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Optimal design of stone columns reinforced soft clay foundation considering design robustness

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Abstract. Stone columns are widely used to treat soft clay ground. Optimizing the design of stone columns based on cost-effectiveness is always an attractive subject in the practice of ground treatment. In this paper, the design of stone columns is optimized using the concept of robust geotechnical design. Standard deviation of failure probability, which is a system response of concern of the stone column-reinforced foundation, is used as a measure of the design robustness due to the uncertainty in the coefficient of variation (COV) of the noise factors in practice. The failure probability of a stone column-reinforced foundation can be readily determined using Monte Carlo simulation (MCS) based on the settlements of the stone column-reinforced foundation, which are evaluated by a deterministic method. A framework based on the concept of robust geotechnical design is proposed for determining the most preferred design of stone columns considering multiple objectives including safety, cost and design robustness. This framework is illustrated with an example, a stone column-reinforced foundation under embankment loading. Based on the outcome of this study, the most preferred design of stone columns is obtained.

Keywords: soft clay ground; stone columns; failure probability; Monte Carlo simulation; robust geotechnical design

1. Introduction

Construction of embankments and structures on soft clay, which has the disadvantages of high compressibility, low permeability, and low bearing capacity, has been a challenging task for geotechnical engineers. Therefore, ground improvement methods are always required in practice (Deb *et al.* 2012, Miranda *et al.* 2015). As one of the best effective and economical soft clay ground treatment method, stone column (or gravel pile) has been widely used in the last decades (Chen *et al.* 2008, Lu *et al.* 2010, Zhang *et al.* 2013, Demir and Sarici 2017, Etezad *et al.* 2018). The stone columns improve the bearing capacity and reduce the post-construction settlement of the soft clay foundation, which is of importance for the safety and serviceability of the infrastructures built on it (Ambily and Gandhi 2007, Xie *et al.* 2009, Keykhosropur *et al.* 2012, Miranda *et al.* 2015, Zhou *et al.* 2019). In the design of stone columns, the area replacement ratio and column length of the stone column are the key parameters. However, these two parameters are often selected based on designer's experience and the construction arrangement due to the lack of guidance for stone column design. Thus, how to optimize the design of stone columns (e.g., area replacement ratio and column length) still needs to be further investigated.

Optimizing the design of stone columns had been an attractive subject in geotechnical engineering. Based on parameter sensitivity analysis, Liu and Hutchinson (2018) investigated the impact of area replacement ratio and column length of the stone column on the seismic response of the composite foundation. Dash and Bora (2013) carried out a series of experiments, which indicated the optimal length and spacing of stone columns giving maximum performance improvement were, respectively, 5 times and 2.5 times of their diameter. Madun *et al.* (2018) investigated the effect of column diameter and length on loading capacity and settlement, based on which the optimal diameter and length of stone columns were determined using response surface. Black *et al.* (2011) found in physical model tests that the preferred area replacement ratio was in the range of 30% to 40% from the perspective of settlement control. These researches assume that the geotechnical parameters are deterministic. Few studies involve the uncertainties of the geotechnical parameters in the design of stone columns. The method of robust geotechnical design (RGD), which has been successfully applied in the field of geotechnical engineering, provides a possible way to solve this problem.

The RGD was first proposed by Juang *et al.* (2013a, b). In the RGD, the input parameters are divided into two categories, the design parameters and noise factors. The design parameters could be specified by the designer, such as area replacement ratio and column length of the stone column. The noise factors have significant uncertainties that cannot be completely eliminated, such as soil parameters. A

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design is considered robust if the variation of the geotechnical system response is insensitive to the variation of noise factors (Juang *et al.* 2014, Khoshnevisan *et al.* 2014, Yu *et al.* 2019a, b). As the variation of the noise factors cannot be fully eliminated, the essence of RGD is to reduce the variation in the system response by adjusting design parameters (Juang *et al.* 2014).

With the RGD approach, the focus is to satisfy three design requirements, namely safety, cost, and robustness. The goal of RGD is to seek a most preferred design in the design space such that the cost is minimized and the design robustness is maximized, while the safety requirements are satisfied (Gong *et al.* 2016, 2017). In general, the desire to maximize the design robustness and the desire to minimize the cost are two conflicting design objectives in the RGD. Thus, the Pareto front (Deb *et al.* 2002), which consists a set of non-dominated designs that present a trade-off between design robustness and cost, can help render an informed design decision. Further, the knee point (Deb and Gupta, 2011) on the Pareto front, which yields the best-compromise solution with respect to the conflicting objectives, may be identified and taken as the most preferred design in the design space.

Based on the concept of robust design for geotechnical engineering, this paper considered the uncertainty of mechanical parameters of soft clay foundation, and established the comprehensive optimization method for design parameters of stone columns with multi objectives of cost, safety and robustness. The rest of this paper is arranged as follows. A deterministic method for evaluating the settlement of a stone column-reinforced foundation is introduced in section 2. Methodology for robust geotechnical design of the stone columns, in which the system response (failure probability) of the stone column-reinforced foundation is considered, is presented in section 3. In section 4, the framework for determining the most preferred design of stone columns is summarized. The proposed method and framework are illustrated with an example in section 5 and discussed in section 6.

2. Soft clay foundation reinforced with stone columns

2.1 Design concept

Fig. 1 shows the conceptual model of stone columns analyzed in this paper. Stone columns, which are formed by filling with crushed stones or granular soils in vertical boreholes in the soft clay and then being compacted by means of a vibrator (Castro 2017). The stone column materials are always stiffer and stronger than the soft clay. Therefore, the stone columns sustain larger proportion of the applied load than the counterpart of soft clay, which leads to significant improvement of bearing capacity and reduction of post-construction settlement of the foundation (Ambily and Gandhi 2007, Dash and Bora 2013). In this paper, we assume that the bearing capacity of the stone column-reinforced foundation could reach its target value, and the focus is to optimize the design of stone columns from the perspective of settlement control.

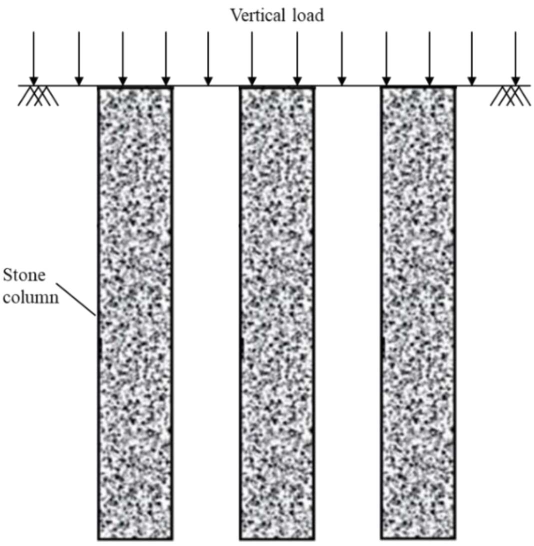


Fig. 1 Illustration of design concept of stone columns

2.2 Deterministic method for settlement evaluation

The settlement of a stone column-reinforced foundation subjected to vertical loads consists two components (as shown in Fig. 2):

