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Using grain size to predict engineering properties of natural sands in Pakistan

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Abstract. Laboratory determination of strength and deformation behavior of clean sands and gravels has always been challenging due to the difficulty in obtaining their undisturbed samples. An alternative solution to this problem is to develop correlations between mechanical properties of cohesionless soils and their gradation characteristics. This study presents database of 3 natural sands with 11 varying particle size gradation curves to allow investigating relationships between mean particle size, maximum and minimum void ratio, relative density and shear strength of the test soils. Direct shear tests were performed at relative densities of 50, 75 and 95% to explore the effects of gradation and density on the angle of internal friction of the modeled sand samples. It is found that the mean grain size D_{50} bears good correlations with void ratio range (e_{max} - e_{min}) and peak angle of internal friction ϕ'_{peak}. The generated regression models are in good agreement with published literature and can be considered as reliable for natural sands in Pakistan. These empirical correlations can save considerable time and efforts involved in laboratory and field testing.

Keywords: natural sands; gradation; relative density; void ratio; direct shear test; friction angle

1. Introduction

Numerous studies have been performed to correlate engineering properties and to produce semi-empirical methods for estimating geotechnical parameters when sufficient laboratory or field measurements are not available (Arvanitidis *et al.* 2019). Likewise, laboratory determination of strength and deformation behavior of clean sands and gravels has always been a challenging task for geotechnical engineers due to the difficulty in obtaining undisturbed soil samples (Dave and Dasaka 2012). An alternative solution to this problem is to develop correlations between mechanical properties of cohesionless soils and their gradation characteristics. It is a well-known fact that among many important parameters controlling the engineering behavior of granular soils, particle size and shape play very important role which should be carefully identified. That is why, the gradation of granular material has been traditionally identified as on the most important factors affecting the properties of gravels and sands and has been essentially included in USCS and AASHTO soil classification systems (Tsomokos and Georgiannou 2010, Tong *et al.* 2018).

In a comprehensive study conducted by Cubrinovski and Ishihara (2002), maximum and minimum void ratio (index densities) characteristics of natural sandy soils and their possible use for material characterization were investigated. They included data of over 300 natural sandy soils including clean sands, sands with fines and sands containing small amount of clay-size particles to examine the influence of fines, grain-size composition and particle shape on e_{max}, e_{min} and void ratio range (e_{max} - e_{min}). The empirical correlations presented by Cubrinovski and Ishihara (2002) clearly demonstrate the relationship between the index densities and material properties of sands. They concluded that as compared to the conventional material parameters such as fine contents and mean particle size, D_{50}, void ratio range (e_{max} - e_{min}) is an indicative of the overall grain-size composition and particle characteristics of a given sand and that it comprehends the combined influence of relevant material factors.

Several studies including Fredlund *et al.* (2000), Cho *et al.* (2006), Georgiannou (2006), Guo and Su (2007), Georgiannou and Tsomokos (2008), Monkul (2013), Hsiao and Phan (2014), Ameratunga *et al.* (2016), Suh *et al.* (2016), Hyodo *et al.* (2017) and Sonmezer *et al.* (2020) have presented regression models to characterize strength and deformation response of cohesionless materials based on their particle size and index densities. Table 1 presents list of relevant empirical correlations available in the literature which are commonly used to predict engineering properties of natural sands based on their gradation characteristics.

Among other factors the shear strength of sand is controlled by the void ratio or relative density which strongly depends on the shape and size, surface roughness and angularity of the particles. Igwe *et al.* (2007) have demonstrated that the friction angle increases as dry density and average particle size of each type of sand (fine, medium, or coarse) increases. They also observed that increasing the percentage of coarse fraction of loose sands also increases the angle of internal friction. Similarly, for medium or dense states of sands, increasing the summation of percentages of fine and medium sand results in an increase in the angle of internal friction due to insertion of

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Table 1 Gradation-based models to predict engineering properties of granular soils

