(\dfrac{x}{a}\right)^2 & \text{Parabolic} \\ 1+\zeta\sin\left(\pi\dfrac{x}{a}\right) & \text{Sinusoidal} \end{cases}
\tag{8}
$$

where J_1 is a constant and ζ is a varied parameter. Where that if ζ is zero, the foundation becomes uniform Winkler and if the rigidity of the shear layer is neglected, the foundation of Pasternak becomes the Winkler foundation.

2.4 Kinematics and strains

Based on assumption higher shear deformation theory for advanced composite plates is used to describe her field displacement as follows:

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\,f(z)\int\theta(x,y,t)dx, \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\,f(z)\int\theta(x,y,t)dx, \\
w(x,y,z,t) &= w_0(x,y,t) + g(z)\varphi(x,y,t)
\end{aligned}
\tag{9}
$$

where, u_0 and v_0 are the mid-plane displacement of the plate in the x and y directions, w_0, θ and φ are the bending , shear and stretching components of transverse displacement, respectively. $f(z)$ is a new shape function determining the distribution of the transverse shear are

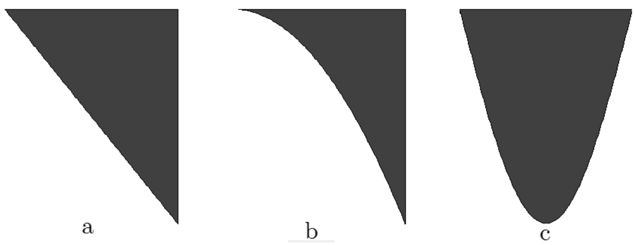

Fig. 2 distributions types of Winkler elastic foundation along the axial direction of the FG plate: (a) linear type, (b) parabolic type and (c) sinusoidal type taken to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate, as follows:

$$f(z)=\frac{2}{3\pi}\left(z\ \cosh\left(\frac{\pi}{2}\right)-h\sinh\left(\frac{z\pi}{h}\right)\right)\quad and \quad g(z)=\frac{\partial f(z)}{\partial z} \tag{10}$$

The linear strain is definite by deriving from the kinematic of Eq. (9), based the application of the linear, small-strain elasticity theory, valid for thin, moderately thick and thick plates under consideration are as expressed:

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}K_x^b\\ K_y^b\\ K_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}K_x^s\\ K_y^s\\ K_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix},\quad \varepsilon_z=g'(z)\varepsilon_z^0 \tag{11}$$

where

$$\begin{pmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{pmatrix}=\begin{pmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial y}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{pmatrix},\quad \begin{pmatrix}K_x^b\\ K_y^b\\ K_{xy}^b\end{pmatrix}=\begin{pmatrix}-\frac{\partial^2 w}{\partial^2 x}\\ -\frac{\partial^2 w}{\partial^2 y}\\ -2\frac{\partial^2 w}{\partial^2 x\partial^2 y}\end{pmatrix} \tag{12a}$$

$$\begin{pmatrix}K_x^b\\ K_y^b\\ K_{xy}^b\end{pmatrix}=\begin{pmatrix}k_1\theta\\ k_2\theta\\ k_1\frac{\partial}{\partial y}\int\theta dx+k_2\frac{\partial}{\partial x}\int\theta dy\end{pmatrix},$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta dy+\frac{\partial\varphi_z}{\partial y}\\ k_1\int\theta dx+\frac{\partial\varphi_z}{\partial x}\end{Bmatrix},\ \varepsilon_z^0=\varphi_z \quad \text{and} \quad g'(z)=\frac{\partial g(z)}{\partial z} \tag{12b}$$

The undefined integral that is found in strains relation, it can be simplified in format derivation by using Navier techniques, rewritten as follows:

$$\frac{\partial}{\partial y}\int\theta dx=A'\frac{\partial^2\theta}{\partial y\partial x},\quad \frac{\partial}{\partial x}\int\theta dy=B'\frac{\partial^2\theta}{\partial y\partial x},\quad \int\theta dx=A'\frac{\partial\theta}{\partial x},\quad \int\theta dy=B'\frac{\partial\theta}{\partial y} \tag{13}$$

where A' and B' coefficients are assumed according to the technique of solution; in this case, the technique is used Navier methods Consequently, A', B', k_1 and k_2 are written as follows:

$$A'=-\frac{1}{\alpha^2},\quad B'=-\frac{1}{\beta^2},\quad k_1=\alpha^2,\quad k_2=\beta^2 \tag{14}$$

2.4 Constitutive equations

The stress-strain relationships, which take into account transverse shear and normal deformations, can be expressed as follows:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&0&0&0\\ Q_{21}&Q_{22}&Q_{23}&0&0&0\\ Q_{31}&Q_{32}&Q_{33}&0&0&0\\ 0&0&0&Q_{66}&0&0\\ 0&0&0&0&Q_{44}&0\\ 0&0&0&0&0&Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \tag{15}$$

where, $(\sigma_x,\sigma_y,\sigma_z,\tau_{xy},\tau_{yz},\tau_{xz})$ and $(\varepsilon_{xx},\varepsilon_{yy},\varepsilon_{zz}\gamma_{xy},\gamma_{yz},\gamma_{xz})$ are the stress and strain components, respectively. The constitutive coefficients Q_{ij} may be expressed in terms of the engineering isotropic characteristics as:

$$Q_{11}=Q_{22}=Q_{33}=\frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{16a}$$

$$Q_{12}=Q_{13}=Q_{23}=\frac{\nu(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{16b}$$

$$Q_{44}=Q_{55}=Q_{66}=\frac{E(z)}{2(1+\nu)} \tag{16c}$$

2.4 Equations of motion

Hamilton's principle is used to guide the equations that govern and can be expressed by the following relation:

$$\int_0^t(\delta U+\delta U_{ef}-\delta K)dt=0 \tag{17}$$

where δ indicates a variation, and U, U_{ef} and K represent the strain energy of FG-plate, the strain energy of elastic foundations and the kinetic energy, respectively.

The variation in kinetic energy based on higher-order shear deformation theory can be expressed as follows:

$$\delta K=\int_V\rho(z)\left(\dot u\delta\dot u+\dot v\delta\dot v+\dot w\delta\dot w\right)dV \tag{18a}$$

$$=\int_A\left\{\begin{array}{l}I_0\left(\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+\dot w_0\delta\dot w_0\right)\\ -I_1\begin{pmatrix}\dot u_0\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial x}\delta\dot u_0\\ +\dot v_0\frac{\partial\delta\dot w_0}{\partial y}+\frac{\partial\dot w_0}{\partial y}\delta\dot v_0\end{pmatrix}\\ +J_1\begin{pmatrix}(k_1A')\left(\dot u_0\frac{\partial\delta\dot\theta}{\partial x}+\frac{\partial\dot\theta}{\partial x}\delta\dot u_0\right)+\\ (k_2B')\left(\dot v_0\frac{\partial\delta\dot\theta}{\partial y}+\frac{\partial\dot\theta}{\partial y}\delta\dot v_0\right)\end{pmatrix}\\ +I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right)\\ +K_2\left((k_1A')^2\left(\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}\right)+(k_2B')^2\left(\frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}\right)\right)\\ -J_2\begin{pmatrix}(k_1A')\left(\frac{\partial\dot w}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}+\frac{\partial\delta\dot w}{\partial x}\frac{\partial\dot\theta}{\partial x}\right)+(k_2B')\\ \left(\frac{\partial\dot w}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}+\frac{\partial\delta\dot w}{\partial y}\frac{\partial\dot\theta}{\partial y}\right)\end{pmatrix}\\ +J_0\left(\dot\varphi\delta\dot w_0+\dot w_0\delta\dot\varphi\right)+K_3\left(\dot\varphi\ \delta\dot\varphi\right)\end{array}\right\}dA \tag{18b}$$

where, the point-exponent convention indicates the differentiation with respect to the time variable t, $\rho(z)$ is the density. Mass inertias is defined as:

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} (1, g(z), z, z^2)\rho(z)dz$$
$$\left(J_1, J_2, K_2, K_3\right) = \int_{-h/2}^{h/2} (f(z), z\ f(z), f(z)^2, g(z)^2)\rho(z)dz \tag{19}$$

The variation of the deformation energy of the plate written by:

$$\delta U = \int_V \begin{pmatrix} \sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{zz}\delta\varepsilon_{zz} \\ +\tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz} \end{pmatrix} dV \tag{20a}$$

$$= \int_A \begin{pmatrix} N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_z\delta\varepsilon_z^0 + N_{xy}\delta\varepsilon_{xy}^0 \\ +M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s \\ +M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + R_{yz}^s\delta R_{yz}^s + R_{xz}^s\delta R_{xz}^s \end{pmatrix} dA \tag{20b}$$

where,

$$\begin{Bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^b & M_{xy}^b \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz,$$
$$N_z = \int_{-h/2}^{h/2} (\sigma_z) g'(z)\ dz, \quad \left(R_{xz}^s, R_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z)\, dz \tag{21}$$

The variation of the deformation energy of the variable elastic foundations indicated as

$$\delta U_{ef} = \int_A (K_w(x)\ w_0\delta w_0 - K_s\left(\frac{d^2 w_0}{dx^2} + \frac{d^2 w_0}{dy^2}\right)\delta w) dA \tag{22}$$

By substituting the expressions for, δK , δU ,and δU_{ef} from the Eqs. (18), (20) and (21) in Eq. (17) and integrating by parts and collecting the coefficients of ($\delta u_0, \delta v_0, \delta w_0$, $\delta\theta$ and $\delta\varphi$), the following equations of advanced composite plate motion are obtained as

$$\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial \ddot{w}_0}{\partial x} + J_1 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \tag{23a}$$

$$\delta v_0: \ \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0\ddot{v}_0 - I_1\frac{\partial \ddot{w}_0}{\partial y} + J_1 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \tag{23b}$$

$$\begin{aligned}\delta w_0: \ & \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + K_w w_0 - K_s \\ & \times\left(\frac{d^2 w_0}{dx^2} + \frac{d^2 w_0}{dy^2}\right) = J_0\ddot{\varphi} - I_0\ddot{w}_0 \\ & + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 \\ & + J_2\left(k_1 A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)\end{aligned} \tag{23c}$$

$$\begin{aligned}\delta\theta: \ & -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} \\ & + k_1 A'\frac{\partial R_{xz}^S}{\partial x} + k_2 B'\frac{\partial R_{yz}^S}{\partial y} = \\ & - J_1\left(k_1 A'\frac{\partial \ddot{u}_0}{\partial x} + k_2 B'\frac{\partial \ddot{v}_0}{\partial y}\right) \\ & - K_2\left(\left(k_1 A'\right)^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + \left(k_2 B'\right)^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\ & + J_2\left(k_1 A'\frac{\partial^2\ddot{w}}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{w}}{\partial y^2}\right)\end{aligned} \tag{23d}$$

$$\delta\varphi: \frac{\partial R_{xz}^S}{\partial x} + \frac{\partial R_{yz}^S}{\partial y} - N_z = J_0\ddot{w}_0 + K_3\ddot{\varphi} \tag{23e}$$

Substituting Eq. (11) into Eq. (15) and integrating across the thickness of the advanced composite plate, the stress resultants are expressed as:

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{Bmatrix} X \\ Y \\ Y^s \end{Bmatrix} \varepsilon_z^0,$$
$$S = A^s\gamma \tag{24a}$$

$$\begin{aligned} N_z &= X\left(\varepsilon_x^0 + \varepsilon_y^0\right) + Y\left(k_x^b + k_y^b\right) \\ &+ Y^s\left(k_x^s + k_y^s\right) + Z\left(\varepsilon_z^0\right)\end{aligned} \tag{24b}$$

where

$$N = \left\{N_x, \quad N_y, \quad N_{xy}\right\}^t, M^b = \left\{M_x^b \quad M_y^b \quad M_{xy}^b\right\}^t,$$
$$M^s = \left\{M_x^s \quad M_y^s \quad M_{xy}^s\right\}^t \tag{25a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \quad \varepsilon_x^0, \quad \varepsilon_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b \quad k_y^b \quad k_{xy}^b\right\}^t, *$$
$$k^s = \left\{k_x^s \quad k_y^s \quad k_{xy}^s\right\}^t \tag{25b}$$

and

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},$$
$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{26a}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{21}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{21}^s & B_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix},$$
$$H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{21}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{26b}$$

$$\begin{Bmatrix} Z \\ X \\ Y \\ Y^S \end{Bmatrix} = \int_{-h/2}^{h/2} Q_{ij} \begin{pmatrix} 1 \\ z \\ f'(z) \\ g(z) \end{pmatrix} g'(z)\ dz \tag{26c}$$

$$S=\left\{R_{xz}^{s},\quad R_{yz}^{s}\right\},\quad \gamma=\left\{\gamma_{xz}^{0}\quad \gamma_{yz}^{0}\right\}$$

$$A^{s}=\begin{pmatrix} A_{44}^{s} & 0 \\ 0 & A_{55}^{s}\end{pmatrix} \tag{26d}$$

In addition, the stiffness components are given as follows

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^{s} & D_{11}^{s} & H_{11}^{s} \\ A_{12} & B_{12} & D_{12} & B_{12}^{s} & D_{12}^{s} & H_{12}^{s} \\ A_{66} & B_{66} & D_{66} & B_{66}^{s} & D_{66}^{s} & H_{66}^{s}\end{Bmatrix} = \int_{-h/2}^{h/2} Q_{11}(1, z, z^2, f(z), zf(z), f(z)^2)\begin{Bmatrix}1\\ z\\ f(z)\end{Bmatrix}dz \tag{27a}$$

$$\left(A_{22},\ B_{22},\ D_{22},\ B_{22}^{s},\ D_{22}^{s},\ H_{22}^{s}\right) = \left(A_{11},\ B_{11},\ D_{11},\ B_{11}^{s},\ D_{11}^{s},\ H_{11}^{s}\right) \tag{27b}$$

$$A_{44}^{s}=A_{55}^{s}=\int_{-h/2}^{h/2} Q_{44}\left[g(z)\right]^2 dz \tag{27c}$$