$$S = S_1 + S_2 \tag{1}$$

where S, S_1, and S_2 are total settlement of the stone column-reinforced foundation, settlement of reinforcement zone, and settlement of subjacent unreinforced layer, respectively. As the stone columns and the surrounding soil are herein regarded as a composite material. Dividing the composite material into many sublayers, the corresponding settlement, S_1, is usually calculated by the composite modulus method as follows:

$$S_1 = \sum_{1}^{n} \frac{\Delta P_i}{E_{csi}} H_i \tag{2}$$

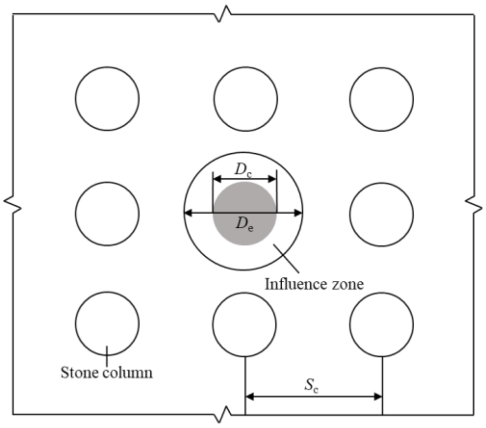
where P_i is the stress increment at the middle of the ith sublayer; H_i is the thickness of the ith sublayer; and E_{csi} is the composite modulus of soil and stone columns of the ith sublayer.

In recent years, several methods for calculating composite modulus have been proposed (e.g., Zhang 1992, Zheng *et al.* 2018), which will be compared in the section of discussion. While in this context, the composite modulus is calculated by the area weighted method (Technical code for ground treatment of buildings 2012) that commonly used in practice:

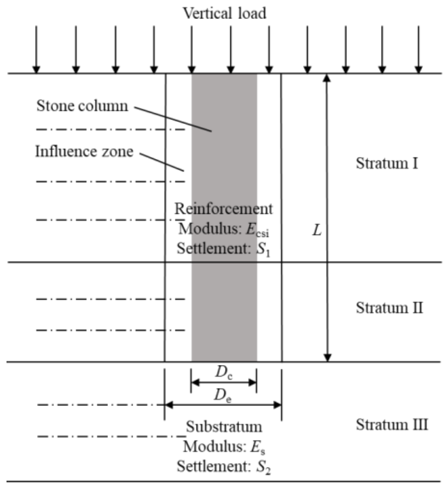
$$E_{csi} = mE_c + (1-m)E_{si} \tag{3}$$

where E_c is the constrained modulus of stone columns; E_{si} is the constrained modulus of soil between columns in the ith sublayer; and m is the area replacement ratio of the stone column, which is defined as:

$$m = \frac{A_c}{A} = \frac{D_c^2}{D_e^2} \tag{4}$$



(a) Cross section



(b) Vertical profile

Fig. 2 Schematic diagram of composite foundation reinforced with stone columns

$$D_e = \alpha S_c \tag{5}$$

where A_c and A are the area of the stone column and total influence zone, respectively; D_c is the diameter of the stone column; D_e is the equivalent diameter of the influence zone; S_c is the center to center spacing between two stone columns; and α is the geometry-dependent constant equal to 1.05, 1.13, and 1.29 for triangular, square, and hexagon patterns, respectively.

According to the layerwise summation method (Code for design of building foundations 2011), the settlement of subjacent unreinforced layer, S_2, can be expressed as:

$$S_2 = \sum_1^n \frac{\Delta P_i}{E_{si}} H_i \tag{6}$$

When the stone columns are designed to rest on a hard stratum, the value of S_2 is equal to zero.

3. Methodology for robust geotechnical design of the stone columns

3.1 Robust geotechnical design concept and parameter setting

Based on the settlement calculation method of stone column-reinforced soft clay foundation mentioned in section 2, the area replacement ratio, column length (the depth of the composite material), constrained modulus of stone columns, and the constrained modulus of soil between columns are the main factors affecting the settlement of the stone column-reinforced foundation. Among them, the area replacement ratio (m) and column length (L) are commonly determined by designers, so that they are treated as the "design parameters" here. While the constrained modulus of stone columns (E_c) and constrained modulus of soil between columns (E_s), are usually quite uncertain due to the variation of geotechnical parameters, thus they are treated as "noise factors" in the framework of RGD.

As the noise factors (E_c and E_s) exhibit significant uncertainty, which leads to the uncertainty in the settlement of the stone column-reinforced soft clay foundation evaluated by the deterministic method in section 2. This makes it difficult for designers to determine an optimal design of stone columns. Therefore, the RGD is adopted here to carefully select design parameters of the stone columns, which can make the settlement of the stone column-reinforced soft clay foundation insensitive or robust to the variation of noise factors, while the safety requirements are satisfied.

3.2 Method for evaluating design robustness

In the RGD, an effective measure of design robustness is required. The variation in failure probability, feasibility robustness, signal-to-noise ratio, and sensitive index based on gradient of the system response are suggested to measure the design robustness (Khoshnevisan *et al.* 2014). In this paper, the variation in failure probability, which is a system response of concern and has been used frequently in geotechnical applications (Juang and Wang 2013, Wang *et al.* 2013, Xu *et al.* 2014), is adopted as a measure of design robustness. Using variation in failure probability to measure robustness can overcome the difficulty of evaluating the coefficient of variation (COV) of the noise factors. In other words, the uncertainty of the COV of the noise factors could be considered in the variation in failure probability. A smaller variation (in terms of standard deviation) of the failure probability for a given design (i.e., a pair of m and L) indicates a better design robustness.

Monte Carlo simulation (MCS) is adopted in order to evaluate the standard deviation of failure probability of a stone column-reinforced foundation. Assuming N_s sets of COVs for noise factors are randomly generated. For each set of COVs, N_t sets of noise factors are randomly generated by MCS. For a given design of stone column, which is denoted as $\mathbf{d}$, the settlement of the stone column-reinforced foundation corresponding to each of the N_t sets of noise factors can be evaluated using the deterministic method in section 2. Let $\mathbf{X}_{k,j}$ ($k = 1, 2, \ldots, N_s$; $j = 1, 2, \ldots, N_t$) be the jth set of the noise factors generated based on the kth set of

COVs, the corresponding probability for the settlement exceeding its allowable value, which is defined as the failure probability of the give design **d**, could be obtained by:

$$P_{\mathrm{f}}(\mathbf{d},\mathbf{X}_{\mathrm{k,j}})=\frac{1}{N_{\mathrm{t}}}\sum_{j=1}^{N_{\mathrm{t}}}I(\mathbf{d},\mathbf{X}_{\mathrm{k,j}}) \tag{7}$$

where $I(\mathbf{d}, \mathbf{X}_{\mathrm{k,j}})$ is an indicator function, which is defined as follows: if $G(\mathbf{d}, \mathbf{X}_{\mathrm{k,j}}) < 0$ then $I(\mathbf{d}, \mathbf{X}_{\mathrm{k,j}}) = 1$, otherwise $I(\mathbf{d}, \mathbf{X}_{\mathrm{k,j}}) = 0$. $G(\mathbf{d}, \mathbf{X}_{\mathrm{k,j}})$ is the system performance function of the given design **d**, which can be expressed as:

$$G(\mathbf{d},\mathbf{X}_{\mathrm{k,j}})=S_0-S_{\mathrm{k,j}} \tag{8}$$

where $S_{\mathrm{k,j}}$ is the settlement of the given design calculated using deterministic method, in which $\mathbf{X}_{\mathrm{k,j}}$ is the input parameters; and S_0 is the allowable settlement.