Existing Models	Applicability/limitations	Reference
(a) $\phi'_{peak} = 5.697\ \ln(c/f) + 33.401$ (b) $\phi'_{peak} = 4.269\ \ln(c/f) + 35.512$	(a) Loose granular soil (b) Dense granular soil where; *c/f = coarse-to-fines weight ratio*	Arvanitidis *et al.* (2019)
(a) $e_{max} = 0.072 + 1.53e_{min}$ (b) $e_{max} = 0.25 + 1.37e_{min}$ (c) $e_{max} = 0.44 + 1.21e_{min}$ (d) $e_{max} = 0.44 + 1.32e_{min}$	(a) Clean sands with 0-5% fines (b) Sands with 6-15% fines (c) Sands with 16-30% fines and 5-20% clay (d) Silty soils with 31-70% fines and 5-20% clay	Cubrinovski and Ishihara (2002)
(a) $e_{max} - e_{min} = 0.0087F_c + 0.43$ (b) $e_{max} - e_{min} = 0.004F_c + 0.57$	(a) Sands with maximum 30% fines and 5-20% clay (b) Sands with 31 - 70% fines and 5 – 20% clay where; *F_c = fine contents*	Cubrinovski and Ishihara (2002)
$e_{max} - e_{min} = 0.23 + \frac{0.06}{D_{50}}$	•All sands with maximum 70% fines, 20% clay and 36% gravel contents	Cubrinovski and Ishihara (1999)
$\frac{N_1}{D_r^2} = \frac{9}{(e_{max} - e_{min})^{1.7}}$	• Sands with 5-14% fines • Clean sands with less than 5% fines •Gravels with gravel contents greater than 50% where: *N_1 = SPT value and Dr = relative density*	Cubrinovski and Ishihara (1999)
$e_{max} = 0.642R^{-0.354}$	•Uniform sands where; *R = roundness*	Shimobe and Moroto (1995)

these particles between coarse particle to increase contact shear surface. Similarly, Dai *et al.* (2019) have presented the concept of skeleton void ratio to explain the role of fine and coarse particles of silty sands in the force chains to carry external load.

The collection of subsoil information contributes substantially to the cost of engineering projects due to the increasing cost of field and laboratory investigations. Therefore, published literature on geotechnical database of a region can save considerable time and expense to decide the scope and extent of ground exploration for proposed projects in that region. To author's knowledge, making geotechnical database available to practicing engineers through empirical correlations for quick identification and characterization of local geomaterials is still an under-explored area of research in Pakistan. Some of the studies available in this regard are Hayat (2003), Akbar (2006), Aziz and Akbar (2017), Faruq and Khan (2015), Khan *et al.* (2017), Aziz *et al.* (2017) and Mujtaba *et al.* (2018). However, more efforts are still needed to develop empirical or semi-empirical correlations for the local soils.

This study is an attempt to gather database of common natural sands in Pakistan (Ravi, Chenab and Lawrencepur) to allow investigating relationships between index properties and mechanical behavior of these soils. The obtained empirical correlations can be considered as reliable for natural sands in Pakistan and can be utilized to save time and effort involved in laboratory and field testing. The current study was mainly focused on the following objectives:

- The effects of gradation on index density characteristics (e_{max}, e_{min}) of natural and modeled alluvial sands of Pakistan.
- Direct shear behavior (peak friction angles) of these sands tested at different gradations and relative densities.
- Exploring the possible correlations and regression models between gradation and shear strength characteristics of the test sands.

2. Materials and methods

The detailed testing plan adopted in this study is presented in Fig. 1. In order to prepare the modeled gradations, first the sieve analyses were performed on the original sand samples (Fig. 2). The gradation curves of these sand samples is plotted in Fig. 3(a). Hit and trial mixing was used to produce different modeled gradations using the original sand samples. The gradation curves thus produced are shown in Fig. 3(b). The index properties of all samples are summarized in Table 2.

