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2D and quasi 3D computational models for thermoelastic bending of FG beams on variable elastic foundation: Effect of the micromechanical models

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Abstract. This paper is concerned with the thermoelastic bending of FG beams resting on two-layer elastic foundations. One of these layers is Winkler springs with a variable modulus while the other is considered as a shear layer with a constant modulus. The beams are considered simply supported and subjected to thermo-mechanical loading. Temperature-dependent material properties are considered for the FG beams, which are assumed to be graded continuously across the panel thickness. The used theories contain undetermined integral terms which lead to a reduction of unknowns functions. Several micromechanical models are used to estimate the effective two-phase FG material properties as a function of the particles' volume fraction considering thermal effects. Analytical solutions for the thermo-mechanical bending analysis are obtained based on Navier's method that satisfies the boundary conditions. Finally, the numerical results are provided to reveal the effect of explicit micromechanical models, geometric parameters, temperature distribution and elastic foundation parameters on the thermoelastic response of FG beams.

Keywords: FG beams; 2D theory; quasi 3D theory; undetermined integral forms; variable elastic foundation; thermoelastic; micromechanical models; temperature-dependent material

1. Introduction

In the last years, functionally graded materials (FGM) as a new class of composite materials has attracted the intention in different sectors like engineering, aerospace and other industries due to the numerous advantages offered and their benefits of possessing supreme mechanical properties (Kar and Panda 2015ab and 2016, Kolahchi *et al.* 2016, Mahapatra *et al.* 2017, Dash *et al.* 2019, Barati 2017, Kar and Panda 2020, Khiloun *et al.* 2020, Kaddari *et al.* 2020, Hussain *et al.* 2020a). Beams have been widely used in different important systems and devices. Functionally graded materials (FGMs) are used in beam forms (Gul *et al.* 2019, Tounsi *et al.* 2019, Ahmed *et al.* 2019, Gafour *et al.* 2020, Al-Maliki *et al.* 2020, Rachedi *et al.* 2020). However, FG beams are often exposed to mechanical, thermal and electrical loads during their operational life. Therefore, considering mechanic and thermal behavior of FG beams under different configurations is very important.

The behavior of FG structures subjected to mechanical and thermal loads with or without considering the interaction between structure–foundation have been studied by many researchers and numerous papers have been presented (Balubaid *et al.* 2019, Batou *et al.* 2019, Zine *et al.* 2020, Rabhi *et al.* 2020, Chikr *et al.* 2020, Refrafi *et al.* 2020, Tounsi *et al.* 2020, Rahmani *et al.* 2020).

Fallah and Aghdam (2012) have used Euler-Bernoulli theory together with von Kármán's assumptions to study the thermo-mechanical buckling and large amplitude free vibration analysis of FG beams on nonlinear elastic foundation. Wang and Wu (2016) used the classical beam theory (CBT) and the Timoshenko beam theory (TBT) to analyze the dynamic response of an axially functionally graded (AFG) beam under thermal environment and subjected to a moving harmonic load. Avcar and Mohammed (2018) examined the free vibration of FG beams resting on elastic foundation using the CBT. Yas *et al.* (2017) analyze the free vibration of FG beams on variable elastic foundation by using Euler-Bernoulli theory and by means of Generalized Differential Quadrature (GDQ) method.

Nguyen *et al.* (2013) used the first-order shear deformation beam theory (FSDBT) for static and free vibration of axially loaded rectangular FG beams. Pradhan and Chakraverty (2013) presented an investigation for the free vibration analysis of FG beams subjected to different sets of boundary conditions. The analysis is based on the CBT and FSDBT. Sun *et al.* (2016) studied the thermomechanical buckling and post-buckling deformations of a FG Timoshenko beam on nonlinear elastic foundation and subjected to only a temperature rise. Esfahani *et al.* (2013) studied nonlinear thermal buckling of temperature dependent FG Timoshenko beams on non-linear hardening elastic foundations with general boundary conditions. Dynamic buckling and imperfection sensitivity of the FG Timoshenko beam resting over a conventional three-parameter elastic foundation and subjected to sudden

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uniform temperature rise has been studied by Ghiasian *et al.* (2015).

Based on the third-order shear deformation beam theory (TSDBT), Trinh *et al.* (2016) presented an analytical method for vibration and buckling analysis for FG beams subjected to thermo-mechanical loads. In the same framework, Wattanasakulpong *et al.* (2011) employed an improved TSDBT to study the free vibration and the thermal buckling of the FG beams. Also, The TSDBT was used by Zahedinejad (2015) for the free vibration of FG beams with various boundary conditions resting on a two-parameter elastic foundation in the thermal environment.

Based on the high order shear deformation beam theory (HSDBT) and the concept of physical neutral surface, Zhang (2014) analyzed the thermal post-buckling and nonlinear vibration behaviors of FG beams with temperature dependent materials. Shen and Wang (2014) studied the nonlinear bending and thermal postbuckling of FG beams on elastic foundation in thermal environments using the HSDBT. They consider two kinds of micromechanics models, Voigt and Mori-Tanaka. Ebrahimi and Jafari (2018) proposed an analytical solution based on a four-variable shear deformation refined beam theory to study the thermo-mechanical vibration characteristics of porous FG beams subjected to several kinds of thermal loadings. Frikha *et al.* (2016) developed a new finite element for FG beams based on the HSDT.

Mantari and Yarasca (2015) presented a quasi-3D hybrid theory with 4-unknown for the bending analysis of FG beam. Using a quasi 3D theory, Vo *et al.* (2015) presented a finite element model for free vibration and buckling analyses of FG sandwich beams. Karamanlı (2017) analyze the static behaviour of two-directional FG sandwich beams subjected to various sets of boundary conditions by using a quasi-3D shear deformation theory. Chen *et al.* (2019) used the 3D theory in conjunction with the isogeometric analysis (IGA) to investigate the vibration of the FG beams. Fahsi et al (2019) presented a refined quasi-3D shear deformation theory for bending, buckling, and free vibration analyses of a FG porous beam on an elastic foundation.

Kumar and Devi (2017) investigate the thermoelastic FG beam in a modified couple stress theory subjected to a dual-phase-lag model.

Tounsi and his co-workers have presented very interesting research on the behavior of FG beams using several beam theories (Chaabane *et al.* 2019, Bourada *et al.* 2019 and 2020, Arioui *et al.* 2018, Kaci *et al.* 2018, Matouk *et al.* 2020, Bousahla *et al.* 2020).

Most of the research cited above uses a rule of mixture called Voigt model to evaluate the effective material properties of FG element. A suitable micromechanical model should be applied to accurately estimate the effective multiphysical properties (Nemati *et al.* (2019)).

Yahiaoui *et al.* (2018) have investigated the role of the micromechanical models on the bending, buckling and free vibration of FG sandwich beams on elastic foundation. They have used a refined quasi-3D solution. Mahmoudi *et al.* (2018) presented a 2D solution for free vibration of FG plate on elastic foundation. The influence of the several micromechanicals on the fundamental frequencies has been studied. In the same framework, Bachir Bouiadjra *et al.* (2018) studied the impact of the micromechanical models on the bending of FG plates using a refined 3D theory.

Most of the research work in the literature studies plates, beams or FG elements on elastic foundations with constant moduli. However, those who deal FG structures resting on variable elastic foundation are really limited. We cite as an example the works of (Eisenberger and Clastornik 1987, Zhou 1993, Pradhan and Murmu 2009, Sobhy 2015, Attia *et al.* 2018, Al-Furjan *et al.* 2020, Shariati *et al.* 2020ab).

According to authors' best knowledge and the literature search there is no report investigating on the thermoelastic bending of FG beams resting on variable elastic foundation with consideration of the micromechanical models effect. Therefore, the aim of present study is to investigate the thermo-mechanical bending of temperature-dependent FG beams resting on variable elastic foundation using two different theories (2D and quasi-3D shear deformation theories).

Undetermined integral terms are employed in the used displacement field in which the normal stresses effects are considered in the quasi-3D shear deformation theory and omitted in the 2D theory. The mechanical characteristics of the beams that vary through the thickness direction are evaluated using several micromechanical models. The effects of these models, the elastic foundation parameters and the thermo-mechanical loading on the response of FG beams will be analyzed and discussed through a detailed parametric study.

2. Effective properties of FGMs

2.1 Temperature-dependent materials

FGMs are composite materials most often made of ceramic and metal. Since they are used in high temperature environments, the constituents of FGM may possess temperature-dependent properties (Reddy and Chin 1998). Therefore, the properties including Young's modulus E, thermal expansion α and thermal conductivity k are assumed to be temperature-dependent and are expressed as function of temperature (Attia *et al.* 2018, Nemati and Mahmoodabadi 2019).

$$P_f(T,z) = P_0\left(P_{-1}T(z)^{-1} + 1 + P_1 T(z) + P_2 T(z)^2 + P_3 T(z)^3\right) \quad (1)$$

P_{-1}, P_0, P_1, P_2 and P_3 are the coefficients of temperature T expressed in Kelvin and are unique to the constituent materials. ΔT is rise temperature through the thickness direction. $P_f(T,z)$ it is an effective property. In our case, it can be either metal or ceramic. The values of each of the coefficients appearing in the preceding equation are listed in Table 1.

2.2 Micromechanical models

Unlike traditional microstructures, in FGMs the material properties are spatially varying, which is not trivial for a micromechanics model (Jaesang and Addis 2014).

A number of micromechanics models have been proposed for the determination of effective properties of

FGMs.

In this work, various micromechanical models such the Voigt, Reuss, LRVE, Tamura, Mori-Tanaka and Halpin-Tsai models are used to determine the effective material properties of the FG beams.