Repeat the procedure for computing the failure probability for each of N_{s} sets of COVs, N_{s} sets of failure probabilities would be obtained for the given design **d**. Thus, based on the reliability theory (e.g., Ang and Tang 2004, Lü *et al.* 2017, Zhang *et al.* 2017), the standard deviation of N_{s} sets of failure probabilities, which is the measure for design robustness of the given design **d**, could be obtained:

$$\sigma_{\mathrm{p}}(\mathbf{d})=\sqrt{\frac{1}{N_{\mathrm{s}}-1}\sum_{k=1}^{N_{\mathrm{s}}}\left[P_{\mathrm{f}}(\mathbf{d},\mathbf{X}_{\mathrm{k,j}})-\mu_{\mathrm{p}}(\mathbf{d})\right]^2} \tag{9}$$

$$\mu_{\mathrm{p}}(\mathbf{d})=\frac{1}{N_{\mathrm{s}}}\sum_{j=1}^{N_{\mathrm{s}}}P_{\mathrm{f}}(\mathbf{d},\mathbf{X}_{\mathrm{k,j}}) \tag{10}$$

where $\sigma_{\mathrm{p}}(\mathbf{d})$ is the standard deviation of the N_{s} sets of failure probabilities; and $\mu_{\mathrm{p}}(\mathbf{d})$ is the mean of N_{s} sets of failure probabilities. If $\mu_{\mathrm{p}}(\mathbf{d})$ is not larger than the target failure probability (safety requirement), design **d** would be feasible design.

Repeat the above mentioned procedure for each design of the stone columns, the feasible designs and their design robustness, in terms of standard deviation of failure probability, could be obtained.

3.3 Cost evaluation

The cost of stone columns is mainly composed of the material and labor costs, which are a function of the total volume of the stone columns used in the project. It should be noted that the cost function is often dependent on local practice and experience. For simplicity and illustrative purpose, the volume per unit influence zone area of the stone columns is adopted in this paper to evaluate the cost:

$$C=\frac{A_{\mathrm{c}}L}{A}=mL \tag{11}$$

4. Framework for selecting the most preferred design of stone columns

The framework for robust geotechnical design of stone columns is presented by the flowchart shown in Fig. 3. This framework is illustrated in the following steps:

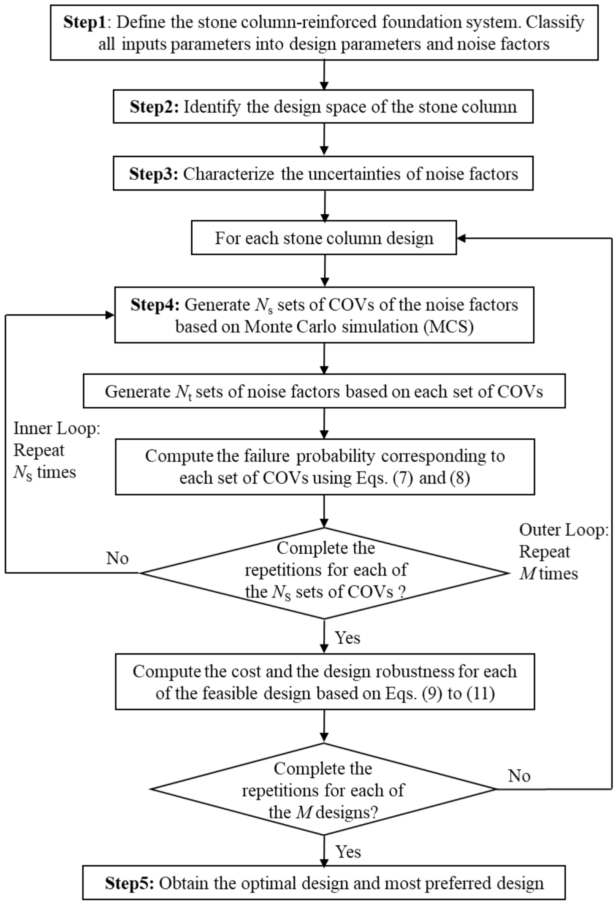


Fig. 3 Framework for robust geotechnical design of the stone columns

Step 1: Define the stone column-reinforced foundation system and then classify all input parameters into two groups, the design parameters and the noise factors. Recall that the design parameters of stone columns are the area replacement ratio (m) and column length (L) of the stone column. The noise factors are the constrained modulus of stone columns (E_{c}) and the constrained modulus of soil between columns (E_{s}).

Step 2: Identify the design space of the stone column. Discrete numbers of design parameters are selected based upon their typical ranges, construction arrangement and experience from similar projects. The combination of different design parameters (i.e., m and L) composes the design space, which is denoted by the number M.

Step 3: Characterize the uncertainties of noise factors. The variation of noise factors might be attributed to the inherent variability, measuring error, and transformation uncertainty. In a typical geotechnical practice, the uncertainty of noise factors (i.e., E_{c} and E_{s}) could be evaluated based on the data from geological survey and geotechnical tests, augmented with literatures or local experiences.

Step 4: Compute the cost and design robustness for each of the feasible designs that meets the safety requirement. As describe in section 3.2, for a given design of stone column,

N_s sets of COVs are generated for the noise factors. Based on each set of COVs, N_t sets of noise factors are generated. The failure probability for the give design and COV is calculated using Eqs. (7) and (8). Repeat this computational process for all the N_s sets of COVs, which completes the inner loop in Fig. 3. Then the standard deviation and mean of the failure probabilities are obtained using Eqs. (9) and (10), based on which the design robustness of a feasible design is obtained. The corresponding cost of the feasible design is obtained using Eq. (11). Repeat this computational process for each of the M designs, which completes the outer loop indicated in Fig. 3.

Step 5: Obtain the optimal design and most preferred design according to the Pareto front and knee point, respectively. Within the feasible designs, a Pareto front could be obtained based on the trade-off relationship between design robustness and cost computed in Step 4 (i.e., outer loop in Fig. 3). On this Pareto front, a knee point could be found by finding the design which has the minimum distance from the utopia point (an unrealistic design with low cost and high design robustness). For more details to establish the Pareto front and knee point, the readers are referred to the literatures (e.g., Deb and Gupta 2011, Khoshnevisan *et al.* 2014).

This entire framework will be illustrated with a stone column-reinforced soft clay foundation in the following section.

5. Illustrative examples

5.1 Brief summary of the foundation

The stone column-reinforced soft clay foundation applied as the ground treatment technical at Gaoyao-Haikou expressway in Yunnan Province, China, which is reported by Sun (2005), is adopted and modified as an example to illustrate the framework proposed in section 4. The embankment geometry, subsoil properties and the layout of stone columns are shown in Fig. 4. The dimensions of the embankment are as follows: the crest width is 25 m; the height is 5 m; and the side slope is 1:1.5 on both sides. The distributed load of embankment is 95 kPa. The stone column is 15 m in length with a diameter of 1 m, and spacing of 1.8 m. They are arranged in the form of a triangular, and the area replacement ratio of the stone column is 28%. Parameters of the stone column and soil are summarized in Table 1. It is required that the allowable settlement is 85 cm.