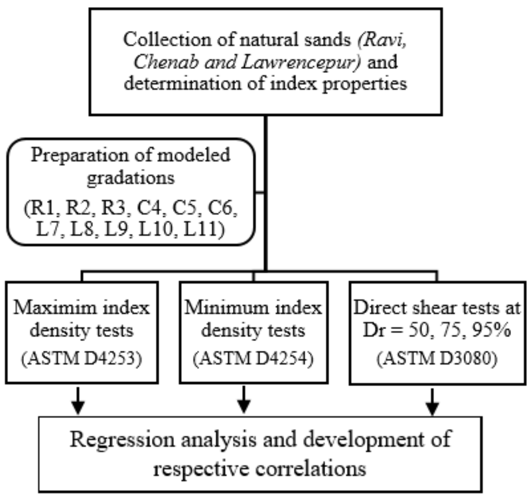
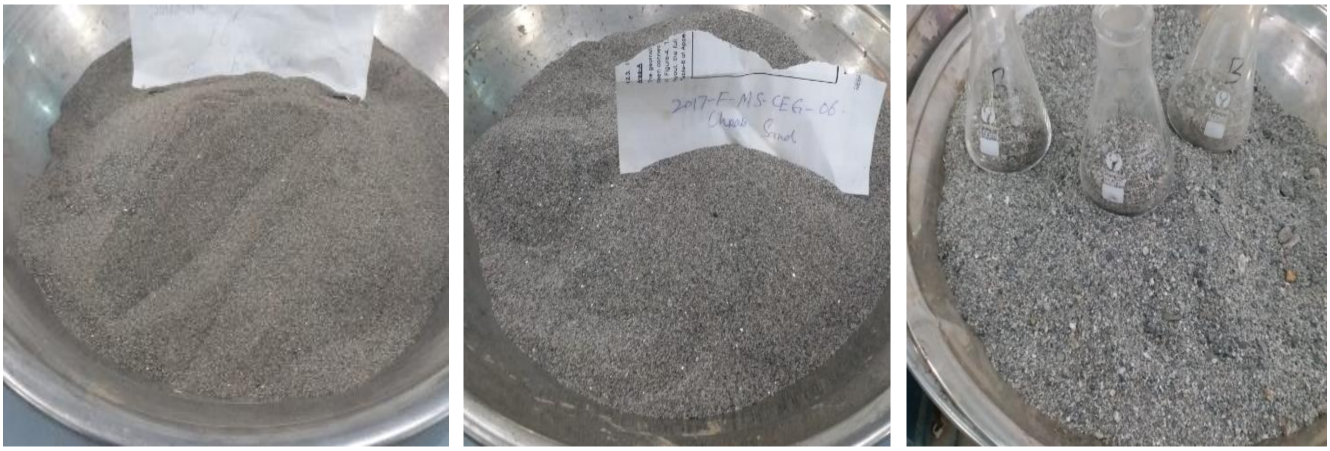