2.5 Equations of motion via terms of displacements

Substituting Eq. (24) Into Eq. (23), the equations of motion of the presented theory can be rewritten in terms of displacements (δu_0, δv_0, δw_0, $\delta\theta_0$ and $\delta\varphi$) as follows:

$$\begin{aligned}&A_{11}\frac{\partial^2}{\partial x^2}u_0+A_{66}\frac{\partial^2}{\partial x^2}v_0+\left(A_{11}+A_{66}\right)\frac{\partial^2}{\partial x\partial y}v_0\\&-B_{11}\frac{\partial^2}{\partial x^2}w_0-\left(B_{12}+2B_{66}\right)\frac{\partial^3}{\partial x\partial y^2}w_0\\&+\left(B_{66}^{s}\left(k_1A'+k_2B'\right)\right)\frac{\partial^3}{\partial x\partial y^2}\theta+\left(B_{11}^{S}k_1+B_{12}^{s}k_2\right)\\&\times\frac{\partial}{\partial x}\theta=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_0}{\partial x}+J_1k_1A'\frac{\partial\ddot{\theta}}{\partial x}\end{aligned} \tag{28a}$$

$$\begin{aligned}&A_{22}\frac{\partial^2}{\partial y^2}v_0+A_{66}\frac{\partial^2}{\partial x^2}v_0+\left(A_{12}+A_{66}\right)\frac{\partial^2}{\partial x\partial y}u_0\\&-B_{22}\frac{\partial^3}{\partial y^3}w_0-\left(B_{12}+2B_{66}\right)\frac{\partial^3}{\partial x^2\partial y}w_0\\&+\left(B_{66}^{s}\left(k_1A'+k_2B'\right)\right)\frac{\partial^3}{\partial x^2\partial y}\theta+\left(B_{22}^{s}k_2+B_{12}^{s}k_1\right)\\&\times\frac{\partial^2}{\partial y^2}\theta=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y}+J_1k_2B'\frac{\partial\ddot{\theta}}{\partial y}\end{aligned} \tag{28b}$$

$$\begin{aligned}&B_{11}\frac{\partial^3}{\partial x^3}u_0+\left(B_{12}+2B_{66}\right)\frac{\partial^3}{\partial x\partial y^2}u_0+\left(B_{12}+2B_{66}\right)\\&\times\frac{\partial^3}{\partial x^2\partial y}v_0+B_{22}\frac{\partial^3}{\partial y^3}v_0-D_{11}\frac{\partial^3}{\partial y^3}w_0\\&-2\left(D_{12}+2D_{66}\right)\frac{\partial^4}{\partial x^2\partial y^2}w_0-D_{22}\frac{\partial^4}{\partial y^4}w_0\\&-K_w(x)w_0+K_s\left(\frac{\partial^2}{\partial x^2}w_0+\frac{\partial^2}{\partial y^2}w_0\right)\\&+\left(D_{11}^{s}k_1+D_{12}^{s}k_2\right)\frac{\partial^2}{\partial x^2}\theta+2\left(D_{66}^{s}\left(k_1A'+k_2B'\right)\right)\\&\times\frac{\partial^4}{\partial x^2\partial y^2}\theta+\left(D_{12}^{s}k_1+D_{22}^{s}k_2\right)\frac{\partial^2}{\partial y^2}\theta=J_0\ddot{\varphi}-I_0\ddot{w}_0\\&+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_0+J_2\begin{pmatrix}k_1A'\dfrac{\partial^2\ddot{\theta}}{\partial x^2}\\+k_2B'\dfrac{\partial^2\ddot{\theta}}{\partial y^2}\end{pmatrix}\end{aligned} \tag{28c}$$

$$\begin{aligned}&\left(-B_{11}^{s}k_1+B_{12}^{s}k_2\right)\frac{\partial}{\partial x}u_0-\left(B_{66}^{s}\left(A'k_1+B'k_2\right)\right)\\&\times\frac{\partial^3}{\partial x\partial y^2}u_0-\left(B_{66}^{s}\left(A'k_1+B'k_2\right)\right)\frac{\partial^3}{\partial x^2\partial y}v_0\\&-\left(B_{12}^{s}k_1+B_{22}^{s}k_2\right)\frac{\partial^2}{\partial x^2}v_0-\left(D_{11}^{s}k_1+D_{12}^{s}k_2\right)\\&\times\frac{\partial^2}{\partial x^2}w_0+2\left(D_{66}^{s}\left(A'k_1+B'k_2\right)\right)\frac{\partial^4}{\partial x^2\partial y^2}w_0\\&+\left(D_{12}^{s}k_1+D_{22}^{s}k_2\right)\frac{\partial^2}{\partial y^2}w_0-H_{11}^{s}k_1^2\theta-H_{12}^{s}k_2^2\theta\\&-2H_{12}^{s}k_1k_2\theta-\left(H_{66}^{s}\left(A'k_1+B'k_2\right)^2\right)\frac{\partial^4}{\partial x^2\partial y^2}\theta\\&+A_{44}^{s}\left(B'k_2\right)^2\frac{\partial^2}{\partial y^2}\theta+A_{55}^{s}\left(A'k_1\right)^2\frac{\partial^2}{\partial x^2}\theta\\&=-J_1\left(k_1A'\frac{\partial\ddot{u}_0}{\partial x}+k_2B'\frac{\partial\ddot{v}_0}{\partial y}\right)-K_2\begin{pmatrix}\left(k_1A'\right)^2\dfrac{\partial^2\ddot{\theta}}{\partial x^2}\\+\left(k_2B'\right)^2\dfrac{\partial^2\ddot{\theta}}{\partial y^2}\end{pmatrix}\\&+J_2\left(k_1A'\frac{\partial^2\ddot{w}}{\partial x^2}+k_2B'\frac{\partial^2\ddot{w}}{\partial y^2}\right)\end{aligned} \tag{28d}$$

$$\begin{aligned}&-X_{13}\frac{\partial}{\partial x}u_0-X_{23}\frac{\partial}{\partial y}v_0+Y_{13}\frac{\partial^2}{\partial x^2}w_0+Y_{23}\\&\times\frac{\partial^2}{\partial y^2}w_0-Y_{13}^{s}k_1\theta-Y_{23}^{s}k_2\theta-A_{44}^{S}\left(k_2B'\right)\\&\times\frac{\partial^2}{\partial y^2}\theta-A_{55}^{S}\left(k_1A'\right)\frac{\partial^2}{\partial x^2}\theta+A_{44}^{S}\frac{\partial^2}{\partial y^2}\varphi\\&+A_{55}^{S}\frac{\partial^2}{\partial x^2}\varphi-Z_{33}\varphi_z=J_0\ddot{w}_0+K_3\ddot{\varphi}\end{aligned} \tag{28e}$$

3. Analytical solution

The Navier technique from analytic solution types is purposed to solve a differential equation of problem for the free vibration of advanced composite plates resting two-parameter variables elastic foundations. The advanced composite plates are considered to have simply supported edge in all sides. The solution is assumed to be from a series of Fourier as follows:

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\\ \varphi\end{Bmatrix}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\ x)\sin(\beta\ y)\ e^{i\omega t}\\ V_{mn}\sin(\alpha\ x)\cos(\beta\ y)\ e^{i\omega t}\\ W_{mn}\sin(\alpha\ x)\sin(\beta\ y)\ e^{i\omega t}\\ X_{mn}\sin(\alpha\ x)\sin(\beta\ y)\ e^{i\omega t}\\ Y_{mn}\sin(\alpha\ x)\sin(\beta\ y)\ e^{i\omega t}\end{Bmatrix} \tag{29}$$

U; *V*; *W*; *X* and *Y* are arbitrary unknown parameters to be determined and *ω* is the natural frequency. $\sqrt{i}=-1$ is the imaginary unit.