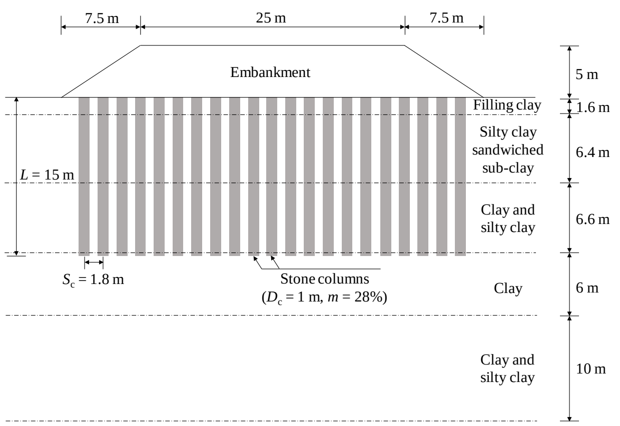


Fig. 4 An embankment supported by the stone column-reinforced soft clay foundation (after Sun (2005))

Table 1 Column and soil parameters of the stone column-reinforced foundation

Column parameters			Soil parameters				
			Filling clay	Silty clay sandwiched sub-clay	Clay and silty clay	Clay	Clay and silty clay
m	L (m)	E_c (MPa)	E_s (MPa)				
28%	15	17.5	2.98	2.58	2.82	5.3	2.16

Table 2 Statistics of the COV of noise factors

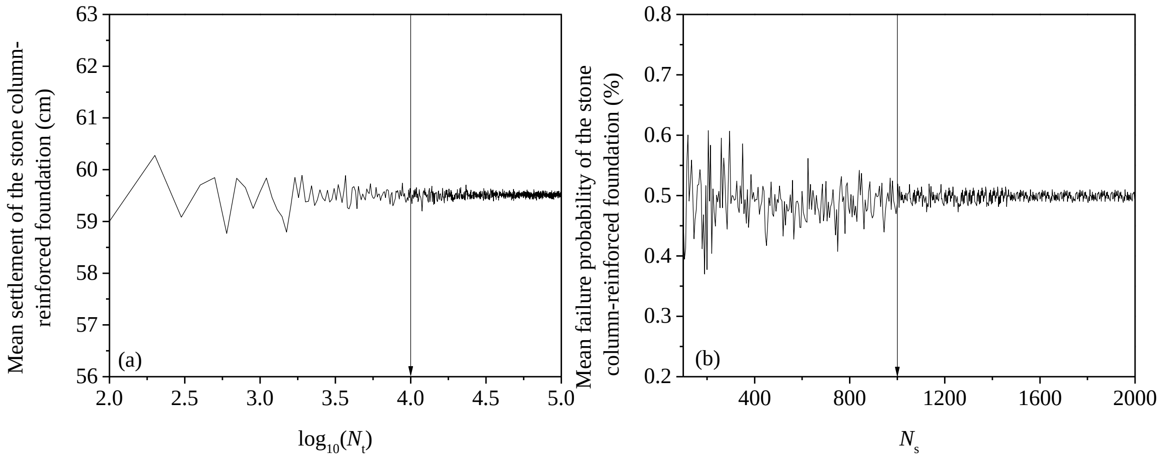
Parameters	Range	Mean	Coefficient of variation	Distribution
$COV[E_c]$	20% - 44%	0.32	19%	Lognormal
$COV[E_s]$	10% - 50%	0.3	33%	Lognormal

5.2 Geotechnical characterization

The noise factors are the uncertain parameters that exhibit a significant effect on the performance of the stone column-reinforced foundation; for this example, they are the constrained modulus of stone columns (E_c) and constrained modulus of soil between columns (E_s). The mean values of E_c and E_s are listed in Table 1 according to Sun (2005). In geotechnical practice, the statistics of geotechnical properties (i.e., COVs of E_c and E_s) are usually difficult to ascertain. However, with the aid of published COVs of geotechnical properties and engineering judgment, the statistics may be estimated as a range for the design example of a stone column-reinforced foundation. For instance, the COV of E_c, denoted as $COV[E_c]$, typically ranges from 20% to 44% (Zheng *et al.* 2018); the COV of E_s, denoted as $COV[E_s]$, typically ranges from 10% to 50% (Phoon and Kulhawy, 1999).

As an example to illustrate the framework of RGD presented herein, the statistics of the COV of noise factors may be estimated using the three-sigma rule (Duncan, 2000). For example, the mean of $COV[E_c]$, denoted as $\mu_{COV[E_c]}$, is assumed to be 0.32, and the coefficient of variation of $COV[E_c]$, denoted as $\delta_{COV[E_c]}$, is assumed to be 19% (roughly to cover the typical range of $COV[E_c]$). Similarly, the mean of $COV[E_s]$, denoted as $\mu_{COV[E_s]}$, is assumed to be 0.3, and the coefficient of variation of $COV[E_s]$, denoted as $\delta_{COV[E_s]}$, is assumed to be 33% (roughly to cover the typical range of $COV[E_s]$). Furthermore, all random variables mentioned above are assumed to follow lognormal distribution, and these random variables are further assumed independent from each other. The statistics of the COV of noise factors are listed in Table 2.

The m and L (in terms of design parameters) of the stone column are 28% and 15 m, respectively. As an illustrative



(a) (b)

Fig. 5 Determination for the proper number of samples generated in MCS: (a) N_t and (b) N_s

example, we assume m ranging from 8% to 40% with an increment of 4%, and L ranging from 10 m to 20 m with an increment of 1 m. Thus, 99 designs are obtained based upon the combination of the design parameters (m and L), and compose the design space.

5.3 Selection of the preferred design of the stone column

Because MCS is used to evaluate the design robustness in this paper. To do this, N_s (the number of randomly generated COVs) and N_t (the number of randomly generated noise factors) should be determined firstly. Fig. 5(a) depicts the effect of N_t on the variation of mean settlement of the stone column-reinforced foundation under a given design with m = 28%, L = 15 m, $COV[E_c]$ = 0.32 and $COV[E_s]$ = 0.3. The result shows that, at a MCS run of 10,000, the variation of mean settlement of the stone column-reinforced foundation tends to be stable. As a result, N_t = 10,000 is used for each possible design here. Based on these 10,000 sets of noise factors, the effects of N_s on mean failure probability of the stone column-reinforced foundation are evaluated and shown in Fig. 5(b). It can be seen that the variation of mean failure probability of the stone column-reinforced foundation tends to be stable with N_s = 1,000. For each design of stone column in the design space, 1,000 sets of COVs are generated for the noise factors. Based on each set of COVs, 10,000 sets of noise factors are generated. To complete the loop calculation of robust design of all designs, the framework proposed in this paper is implemented using Matlab code to process the data efficiently. The total computational time required to run the code once is 2428 s (about 40 minutes), which is recorded using the Windows 7® PC equipped with a 8.0 GB RAM and an Intel® Core™ i7-3537U CPU running at 2.50 GHz.