Fig. 1 Testing plan and methodology

Table 2 Gradation characteristics of the sand samples

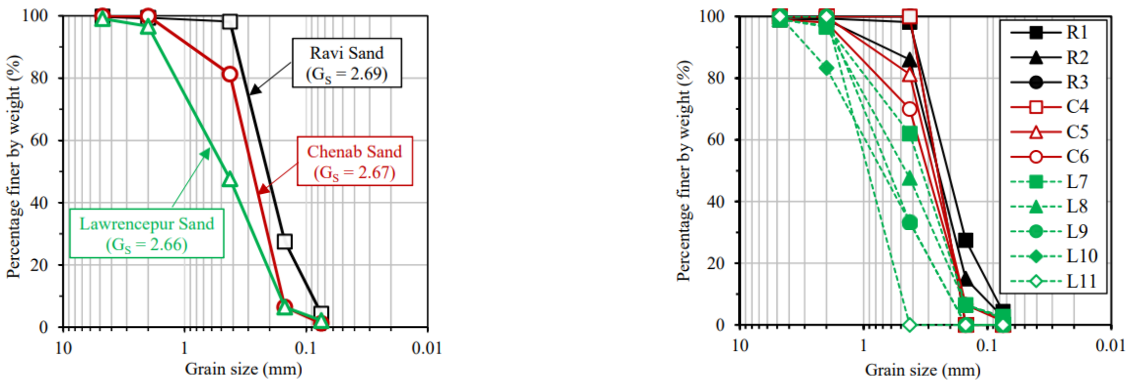
Property	Ravi sand			Chenab sand			Lawrencepur sand				
	R1	R2	R3	C4	C5	C6	L7	L8	L9	L10	L11
D_{10}	0.09	0.12	0.18	0.17	0.17	0.18	0.18	0.18	0.21	0.21	0.5
D_{30}	0.17	0.19	0.21	0.20	0.21	0.22	0.28	0.28	0.39	0.41	0.69
D_{50}	0.21	0.22	0.25	0.26	0.29	0.30	0.37	0.45	0.64	0.73	0.9
D_{60}	0.25	0.27	0.28	0.29	0.31	0.38	0.43	0.61	0.8	1.0	1.1
Cc	1.28	1.11	0.88	0.81	0.84	0.71	1.01	0.71	0.91	0.80	0.87
Uc	2.78	2.25	1.56	1.71	1.82	2.11	2.39	3.39	3.81	4.76	2.20

Particle size, *D* in mm; Coefficient of curvature, $Cc = D_{30}^2/(D_{10} \times D_{60})$; Uniformity coefficient, $Uc = D_{60}/D_{10}$



(Ravi sand) (Chenab sand) (Lawrencepur sand)

Fig. 2 Natural sand samples used in this study *(Photo credit: Muhammad Shah Behram)*



(a) Natural sand samples (b) Modeled gradations

Fig. 3 Particle size distribution curves of the test sands

Table 3 Relative density test results

Property	Ravi sand			Chenab sand			Lawrencepur sand				
	R1	R2	R3	C4	C5	C6	L7	L8	L9	L10	L11
e_{max}	1.05	1.05	1.07	0.99	0.93	0.92	0.85	0.81	0.84	0.87	0.92
e_{min}	0.96	0.93	0.97	0.91	0.82	0.84	0.78	0.76	0.78	0.80	0.86
e_{max} - e_{min}	0.09	0.12	0.10	0.08	0.11	0.08	0.07	0.05	0.06	0.07	0.06
F	0.09	0.13	0.10	0.09	0.13	0.10	0.09	0.07	0.08	0.09	0.07
ε_{vr} (%)	4.39	5.85	4.83	4.02	5.70	4.17	3.78	2.76	3.26	3.74	3.13

It is evident from the particle size distribution curves that the Ravi sand can be considered as relatively the finest and the Lawrencepur sand as the coarsest one. The values of coefficient of curvature, *Cc*, and uniformity coefficient, *Uc,* suggest that, as per unified soil classification system (ASTM D2487 2017), all the modeled sand samples used in this study can be classified as poorly graded sands, *SP*.

Maximum and minimum void ratios were determined by the procedures explained in ASTM D4253 (2016) and ASTM D4254 (2016) procedures. The sand samples were poured into a cylindrical mould having 15.2cm diameter and height of 15.6 cm using a pouring device which was kept vertical and the free fall of the sand was kept to 12.5 mm (high enough to maintain continuous flow of sand particles without the spout connecting the already deposited sand). The poring device was moved in a spiral path from the outside towards the center of the mould to form each layer of nearly uniform thickness. The mould was filled approximately 12.5 to 25 mm above the top. Excess soil was screened off by trimming with a straight edge. The weight of the sand filling the mould was measured and by dividing the weight with the volume and minimum index density (γ_{dmin}) or maximum void ratio (e_{max}) was calculated. A vibrating table was used to determine maximum index density (γ_{dmax}) or minimum void ratio (e_{min}). The mould filled with sand was vibrated for 10min. at a frequency of 50Hz. The change in the volume of sand was measured with a dial gauge. Maximum dry density (γ_{dmax}) was calculated by dividing the weight of the sand with the final volume. The index void ratios (e_{max} and e_{min}) and void ratio range (e_{max} - e_{min}) of all the sand samples used in this study are provided in Table 3. The compactibility factor F (Terzaghi and Peck 1967) and volumetric strain range ε_{vr} (Cubrinovski and Ishihara 2002) are computed from index void ratios using Eqs. (1) and (2), respectively.