2.2.1 Voigt model

The Voigt model is relatively simple; this model is frequently used in most FGM analyses estimates properties of FGMs as (Mishnaevsky 2007, Zimmerman 1994):

$$P(T,z)=P_c(T,z)\,V(z)+P_m(T,z)\big(1-V(z)\big) \tag{2}$$

2.2.2 Reuss model

Reuss assumed the stress uniformity through the material and obtained the effective properties as (Mishnaevsky 2007, Zimmerman 1994):

$$P(T,z)=\frac{P_c(T,z)\,P_m(T,z)}{P_c(T,z)\big(1-V(z)\big)+P_m(T,z)V(z)} \tag{3}$$

2.2.3 Tamura model

The Tamura model uses actually a linear rule of mixtures, introducing one empirical fitting parameter known as "stress-to-strain transfer" (Gasik 1995, Zuiker 1995)

$$q=\frac{\sigma_1-\sigma_2}{\varepsilon_1-\varepsilon_2} \tag{4}$$

Estimate for *q=0* correspond to Reuss rule and with $q=\pm\infty$ to the Voigt rule, being invariant to the consideration of with phase is matrix and which is particulate. The effective property is found as:

$$P(T,z)=\frac{(1-V(z))\,P_m(T,z)\big(q-P_c(T,z)\big)+V(z)\,P_c(T,z)\big(q-P_m(T,z)\big)}{(1-V(z))\big(q-P_c(T,z)\big)+V(z)\big(q-P_m(T,z)\big)} \tag{5}$$

2.2.4 Description by a representative volume element (LRVE)

The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures. The effective property is expressed as follows by the LRVE method (Akbarzadeh *et al.* 2015):

$$P(T,z)=P_m(T,z)\left(1+\frac{V(z)}{FE-\sqrt[3]{V(z)}}\right)\quad FE=\frac{1}{1-\dfrac{P_m(T,z)}{P_c(T,z)}} \tag{6}$$

2.2.5 Mori-Tanaka model

According to Mori–Tanaka homogenization scheme, the effective Bulk Modulus (K) and the effective shear modulus (G) are given by Mori and Tanaka (1973):

$$P(T,z)=P_m(T,z)+\big(P_c(T,z)-P_m(T,z)\big)\times \left(\frac{V_C}{1+(1-V_C)\big(P_c(T,z)/P_c(T,z)-1\big)(1+\upsilon)/(3-3\upsilon)}\right) \tag{7a}$$

where

$$V_C=\left(0,5+\frac{z}{h}\right)^p \tag{7b}$$

2.2.6 Halpin-Tsai (H-T) model

The Halpin-Tsai model is a mathematical scheme for prediction of composite materials' elasticity based on the geometry of inclusions and the elastic properties of both matrix and inclusions (Halpin 1969, Nemati and Mahmoodabadi 2019). The estimate of Young's modulus by this model is given by:

$$E(T,z)=\frac{E_m(T,z)\big(1+2s\,q(T,z)V(z)\big)}{1-q(T,z)V(z)} \tag{8a}$$

where

$$q(T,z)=\frac{\dfrac{E_c(T,z)}{E_m(T,z)}-1}{\dfrac{E_c(T,z)}{E_m(T,z)}+2s} \tag{8b}$$

and "*s*" is the aspect ratio of inclusions or particles (*s* = 1 for solid spheres).

In all models outlined above, the subscripts *c* and *m* refer to the ceramic and metal respectively and $P(T,z)$ it's a property that can be, Young's modulus *E*, thermal expansion *α* or thermal conductivity *k* of the FG beams.

The volume fractions of the ceramic and metal phases are related by $V_C+V_m=1$, and V_C is expressed as:

$$V_C=\left(0,5+\frac{z}{h}\right)^p \quad p\geq 0 \tag{9}$$

3. Theoretical developments

Consider a FG beam of thickness *h* and length *L* as shown by the Fig. 1. The beam is assumed to rest on a Winkler-Pasternak elastic foundation. The mechanical characteristics of the beam are assumed to be varied across the thickness.

3.1 Kinematics

The displacement field of the conventional HSDT is given by:

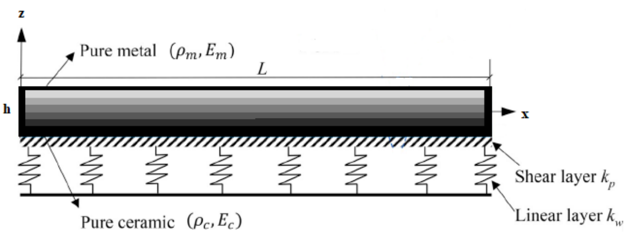

Fig. 1 Coordinate system and geometry for FG beams on elastic foundation

$$u\left(x,z\right)=u_0(x)-z\frac{\partial w_0}{\partial x}+f\left(z\right)\phi_x(x)$$
$$w\left(x,z\right)=w_0(x)+n.g(z)\theta(x) \tag{10}$$

"n" is a coefficient that equal $\begin{cases} n=0 & for\ 2D \\ n=1 & for\ 3D \end{cases}$

$u_0, w_0, \theta_x, \phi_x$ are the four unknown displacement of the mid-plane of the beam. By considering that $\phi_x=\int\theta(x)dx$ and and taking into account the stretching effect, we will have

$$u(x,z)=u_0(x)-z\frac{\partial w_0}{\partial x}+k_1\ f(z)\int\theta(x)dx$$
$$w(x,z)=w_0(x)+n.g(z)\theta(x) \tag{11a}$$

where

$$k_1=-\alpha^2 \tag{11b}$$

The new shape function $f(z)$ is given as follows:

$$f(z)=z[1-\frac{4}{3}(\frac{z}{h})^2] \tag{12a}$$

and

$$\begin{cases} g(z)=\dfrac{2}{15}\dfrac{df(z)}{dz} & for\quad 3D \\ g(z)=\dfrac{df(z)}{dz} & for\quad 2D \end{cases} \tag{12b}$$

The kinematic relations can be obtained as follows:

$$\varepsilon_x=\varepsilon_x^0+zk_x^b+f(z)k_x^s\ ,$$
$$\{\gamma_{xz}\}=f'(z)\{\gamma_{xz}^0\}+g(z)\{\gamma_{xz}^1\}\ ,\quad \varepsilon_z=g'(z)\ \varepsilon_z^0 \tag{13a}$$

where

$$\{\varepsilon_x^0\}=\left\{\frac{\partial u_0}{\partial x}\right\},\quad \begin{Bmatrix} k_x^b \\ k_x^s \end{Bmatrix}=\begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ k_1\theta \end{Bmatrix}, \tag{13b}$$

$$\{\gamma_{xz}^0\}=\left\{k_1\int\theta\ dx\right\},\quad \{\gamma_{xz}^1\}=\left\{\frac{\partial\theta}{\partial x}\right\}\quad \varepsilon_z^0=\theta \tag{13c}$$

The integrals used in the above equations shall be resolved by a Navier type method and can be given as follows:

$$\int\theta\ dx=A'\frac{\partial\theta}{\partial x}$$

where the coefficient A' is expressed according to the type of solution used, in this case via Navier. Therefore, A' and k_1 are expressed as follows:

$$A'=-\frac{1}{\alpha^2},\ k_1=\alpha^2, \tag{14}$$

3.2 Constitutive relations

The linear constitutive relations of a FG beam can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix}=\begin{bmatrix} C_{11} & C_{13} & 0 \\ C_{13} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{bmatrix}\begin{Bmatrix} \varepsilon_x-\alpha\Delta T \\ \varepsilon_z-\alpha\Delta T \\ \gamma_{xz} \end{Bmatrix} \tag{15}$$

The $C_{ij}(i,j$=1,3,5.) expressions in terms of engineering constants are depends on the normal strain ε_z

- If the 2D shear deformation is used, the ε_z=0 , then C_{ij} are:

$$C_{ii}=\frac{E(z,T)}{1-\nu?\ z,T)},\quad (i=1,3). \tag{16a}$$

$$C_{ij}=\frac{E(z,T)\nu(z,T)}{1-\nu?\ z,T)},\quad (i,j=1,3). \tag{16b}$$

$$C_{ii}=\frac{E(z,T)}{2(1+\nu(z,T))}\quad (i=5). \tag{16c}$$

- If the 3D shear deformation is used, the $\varepsilon_z\neq0$, then C_{ij} are:

$$C_{ii}=\frac{\left(1-\nu(z,T)\right)E(z,T)}{\left(1-2\nu(z,T)\right)\left(1+\nu(z,T)\right)},\quad (i=1,3). \tag{17a}$$

$$C_{ij}=\frac{E(z,T)\nu(z,T)}{\left(1-2\nu(z,T)\right)\left(1+\nu(z,T)\right)},\quad (i,j=1,3). \tag{17b}$$

$$C_{ii}=\frac{E(z,T)}{2(1+\nu(z,T))}\quad (i=5). \tag{17c}$$

The beam is assumed to rest on two-parameter elastic foundation model, which consists of closely spaced springs interconnected through a shear layer made of incompressible vertical elements, which deform only by transverse shear. The response equation of this foundation is given:

$$R(x,y)=\overline{K}(x)w(x,y)+\overline{G}\nabla^2 w(x,y) \tag{18}$$

where R is the density of the reaction force of elastic foundation, $\overline{K}$ is Winkler parameter depended on x only. It is assumed to be linear, parabolic or sinusoidal (Sobhy 2015, Attia *et al.* 2018, Pradhan and Murmu 2009):

$$\overline{K}(x)=\frac{J_1h^3}{a^4}\begin{cases} 1+\xi\dfrac{x}{a} & Linear \\ 1+\xi(\dfrac{x}{a})? & Parabolic \\ 1+\xi\sin(\pi\dfrac{x}{a}) & \mathrm{Sin}\,usoidal \end{cases} \tag{19}$$

in which J_1 is a constant and ξ is a varied parameter. G is the shear layer foundation stiffness ∇^2 is the Laplace

operator in x and y, and w is the deflection of the beam.
Note that, if ξ=0, the elastic foundation becomes Pasternak foundation and if the shear layer foundation stiffness is neglected, the Pasternak foundation becomes the Winkler foundation.

3.3 Equations of motion

The principle of virtual work is here in utilized to determine the equations of motion.