The displacement functions are given in Eq. (23) satisfy the kinematic boundary conditions of the simply supported plate, which are given below:

$$\begin{aligned} & v = w = \theta = N_x = M_x = R_x = \varphi = 0 \\ & \quad at \qquad x = 0\ , a, \\ & u = w = \theta = N_y = M_y = R_y = \varphi = 0 \\ & \quad at \qquad y = 0\ , b, \end{aligned} \tag{30}$$

By substituting the Eq. (29) for the equations of motion (28), we obtain below the equation of the eigenvalue for any fixed value of m and n, for a free vibration problem:

$$\left(\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ & & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ & sym. & & \Gamma_{44} & \Gamma_{45} \\ & & & & \Gamma_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ & M_{22} & M_{23} & M_{24} & M_{25} \\ & & M_{33} & M_{34} & M_{35} \\ & sym. & & M_{44} & M_{45} \\ & & & & M_{55} \end{bmatrix} \right) \begin{Bmatrix} U \\ V \\ W \\ X \\ Y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{31}$$

where

$$\begin{aligned}
\Gamma_{11} &= -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \\
\Gamma_{12} &= -\alpha\beta\left(A_{12} + A_{66}\right), \\
\Gamma_{13} &= \alpha(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2), \\
\Gamma_{14} &= \alpha\left(k_1B_{11}^s + k_2B_{12}^s - \left(k_1A^{'} + k_2B^{'}\right)B_{66}^s\beta^2\right), \\
\Gamma_{15} &= \alpha X_{13} \\
\Gamma_{22} &= -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), \\
\Gamma_{23} &= \beta(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2), \\
\Gamma_{24} &= \beta\left(k_2B_{22}^s + k_1B_{12}^s - \left(k_1A^{'} + k_2B^{'}\right)B_{66}^s\alpha^2\right), \\
\Gamma_{25} &= -\beta X_{23} \\
\Gamma_{33} &= -\left(D_{11}\alpha^4 + 2\left(D_{12} + 2D_{66}\right)\alpha^2\beta^2 + D_{22}\beta^4\right) \\
&\quad - \bar{K}_W - K_P(\alpha^2 + \beta^2) \\
\Gamma_{34} &= -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2\left(k_1A^{'} + k_2B^{'}\right)D_{66}^s \\
&\quad \times \alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\
\Gamma_{44} &= -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - \left(k_1A^{'} + k_2B^{'}\right)^2 \\
&\quad \times H_{66}^s\alpha^2\beta^2 - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) \\
&\quad - \left(k_1A^{'}\right)^2 A_{55}^s\alpha^2 - \left(k_2B^{'}\right)^2 A_{44}^s\beta^2 \\
\Gamma_{45} &= k_1Y_{13}^s + k_2Y_{23}^s + \alpha^2k_1A'A_{55}^S + \beta^2k_2B'A_{44}^S \\
\Gamma_{55} &= \alpha^2A_{55}^S + \beta^2A_{44}^S + Z_{33}
\end{aligned} \tag{32a}$$

$$\begin{aligned}
& M_{11} = -I_0, \quad M_{13} = \alpha I_1, \quad M_{14} = -J_1k_1A^{'}\alpha, \\
& M_{22} = -I_0, \quad M_{23} = \beta I_1, \quad M_{24} = -J_1k_2B^{'}\beta, \\
& M_{33} = -I_0 - I_2\left(\alpha^2 + \beta^2\right) \qquad , \\
& M_{34} = J_2\left(A^{'}k_1\alpha^2 + B^{'}k_2\beta^2\right), \\
& M_{44} = -K_2\left(\left(A^{'}k_1\right)^2\alpha^2 + \left(B^{'}k_2\right)^2\beta^2\right) \\
& M_{55} = K_0, M_{12} = M_{15} = M_{25} = M_{45} = 0
\end{aligned} \tag{32b}$$

4. Numerical results

In this part of the research paper, the various present examples and validations are presented to verify the accuracy of the current method studying free vibration of the advanced composite plates resting on variables elastic foundations. We should be indicated that this analytical method is employed a quasi -3D shear deformation theory. The materials proprieties are assumed to be changes through the thickness of plates following the Eqs. (2), (4) and (5) and are given in Table 1.

Table 1 Material properties of the functionally graded plates

Material	Properties		
	E(GPa)	ν	$\rho(Kg/m^3)$
Aluminum (Al)	70	0.3	2702
Alumina (Al2O3)	380	0.3	3800
Zirconia (ZrO_2)	200	0.3	5700
Ti–6Al–4V	105.7	0.2981	4429
aluminum oxide	320.24	0.26	3750

The used non-dimensional parameters are:

$$D_c = \frac{E_c h^3}{12\left(1 - 2\nu^2\right)}, \; D_m = \frac{E_m h^3}{12\left(1 - 2\nu^2\right)}$$

$$J = w\left(a^2/h\right)\sqrt{\frac{r_m}{E_m}}, \; v = wa^2\sqrt{\frac{rh}{D_c}}, \; \bar{W} = wh\sqrt{\frac{r_m}{E_m}}, \; \bar{v} = wh\sqrt{\frac{r_c}{E_c}}$$

$$K_S = \frac{k_S D_i}{a^2}, \quad i = m, c$$

4.1 Validations

4.1.1 Example 1: Analysis of FGM plates

The functionally graded plates are made of Al/Al2O3 that has materials proprieties as shown in table (1). The plates have(a/b=1), (a/b=2) and (h/a =10), (h/a =5) and (h/a =2) and are used Eq. (6) to describe the variation of materials properties. The comparison of non-dimensional fundamental frequencies isgiven in the Table 2. The current method is compared with the 3D exact solution provided by Jin *et al.* (2014), the solution of Mantari (2015) where based quasi-3D shear deformation theory and analytical solution of Zaoui *et al.* (2019) based quasi-3D four variables higher shear deformation theory. It can be seen that present results are closure to preceding studies. It can be seen that, for a given relation (b/a) and an index (p), as the side-to-thickness ratio (a/h) decreases, the present results diverge with those of the literature.

4.1.2 Example 2: Analysis of advanced composite plates

The advanced composite plates are prepared from the aluminum oxide and Ti–6Al–4V. The distributions of material properties use Eq. (1) of the volume fraction and Eq. (2) are used for the Voigt model and Eq. (3) of the More-Tanaka model. The advanced composite square plate has a = b = 0.4 m and h = 5 mm. Table 3 presents the first six dimensional natural frequencies (Hz) for the two cases, i.e., volume fraction index p = 0 and 2000. The present

Table 2 Comparison of non-dimensional frequencies $\overline{w}$ for P-FG plates

b/a	a/h	p	(Jin *et al.* 2014)	(Mantari 2015)	(Zaoui *et al.* 2019)	Present
1	10	0	0.1135	0.1135	0.1137	0.11359
		1	0.0870	0.0882	0.0883	0.08824
		2	0.0789	0.0806	0.0807	0.08059
		5	0.0741	0.0755	0.0756	0.07562
	5	0	0.4169	0.4169	0.4178	0.41779
		1	0.3222	0.3261	0.3267	0.32670
		2	0.2905	0.2962	0.2968	0.29677
		5	0.2676	0.2722	0.2725	0.27249
	2	0	1.8470	1.8510	1.8583	1.85830
		1	1.4687	1.4778	1.483	1.48306
		2	1.3095	1.3223	1.3269	1.32688
		5	1.1450	1.1557	1.1576	1.15761
2	10	0	0.0719	0.0718	0.0719	0.07193
		1	0.0550	0.0557	0.0558	0.05581
		2	0.0499	0.0510	0.0511	0.05107
		5	0.0471	0.0479	0.048	0.04799
	5	0	0.2713	0.2713	0.2718	0.27180
		1	0.2088	0.2115	0.2119	0.21189
		2	0.1888	0.1926	0.193	0.19301
		5	0.1754	0.1786	0.1788	0.17879
	2	0	0.9570	1.3044	1.3086	1.30865
		1	0.7937	1.0348	1.0378	1.03785
		2	0.7149	0.9296	0.9322	0.93224
		5	0.6168	0.8241	0.825	0.82499