$$F = \frac{e_{max} - e_{min}}{e_{min}} \tag{1}$$

$$\varepsilon_{vr} = \frac{e_{max} - e_{min}}{1 + e_{max}} \tag{2}$$

The direct shear apparatus enables us to measure peak and critical state friction angles of sand specimens of distinct relative densities. Friction angle of sands was determined by performing direct shear tests. Direct shear tests were performed according to ASTM D3080 (2011) at horizontal displacement rate of 1 mm/min. The tests were performed on dry specimens at relative densities of 50, 75 and 95% against three different normal stresses of 12.7, 25.7 and 39.1 kPa. Peak angles of internal friction were computed from the respective shear stress – normal stress plots of each sample.

3. Results and discussions

3.1 Gradation versus index densities

A relationship between maximum and minimum void ratio is shown in Fig. 4 which yielded the regression model given as Eq. (3). This linear relationship between e_{max} and e_{min} is consistent with the findings of Cubrinovski and Ishihara (2002).

$$e_{max} = 1.188e_{min} - 0.08 \tag{3}$$

The gradation-based characterises such as mean particle size (D_{50}), void ratio range (e_{max}-e_{min}), compactibility factor (F) and volumetric strain range (ε_{vr}) have been plotted in Fig. 5. According to Fang (1991), for well-graded cohesionless sands or gravels, void ratio range is large and e_{min} is small; hence compactibility factor F is large (i.e., soil can be easily compacted). Conversely, when F is small (i.e., void ratio range is small and e_{min} is large) the soils are more difficult to compact and this is true for uniform soils such as poorly graded sands or gravels. Similarly, Cubrinovski and Ishihara (2002) stated that the void ratio range (e_{max} - e_{min}) is an indicative of the degree of possible variation in the packing of cohesionless soil. They suggested that this variation in the packing of sand can also be expressed by volumetric strain range which represents volumetric strain induced in the soil while its densification from the loosest (e_{max}) to the densest (e_{min}) state. Fig. 5(b) shows that ε_{vr} increases linearly with the increase in compactibility factor and Eq. (4) shows the respective regression model developed from the data of all sand samples used in this study.

$$\varepsilon_{vr}\ (\%) = 0.442F \tag{4}$$

3.2 Gradation versus shear strength

Angle of shearing resistance of all modeled sand sample was determined by performing direct shear tests at loading rate of 1 mm/min under dry conditions. The tests were performed at relative densities Dr = 50, 75 and 95% against three different normal loads of 4.54, 9.07 and 13.61 kg. Peak angles of internal friction were computed from the respective shear stress – normal stress plot of each specimen. A typical test data for dry C4 sample at Dr = 75%