With N_s = 1,000 and N_t = 10,000, the mean of failure probabilities for a given design ($\mu_p(\mathbf{d})$) can be readily computed using Eq. (10). The target failure probability adopted in this example is P_T = 0.01. For each of the 99 designs, the cost and standard deviation of failure probabilities ($\sigma_p(\mathbf{d})$) of designs with $\mu_p(\mathbf{d}) \leq 0.01$ are calculated based on Eq. (11) and Eq. (9). Thus, out of the 99

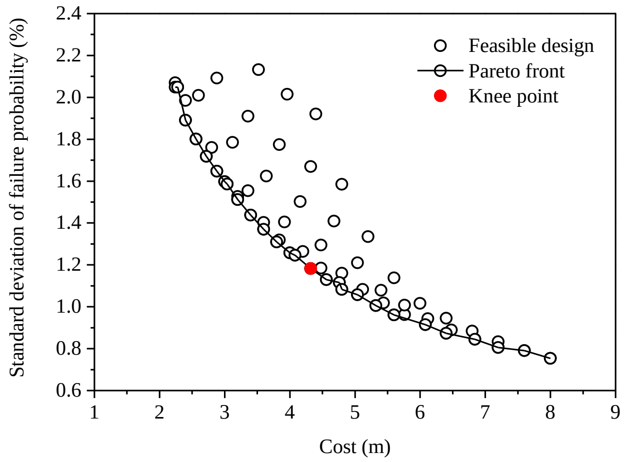


Fig. 6 Feasible designs, Pareto front and knee point with bi-objectives (Cost and design robustness)

designs in design space, 65 feasible designs are obtained and presented in Fig. 6.

Following the procedure proposed by Deb and Gupta (2011) and Khoshnevisan *et al.* (2014), the Pareto front and knee point are obtained and presented in Fig. 6. The Pareto front includes 29 non-dominated designs, among which none design is superior to another on both objectives of cost and design robustness. As expected, a higher cost is required in order to increase the design robustness (identified by a reduction in $\sigma_p(\mathbf{d})$). In practice, the designer can select an optimal design on the Pareto front based on the perspective of the project (i.e., designated robustness or cost).

If the perspectives of robustness and/or cost are not designated, the knee point could be used to find the most preferred design, which can find the balance between the design robustness and cost. In the context of RGD, the knee point may be obtained based on the minimum distance with respect to the utopia point, which is an unrealistic design that has the lowest cost and the highest design robustness (Khoshnevisan *et al.* 2014). To begin with, a transformation, which normalizes the objective function into a value ranging from 0 to 1, is usually taken:

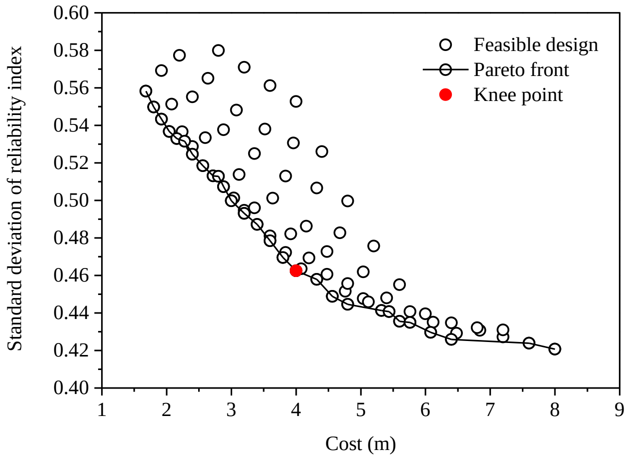


Fig. 7 Result of the RGD with $\sigma_\beta(\mathbf{d})$ as the robustness measure

$$X_N = \frac{X_b - [X_b]_{min}}{[X_b]_{max} - [X_b]_{min}} \tag{12}$$

where $[X_b]_{max}$ and $[X_b]_{min}$ are the maximum and minimum values of the bth objective function X_b, respectively; and X_N is the normalized value of the bth objective function X_b.

After the normalization, the coordinates of the utopia point are all equal to 0 or 1. When the cost and $\sigma_p(\mathbf{d})$ are the two adopted objective functions, the coordinates of the utopia point are (0, 0) (lowest cost and lowest $\sigma_p(\mathbf{d})$). With the minimum distance approach (Khoshnevisan *et al.* 2014), the Euler distance from the normalized utopia point to the normalized objective function for each non-dominated design on the Pareto front is computed as:

$$l_e^n = \sqrt{\left(x_1^n - 0\right)^2 + \left(x_2^n - 0\right)^2} \tag{13}$$

where l_e^n is the Euler distance between the nth non-dominated design and the utopia point; and x_i^n is the value of the ith objective for the nth non-dominated design. For example, x_1^n is the cost of the nth non-dominated design; and x_2^n is the value of $\sigma_p(\mathbf{d})$ of the nth non-dominated design.

Based on the calculated Euler distances, the non-dominated design that yields the minimum Euler distance is taken as the knee point. As can be seen in Fig. 6, on the upper side of the knee point (marked as red ball), a slight improvement of design robustness requires a large increase in cost, which is not desirable. On the other side of the knee point, a slight reduction in cost yields a large reduction of design robustness, which is also not desirable (Khoshnevisan *et al.* 2014). Therefore, the knee point with design parameters m = 24% and L = 18 m, which represents the best compromise between these two objectives (design robustness and cost), is considered the most preferred design among all non-dominated designs on the Pareto front.

It is worth noting that the reliability index, β has a direct relation with the failure probability, p_f. To be specific, $p_f = 1 - \Phi(\beta)$, where Φ is the cumulative distribution function of a standard normal distribution. The optimal design associated with the robustness measure, $\sigma_p(\mathbf{d})$, may be different from that associated with the robustness measure, standard deviation of reliability index $\sigma_\beta(\mathbf{d})$. Fig. 7 depicts the result of the RGD with $\sigma_\beta(\mathbf{d})$ as the robustness measure, in which the target reliability index of 2.33 (corresponding to P_T = 0.01) is adopted as the safety requirement. The optimal design, which is m = 20% and L = 20 m with a cost =4 m, is obtained out of the 80 feasible designs. Compared to the optimal design (m = 24% and L = 18 m with a cost = 4.32 m) obtained with $\sigma_p(\mathbf{d})$ as the robustness measure, it can be seen that a little higher cost is required for the optimal design if the standard deviation of reliability index is used to measure design robustness.

6. Further discussions

6.1 Influence of allowable settlement on the preferred designs

As the allowable settlement (S_0) of the stone column-reinforced foundation may be different in different projects. In this section, S_0 = 85 cm, 80 cm, 75 cm, and 70 cm are assumed to investigate its influence on the most preferred design of the stone column.

Fig. 8 shows the Pareto fronts and knee points with the assumed allowable settlements. As the allowable settlement decreases, the failure probabilities of all feasible designs increase. The stone column-reinforced foundations corresponding to the designs with smaller m and L have larger settlements, the mean of failure probability ($\mu_p(\mathbf{d})$) corresponding to each of these designs (i.e., designs with smaller m and L) increases, which results in $\mu_p(\mathbf{d}) > P_T$ (P_T = 0.01). Thus, higher settlement requirement (smaller S_0) results in the decrease of the number of feasible designs. The most preferred designs with different allowable settlements are listed in Table 3. It can be seen that the cost of the most preferred design increases as the allowable settlement decreases. This means that more costs are needed in order to fulfill the stricter settlement requirement.