The variation of strain energy of the plate is calculated by (Draiche *et al.* 2019, Abualnour *et al.* 2019, Sahla *et al.* 2019, Berghouti *et al.* 2019, Zarga *et al.* 2019, Salah *et al.* 2019):

$$\begin{aligned}\delta U &= \int_A [N_x \delta\varepsilon_x^0 + N_z \delta\varepsilon_z^0 + M_x^b \delta k_x^b \\ &+ M_x^s \delta k_x^s + Q_{xz}\delta\gamma_{xz}^0 + S_{xz}\delta\gamma_{xz}^1] dA = 0\end{aligned} \tag{20}$$

where $\boldsymbol{A}$ is the surface; and stress resultants N, M, Q, and S are defined by

$$\begin{Bmatrix} N_x \\ M_x^b \\ M_x^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{21a}$$

$$N_z = \int_{-h/2}^{h/2} \sigma_z \, g'(z)\, dz \quad (S_{xz}) = \int_{-h/2}^{h/2} (\tau_{xz})\, g(z)\, dz \quad (Q_{xz}^s) = \int_{-h/2}^{h/2} (\tau_{xz})\, f'(z)\, dz \tag{21b}$$

The variation of potential energy of the applied loads can be expressed as

$$\delta V = -\int_A q\delta(w_0(x) + g(z)\theta(x)) dA \tag{22}$$

The variation of potential energy of the foundation can be expressed as

$$\delta U_R = \int_A R\delta(w_0(x) + g(z)\theta(x)) dA \tag{23}$$

Substituting the expressions of δU, δV and δU_R :

$$\begin{aligned}\delta U - \delta V + \delta U_R &= \int_A [N_x \delta\varepsilon_x^0 + N_z \delta\varepsilon_z^0 + M_x^b \delta k_x^b + M_x^s \delta k_x^s \\ &+ Q_{xz}\delta\gamma_{xz}^0 + S_{xz}\delta\gamma_{xz}^1] dA - \int_A q\delta w_0 dA - \int_A q g(z)\delta\theta dA \\ &+ \int_A R\delta w_0 dA + \int_A R g(z)\delta\theta dA = 0\end{aligned} \tag{24}$$

Integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_0, and $\delta\theta$, and the following equations of motion of the plate are obtained:

$$\begin{aligned}&\delta u_0 : \ \frac{\partial N_x}{\partial x} = 0 \\ &\delta w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} + q - R = 0 \\ &\delta\theta : \ -\mathrm{N}_z - k_1 M_x^s + k_1 A' \frac{\partial Q_{xz}}{\partial x} + \frac{\partial S_{xz}}{\partial x} + q g(z) - R g(z) = 0\end{aligned} \tag{25}$$

The stress resultants are obtained as:

$$\begin{Bmatrix} N \\ M_x^b \\ M_x^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^b \\ k_x^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_0^z \ , \tag{26a}$$

$$\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} F^s & X^s \\ X^s & A^s \end{bmatrix} \begin{Bmatrix} \gamma^0 \\ \gamma^1 \end{Bmatrix} \tag{26b}$$

$$N_z = R^a \varepsilon_z^0 + L(\varepsilon_x^0) + L^a(k_x^b) + R(k_x^s)$$

where

$$S = \{S_{xz}\},\ Q = \{Q_{xz}\} \tag{27a}$$

$$\begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} C_{ii} \\ C_{ii} z \\ C_{ii} f(z) \\ C_{ij} g'(z) \end{Bmatrix} g'(z) dz \tag{27b}$$

$$\begin{aligned}&\{A \ \ B \ \ D \ \ B^s \ \ D^s \ \ H^s\} = \\ &\int_{-h/2}^{h/2} C_{ii}\left[1,\ \ z,\ \ z^2,\ \ f(z),\ \ zf(z),\ \ f^2(z)\right] dz\end{aligned} \tag{27c}$$

$$F^S = F_{44}^s \ , A^s = A_{44}^s, X^s = X_{44}^s \tag{27d}$$

$$\begin{aligned}&(F_{44}^s, X_{44}^s, A_{44}^s) = \\ &\int_{-h/2}^{h/2} \left(\frac{E(z,T)}{2(1+v)}\ [f'?\ z), f'(z)g(z), g?\ z)]\right) dz\end{aligned} \tag{27e}$$

The equations of motion can be expressed in terms of displacements (δu_0, δw_0, $\delta\theta$) as:

$$\begin{aligned}&\delta u_0 : A\frac{\partial^2 u_0}{\partial x^2} - B\frac{\partial^3 w_0}{\partial x^3} + (B^s k_1 + L)\frac{\partial \theta}{\partial x} = 0 \\ &\delta w_0 : B\frac{\partial^3 u_0}{\partial x^3} - D\frac{\partial^4 w_0}{\partial x^4} + (D_{11}^s k_1 + L^a)\frac{\partial^2 \theta}{\partial x^2} + q - R = 0 \\ &\delta\theta : -(L + k_1 B_{11}^s)\frac{\partial u_0}{\partial x} + (L^a + k_1 D_{11}^s)\frac{\partial^2 w_0}{\partial x^2} - (k_1^2 H_{11}^s + 2k_1 R + R^a)\theta \\ &+ (k_1^2 A'^2 F_{44}^s + k_1 A' X_{44}^s)\frac{\partial^2 \theta}{\partial x^2} + (A_{44}^s + k_1 A' X_{44}^s)\frac{\partial^2 \theta}{\partial x^2} \\ &+ q g(z) - R g(z) = 0\end{aligned} \tag{28}$$

3.4 Temperature field

The nonlinear temperature rise across the thickness of the plate is determined by solving the one-dimensional heat conduction equation. The one dimensional steady-state heat conduction equation in the z-direction is given by:

$$-\frac{d}{dz}\left(k(z)\frac{dT}{dz}\right) = 0 \tag{29}$$

With the boundary condition T(*h/2*) = Tt and T(−*h/2*)=T_b=T_0. Here a stress-free state is assumed to exist at T_0=300 K. The analytical solution of Eq:

$$T(z)=T_b-(T_t-T_b)\frac{\int_{-h/2}^{z}\frac{1}{k(z)}dz}{\int_{-h/2}^{h/2}\frac{1}{k(z)}dz} \tag{30}$$

In the case of power-law FG plate, the solution of Eq. (29) also can be expressed by means of a polynomial series:

$$T(z)=T_b+\frac{(T_t-T_b)}{C_{tb}}\begin{bmatrix}\left(\frac{2z+h}{2h}\right)-\frac{k_{tb}}{(p+1)k_b}\left(\frac{2z+h}{2h}\right)^{p+1}\\+\frac{k_{tb}^2}{(2p+1)k_b^2}\left(\frac{2z+h}{2h}\right)^{2p+1}\\-\frac{k_{tb}^3}{(3p+1)k_b^3}\left(\frac{2z+h}{2h}\right)^{3p+1}\\+\frac{k_{tb}^4}{(4p+1)k_b^4}\left(\frac{2z+h}{2h}\right)^{4p+1}\\-\frac{k_{tb}^5}{(5p+1)k_b^5}\left(\frac{2z+h}{2h}\right)^{5p+1}\end{bmatrix} \tag{31a}$$

with

$$C_{tb}=\begin{bmatrix}1-\frac{k_{tb}}{(p+1)k_b}+\frac{k_{tb}^2}{(2p+1)k_b^2}-\frac{k_{tb}^3}{(3p+1)k_b^3}\\+\frac{k_{tb}^4}{(4p+1)k_b^4}-\frac{k_{tb}^5}{(5p+1)k_b^5}\end{bmatrix} \tag{31b}$$

where k_{tb}=k_t−k_b, with k_t and k_b are the thermal conductivity of the top and bottom faces of the plate, respectively.

4. Exact solution for simply supported FG Plate

Beams are generally classified according to the type of support used. This paper is concerned with the exact solutions of Eq. (28) for a simply supported FG beam.

The following boundary conditions are imposed at the edges:

$$u_0=w_0=\theta=\frac{\partial\theta}{\partial y}=N_x=M_x^b=M_x^s=0 \quad \text{at } x=0,a \tag{32}$$

Following the Navier solution procedure, the authors assume the following solution from for u_0, w_0, and θ that satisfies the boundary conditions given in:

$$\begin{Bmatrix}u_0\\w_0\\\theta\end{Bmatrix}=\begin{Bmatrix}U_{mn}\cos(\alpha\ x))\\W_{mn}\sin(\alpha\ x)\\X_{mn}\sin(\alpha\ x)\end{Bmatrix} \tag{33}$$

where U_{mn}, W_{mn} and X_{mn} and are arbitrary parameters to be determined and α are defined as:

$$\alpha=m\pi/a \tag{34}$$

The transverse load q is also expanded in the double-Fourier sine series as

$$q(x)=q_{mn}\sin(\alpha\ x) \tag{35}$$

The analytical solutions can be obtained from:

$$\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{12}&a_{22}&a_{23}\\a_{13}&a_{23}&a_{33}\end{bmatrix}\begin{Bmatrix}U_{mn}\\W_{mn}\\X_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{36}$$

in which:

$$a_{11}=A_{11}\alpha^2$$
$$a_{12}=-B\alpha^3$$
$$a_{13}=\alpha^3B^sk_1-L\alpha$$
$$a_{33}=\alpha^4D+\overline{K}+\overline{G}(\alpha^2) \tag{37}$$
$$a_{34}=D^sk_1\alpha^2+L_a\alpha^2$$
$$a_{44}=\alpha^4H^sk_1^2A^2+\alpha^2(A^2F_{44}^sk_1^2+2AX_{44}^SK_1+A_{44}^S)$$
$$+Ra-2\alpha^2AK_1R$$

5. Numerical results and discussion

In this section, thermal bending response of FG beam resting on variable elastic foundation is analyzed based on 2D and quasi 3D computational models. The FG beam is subjected to thermo-mechanical loading.

As discussed before, and in the absence of previous works that addresses the same thematic, the results of the present models have not been validated against data in the literature.

The mechanical characteristics of metal and ceramics used in the FG beam are listed in Table 1.