Table 3 Comparison of natural frequencies $f_{ij}=\omega/2\pi$ for FG plates with Mori-Tanaka's mixture

p	Source	f_{11}	$f_{12}=f_{21}$	f_{22}	$f_{13}=f_{31}$	$f_{32}=f_{32}$	$F_{14}=f_{41}$
0	(Bishop 1969)	145.04	362.61	580.18	725.22	942.79	1233
	(He *et al.* 2001)	144.66	360.53	569.89	720.57	919.74	1225.72
	(Kitipornchai *et al.* 2006)	143.96	360.07	568.87	718.22	916.4	1207.06
	(Park and Kim 2006)	145.06	362.41	579.39	724.62	–	–
	(Shen and Wang 2012)(M–T)	144.97	362.13	578.94	723.29	939.53	1227.31
	(Shen and Wang 2012)(Voigt)	144.97	362.13	578.94	723.29	939.53	1227.31
	Present (M–T model)	145.1143	362.4875	579.5042	723.9849	940.4112	1228.4323
	Present (Voigt model)	145.1143	362.4875	579.5042	723.9849	940.4112	1228.4323
2000	(Bishop 1969)	271.23	678.06	1084.9	1356.1	1763	2305.4
	(He *et al.* 2001)	268.92	669.4	1052.49	1338.52	1695.23	2280.95
	(Kitipornchai *et al.* 2006)	261.46	653.13	1044.31	1304.79	1694.98	2214.32
	(Park and Kim 2006)	274.23	685.18	1095.4	1369.98	–	–
	(Shen and Wang 2012)(M–T)	271.05	677.1	1082.53	1352.47	1756.87	2295.1
	(Shen and Wang 2012) (Voigt)	271.06	677.12	1082.57	1352.52	1756.93	2295.19

Table 3 Continued

p	Source	f_{11}	$f_{12}=f_{21}$	f_{22}	$f_{13}=f_{31}$	$f_{32}=f_{32}$	$F_{14}=f_{41}$
2000	Present (M–T model)	271.0842	677.1739	1082.6224	1352.5665	1756.9526	2295.1488
	Present (Voigt model)	271.1141	677.2487	1082.7419	1352.7158	1757.1463	2295.4015

Table 4 Comparison of the non-dimensional natural frequency parameters ϖ/π of isotropic plates resting on an elastic foundation

h/a	Kw	Methods	ϖ_{11}	ϖ_{12}	ϖ_{22}
0.001	100	Present	2.2439	5.1067	8.0721
		(Zhang *et al.* 2015)	2.2260	5.0860	8.0478
		(Zhou *et al.* 2004)	2.2413	5.0973	8.0527
		(Leissa 1973)	2.2420	5.1016	8.0639
		(Dehghan and Baradaran 2011)	2.2450	5.1643	8.1338
	500	Present	3.0235	5.4941	8.3226
		(Zhang *et al.* 2015)	3.0213	5.4832	8.2923
		(Zhou *et al.* 2004)	3.0214	5.4850	8.3035
		(Leissa 1973)	3.0221	5.4894	8.3146
		(Dehghan and Baradaran 2011)	3.0242	5.5474	8.3821
0.1	200	Present	2.3966	4.8290	7.2368
		(Zhang *et al.* 2015)	2.3791	4.7912	7.1637
		(Zhou *et al.* 2004)	2.3951	4.8262	7.2338
		(Dehghan and Baradaran 2011)	2.3903	4.8098	7.2186
	1000	Present	3.7047	5.5705	7.7376
		(Zhang *et al.* 2015)	3.7059	5.5510	7.7251
		(Zhou *et al.* 2004)	3.7008	5.5661	7.7335
		(Dehghan and Baradaran 2011)	3.6978	5.5521	7.7193

Table 5 Non-dimensional fundamental frequencies ϖ for isotropic plates resting on elastic foundation (a/h=5)

kw	*kp*	Frequencies	(Akavci 2014)	(Thai and Choi 2012)	(Mantari *et al.* 2014)	(Zaoui *et al.* 2019)	Present
0	0	ϖ_{11}	17.5149	17.4523	17.4537	17.5677	17.5332
10			17.7859	17.7248	17.7257	17.8261	17.7919
10^2			20.0603	20.0076	20.0084	19.9988	19.9677
10^3			35.5261	35.5039	35.5044	34.8113	34.7905
10^4			45.526	45.5255	45.526	45.526	45.5260
10^5			45.526	45.5255	45.526	45.526	45.5260
0	10		22.2607	22.2145	22.2154	22.1062	22.0774
10			22.4745	22.4286	22.4297	22.3111	22.2825
10^2			24.3133	24.2723	24.2731	24.0743	24.0473
10^3			38.0839	38.065	38.0651	37.2488	37.2290
10^4			45.526	45.5255	45.526	45.526	45.5260
10^5			45.526	45.5255	45.526	45.526	45.5260
0	0	ϖ_{12}	38.4722	38.1883	38.1966	38.6161	38.4856
10			38.5929	38.3098	38.3184	38.7262	38.5959
10^2			39.662	39.3895	39.3975	39.702	39.5737

Table 5 Continued

kw	*kp*	Frequencies	(Akavci 2014)	(Thai and Choi 2012)	(Mantari *et al.* 2014)	(Zaoui *et al.* 2019)	Present
10^3	0	ϖ_{12}	47.0757	48.8772	48.8829	48.28	48.1667
10^4			71.9829	71.9829	71.9829	71.9829	71.9829
10^5			71.983	71.9829	71.9829	71.9829	71.9829
0	10		44.0294	43.7943	43.8014	43.6871	43.5663
10			44.1347	43.9009	43.9075	43.7831	43.6625
10^2			45.0711	44.8445	44.8509	44.6367	44.5175
10^3			53.5296	53.358	53.363	52.3134	52.2062
10^4			71.9829	71.9829	71.9829	71.9829	71.9829
10^5			71.9829	71.9829	71.9829	71.9829	71.9829
0	0	ϖ_{13}	66.1207	65.3135	65.3447	66.3436	66.0105
10			66.1899	65.3841	65.415	66.4043	66.0713
10^2			66.8087	66.0138	66.0445	66.9471	66.6150
10^3			72.6997	72.0036	72.0298	72.0778	71.7543
10^4			101.799	101.799	101.7992	101.7992	101.7992
10^5			101.799	101.799	101.7992	101.7992	101.7992
0	10		72.6178	71.9198	71.9467	72.007	71.6834
10			72.6806	71.9839	72.0104	72.0613	71.7378
10^2			73.243	72.5554	72.5812	72.5474	72.2248
10^3			78.6389	78.029	78.0519	77.1736	76.8599
10^4			101.799	101.799	101.7992	101.799	101.7992
10^5			101.799	101.799	101.7992	101.799	101.7992