6.2 Influence of target failure probability on the preferred designs

As the resulting feasible designs, Pareto front, and knee point are all affected by the choice of the target failure probability (P_T). P_T = 0.01, 0.005, and 0.001 are adopted to examine effect of target failure probability on the most preferred design.

Table 3 Results of RGD with five different levels of S_0

S_0 (cm)	Number of non-dominated designs	Knee point on the Pareto front		Two objectives at the knee point	
		m	L (m)	Robustness	Cost (m)
85	29	24%	18	0.0123	4.32
80	20	24%	20	0.0154	4.8
75	14	28%	20	0.0171	5.6
70	8	32%	20	0.0183	6.4

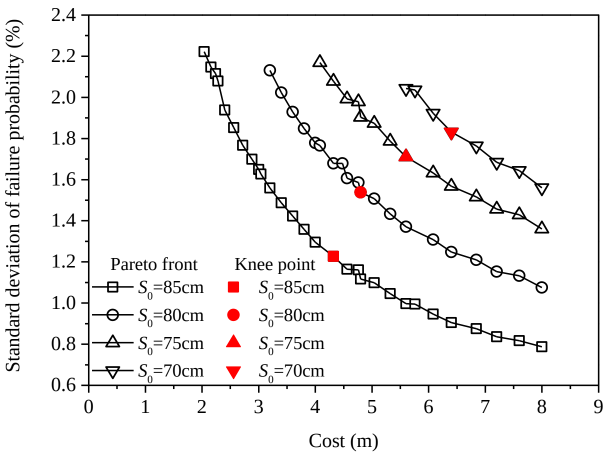


Fig. 8 Pareto fronts and knee points with five different allowable settlements

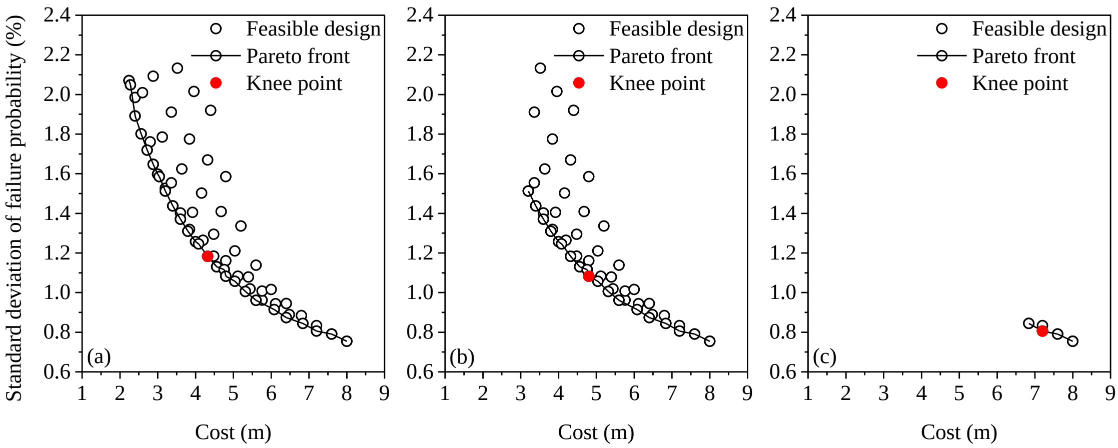


Fig. 9 Feasible designs, Pareto fronts and knee points with five different levels of P_T

Table 4 Results of RGD with five different levels of P_T

P_T	Number of non-dominated designs	Knee point on the Pareto front		Two objectives at the knee point	
		m	L (m)	Robustness	Cost (m)
0.01	29	24%	18	0.0118	4.32
0.005	19	24%	20	0.0108	4.8
0.001	4	36%	20	0.0081	7.2

The feasible designs, Pareto fronts and knee points with the assumed target failure probabilities are shown in Fig. 9. The result indicates that as the target failure probability decreases (or a higher safety level is demanded), fewer feasible designs and non-dominated designs on the Pareto front could be secured. This could be explained that as the P_T decreases, the designs with lower cost cannot meet the safety requirement ($\mu_p(\mathbf{d}) < P_T$) anymore. Table 4 presents the most preferred designs with different levels of P_T. It is shown that the most preferred design will have a higher cost and higher design robustness as the P_T decreases. This means that if a higher safety requirement is demanded, more costs are need.

6.3 Influence of allowable settlement on the preferred designs

As mentioned previously, a stone column-reinforced foundation is usually considered as a composite foundation, in which the composite modulus is used to calculate the settlement of reinforcement zone. At present, the area weighted method (Technical code for ground treatment of buildings, 2012) adopted in this paper is commonly used in practice due to its simplicity. However, other methods may also be used. To investigate the influence of different methods for composite modulus calculation on the preferred designs, the methods proposed by Zhang (1992) and Zheng *et al.* (2018) are introduced here. Zhang (1992) proposed a composite modulus formula based on the elastic theory and considering the interaction between stone column and soil, which is not considered in area weighted method (Technical code for ground treatment of buildings, 2012). Recently, Zheng *et al.* (2018) introduced the plasticity theory into the composite modulus solution. A detailed description of these two methods proposed by Zhang (1992) and Zheng *et al.* (2018) is presented in the Appendix.

In the methods proposed by Zhang (1992), the constrained modulus of stone columns (E_c), Poisson's ratio of stone columns (ν_c), constrained modulus of soil between columns (E_s), and Poisson's ratio of soil between columns (ν_s) are the four noise factors. The mean values of Poisson's ratio are assumed to be 0.25 for ν_c and 0.4 for ν_s according

Table 5 The statistics of COV of noise factors with three different methods

Method in Code (2012)		Method of Zhang (1992)		Method of Zheng *et al.* (2018)	
Mean of $COV[E_c]$	0.32	Mean of $COV[E_c]$	0.32	Mean of $COV[E_c]$	0.32
Mean of $COV[E_s]$	0.3	Mean of $COV[E_s]$	0.3	Mean of $COV[E_s]$	0.3
Coefficient of variation of $COV[E_c]$	19%	Mean of $COV[v_c]$	0.18	Mean of $COV[v_s]$	0.3
Coefficient of variation of $COV[E_s]$	33%	Mean of $COV[v_s]$	0.3	Mean of $COV[c]$	0.3
		Coefficient of variation of $COV[E_c]$	19%	Mean of $COV[\varphi]$	0.08
		Coefficient of variation of $COV[E_s]$	33%	Coefficient of variation of $COV[E_c]$	19%
		Coefficient of variation of $COV[v_c]$	21%	Coefficient of variation of $COV[E_s]$	33%
		Coefficient of variation of $COV[v_s]$	33%	Coefficient of variation of $COV[v_s]$	33%
				Coefficient of variation of $COV[c]$	17%
				Coefficient of variation of $COV[\varphi]$	25%

to Sun (2005). The COV of v_c, denoted as $COV[v_c]$, typically ranges from 10% to 25% (Deng *et al.* 2003); the COV of v_s, denoted as $COV[v_s]$, typically ranges from 10% to 50% (Jimenez and Sitar, 2009). In the study of Zheng *et al.* (2018), there are five noise factors, which include the constrained modulus of stone columns (E_c), the constrained modulus (E_s), Poisson's ratio (v_s), cohesion (c), and friction angle (φ) of soil between columns. The mean values of c and φ are assumed to be 10 kPa and 15° (Sun, 2005), respectively. According to the literature (Juang and Wang, 2013), the COV of c, denoted as $COV[c]$, typically ranges from 20% to 40%; the COV of φ, denoted as $COV[\varphi]$, typically ranges from 4% to 11%. For comparison purpose, equal values of E_c and E_s in the three methods are assumed. The statistics of the COV of noise factors with the three different methods are presented in Table 5. For simplicity, all random variables listed in Table 5 are assumed to be independent and lognormally distributed.