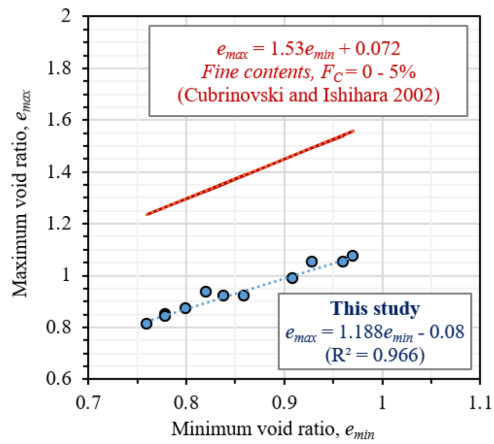
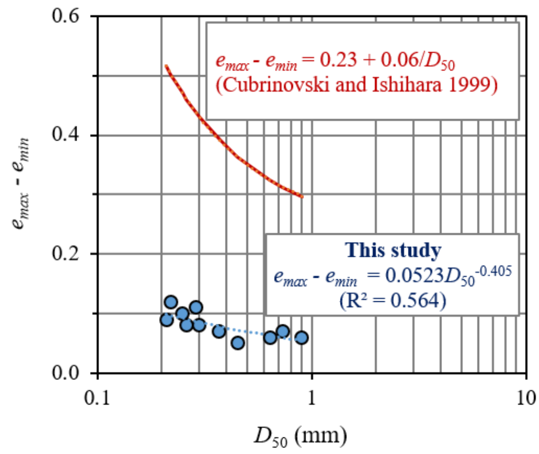
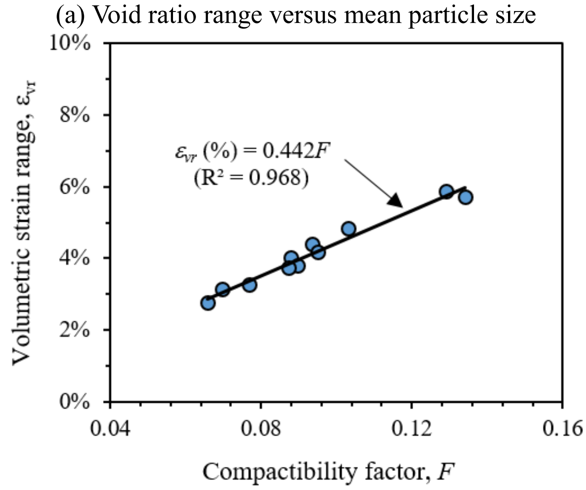


Fig. 4 Relationship between e_{max} and e_{min}



(a) Void ratio range versus mean particle size

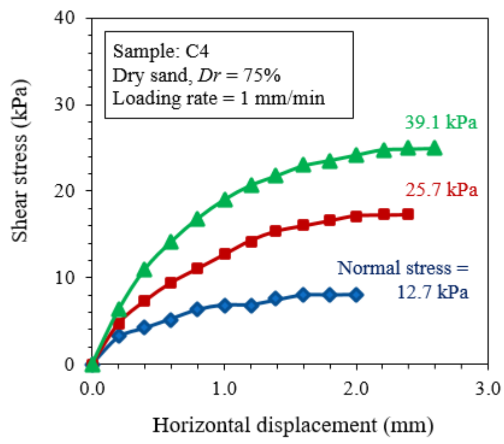


(b) Plot of F versus ε_{vr}

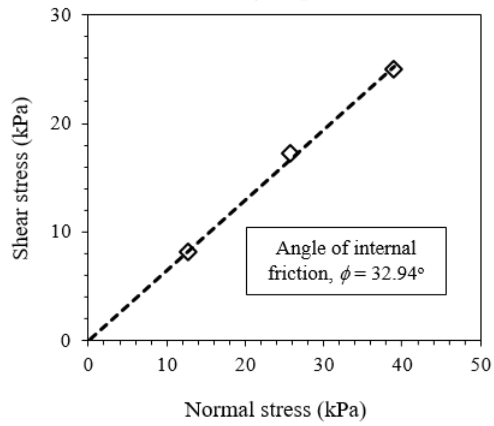
Fig. 5 Gradation-based relationships of the sand samples

Table 4 Comparison of increase in friction angle with particle size and relative density

Dr (%)	Lowest ϕ_{peak}		Highest ϕ_{peak}		% increase
50	R1	29.86	L10	35.21	17.9%
75	R1	31.34	L11	36.5	16.5%
95	R1	31.38	L11	37.0	17.9%
% increase in ϕ_{peak} with increase in Dr from 50 to 95%		5.1%	-	5.1%	-