Table 6 Comparison of the non-dimensional natural frequency $\hat{W}$ parameters of P-FG plates resting on an elastic foundation

(Kw,Kp)	a/h	p	Theories				
			(Hasani Baferani *et al.* 2011)	(Hosseini-Hashemi *et al.* 2010)	(Akavci 2014)	(Mantari 2015)	Present
(0,0)	20	0	–	0.02392	0.02393	0.02393	0.02395
		0.25	–	0.02269	0.02309	0.02293	0.02314
		1	–	0.02156	0.02202	0.02218	0.02219
		5	–	0.0218	0.02244	0.0226	0.02262
	10	0	–	0.09188	0.09203	0.09207	0.09213
		0.25	–	0.08603	0.08895	0.08888	0.08916
		1	–	0.08155	0.08489	0.0855	0.08533
		5	–	0.08171	0.08576	0.08639	0.08645
	5	0	–	0.32284	0.32471	0.32504	0.32498
		0.25	–	0.31003	0.31531	0.31424	0.31594
		1	–	0.29399	0.30152	0.30354	0.30349
		5	–	0.29099	0.3186	0.29987	0.30001
(250,25)	20	0	0.03421	0.03421	0.03422	0.03417	0.03419
		0.25	0.03321	0.03285	0.03312	0.03296	0.03311
		1	0.03249	0.03184	0.03213	0.0322	0.03221
		5	0.03314	0.03235	0.03277	0.03283	0.03285
	10	0	0.13365	0.13365	0.13375	0.13302	0.13307
		0.25	0.13004	0.12771	0.12959	0.1288	0.12900

Table 6 Comparison of the non-dimensional natural frequency $\hat{W}$ parameters of P-FG plates resting on an elastic foundation

(Kw,Kp)	a/h	p	Theories				
			(Hasani Baferani *et al.* 2011)	(Hosseini-Hashemi *et al.* 2010)	(Akavci 2014)	(Mantari 2015)	Present
(250,25)	10	1	0.12749	0.12381	0.12585	0.12557	0.12635
		5	0.1295	0.12533	0.12778	0.12756	0.12761
	5	0	0.43246	0.49945	0.50044	0.48949	0.48956
		0.25	0.42868	0.48327	0.48594	0.47498	0.47545
		1	0.46406	0.46997	0.47298	0.46405	0.46412
		5	0.44824	0.474	0.47637	0.46836	0.46862

results are compared with previous results that are reported in Table 3. The classical analytical solutions using on the classical plate theory (CPT) of Bishop (1969), the linear FEM results employing on the CPT of He *et al.* (2001), the semi-numerical results based on the higher order shear deformation plate theory (HSDT) of Kitipornchai *et al.* (2006), and the nonlinear FEM results based on the first shear deformation plate theory (FSDT) of Park and Kim (2006), the solutions of Shen and Wang (2012) based on a higher order shear deformation theory are listed of comparison. It can be observed that current Voigt and Mori–Tanaka models are similar for an isotropic plate (p = 0), but there is a small difference in (p=2000). It is clear that the results of the two models are almost similar to the studies of Shen and Wang (2012) and those of Bishop (1969), but they are further from the results given by Kitipornchai *et al.* (2006), Park and Kim (2006) and He *et al.* (2001).

4.1.3 Example 3: Analysis of homogenous plates resting on elastic foundations

The homogenous isotropic plates are made by aluminum (Al). The square plates have a/h=1000, 10 sides to thickness ratio. The Winkler foundation parameters are defined as $K_W = K_W D_C / 2\pi a^4$. In Table 4, the non-dimensional frequency parameters of thin and moderately thick plates resting on an elastic foundation are carried out with different values of Winkler modulus. The results are reported in Table 4 together with the vibration solutions reported by Leissa (1973), Zhou *et al.* (2004), Dehghan and Baradaran (2011) and Zhang *et al.* (2015). It can be seen that the current results are in agreement with the other results, with slight differences, since the theories used in the methods of the other works are not similar.

In another comparison, Table 5 shows the results of the three non-dimensional natural frequencies of a square homogenous thick plate on two-parameter elastic foundation with side to thickness ratio (a/h=5). The current results are compared with the shear deformation theory given by Thai and Choi (2012) and a Higher shear deformation theory proposed by Akavci (2014), quasi-3D shear deformation theory presented by Zaoui *et al.* (2019) and the results presented by Mantari *et al.* (2014). The results are closer to the solutions obtained by Zaoui *et al.*

Table 7 Comparison of the non-dimensional natural frequency parameters *J* of non-homogenous plates resting on an elastic foundation

a/h	*Kw*	*Kp*	*p* = 0		*p* = 1		*p* =5		*p* = 8		*p* = 10	
			Voigt	M-T	Voigt	M-T	Voigt	M-T	Voigt	M-T	Voigt	M-T
10	0	0	5.7784	5.7784	8.2044	7.9947	9.5336	9.3443	9.8620	9.7037	10.0002	9.8591
		10	7.1377	7.1377	9.2920	9.1075	10.5374	10.3665	10.8449	10.7012	10.9741	10.8458
		100	14.4484	14.4484	16.0463	15.9407	17.0959	16.9914	17.3401	17.2508	17.4408	17.3605
	10	0	5.8548	5.8548	8.2630	8.0548	9.5870	9.3987	9.9142	9.7567	10.0518	9.9115
		10	7.1997	7.1997	9.3437	9.1603	10.5857	10.4156	10.8923	10.7492	11.0211	10.8934
		100	14.4791	14.4791	16.0762	15.9709	17.1257	17.0213	17.3698	17.2806	17.4704	17.3903
	100	0	6.5026	6.5026	8.7723	8.5765	10.0547	9.8754	10.3716	10.2212	10.5049	10.3707
		10	7.7355	7.7355	9.7970	9.6222	11.0111	10.8477	11.3102	11.1725	11.4358	11.3128
		100	14.7522	14.7522	16.3435	16.2399	17.3916	17.2888	17.6346	17.5468	17.7347	17.6558
20	0	0	5.9241	5.9241	8.4010	8.1908	9.7431	9.5470	10.0818	9.9167	10.2251	10.0777
		10	7.2811	7.2811	9.4862	9.3007	10.7461	10.5686	11.0633	10.9131	11.1975	11.0630
		100	14.6385	14.6385	16.2693	16.1620	17.3331	17.2237	17.5838	17.4897	17.6876	17.6029
	10	0	6.0002	6.0002	8.4593	8.2507	9.7964	9.6013	10.1338	9.9695	10.2766	10.1299
		10	7.3432	7.3432	9.5379	9.3534	10.7944	10.6178	11.1108	10.9612	11.2445	11.1106
		100	14.6695	14.6695	16.2995	16.1924	17.3631	17.2539	17.6136	17.5198	17.7174	17.6329
	100	0	6.6463	6.6463	8.9672	8.7707	10.2635	10.0775	10.5904	10.4334	10.7288	10.5884
		10	7.8799	7.8799	9.9911	381.5676	11.2201	11.0502	11.5288	11.3847	11.6592	11.5301
		100	14.9453	14.9453	16.5688	16.4634	17.6309	17.5234	17.8802	17.7878	17.9835	17.9002

(2019). It is clear that as the value of (K_w and k_p) increases, the current solution and the results of other theories tend to approach a similar value.