With the same computational procedure as section 5, the Pareto front and knee point corresponding to each method are obtained and presented in Fig. 10. Based on the criterion of design robustness (in terms of $\sigma_p(\mathbf{d})$), the performance of the three composite modulus calculation methods is ranked in sequence (from best to worst): Code (2012) > Zhang (1992) > Zheng *et al.* (2018). Because more noise factors are introduced due to the consideration of the interaction between stone column and soil and plasticity of the soil in the methods of Zhang (1992) and Zheng *et al.* (2018), the robustness of these two methods is not as good as the area weighted method (Technical code for ground treatment of buildings, 2012). Only from the perspective of design robustness, the simple model is preferred than the complex

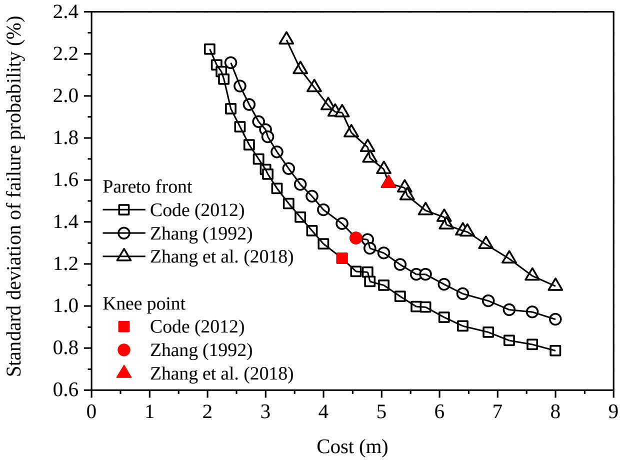


Fig. 10 Pareto fronts and knee points with three different methods

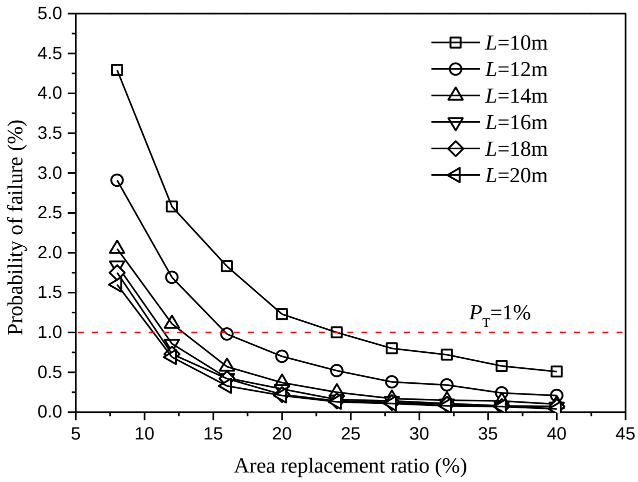


Fig. 11 Failure probabilities for selected designs obtained using system reliability approach with fixed statistics of geotechnical parameters

model. However, it should be noted that accuracy is a very important index in the design of stone column-reinforced foundation. In practice, more efforts should be devoted to the selection of input parameters and the evaluation of the uncertainty of noise factors. Thus, both accuracy and robustness of the complex method could be guaranteed.

6.4 Comparison with the traditional reliability-based design approach

Compared with the deterministic design method, the traditional reliability-based design (RBD) approach, which has been advocated by many scholars, can calculate the structural failure probability and consider the influence of the design parameter uncertainty on the design result. Although the RBD approach can consider uncertainties explicitly, an accurate statistical characterization of uncertainties of geotechnical parameters is necessary, which is often a challenge in geotechnical practice. Underestimation of the variability in geotechnical parameters may lead to a violation of safety constraints even if the design is conducted using the RBD approach.

Table 6 Least-cost designs under various COV assumptions of E_c and E_s

$COV[E_s]$	$COV[E_c]$	m	L (m)	Cost (m)
0.1	0.2	8%	12	0.96
0.1	0.32	8%	12	0.96
0.1	0.44	8%	14	1.12
0.3	0.2	12%	14	1.68
0.3	0.32	12%	16	1.92
0.3	0.44	12%	18	2.16
0.5	0.2	16%	18	2.88
0.5	0.32	20%	16	3.2
0.5	0.44	20%	20	4

However, in the proposed RGD approach, the measure of design robustness (i.e., variation in failure probability) can overcome the difficulty of evaluating the uncertainty of the statistics of geotechnical parameters, which indicate that the RGD can more reasonably reflect the safety level and performance of geotechnical engineering. To validate this, a comparison with the traditional RBD is made herein.

In the traditional RBD, the p_f for each design is computed with the premise that the statistics of geotechnical parameters are fixed values; as a result, p_f is a fixed value. For this case, it is assumed that the estimation of variability in geotechnical parameters is accurate, that is, the mean of COV of geotechnical parameters ($COV[E_c]$ = 0.32 and $COV[E_s]$ = 0.3) in Table 2 is taken in the calculation of reliability. The failure probabilities shown in Fig. 11 can be considered as the results of the traditional RBD; thus, the selected least-cost design constrained with the target reliability requirement would yield the final design, which is m = 16% and L = 12 m with a cost =1.92 m. Compared to the optimal design (m = 24% and L = 18 m with a cost = 4.32 m) obtained from the proposed RGD approach, the design obtained with the traditional RBD appears to be slightly more economical. However, the design obtained from the traditional RBD may violate the safety requirement if the variability in geotechnical parameters is underestimated.

Table 6 lists results of traditional RBD for various parameter uncertainty levels, each representing a combination of variations in E_c and E_s. Here, the COVs of E_c and E_s are assumed to vary within the typical COV ranges listed in Table 2 for these two parameters. It is clear that from these results, the least-cost design obtained from traditional RBD approach is very sensitive to the assumed COVs of E_c and E_s. Under the lowest uncertainty level of E_c and E_s, as shown in Table 6, the least-cost design costs only 0.96 m; whereas under the highest uncertainty level, the least-cost design costs 4 m. Thus, the traditional RBD using least-cost criterion is meaningful only if the statistical parameters of geotechnical properties can be precisely defined. If the COVs of rock properties are underestimated, an initially acceptable design may no longer be satisfactory. Similarly, if the COVs are overestimated, the traditional design may not be cost-effective. Comparing the results of RBD and RGD, it can be found that when the variation of geotechnical parameters is predicted accurately, the traditional RBD alone can meet the requirements of structural safety performance. When the variation of geotechnical parameters cannot be accurately estimated, the proposed RGD approach has obvious advantages, which can effectively avoid the problems such as the design result being too conservative or too dangerous, and obtain better safety performance and economic benefits.