(a) Shearing response



(b) Failure envelope

Fig. 6 Typical direct shear test result on dry sand sample

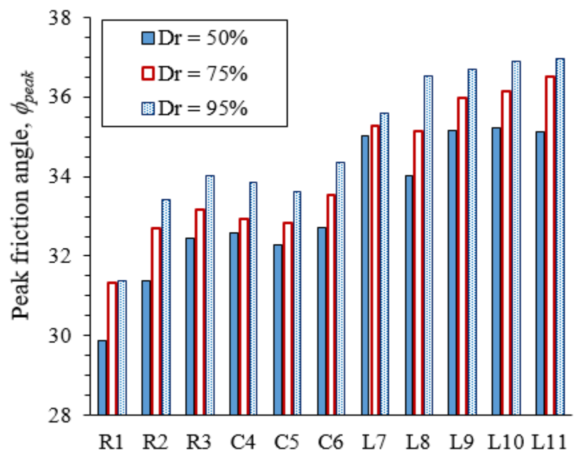


Fig. 7 Effect of relative density on peak friction angle

is shown in Fig. 6. The angle of shearing resistance for all sand samples were determined accordingly.

As shown in Fig. 7, the angle of internal friction increases with increase in relative density. It can be observed that Lawrencepur sand (L7-L11) shows higher friction angle at a given relative density due to relatively larger mean grain size and angular particle shape as compared to Ravi (R1-R3) and Chenab (C4-C6) sands. A comparative summary of maximum and minimum friction angles is given in Table 4. Shahu and Yudhbir

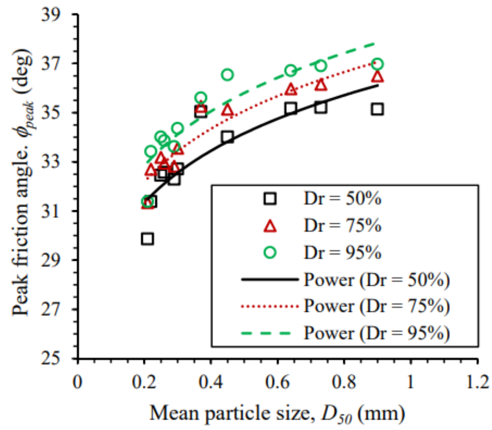


Fig. 8 Correlation between D_{50} and ϕ_{peak} for different states of compaction of sandy soils

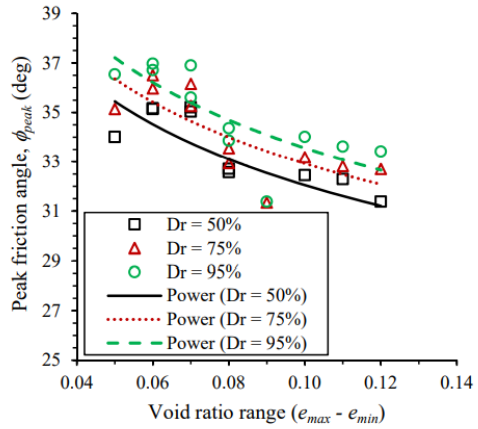


Fig. 9 Correlation between void ratio range and ϕ_{peak} for different states of compaction of sandy soils

(1998) have also reported that angle of shearing resistance is much higher for the angular compressible sands as compared with those for the rounded, low compressibility sand.

It is evident from Table 4 that for a given relative density, the average increase in peak friction angle is 17.4% for an increase in D_{50} from 0.21 mm (sample R1) to 0.9 mm (sample L11). However, the average increase in peak friction angle is 5.1% for a given increase in relatively density from 50 to 95%.

Based on direct shear test data, correlations of mean particle size (D_{50}) and void ratio range (e_{max}-e_{min}) with the peak angle of shearing resistance (ϕ_{peak}) at different states of compaction have been developed as shown in Figs. 8 and 9. The relationships of peak angle of internal friction with mean particle size and void ratio range follow the power law and is expressed as Eqs. (5) and (6).