For convenience, the following expressions to compute the non-dimensional central deflection, stresses and foundation parameters were used:

$$\bar{w}=\frac{100h}{q_0L^2}w\left(\frac{L}{2},0\right),\quad \bar{\sigma}_{xz}=\frac{h}{q_0L}\sigma_{xz}(0,0),$$
$$\bar{\sigma}_{xx}=\frac{10h^2}{q_0L^2}\sigma_{xx}(L/2,h/2),\quad j_2=\frac{\overline{G}L^2}{h^3}$$

Table 1 Material properties used in the FG plate

	P0	P−1	P1	P2	P3
	ZrO2 (Ceramic)				
E	244.27 e^{+9}	0	-1.371 e^{-3}	1.214 e^{-6}	–3.681e^{-10}
α	12.766 e^{-6}	0	-1.491 e^{-3}	1.006 e^{-5}	–6.778e^{-11}
k	1.7	0	0	0	0
υ	0.3	0	0	0	0
	Ti–4V–6Al (Metal)				
E	122.56e^{+9}	0	-4.586e^{-4}	0	0
α	7.5788e^{-6}	0	6.638e^{-4}	–3.147e^{-6}	0
k	1	0	0	0	0
υ	0.3	0	0	0	0

Table 2 The central deflection $\overline{w}$ versus length-to-thickness ratios of FG beam without elastic foundations (ΔT =300)

L/h	Theory	*p*				
		0	1	2	5	10
5	Present 2D (ε_z=0)	2.1224	2.6324	2.7588	2.8862	2.9815
	Present quasi-3D ($\varepsilon_z \neq 0$)	2.0912	2.5934	2.7206	2.8501	2.9441
10	Present 2D (ε_z=0)	7.8420	9.7355	10.1724	10.6108	10.9698
	Present quasi-3D ($\varepsilon_z \neq 0$)	7.7996	9.6843	10.1230	10.5688	10.9277
20	Present 2D (ε_z=0)	30.7198	38.1468	39.8259	41.5075	42.9218
	Present quasi-3D ($\varepsilon_z \neq 0$)	30.6525	38.0749	39.7514	41.4568	42.8775
30	Present 2D (ε_z=0)	68.8493	85.4988	89.2482	93.0021	96.1751
	Present quasi-3D ($\varepsilon_z \neq 0$)	68.7443	85.3979	89.1358	92.9393	96.1300
50	Present 2D (ε_z=0)	190.8638	237.0252	247.3997	257.7845	266.5855
	Present quasi-3D ($\varepsilon_z \neq 0$)	190.6396	236.8335	247.1674	257.6844	266.5393

Table 3 The transverse shear stress $\overline{\sigma}_{xz}$ versus length-to-thickness ratios (*L/h*) of FG beam without elastic foundations (ΔT =300)

L/h	Theory	***p***				
		0	1	2	5	10
5	Present 2D (ε_z=0)	0.4746	0.4775	0.4693	0.4634	0.4694
	Present quasi-3D ($\varepsilon_z \neq 0$)	0.5095	0.5165	0.4968	0.4809	0.4899
10	Present 2D (ε_z=0)	0.4751	0.4780	0.4698	0.4639	0.4699
	Present quasi-3D ($\varepsilon_z \neq 0$)	0.5635	0.5740	0.5448	0.5214	0.5329
20	Present 2D (ε_z=0)	0.4752	0.4781	0.4699	0.4640	0.4700
	Present quasi-3D ($\varepsilon_z \neq 0$)	0.5886	0.6009	0.5669	0.5397	0.5524
30	Present 2D (ε_z=0)	0.4753	0.4781	0.4700	0.4641	0.4700
	Present quasi-3D ($\varepsilon_z \neq 0$)	0.5943	0.6070	0.5718	0.5438	0.5566
50	Present 2D (ε_z=0)	0.4753	0.4781	0.4700	0.4641	0.4700
	Present quasi-3D ($\varepsilon_z \neq 0$)	0.5973	0.6103	0.5744	0.5459	0.5590

Table 4 The normal stress $\overline{\sigma}_{xx}$ versus length-to-thickness ratios (*L/h*) of FG beam without elastic foundations (ΔT =300)

L/h	Theory	***p***				
		0	1	2	5	10
5	Present 2D (ε_z=0)	5.7331	6.6152	6.8754	7.2884	7.6676
	Present quasi-3D ($\varepsilon_z \neq 0$)	5.7495	6.6365	6.8948	7.3058	7.6862
10	Present 2D (ε_z=0)	5.6571	6.5215	6.7749	7.1820	7.5598
	Present quasi-3D ($\varepsilon_z \neq 0$)	5.6643	6.5308	6.7835	7.1896	7.5679
20	Present 2D (ε_z=0)	5.6380	6.4980	6.7498	7.1553	7.5328
	Present quasi-3D ($\varepsilon_z \neq 0$)	5.6402	6.5008	6.7523	7.1576	7.5352
30	Present 2D (ε_z=0)	5.6345	6.4937	6.7451	7.1504	7.5278
	Present quasi-3D ($\varepsilon_z \neq 0$)	5.6355	6.4950	6.7463	7.1515	7.5289
50	Present 2D (ε_z=0)	5.6327	6.4914	6.7427	7.1479	7.5252
	Present quasi-3D ($\varepsilon_z \neq 0$)	5.6331	6.4919	6.7432	7.1483	7.5256

Unless otherwise specified the Voigt model is used in the different calculations.

In Table 2, the central deflection $\overline{w}$ against length-to-thickness ratio (*L/h*) computed with the present 2D and quasi-3D models are presented for different values of the power law index “*p*”.

It can be seen from this figure that the central deflection increases with the increase of the power-law index. It is due

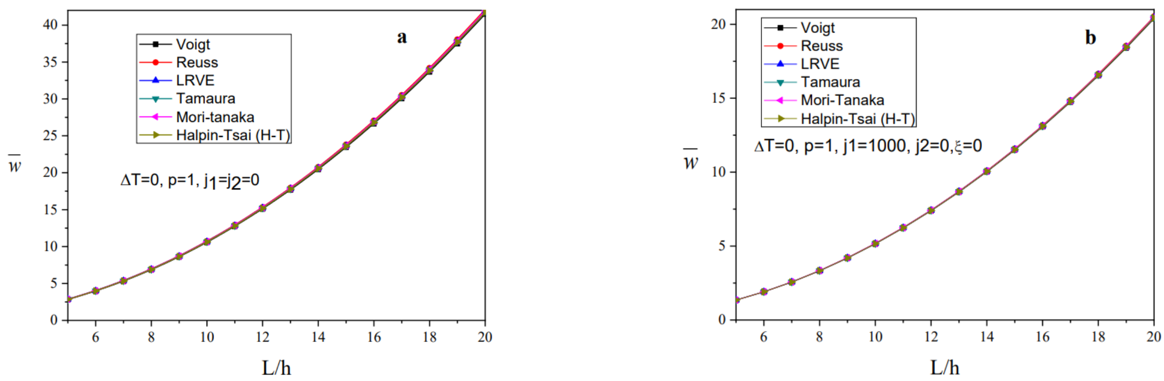

Fig. 2 The deflection $\overline{w}$ of FG beam versus length-to-thickness ratios (L/h) for different micromechanical models (a) beam without elastic foundation and (b) beam on elastic foundation

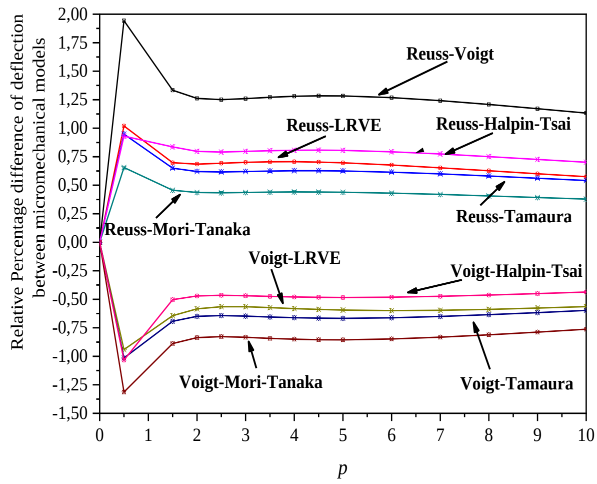

Fig. 3 Relative percentage difference of deflection $\overline{w}$ between micromechanical models versus power law index p (L/h=10, ΔT=300)

Table 5 The deflection $\overline{w}$ of FG beam with or elastic foundations (ΔT=300, p=1, ξ=0)

j_1	j_2	Theory	L/h				
			5	10	20	30	50
0	0	Present 2D (ε_z=0)	2.6324	9.7355	38.1468	85.4988	237.0252
		Present quasi-3D ($\varepsilon_z\neq$0)	2.5934	9.6843	38.0749	85.3979	236.8335
1000	0	Present 2D (ε_z=0)	1.2823	4.9330	19.5257	43.8458	121.6699
		Present quasi-3D ($\varepsilon_z\neq$0)	1.2835	4.9315	19.5191	43.8316	121.6318
1000	1000	Present 2D (ε_z=0)	0.2115	0.8406	3.3562	7.5489	20.9655
		Present quasi-3D ($\varepsilon_z\neq$0)	0.2145	0.8439	3.3596	7.5521	20.9680

Table 6 The transverse shear stress $\overline{\sigma}_{xz}$ in FG beam with or without elastic foundation, (ΔT=300, p=1, ξ=0)

j_1	j_2	Theory	L/h				
			5	10	20	30	50
0	0	Present 2D (ε_z=0)	0.4775	0.4780	0.4781	0.4781	0.4781
		Present quasi-3D ($\varepsilon_z\neq$0)	0.5165	0.5740	0.6009	0.6070	0.6103
1000	0	Present 2D (ε_z=0)	0.2326	0.2422	0.2447	0.2452	0.2454
		Present quasi-3D ($\varepsilon_z\neq$0)	0.2557	0.2923	0.3081	0.3116	0.3134
1000	1000	Present 2D (ε_z=0)	0.0384	0.0413	0.0421	0.0422	0.0423
		Present quasi-3D ($\varepsilon_z\neq$0)	0.0427	0.0500	0.0530	0.0537	0.0540

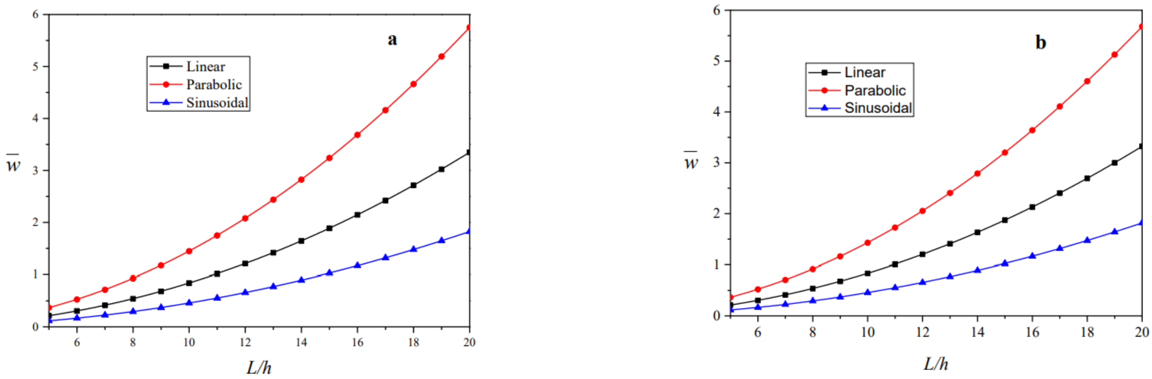

Fig. 4 The deflection $\overline{w}$ of FG beam (p=1, ξ=20) versus to the side- to- thickness ratio L/h under (a) mechanical load and (b) thermomechanical load ΔT=300 for various types of Winkler parameter j_1=1000, j_2=0

to the fact that a higher value of (p) corresponds to lower value of volume fraction of the ceramic phase, and thus makes the beam become the softer ones.