4.1.4 Example 4: Analysis of FG plates resting on elastic foundations

The advanced composite plates are made by FGM of Aluminum (Al) / Zirconia (ZrO2) and Eqs. (6) are used to describe the variation of materials proprieties. The square plates has uniform thickness with different values (a/h=20, 10 and 5). In Table 6, dimensionless fundamental frequencies of FGM plates resting on elastic foundations are presented. The present results are compared with the first shear deformation theory results by Hosseini-Hashemi *et al.* (2010), and the higher shear deformation theory by Hasani Baferani *et al.* (2011), the HSDT by Akavci (2014) and the solution of Mantari (2015) based qausi-3D higher shear theory. It can be seen that the results of the present theory are closer to the pervious results reported in Table 6.

4.2 Parametric study

4.2.1 Example 1: comparison of two advanced composite models plates resting on elastic foundations

In the second part, advanced composite plates made of the aluminum oxide and Ti–6Al–4Vare used. Voigt model and Mori-Tanaka model based on the volume fraction of metal are used. The square plates are considered to be resting two parameters Winkler (Kw)-Pasternak (Kp) with different for Kw and Kp. It has two sides to thickness ratio (a/h=10,20). In Table 7, non-dimensional natural frequency

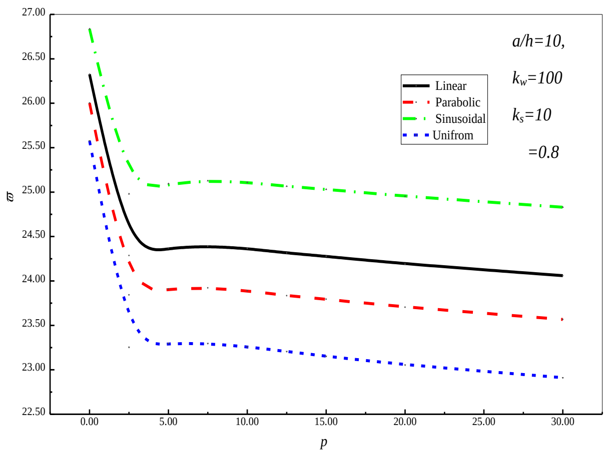

Fig. 3 Influence of the different types of elastic foundation on non-dimensional frequencies *V* of advanced square plates versus index power

parameters *J* of non-homogenous plates resting on a two-parameter elastic foundation with various values of index power (p=0,1,5,8 and 10), is presented. It can be seen that the results for homogenous plates (p=0) are given identical results, however for other differences are not significant. The results of non-dimensional natural frequency between Mori-Tanaka model and Voigt model converge for isotropic plates with p=0 and p= ∞. In addition, the non-dimensional frequency of the plate increase with increase of elastic foundation effect.

4.2.2 Example 2: Analysis of advanced composite plates resting on variables elastic foundations

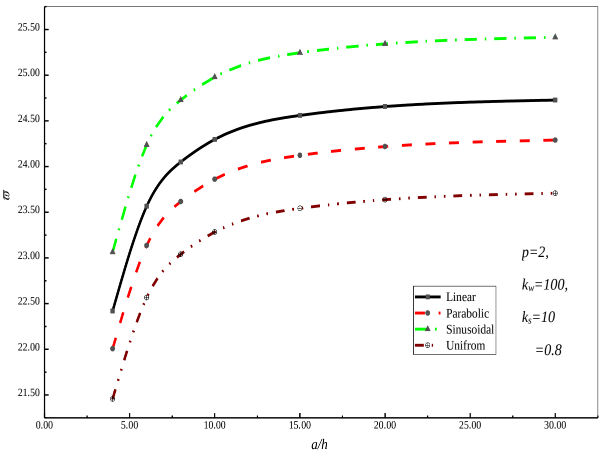

Fig. 4 Influence of the different types of elastic foundation on non-dimensional frequencies *V* of advanced square plates versus a/h ratio

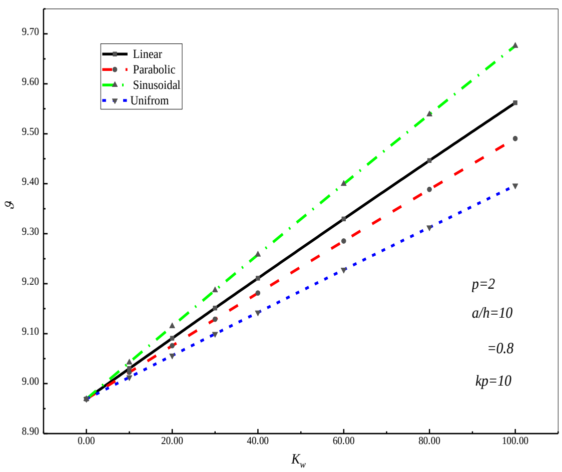

Fig. 7 Influence of the different types of elastic foundation on non-dimensional frequencies *J* of advanced square plates

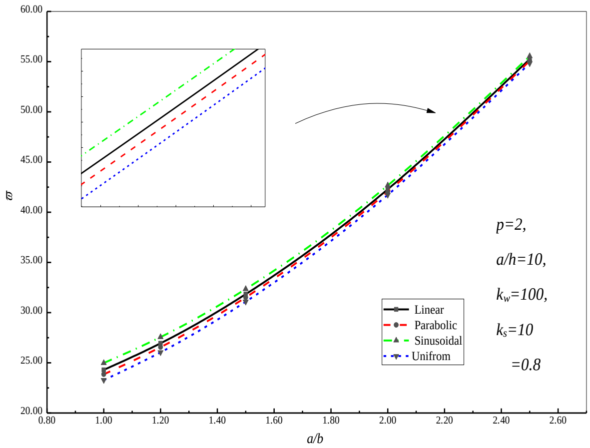

Fig. 5 Influence of the different types of elastic foundation on non-dimensional frequencies *V* of advanced plates versus a/b ratio

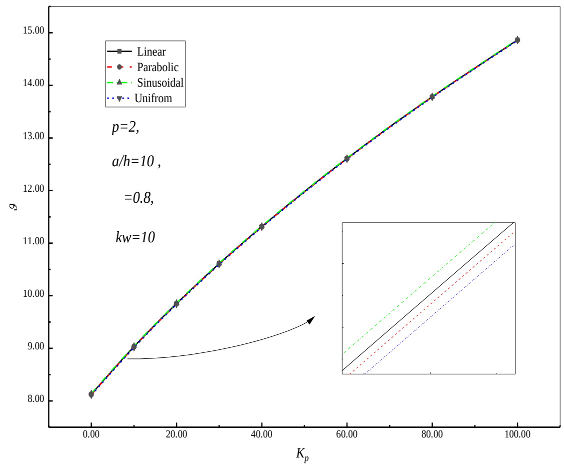

Fig. 8 Influence of the different types of elastic foundation on non-dimensional frequencies *J* of advanced square plates