Table 5 summarize the components of the generated regression models for relative densities of 50, 75 and 95%.

$$\emptyset_{peak} = K D_{50}{}^{\alpha} \tag{5}$$

$$\emptyset_{peak} = K(e_{max} - e_{min})^{\alpha} \tag{6}$$

where,

α = Power-law exponent

K = Arbitrary constant of the equation

Table 5 Components of the generated power-law models

Dr (%)	Component	Values for D_{50} models	Values for (e_{max} - e_{min}) models
50	K	36.469	22.926
	α	0.0943	-0.145
	R^2	0.765	0.530
75	K	37.428	23.70
	α	0.0938	-0.143
	R^2	0.887	0.597
95	K	38.222	23.807
	α	0.0943	-0.149
	R^2	0.820	0.596

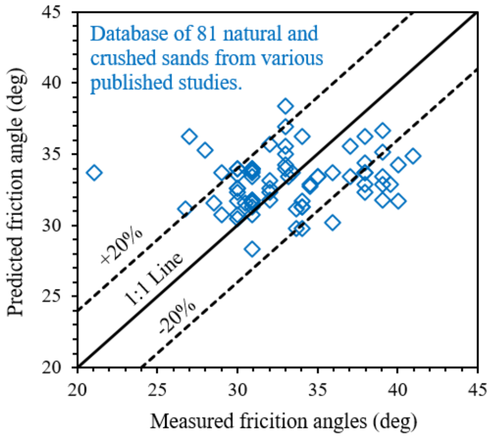


Fig. 10 Validation of D_{50} - ϕ_{peak} regression model

To validate the developed power-law regression model (D_{50} versus ϕ_{peak}) as given in Eq. (5), a database of 81 different sands has been used to compare the measured and predicted friction angles. The database from various published studies (Chu and Lo 1993, Ishihara 1993, Sukumaran and Ashmawy 2001, Ashmawy *et al.* 2003) has been summarized by Cho *et al.* (2006) which comprises of particle size and shape, packing density, strength and stiffness of famous natural and crushed sands such as Nevada sand, Ticino sand, Margaret river sand, ASTM 20/30 sand, Ottawa sand, Michigan dune sand, Sydney sand and Toyoura sand, etc. The friction angles of these sands have been computed using the D_{50} -ϕ_{peak} correlation at a relative density of 75%. It can be observed from Fig. 10 that the measured and predicted friction angles are in good agreement with each other as most of the database is plotted within a scatter of ±20%.

4. Conclusions

A series of tests on sands with different particle gradations were undertaken to correlate grain size with index properties and shearing resistance of natural sands in Pakistan. On the basis of the results, the following conclusions are drawn:

- A linear relationship between maximum and minimum void ratio has been developed. Based on the fact that the void ratio range (e_{max}-e_{min}) provides a general basis for comparative evaluation of material properties of sandy soils, it has been observed that void ratio range decreases with increase in mean particle size (D_{50}). A linear correlation between volumetric strain range (ε_{vr}) and soil compactibility factor (F) has also been developed.
- The direct shear test data shows that for a given relative density, the average increase in peak friction angle is 17.4% for an increase in D_{50} from 0.21mm to 0.9 mm. However, the average increase in peak friction angle is 5.1% for a given increase in relatively density from 50 to 95%.
- The correlations of mean particle size (D_{50}) and void ratio range (e_{max}-e_{min}) with the peak angle of shearing resistance (ϕ_{peak}) at different relative densities follow the power law with a reasonable coefficient of determination (R^2). Thus, the mean particle size and void ratio range can provide a general basis for evaluation of shear strength properties of sandy soils.
- The friction angles of 81 natural and crushed sands computed using the D_{50}-ϕ_{peak} correlation at a relative density of 75% has been compared with the measured friction angles and the developed model is validated as most of the database plot within a scatter of ±20%.

It is concluded that the generated regression models presented in this study are in good agreement with the published literature. Hence, the obtained empirical correlations can be considered as reliable for natural sands in Pakistan and can be utilized to save time and effort involved in laboratory and field testing.

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