7. Conclusions

A method and framework for selecting the most preferred design of the stone column are proposed based on the concept of robust geotechnical design. The proposed method can simultaneously consider safety, cost and design robustness of the stone column-reinforced soft clay foundation, whose geotechnical parameters exist significant uncertainty. The proposed method and framework are illustrated with an example. It is concluded as follows:

- An example of a stone column-reinforced soft clay foundation under embankment loading is used to illustrate and verify the proposed method and framework. Out of this study, the design m = 24% and L = 18 m is deemed the most preferred design.
- The influences of allowable settlement and target failure probability on the most preferred designs are discussed, and the results show that more engineering costs on stone columns are needed to reduce the failure probability of the stone column-reinforced foundation in actual project so as to further meet the safety requirement.
- Three composite modulus calculation methods with different complexities are evaluated. Only from the perspective of design robustness, the simple composite modulus calculation method is preferred than the complex one. However, it should be noted that accuracy is a very important index in the design of stone column-reinforced foundation. In practice, more efforts should be devoted to the selection of input parameters and the evaluation of the uncertainty of noise factors. Thus, both accuracy and robustness of the complex method could be guaranteed.
- The proposed RGD approach and the traditional reliability-based design (RBD) approach are compared. It can be found that when the variation of geotechnical parameters is predicted accurately, the traditional RBD alone can meet the requirements of structural safety performance. When the variation of geotechnical parameters cannot be accurately estimated, the proposed RGD approach has obvious advantages, which can effectively avoid the problems such as the design result being too conservative or too dangerous, and obtain better safety performance and economic benefits.

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CC

Notation

A	area of total influence zone
A_c	area of stone column
C	cost
c	cohesion of soil between columns
$COV[c]$	COV of c
$COV[\varphi]$	COV of φ
$COV[E_c]$	COV of E_c
$COV[E_s]$	COV of E_s
$COV[v_c]$	COV of v_c
$COV[v_s]$	COV of v_s
D_c	diameter of stone column
D_e	equivalent diameter of influence zone
$\mathbf{d}$	a given design
E_1	constrained modulus of soil between columns before yielding
E_2	constrained modulus of soil between columns after yielding
E_c	constrained modulus of stone columns
E_{csi}	composite modulus of soils and columns of the *i*th sublayer
E_s	constrained modulus of soil between columns
E_{si}	constrained modulus of soil between columns in the *i*th sublayer
$G(\mathbf{d}, \mathbf{X}_{k,j})$	system performance function of the given design **d**
H_i	thickness of the *i*th sublayer
$I(\mathbf{d}, \mathbf{X}_{k,j})$	indicator function of the given design **d**
L	column length
l_e^n	Euler distance between the *n*th non-dominated design and the utopia point
M	number of designs in design space
m	area replacement ratio of stone column
N_s	number of randomly generated COVs
N_t	number of randomly generated noise factors
P_T	target failure probability
p	uniform pressure applied to the composite material
p_f	failure probability
S	total settlement of stone column-reinforced foundation
S_0	allowable settlement
S_1	settlement of reinforcement zone
S_2	settlement of subjacent unreinforced layer
S_c	center to center spacing between two stone columns
$S_{k,j}$	settlement of the given design calculated using deterministic method

$\mathbf{X}_{k,j}$	*j*th set of the noise factors generated based on the *k*th set of COVs
X_N	normalized value of *b*th objective function
X_b	*b*th objective function in design space
$[X_b]_{max}$	maximum value of *b*th objective function X_b
$[X_b]_{min}$	minimum value of *b*th objective function X_b
x_1^n	cost of the *n*th non-dominated design
x_2^n	value of $\sigma_p(\mathbf{d})$ of the *n*th non-dominated design
x_i^n	value of the *i*th objective for the *n*th non-dominated design
α	geometry-dependent constant
β	reliability index
P_i	stress increment at the middle of the *i*th sublayer
$\delta_{COV[E_c]}$	coefficient of variation of $COV[E_c]$
$\delta_{COV[E_s]}$	coefficient of variation of $COV[E_s]$
Φ	cumulative distribution function of a standard normal distribution
$\mu_{COV[E_c]}$	mean of $COV[E_c]$
$\mu_{COV[E_s]}$	mean of $COV[E_s]$
$\mu_p(\mathbf{d})$	mean of N_s sets of failure probabilities
v_c	Poisson's ratio of stone columns
v_s	Poisson's ratio of soil between columns
φ	friction angle of soil between columns
σ_0	turning stress of the soil
$\sigma_\beta(\mathbf{d})$	standard deviation of the N_s sets of reliability indices
$\sigma_p(\mathbf{d})$	standard deviation of the N_s sets of failure probabilities

Appendix: Methods proposed by Zhang (1992) and Zheng *et al.* (2018)

In Zhang's (1992) study, the elastic mechanics theory was adopted and the strain between stone column and soil was assumed to be the same. Then based on the principle that the total strain energy of composite foundation is equal to the sum of the strain energy of stone column and that of soil between columns, the formula for the composite modulus is given as follows:

$$E_{\mathrm{csi}} = mE_{\mathrm{c}} + (1-m)E_{\mathrm{si}} + \frac{4(\nu_{\mathrm{c}} - \nu_{\mathrm{s}})^2 K_{\mathrm{c}} K_{\mathrm{s}} G_{\mathrm{s}} (1-m)m}{[mK_{\mathrm{c}} + (1-m)K_{\mathrm{s}}]G_{\mathrm{s}} + K_{\mathrm{c}} K_{\mathrm{s}}} \quad \text{(A1)}$$

$$K_{\mathrm{c}} = \frac{E_{\mathrm{c}}}{2(1+\nu_{\mathrm{c}})(1-2\nu_{\mathrm{c}})} \quad \text{(A2)}$$

$$K_{\mathrm{s}} = \frac{E_{\mathrm{si}}}{2(1+\nu_{\mathrm{s}})(1-2\nu_{\mathrm{s}})} \quad \text{(A3)}$$

$$G_{\mathrm{s}} = \frac{E_{\mathrm{si}}}{2(1+\nu_{\mathrm{s}})} \quad \text{(A4)}$$

where ν_{c} is the Poisson's ratio of stone columns; and ν_{s} is the Poisson's ratio of soil between columns.

Zheng *et al.* (2018) deduced the expression of composite modulus based on the principle of minimum potential energy and the condition of column-soil deformation coordination. Before the soil between columns yields, the calculation formula of composite modulus is the same as Eq. (3). After the yielding of soil between columns, the composite modulus is related to the ratio of modulus of soil between columns before and after yielding and the magnitude of load, which can be expressed as

$$E_{\mathrm{csi}} = \frac{mE_{\mathrm{c}} + (1-m)E_1}{1 + \dfrac{(1-m)\sigma_0}{p}\left(\dfrac{E_1}{E_2} - 1\right)} \quad \text{(A5)}$$

$$\sigma_0 = 2c\sqrt{g}r/(r-g) \quad \text{(A6)}$$

$$E_2 = \alpha E_1 \quad \text{(A7)}$$

where p is the uniform pressure applied to the composite material; σ_0 is the turning stress of the soil, in which the constrained modulus of soil between columns changes; E_1 and E_2 are the constrained modulus of soil between columns before and after yielding, respectively; c and φ are the cohesion and friction angle of soil between columns, respectively; $r = (1-\nu_{\mathrm{s}})/\nu_{\mathrm{s}}$; $g = \tan^2(\pi/4 + \varphi/2)$; and $\alpha = 1 + (g^2 - r)^2/[g^4(r^2-1)]$.