Another remark is noted, when the ratio L/h increases, the results of the 2D and quasi-3D models come closer and this whatever the value of the power index p. This can be explained by the fact that, when the L/h ratio increases the beam becomes thin and the stretching effect taken into account in the quasi-3D model will have no impact.

In Tables 3 and 4 we present respectively shear stress $\overline{\sigma}_{xz}$ and normal stress $\overline{\sigma}_{xx}$ versus length-to-thickness ratios of FG beam without elastic foundations for different values of the power law index "p".

From Table 3, it can be seen that the shear stress calculated by the 2D model is very little influenced by the increase of "p" and "L/h". Indeed, the increase in the power index "p" leads to a very slight decrease of this stress and the increase in the "L/h" ratio to a slight increase that quickly becomes constant when the beam becomes thin. For the results obtained by the quasi-3D model, the same observation is noticed for the variation of the power index "p". As for the "L/ h" ratio, the increase in the stresses is slightly greater than that of the 2D model.

In general, it can be said that this stresses is insensitive to the variation of the two parameters "p" and "L/h".

The results of axial stresses reported in Table 4 reveal an increase in these stresses with the increase of the index "p" and a decrease with the increase in the "L/h" ratio for both 2D and quasi-3D models. Moreover, both models give the same results with the increase of the L/h ratio (beam becomes thin).

Fig. 2 shows the effect of the different micromechanical models on the deflections calculated from the quasi-3D model for two cases of beam with and without elastic foundation.

From this figure it is clear that all the micromechanical models give practically the same result. Moreover, as can be seen from this figure, the displacements increase rapidly with the increase of the "L/h" ratio and that the presence of the elastic foundation allows to reduce the displacements by more than 50%.

Relative Percentage difference of deflection between micromechanical models versus power law index p is shown in Fig. 3. The discrepancy between the estimated

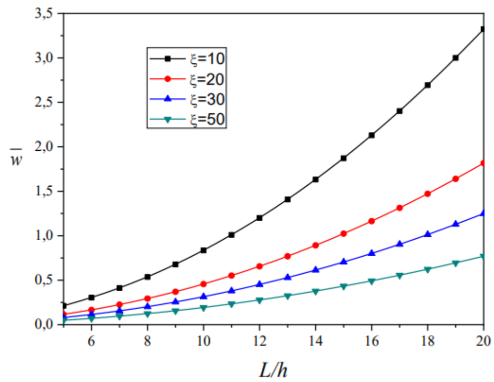

Fig. 5 The deflection $\overline{w}$ of FGM plate (p=1) against the side-to-thickness ratio L/h under thermomechanical load ΔT=300 for different values of the sinusoidal parameter ξ (j_1=1000, j_2=0)

deflection of FG beams by the Voigt, Reuss and other micromechanical models is little influenced by the power law index "p".

According to this figure, all models give practically the same result. The difference in results between the micro-mechanical models reaches a maximum of 2% between the Voigt and Reuss models for a value of the power index less than 1. Then this difference decreases rapidly with the increase of "p".

In general, we can say that the difference between the different micro-mechanical models is minimal and can therefore be neglected.

Another example is to analyze the effect of the elastic foundation on the deflection and stresses of FG beams. Tables 5 and 6 present respectively the deflection $\overline{w}$ and transverse shear stress $\overline{\sigma}_{xz}$ of FG beams with and without elastic foundation for different values of the side-to-thickness ratio a/h. The results are presented for both 2D and quasi-3D models.

As can be seen, the presence of an elastic foundation with one or two parameters strongly influences the response of the FG beam. In fact, taking into account an elastic foundation, especially the two-parameter one (Pasternak's model) increases the rigidity of the beam and consequently leads to a significant reduction of displacements and stresses.

Fig. 4(a) and 4(b) compare the deflection of a simply supported beam subjected to mechanical and thermo-mechanical load respectively for three different types of elastic foundations. It can be seen from this figure that the deflection strongly depends on the type of foundation. Also, beam under thermo-mechanical load gives deflections slightly higher compared to the mechanical load and this whatever the type of elastic foundation.

Fig. 5 depicts the variation of the deflection of simply supported beam under thermo-mechanical load and resting on elastic foundation with different values of parabolic parameter. It is clear from this figure that the deflection is strongly influenced by this parameter. Its increase leads to a reduction in deflections. This can be explained by the fact that the increase in this parameter makes the foundation more rigid and consequently it prevents the beam from deforming.

6. Conclusions

The present work is focused on thermomechanical bending of FG beams in thermal environment and resting on variable elastic foundation by using 2D and quasi-3D HSDT. Various micromechanical models have been employed to obtain the effective material properties of the FG beams which vary across thickness direction and they are considered to be temperature-dependent. The equations of motion are obtained through the Hamilton's principle. These equations are solved by employing Navier's procedure. A parametric study has been carried out to highlight the effect the side-to-thickness ratio, the parabolic parameter and the micromechanical models on the deflection and stress of beam under thermomechanical loading and resting on variable elastic foundation.

The present model can be used as a reference to check the efficiency of approximate numerical methods. The extension of this study is also envisaged by considering other types of materials and other models with shear deformation effect. Other works can be carried out in future by considering other types of materials and other models with shear deformation effect (Kar *et al.* 2015, Mahapatra *et al.* 2016, Mehar and Panda 2017a,b,c, 2018a,b,c and 2019, Panjehpour *et al.* 2018, Othman and Fekry 2018, Behera and Kumari 2018, Mehar *et al.* 2018a, b, Mehar *et al.* 2019, Alimirzaei *et al.* 2019, Gupta and Anandkumar 2019, Karami *et al.* 2019a,b,c, Bedia *et al.* 2019, Semmah *et al.* 2019, Draoui *et al.* 2019, Boutaleb *et al.* 2019, Shadravan *et al.* 2019, Medani *et al.* 2019, Selmi 2019, Timesli 2020, Bellal *et al.* 2020, Mehar and Panda 2020, Kim *et al.* 2020, Khosravi *et al.* 2020, Al-Basyouni *et al.* 2020, Shokrieh and Kondori 2020, Hussain *et al.* 2019 and 2020b, Taj *et al.* 2020, Asghar *et al.* 2020).

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References

Abualnour, M., Chikh, A., Hebali, H., Kaci, A., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2019), "Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory", *Comput. Concrete*, **24**(6), 489-498. https://doi.org/10.12989/cac.2019.24.6.489.

Ahmed, R.A., Fenjan, R.M. and Faleh, N.M. (2019), "Analyzing post-buckling behavior of continuously graded FG nanobeams with geometrical imperfections", *Geomech. Eng.*, **17**(2), 175-180. https://doi.org/10.12989/gae.2019.17.2.175.

Akbarzadeh, A.H., Abedini, A. and Chen, Z.T. (2015), "Effect of micromechanical models on structural responses of functionally graded plates", *Compos. Struct.*, **119**, 598-609. http://doi.org/10.1016/j.compstruct.2014.09.031.

Al-Basyouni, K.S., Ghandourah, E., Mostafa, H.M. and Algarni, A. (2020), "Effect of the rotation on the thermal stress wave propagation in non-homogeneous viscoelastic body", *Geomech. Eng.*, **21**(1), 1-9. https://doi.org/10.12989/gae.2020.21.1.001.

Al-Furjan, M.S.H., Safarpour, H., Habibi, M., Safarpour, M. and Tounsi, A. (2020), "A comprehensive computational approach for nonlinear thermal instability of the electrically FG-GPLRC disk based on GDQ method", *Eng. Comput.*, 1-8. https://doi.org/10.1007/s00366-020-01088-7

Al-Maliki, A.F.H., Ahmed, R.A., Moustafa, N.M. and Faleh, N.M. (2020), "Finite element based modeling and thermal dynamic analysis of functionally graded graphene reinforced beams", *Adv. Comput. Des.*, **5**(2), 177-193. https://doi.org/10.12989/acd.2020.5.2.177.

Alimirzaei, S., Mohammadimehr, M. and Tounsi, A. (2019), "Nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions", *Struct. Eng. Mech.*, **71**(5), 485-502. https://doi.org/10.12989/sem.2019.71.5.485.

Arioui, O., Belakhdar, K., Kaci, A. and Tounsi, A. (2018), "Thermal buckling of FGM beams having parabolic thickness variation and temperature dependent materials", *Steel Compos. Struct.*, **27**(6), 777-788. https://doi.org/10.12989/scs.2018.27.6.777.

Asghar, S., Naeem, M.N., Hussain, M., Taj, M. and Tounsi, A. (2020), "Prediction and assessment of nonlocal natural frequencies of DWCNTs: Vibration analysis", *Comput. Concrete*, **25**(2), 133-144. https://doi.org/10.12989/cac.2020.25.2.133

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R and Alwabli, S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundation", *Struct. Eng. Mech.*, **65**(4), 453-464. https://doi.org/10.12989/sem.2018.65.4.453.