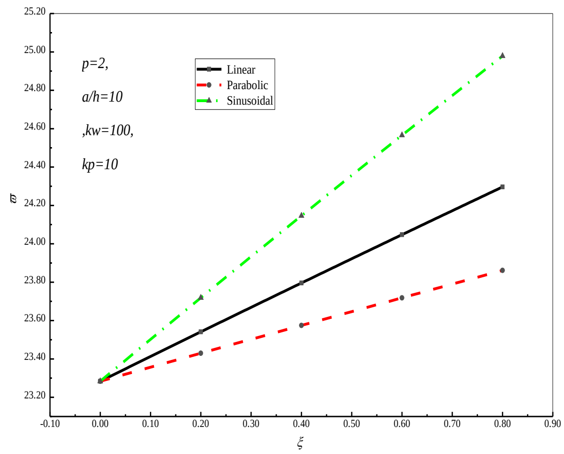

Fig. 6 Influence of the different types of elastic foundation on non-dimensional frequencies *V* of advanced plates versus *z* coefficient

The second example, it is functionally graded plates prepared from Al/Al_2O_3 where the Voigt model is used to describe the material distribution according to the thickness direction. Figs. 3, 4 and 5, show the variation of non-dimensional frequencies of advanced plates sitting on different types (linear, parabolic, sinusoidal and uniform) of elastic foundation with some parameters like index power (p), side-to-thickness ratio (a/h) and side-to-length (a/b). In Fig. 3, it can be seen that non-dimensional frequencies are decreased with increased values of index material p. In Figs. 4 and 5, non-dimensional frequencies are increased with increased of (a/h) and (a/b) ratios. Fig. 6 shows the influence of ξ coefficient with non-dimensional frequencies of advanced square plates. It can be observed that frequency increased with an increase of ξ coefficient. The effect of Winkler (Kw) and Pasternak (Kp) parameters on non-dimensional frequencies of advanced composite square plates are presented in Figs. 7 and 8 respectively. It is clear those current results are increased with an increase in the parameter of the foundation. In all Figs. 3-8, we can easily classify the effect types variables elastic foundation from grated to down in this order uniform, parabolic, linear and sinusoidal. It can be concluded that the present method can predict the effect variables elastic foundation on

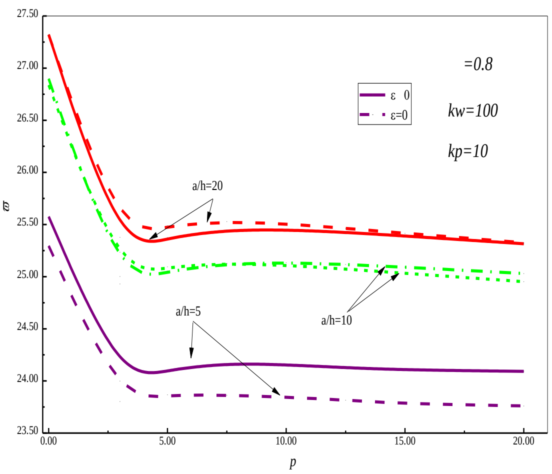

Fig. 9 Comparison of non-dimensional frequencies *V* of advanced square plates on variable elastic foundations versus a/h ratios

Table 8 Comparison of dimensional frequencies *f*=*w*/2*p* of advanced square plates with different mass density versus a/h ratios, (*z*=0.8, kw=100 and kp=10)

	Material properties ρ1=1500 kg/m^3		Material properties ρ2=3500 kg/m^3	
a/h	e_z=0	e_z^1=0	e_z=0	e_z^1=0
4	1984.487465	1948.932440	1299.15200	1275.875777
6	1387.268698	1377.060008	908.1805453	901.4973889
8	1062.579939	1058.475291	695.6218581	692.9347351
10	859.3066164	857.3576419	562.5482309	561.2723271
15	579.4788533	579.0712392	379.3579584	379.0911125
20	436.4464187	436.3776090	285.7212500	285.6762034
30	291.8626891	291.9303145	191.0689808	191.1132522

Table 9 Comparison of dimensional frequencies *f*=*w*/2*p* of advanced square plates with different young modulus versus a/h ratios (*z*=0.8, kw=100 and kp=10)

	Material properties E=150 Gpa		Material properties E=66 Gpa	
a/h	e_z=0	e_z^1=0	e_z=0	e_z^1=0
4	1958.5453	1923.4551	1299.1520	1275.8757
6	1369.1336	1359.0584	908.1805	901.4973
8	1048.6894	1044.6384	695.6218	692.9347
10	848.07336	846.1498	562.5482	561.2723
15	571.9036	571.5013	379.3579	379.09111
20	430.74099	430.6730	285.7212	285.6762
30	288.0473	288.1140	191.0689	191.1132

vibrational of advanced plates.

4.2.3 Example 3: effect of the normal deformation on the plates resting on elastic foundations

In order to verify the effect of the normal deformation along the z direction, we carried out a comparative study of evolution of the frequency according to the power index of FG plate resting variables elastic foundation (parabolic variation type) with and without stretching effect for different values of (a/h). The results of this parametric study are presented in Fig. 9. It should be noted that for all the values of p the effect of the deformation becomes significant when the plate becomes thick.

It should be noted that the effect of the variation of the density and the young modulus cannot be seen in Fig. 9; so another purely theoretical study was done to show the effect of density and Young modulus on the stretching effect.

To study the effect of the normal deformation according to z with respect to the density we took two materials with a constant young modulus of 66 Gpa the densities are ρ1= 1500 kg/m^3 and ρ2= 3500 kg/m^3. The results of this study are grouped in Table 8.

Even with the change in density the rate of change in frequency for thin plates is minimal, while for thick plates this rate becomes more significant, since the difference between the results of frequencies with and without stretching effect is greater when the density is small.

To study the effect of the normal deformation along z with respect to the Young E module, we took two materials with a constant density of ρ= 3500 kg/m^3, the young modules are E1 = 150 Gpa and E2 = 66 Gpa. The results of this study are grouped in Table 9.

Even with the change of the module of young the rate of change of the frequency for the thin plates is almost zero, while for the thick plates this rate becomes more significant, since the difference between the results of the frequencies with and without stretching effect is greater when the module of young is large.

5. Conclusions

Free vibration of advanced composite plates resting variables elastic foundations are investigated using new quasi-3D higher shear deformation theory with success. The present qausi-3D HSDT has only five unknowns without needing shear corrector coefficient, which means it's better than others similar qausi-3D HSDTs found in literature. Consequently, the present qausi-3D HSDT was allowed to reduce time of calculates. The advanced composite plates are supposed to resting on tow parameter elastic foundation (Pasternak- Winkler). We supposed to have variation in Winkler modulus, however Pasternak modulus are to be constant. Hamilton principle are employed to derived the equations motions for dynamics of advanced composite plates resting variables elastic foundations. The effect variables elastic foundation, power index, thickness-to-side, length to side are present in part of numerical results. Finally, it can be conclude that the present methods are efficient to predict the effect of elastic foundation on vibrational analysis of advanced composite plates. An improvement of the current analytical formulation will be considered in the future work to consider other type of structures and materials (Hirwani *et al.* 2017c, Narwariya *et al.* 2018, Ayat *et al.* 2018, Behera and Kumari 2018, Jamali *et al.* 2019, Hussain *et al.* 2019, Alimirzaei *et al.* 2019, Medani *et al.* 2019, Draiche *et al.* 2019, Al-Furjan *et al.* 2020, Khosravi *et al.* 2020, Bourada *et al.* 2020, Shariati *et al.* 2020, Bousahla *et al.* 2020, Bellal *et al.* 2020, Asghar *et al.* 2020, Mehar *et al.* 2020, Dewangan *et al.* 2020).

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