Avcar, M. and Mohammed, W.K.M. (2018), "Free vibration of functionally graded beams resting on Winkler-Pasternak foundation", *Arab. J. Geosci.*, **11**(10), 232. https://doi.org/10.1007/s12517-018-3579-2.

Bachir Bouiadjra, R., Mahmoudi, A., Benyoucef, S., Tounsi, A. and Bernard, F. (2018), "Analytical investigation of bending response of FGM plate using a new quasi 3D shear deformation theory: Effect of the micromechanical models", *Struct. Eng. Mech.*, **66**(3), 317-328. https://doi.org/10.12989/sem.2018.66.3.317.

Balubaid, M., Tounsi, A., Dakhel, B. and Mahmoud, S.R. (2019), "Free vibration investigation of FG nanoscale plate using nonlocal two variables integral refined plate theory", *Comput. Concrete*, **24**(6), 579-586. https://doi.org/10.12989/cac.2019.24.6.579.

Barati, M.R. (2017), "Investigating dynamic response of porous inhomogeneous nanobeams on hybrid Kerr foundation under hygro-thermal loading", *Appl. Phys. A.*, **123**(5), 332. https://doi.org/10.1007/s00339-017-0908-3.

Batou, B., Nebab, M., Bennai, R., Ait Atmane, H., Tounsi, A. and Bouremana, M. (2019), “Wave dispersion properties in imperfect sigmoid plates using various HSDTs”, *Steel Compos. Struct.*, **33**(5), 699-716. https://doi.org/10.12989/scs.2019.33.5.699

Bedia, W.A., Houari, M.S.A., Bessaim, A., Bousahla, A.A., Tounsi, A., Saeed, T. and Alhodaly, M.Sh. (2019), “A new hyperbolic two-unknown beam model for bending and buckling analysis of a nonlocal strain gradient nanobeams”, *J. Nano Res.*, **57**, 175-191. https://doi.org/10.4028/www.scientific.net/JNanoR.57.175.

Behera, S. and Kumari, P. (2018), “Free vibration of Levy-type rectangular laminated plates using efficient zig-zag theory”, *Adv. Comput. Des.*, **3**(3), 213-232. https://doi.org/10.12989/acd.2017.2.3.165.

Bellal, M., Hebali, H., Heireche, H., Bousahla, A.A., Tounsi, A., Bourada, F., Mahmoud, S.R., Adda Bedia, E.A. and Tounsi, A. (2020), “Buckling behavior of a single-layered graphene sheet resting on viscoelastic medium via nonlocal four-unknown integral model”, *Steel Compos. Struct.*, **34**(5), 643-655. https://doi.org/10.12989/scs.2020.34.5.643.

Berghouti, H., Adda Bedia, E.A., Benkhedda, A. and Tounsi, A. (2019), “Vibration analysis of nonlocal porous nanobeams made of functionally graded material”, *Adv. Nano Res.*, **7**(5), 351-364. https://doi.org/10.12989/anr.2019.7.5.351.

Bourada, F., Bousahla, A.A., Bourada, M., Azzaz, A., Zinata, A. and Tounsi, A. (2019), “Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory”, *Wind Struct.*, **28**(1), 19-30. https://doi.org/10.12989/was.2019.28.1.019.

Bourada, F., Bousahla, A.A., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H. and Tounsi, A. (2020), “Stability and dynamic analyses of SW-CNT reinforced concrete beam resting on elastic-foundation”, *Comput. Concrete*, **25**(6), 485-495. https://doi.org/10.12989/cac.2020.25.6.485.

Bousahla, A.A., Bourada, F., Mahmoud, S.R., Tounsi, A., Algarni, A., Adda Bedia, E.A. and Tounsi, A. (2020), “Buckling and dynamic behavior of the simply supported CNT-RC beams using an integral-first shear deformation theory” *Comput. Concrete*, **25**(2), 155-166. https://doi.org/10.12989/cac.2020.25.2.155.

Boutaleb, S., Benrahou, K.H., Bakora, A., Algarni, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Tounsi, A. (2019), “Dynamic Analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT”, *Adv. Nano Res.*, **7**(3), 191-208. https://doi.org/10.12989/anr.2019.7.3.191.

Chaabane L.A., Bourada, F., Sekkal, M., Zerouati, S., Zaoui, F.Z., Tounsi, A., Derras, A., Bousahla, A.A. and Tounsi, A. (2019), “Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation”, *Struct. Eng. Mech.*, **71**(2), 185-196. https://doi.org/10.12989/sem.2019.71.2.185.

Chen, M., Jin, G., Zhang, Y., Niu, F. and Liu, Z. (2019), “Three-dimensional vibration analysis of beams with axial functionally graded materials and variable thickness”, *Compos. Struct.*, **207**, 304-322. https://doi.org/10.1016/j.compstruct.2018.09.029.

Chikr, S.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, S.R. and Tounsi, A. (2020), “A novel four-unknown integral model for buckling response of FG sandwich plates resting on elastic foundations under various boundary conditions using Galerkin’s approach”, *Geomech. Eng.*, **21**(5), 471-487. https://doi.org/10.12989/gae.2020.21.5.471.

Dash, S., Mehar, K., Sharma, N., Mahapatra, T.R. and Panda, S.K. (2019), “Finite element solution of stress and flexural strength of functionally graded doubly curved sandwich shell panel”, *Earthq. Struct.*, **16**(1), 55-67. https://doi.org/10.12989/eas.2019.16.1.055.

Draiche, K., Bousahla, A.A., Tounsi, A., Alwabli, A.S., Tounsi, A. and Mahmoud, S.R. (2019), “Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory”, *Comput. Concrete*, **24**(4), 369-378. https://doi.org/10.12989/cac.2019.24.4.369.

Draoui, A., Zidour, M., Tounsi, A. and Adim, B. (2019), “Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)”, *J. Nano Res.*, **57**, 117-135. https://doi.org/10.4028/www.scientific.net/JNanoR.57.117.

Ebrahimi, F. and Jafari, A., (2018), “A four-variable refined shear-deformation beam theory for thermo-mechanical vibration analysis of temperature-dependent FGM beams with porosities”, *Mech. Adv. Mater. Struct.*, **25**(3), 212-224. https://doi.org/10.1080/15376494.2016.1255820.

Eisenberger, M. and Clastornik, J. (1987), “Vibrations and buckling of a beam on a variable Winkler elastic foundation”, *J. Sound Vib.*, **115**(2), 233-241. https://doi.org/10.1016/0022-460X(87)90469-X.

Esfahani, S., Kiani, Y. and Eslami, M. (2013), “Non-linear thermal stability analysis of temperature dependent FGM beams supported on non-linear hardening elastic foundations”, *Int. J. Mech. Sci.*, **69**, 10-20. https://doi.org/10.1016/j.ijmecsci.2013.01.007.

Fahsi, B., Bouiadjra, R.B., Mahmoudi, A., Benyoucef, S. and Tounsi, A. (2019), “Assessing the effects of porosity on bending, buckling and vibration of FG beam resting on elastic foundation using a new refined quasi-3d theory”, *Mech. Compos. Mater.*, **55**(2), 219-230. https://doi.org/10.1007/s11029-019-09805-0.

Fallah, A. and Aghdam, M.M. (2012), “Thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded beams on nonlinear elastic foundation”, *Compos. Part B Eng.*, **43**(3), 1523-1530. https://doi.org/10.1016/j.compositesb.2011.08.041.

Frikha, A., Hajlaoui, A., Wali, M. and Dammak, F., (2016), “A new higher order *C0* mixed beam element for FGM beams analysis”, *Compos. Part B Eng.*, **106**, 181-189. https://doi.org/10.1016/j.compositesb.2016.09.024.

Gafour, Y., Hamidi, A., Benahmed, A., Zidour, M. and Bensattalah, T. (2020), “Porosity-dependent free vibration analysis of FG nanobeam using non-local shear deformation and energy principle”, *Adv. Nano Res.*, **8**(1), 49-58. https://doi.org/10.12989/anr.2020.8.1.049.

Gasik, M. (1995), “Scand. Ch226”, *Acta Polytech.*, **72**.

Ghiasian, S.E., Kiani, Y. and Eslami, M.R. (2015), “Nonlinear thermal dynamic buckling of FGM beams”, *Eur. J. Mech. A Solids*, **54**, 232-242. https://doi.org/10.1016/j.euromechsol.2015.07.004.

Gul, U., Aydogdu, M. and Karacam, F. (2019), “Dynamics of a functionally graded Timoshenko beam considering new spectrums”, *Compos. Struct.*, **207**, 273-291. https://doi.org/10.1016/j.compstruct.2018.09.021.

Gupta, V. and Anandkumar, J. (2019), “Phenol removal by tailor-made polyamide-fly ash composite membrane: Modeling and optimization”, *Membr. Water Treat.*, **10**(6), 431-440. https://doi.org/10.12989/mwt.2019.10.6.431.

Halpin, J.C. (1969), “Effects of environmental factors on composite materials”, Technical Report AFML-TR Ohio, U.S.A., 67-423.

Hussain, M., Naeem, M.N., Khan, M.S. and Tounsi, A. (2020a), “Computer-aided approach for modelling of FG cylindrical shell sandwich with ring supports”, *Comput. Concrete*, **25**(5), 411-425. https://doi.org/10.12989/cac.2020.25.5.411.

Hussain, M., Naeem, M.N., Taj, M. and Tounsi, A. (2020b), “Simulating vibrations of vibration of single-walled carbon nanotube using Rayleigh-Ritz's method", *Adv. Nano Res.*, **8**(3), 215-228. https://doi.org/10.12989/anr.2020.8.3.215.

Hussain, M., Naeem, M.N., Tounsi, A. and Taj, M. (2019), "Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity", *Adv. Nano Res.*, **7**(6), 431-442. https://doi.org/10.12989/anr.2019.7.6.431

Jaesang, Y. and Addis, K. (2014), "Modeling functionally graded materials containing multiple heterogeneities", *Acta Mech.*, **225**(7), 1931-1943. https://doi.org/10.1007/s00707-013-1033-9.

Kaci, A., Houari, M.S.A, Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech.*, **65**(5), 621-631. https://doi.org/10.12989/sem.2018.65.5.621.

Kaddari, M., Kaci, A., Bousahla, A.A., Tounsi, A., Bourada, F., Tounsi, A., Adda Bedia, E.A. and Al-Osta, M.A. (2020), "A study on the structural behaviour of functionally graded porous plates on elastic foundation using a new quasi-3D model: Bending and Free vibration analysis", *Comput. Concrete*, **25**(1), 37-57. https://doi.org/10.12989/cac.2020.25.1.037.

Kar, V.R. and Panda, S.K. (2015a), "Large deformation bending analysis of functionally graded spherical shell using FEM", *Struct. Eng. Mech.*, **53**(4), 661-679. https://doi.org/10.12989/sem.2015.53.4.661.

Kar, V.R. and Panda, S.K. (2015b), "Thermoelastic analysis of functionally graded doubly curved shell panels using nonlinear finite element method", *Compos. Struct.*, **129**, 202-212. https://doi.org/10.1016/j.compstruct.2015.04.006.

Kar, V.R. and Panda, S.K. (2016), "Nonlinear thermomechanical behavior of functionally graded material cylindrical/hyperbolic/elliptical shell panel with temperature-dependent and temperature-independent properties", *J. Press. Vess. T.*, **138**(6), 061202. https://doi.org/10.1115/1.4033701.

Kar, V.R. and Panda, S.K. (2020), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct.*, **18**(3), 693-709. https://doi.org/10.12989/scs.2015.18.3.693.

Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2015), "Nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading", *Steel Compos. Struct.*, **19**(4), 1011-1033. http://doi.org/10.12989/scs.2015.19.4.1011.

Karamanlı, A. (2017), "Bending behaviour of two directional functionally graded sandwich beams by using a quasi-3d shear deformation theory", *Compos. Struct.*, **174**, 70-86. https://doi.org/10.1016/j.compstruct.2017.04.046.

Karami, B., Janghorban, M. and Tounsi, A. (2019a), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, **35**, 1297-1316. https://doi.org/10.1007/s00366-018-0664-9.

Karami, B., Janghorban, M. and Tounsi, A. (2019c), "On pre-stressed functionally graded anisotropic nanoshell in magnetic field", *J. Brazil. Soc. Mech. Sci. Eng.*, **41**(11), 495. https://doi.org/10.1007/s40430-019-1996-0.

Karami, B., Shahsavari, D., Janghorban, M. and Tounsi, A. (2019b), "Resonance behavior of functionally graded polymer composite nanoplates reinforced with grapheme nanoplatelets", *Int. J. Mech. Sci.*, **156**, 94-105. https://doi.org/10.1016/j.ijmecsci.2019.03.036.

Khiloun, M., Bousahla, A.A., Kaci, A., Bessaim, A., Tounsi, A. and Mahmoud, S.R. (2020), "Analytical modeling of bending and vibration of thick advanced composite plates using a four-variable quasi 3D HSDT", *Eng. Comput.*, **36**(3), 807-821. https://doi.org/10.1007/s00366-019-00732-1.

Khosravi, F., Hosseini, S.A. and Tounsi, A. (2020), "Forced axial vibration of a single-walled carbon nanotube embedded in elastic medium under various moving forces", *J. Nano Res.*, **63**, 112-133. https://doi.org/10.4028/www.scientific.net/jnanor.63.112.

Kim, I., Zhu, T., Jeon, C.H. and Lawler, D.F. (2020), "Detachment of nanoparticles in granular media filtration", *Membr. Water Treat.*, **11**(1), 1-10. https://doi.org/10.12989/mwt.2020.11.1.001.

Kolahchi, R., Safaria, M. and Esmailpour, M. (2016), "Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium", *Compos. Struct.*, **150**, 255-265 .https://doi.org/10.1016/j.compstruct.2016.05.023.

Kumar, R. and Devi, S. (2017), "Response of thermoelastic functionally graded beam due to ramp type heating in modified couple stress with dual-phase-lag model", *Multidisciplin. Model. Mater. Struct.*, **13**(3), 471-488. https://doi.org/10.1108/MMMS-05-2017-0034.

Mahapatra, T.R., Kar, V.R., Panda, S.K. and Mehar, K. (2017), "Nonlinear thermoelastic deflection of temperature-dependent FGM curved shallow shell under nonlinear thermal loading", *J. Therm. Stresses*, **40**(9), 1184-1199. https://doi.org/10.1080/01495739.2017.1302788.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016), "Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel", *Int. J. Mech. Mater. Des.*, **12**(2), 153-171. https://doi.org/10.1007/s10999-015-9299-9.

Mahmoudi, A., Benyoucef, S., Tounsi, A., Benachour, A. and Adda Bedia, E.A. (2018), "On the effect of the micromechanical models on the free vibration of rectangular FGM plate resting on elastic foundation", *Struct. Eng. Mech.*, **14**(2), 117-128. https://doi.org/10.12989/eas.2018.14.2.117.

Mantari, J.L. and Yarasca, J. (2015), "A simple and accurate generalized shear deformation theory for beams", *Compos. Struct.*, **134**, 593-601. http://doi.org/10.1016/j.compstruct.2015.08.073.

Matouk, H., Bousahla, A.A., Heireche, H., Bourada, F., Adda Bedia, E.A., Tounsi, A., Mahmoud, S.R., Tounsi, A. and Benrahou, K.H. (2020), "Investigation on hygro-thermal vibration of P-FG and symmetricS-FG nanobeam using integral Timoshenko beam theory", *Adv. Nano Res.*, **8**(4), 293-305. https://doi.org/10.12989/anr.2020.8.4.293.

Medani, M., Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate using energy principle", *Steel Compos. Struct.*, **32**(5), 595-610. https://doi.org/10.12989/scs.2019.32.5.595.

Mehar, K. and Panda, S.K. (2017a), "Numerical investigation of nonlinear thermomechanical deflection of functionally graded CNT reinforced doubly curved composite shell panel under different mechanical loads", *Compos. Struct.*, **161**, 287-298. https://doi.org/10.1016/j.compstruct.2016.10.135.

Mehar, K. and Panda, S.K. (2017b), "Thermoelastic analysis of FG-CNT reinforced shear deformable composite plate under various loadings", *Int. J. Comput. Meth.*, **14**(2), 1750019. https://doi.org/10.1142/S0219876217500190.

Mehar, K. and Panda, S.K. (2017c), "Nonlinear static behavior of FG-CNT reinforced composite flat panel under thermomechanical load", *J. Aerospace Eng.*, **30**(3), 04016100. https://doi.org/10.1061/(ASCE)AS.1943-5525.0000706.

Mehar, K. and Panda, S.K. (2018b), "Nonlinear finite element solutions of thermoelastic flexural strength and stress values of temperature dependent graded CNT-reinforced sandwich shallow shell structure", *Struct. Eng. Mech.*, **67**(6), 565-578. http://doi.org/10.12989/sem.2018.67.6.565.

Mehar, K. and Panda, S.K. (2018c), "Thermoelastic flexural analysis of FG-CNT doubly curved shell panel", *Aircr. Eng. Aerosp. Tech.*, **90**(1), 11-23. https://doi.org/10.1108/AEAT-11-2015-0237.

Mehar, K. and Panda, S.K. (2019), “Theoretical deflection analysis of multi-walled carbon nanotube reinforced sandwich panel and experimental verification”, *Compos. Part B Eng.*, **167**, 317-328. https://doi.org/10.1016/j.compositesb.2018.12.058.

Mehar, K. and Panda, S.K. (2020), “Nonlinear deformation and stress responses of a graded carbon nanotube sandwich plate structure under thermoelastic loading”, *Acta Mech.*, **231**(3), 1105-1123. https://doi.org/10.1007/s00707-019-02579-5.

Mehar, K. and Panda, S.K. (2018a), “Elastic bending and stress analysis of carbon nanotube-reinforced composite plate: Experimental, numerical, and simulation”, *Adv. Polym. Tech.*, **37**(6), 1643-1657. https://doi.org/10.1002/adv.21821.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2018b), “Thermoelastic deflection responses of CNT reinforced sandwich shell structure using finite-element method”, *Scientia Iranica T. B Mech. Eng.*, **25**(5), 2722-2737. https://doi.org/10.24200/SCI.2017.4525.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2019), “Large deformation bending responses of nanotube-reinforced polymer composite panel structure: Numerical and experimental analyses”, *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.*, **233**(5), 1695-1704. https://doi.org/10.1177/0954410018761192.

Mehar, K., Panda, S.K. and Patle, B.K. (2018a), “Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach”, *Polym. Composites*, **39**(10), 3792-3809. https://doi.org/10.1002/pc.24409.

Mishnaevsky, J.L. (2007), *Computational Mesomechanics of Composites*, John Wiley & Sons, U.K.

Mori, T. and Tanaka, K. (1973), “Average stress in matrix and average elastic energy of materials with misfitting inclusions”, *Acta Metall.*, **21**(5), 571-574. https://doi.org/10.1016/0001-6160(73)90064-3.

Nemati, A.R. and Mahmoodabadi, M.J. (2019), “Effect of micromechanical models on stability of functionally graded conical panels resting on Winkler-Pasternak foundation in various thermal environments”, *Arch. Appl. Mech.*, 1-33. https://doi.org/10.1007/s00419-019-01646-6.

Nguyen, T.K., Vo, T.P. and Thai, H.T. (2013), “Static and free vibration of axially loaded functionally graded beams based on the first-order shear deformation theory”, *Compos. Part B Eng.*, **55**, 147-157. http://doi.org/10.1016/j.compositesb.2013.06.011.

Othman, M. and Fekry, M. (2018), “Effect of rotation and gravity on generalized thermo-viscoelastic medium with voids”, *Multidisciplin. Model. Mater. Struct.*, **14**(2), 322-338. http://doi.org/10.1108/MMMS-08-2017-0082.

Panjehpour, M., Loh, E.W.K. and Deepak, TJ. (2018), “Structural Insulated Panels: State-of-the-Art”, *Trends Civil Eng. Architect.*, **3**(1) 336-340. http://doi.org/10.32474/TCEIA.2018.03.000151.

Pradhan K.K. and Chakraverty, S. (2013), “Free vibration of Euler and Timoshenko functionally graded beams by Rayleigh-Ritz method”, *Compos. Part B Eng.*, **51**, 175-184. http://doi.org/10.1016/j.compositesb.2013.02.027.

Pradhan, S.C. and Murmu, T. (2009), “Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method”, *J. Sound Vib.*, **321**, 342-362. http://doi.org/10.1016/j.jsv.2008.09.018.

Rabhi, M., Benrahou, K.H., Kaci, A., Houari, M.S.A., Bourada, F., Bousahla, A.A., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R. and Tounsi, A. (2020), “A new innovative 3-unknowns HSDT for buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions”, *Geomech. Eng.*, **22**(2). 119-132. https://doi.org/10.12989/gae.2020.22.2.119

Rachedi, M.A., Benyoucef, S., Bouhadra, A., Bachir Bouiadjra, R., Sekkal, M. and Benachour, A. (2020), “Impact of the homogenization models on the thermoelastic response of FG plates on variable elastic foundation”, *Geomech. Eng.*, **22**(1), 65-80. http://doi.org/10.12989/gae.2020.22.1.065.

Rahmani, M.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H. and Tounsi, A. (2020), “Influence of boundary conditions on the bending and free vibration behavior of FGM sandwich plates using a four-unknown refined integral plate theory”, *Comput. Concrete*, **25**(3), 225-244. https://doi.org/10.12989/cac.2020.25.3.225.

Reddy, J.N. and Chin, C.D. (1998), “Thermomechanical analysis of functionally graded cylinders and plates”, *J. Therm. Stresses*, **21**(6), 593-629. http://doi.org/10.1080/01495739808956165.

Refrafi, S., Bousahla, A.A., Bouhadra, A., Menasria, A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H. and Tounsi, A. (2020), “Effects of hygro-thermo-mechanical conditions on the buckling of FG sandwich plates resting on elastic foundations”, *Comput. Concrete*, **25**(4), 311-325. https://doi.org/10.12989/cac.2020.25.4.311.

Sahla, F., Saidi, H., Draiche, K., Bousahla, A.A., Bourada, F. and Tounsi, A. (2019), “Free vibration analysis of angle-ply laminated composite and soft core sandwich plates”, *Steel Compos. Struct.*, **33**(5), 663-679. https://doi.org/10.12989/scs.2019.33.5.663.

Salah, F., Boucham, B., Bourada, F., Benzair, A., Bousahla, A.A. and Tounsi, A. (2019), “Investigation of thermal buckling properties of ceramic-metal FGM sandwich plates using 2D integral plate model”, *Steel Compos. Struct.*, **33**(6), 805-822. https://doi.org/10.12989/scs.2019.33.6.805.

Selmi, A. (2019), “Effectiveness of SWNT in reducing the crack effect on the dynamic behavior of aluminium alloy”, *Adv. Nano Res.*, **7**(5), 365-377. https://doi.org/10.12989/anr.2019.7.5.365.

Semmah, A., Heireche, H., Bousahla, A.A. and Tounsi, A. (2019), “Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT”, *Adv. Nano Res.*, **7**(2), 89-98. DOI: https://doi.org/10.12989/anr.2019.7.2.089.

Shadravan, S., Ramseyer, C.C. and Floyd, R.W. (2019), “Comparison of structural foam sheathing and oriented strand board panels of shear walls under lateral load”, *Adv. Comput. Des.*, **4**(3), 251-272. https://doi.org/10.12989/acd.2019.4.3.251.

Shariati, A., Ghabussi, A., Habibi, M., Safarpour, H., Safarpour, M., Tounsi, A. and Safa, M. (2020a), “Extremely large oscillation and nonlinear frequency of a multi-scale hybrid disk resting on nonlinear elastic foundations”, *Thin-Walled Struct.*, **154**, 106840. https://doi.org/10.1016/j.tws.2020.106840.

Shariati, A., Habibi, M., Tounsi, A., Safarpour, H. and Safa, M. (2020b), “Application of exact continuum size-dependent theory for stability and frequency analysis of a curved cantilevered microtubule by considering viscoelastic properties”, *Eng. Comput.* https://doi.org/10.1007/s00366-020-01024-9

Shen, H.S. and. Wang, Z.X. (2014), “Nonlinear analysis of shear deformable FGM beams resting on elastic foundation in thermal environment”, *Int. J. Mech. Sci.*, **81**, 195-206. http://doi.org/10.1016/j.ijmecsci.2014.02.020.

Shokrieh, M.M. and Kondori, M.S. (2020), “Effects of adding graphene nanoparticles in decreasing of residual stresses of carbon/epoxy laminated composites”, *Compos. Mater. Eng.*, **2**(1), 53-64. https://doi.org/10.12989/cme.2020.2.1.053.

Sobhy, M. (2015), “Thermoelastic response of FGM plates with temperature-dependent properties resting on variable elastic foundations”, *J. Appl. Mech.*, **7**(6), 1550082. https://doi.org/10.1142/S1758825115500829.

Sun, Y., Li, S.R. and Batra, R.C. (2016), “Thermal buckling and post-buckling of FGM Timoshenko beams on nonlinear elastic foundation”, *J. Therm. Stresses*, **39**(1), 11-26. http://doi.org/10.1080/01495739.2015.1120627.

Taj, M., Majeed, A., Hussain, M., Naeem, M.N., Safeer, M., Ahmad, M., Khan, H.U. and Tounsi, A. (2020), "Non-local orthotropic elastic shell model for vibration analysis of protein microtubules", *Comput. Concrete*, **25**(3), 245-253. https://doi.org/10.12989/cac.2020.25.3.245.

Timesli, A. (2020), "An efficient approach for prediction of the nonlocal critical buckling load of double-walled carbon nanotubes using the nonlocal Donnell shell theory", *SN Appl. Sci.*, **2**(3), 1-12. https://doi.org/10.1007/s42452-020-2182-9.

Tounsi, A., Ait Atmane, H., Khiloun, M., Sekkal, M., Taleb, O. and Bousahla, A.A. (2019), "On buckling behavior of thick advanced composite sandwich plates", *Compos. Mater. Eng.*, **1**(1), 1-19. https://doi.org/10.12989/cme.2019.1.1.001.

Tounsi, A., Al-Dulaijan, S.U., Al-Osta, M.A., Chikh, A., Al-Zahrani, M.M., Sharif, A. and Tounsi, A. (2020), "A four variable trigonometric integral plate theory for hygro-thermo-mechanical bending analysis of AFG ceramic-metal plates resting on a two-parameter elastic foundation", *Steel Compos. Struct.*, **34**(4), 511-524. https://doi.org/10.12989/scs.2020.34.4.511.

Trinh, L.C., Vo, T.P., Thai, H.T. and Nguyen, T.K. (2016), "An analytical method for the vibration and buckling of functionally graded beams under mechanical and thermal loads", *Compos. Part B Eng.*, **100**, 152-163. https://doi.org/10.1016/j.compositesb.2016.06.067.

Vo, T.P., Thai, H.T., Nguyen, T.K., Inam, F. and Lee, J. (2015), "A quasi-3D theory for vibration and buckling of functionally graded sandwich beams", *Compos. Struct.*, **119**, 1-12. https://doi.org/10.1016/j.compstruct.2014.08.006.

Wang, Y. and Wu, D. (2016), "Thermal effect on the dynamic response of axially functionally graded beam subjected to a moving harmonic load", *Acta Astronaut.*, **127**, 171-181. http://doi.org/10.1016/j.actaastro.2016.05.030.

Wattanasakulpong, N., Prusty, B.G. and Kelly, D.W. (2011), "Thermal buckling and elastic vibration of third-order shear deformable functionally graded beams", *Int. J. Mech. Sci.*, **53**(9), 734-743. https://doi.org/10.1016/j.ijmecsci.2011.06.005.

Yahiaoui, M., Tounsi, A., Fahsi, B., Bouiadjra Bouiadjra, R. and Benyoucef, S. (2018), "The role of micromechanical models in the mechanical response of elastic foundation FG sandwich thick beams", *Struct. Eng. Mech.*, **68**(1), 53-66. https://doi.org/10.12989/sem.2018.68.1.053.

Yas, M. H., Kamarian, S. and Pourasghar, A. (2017), "Free vibration analysis of functionally graded beams resting on variable elastic foundations using a generalized power-law distribution and GDQ method", *Ann. Solid Struct. Mech.*, **9**(1-2), 1-11. https://doi.org/10.1007/s12356-017-0046-9.

Zahedinejad, P. (2015), "Free vibration analysis of functionally graded beams resting on elastic foundation in thermal environment", *Int. J. Struct. Stabil. Dyn.*, **16**(7), 1550029. https://doi.org/10.1142/S0219455415500297.

Zarga, D., Tounsi, A., Bousahla, A.A., Bourada, F. and Mahmoud, S.R. (2019), "Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory", *Steel Compos. Struct.*, **32**(3), 389-410. https://doi.org/10.12989/scs.2019.32.3.389.

Zhang, D.G. (2014), "Thermal Post-buckling and nonlinear vibration Analysis of FGM beams based on physical neutral Surface and high order shear deformation theory", *Meccanica*, **49**(2), 2014, 283-293. https://doi.org/10.1007/s11012-013-9793-9.

Zhou, D. (1993), "A general solution to vibrations of beams on variable Winkler elastic foundation", *Comput. Struct.*, **47**(1), 83-90. https://doi.org/10.1016/0045-7949(93)90281-H.

Zimmerman, R.W. (1994), "Behavior of the Poisson ratio of a two-phase composite material in the high-concentration limit", *Appl. Mech. Rev.*, **47**(1), 38-44. https://doi.org/10.1115/1.3122819.

Zine, A., Bousahla, A.A., Bourada, F., Benrahou, K.H., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R. and Tounsi, A. (2020), "Bending analysis of functionally graded porous plates via a refined shear deformation theory", *Comput. Concrete*, **26**(1), In Press.

Zuiker, J.R. (1995), "Functionally graded materials-choice of micromechanics model and limitations in property variation", *Compos. Eng.*, **5**(7), 807-819. https://doi.org/10.1016/0961-9526(95)00031-H.

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