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Deformation in a nonlocal magneto-thermoelastic solid with hall current due to normal force

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Abstract. The present article is concerned about the study of disturbances in a homogeneous nonlocal magneto-thermoelastic medium under the combined effects of hall current, rotation and two temperatures. The model under assumption has been subjected to normal force. Laplace and Fourier transform have been used for finding the solution to the field equations. The analytical expressions for conductive temperature, stress components, normal current density, transverse current density and displacement components have been obtained in the physical domain using a numerical inversion technique. The effects of hall current and nonlocal parameter on resulting quantities have been depicted graphically. Some particular cases have also been figured out from the current work. The results can be very important for the researchers working in the field of magneto-thermoelastic materials, nonlocal thermoelasticity, geophysics etc.

Keywords: thermoelasticity; nonlocality; nonlocal theory of thermoelasticity; normal force; hall current; rotation; Laplace and Fourier transform

1. Introduction

Thermoelasticity is the theory of study of stresses and strains due to temperature changes. Hall effect is produced due to the electric current flowing along a conducting material with an attached magnetic field to it. As most of the natural bodies like earth are rotating with an angular velocity and have magnetic fields associated with them, so the hall current and thus hall effect are very important research fields. The effects have been correlated with nonlocality in this study. The concept of nonlocality is well established. It considers that the stress at a point is not just dependent upon strain at that point only but the strain due to the points all over the whole body.

Two temperature theory was developed by Chen and Gurtin (1968). Edelen *et al.* (1971) and Edelen and Law (1971) developed the concept of nonlocal continuum mechanics. Eringen (2002) derived nonlocal continuum field theories. Youssef (2005) gave the theory of two-temperature-generalized thermoelasticity. Youssef and Al-Lehaibi (2007) studied the state space approach of two-temperature generalized thermoelasticity. Marin (2010) discussed the concept of thermoelasticity in detail for dipolar bodies. Abbas *et al.* (2011) studied the propagation of plane waves in a fiber-reinforced, anisotropic thermoelastic half-space under the effect of a magnetic field. Abbas and Othman (2012) studied thermoelastic interaction in a fiber-reinforced anisotropic half-space. Othman and Abbas (2012) studied generalized thermoelasticity of thermal-shock problem. Abbas (2014) constructed a model based upon two temperature generalized thermoelastic theory. Atwa and Jahangir (2014) investigated the two temperature effects on plane waves in generalized thermo-microstretch elastic solid. Marin and Florea (2014) investigated behaviour of solutions in thermoelasticity of porous micropolar bodies Mokhtar *et al.* (2015) used hyperbolic shear deformation theory for post buckling analysis of beams. Sharma *et al.* (2016) and Kumar *et al.* (2016) described the effects of hall current in a transversely isotropic magneto-thermoelastic two temperature medium with rotation due to normal force. Marin and Nicaise (2016) and Marin *et al.* (2016) extended the thermoelasticity concepts to porous micropolar bodies. Rakrak *et al.* (2016) used nonlocal elasticity theory for analyzing free vibration in a carbon nanotube. Ebrahimi *et al.* (2016) applied Eringen's nonlocal elasticity theory for vibration analysis of FG nanobeams. Othman and Marin (2017) studied effect of thermal loading due to laser pulse on thermoelastic porous medium. Ezzat and El-Barrry (2017) studied the magneto-thermoelasticity based on memory dependent derivatives. Abdelmalek *et al.* (2017) discussed the hygrothermal effects on the free vibration behavior of a composite plate.

Belmahi *et al.* (2018) discussed the vibrations of nanobeams under different boundary conditions. Belkacem *et al.* (2018) investigated buckling of plates under different boundary conditions. Dihaj *et al.* (2018) used nonlocal elasticity theory for analyzing free vibrations of a carbon nanotube embedded in an elastic medium. Karami *et al.* (2018) developed a nonlocal strain gradient theory. Hassan *et al.* (2018) studied convective heat transfer and Zenkour

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(2018) studied generalized thermoelastic problem of a thermo-mechanically loaded beam. Abouelregal (2019) studied the rotating magneto-thermoelastic rod due to moving heat sources via Eringen's nonlocal model. Abualnour *et al.* (2019) analyzed antisymmetric laminated reinforced composite plates. Balubaid *et al.* (2019) used nonlocal two variable refined plate theory for investigating free vibrations of a plate. Belmahi *et al.* (2019) used nonlocal elasticity theory in their study. Belbachir *et al.* (2019) analyzed bending of cross laminated plates under thermal and mechanical loadings. Lata and Singh (2019) discussed nonlocal effects for their study. Soleimani *et al.* (2019) also used nonlocal elasticity theory to prove their results. Marin *et al.* (2019) discussed energy partition concept for thermoelastic materials. Medani *et al.* (2019) discussed the behavior of a plate and used energy principle for the study. Jahangir *et al.* (2020) studied reflection of photothermoelastic waves. Lata and Kaur (2019) studied effects of hall current in transversely isotropic magneto thermoelasic rotating medium due to normal force. Tildji *et al.* (2019) studied the vibrations for a microbeam. Mahmoudi *et al.* (2019) proposed a new refined quasi shear deformation theory. Bensattalah *et al.* (2020) theoretically analyzed the effects on critical buckling load of triple walled carbon nanotubes under the effects of nonlocal elasticity theory. Bousahla *et al.* (2020) studied the behavior of CNT beams using shear deformation theory. Chikr *et al.* (2020) proposed a new four unknown integral model for plates resting on elastic foundations using Galerkin's approach. Gafour *et al.* (2020) used nonlocal shear deformation theory for their study. Kaddari *et al.* (2020) gave a new model and used it to study structural behavior of plates on elastic foundation. Lata and Singh (2020) discussed time harmonic interactions in a nonlocal thermoelastic medium. Zenkour (2020) investigated the effects of magnetic field parameter and different thermoelasticity theories. Bellal *et al.* (2020) studied buckling behavior of a graphene sheet using a nonlocal integral model. Hosseini (2020) investigated a size-dependent coupled thermoelasticity analysis using a new modified nonlocal model of heat conduction, based on the GN theory and nonlocal Eringen theory of elasticity. Rahmani *et al.* (2020) studied the influences of boundary conditions on bending and free vibration behavior of plates. Refrafi *et al.* (2020) discussed the effects of hygro-thermo-mechanical conditions on FG plates. Tounsi *et al.* (2020) proposed a four variable plate theory for studying AFG plates resting on a two-parameter elastic foundation.

From above discussion, it has been observed that work has already been carried out in recent years on hall current effects. But the effects of hall current on the variations in a nonlocal thermoelastic solid have not been examined yet. As, we are already aware of the fact that nonlocal theory of thermoelasticity is a vital theory due to its dependence for properties on all the points of a body rather than being concentrated on a single point as most other theories do. Also, hall current is produced in a rotating medium with a magnetic field attached to it and as such effects are prevalent in heavenly bodies such as earth, moon etc. and thus it is of utmost importance for researchers. So, in this paper an effort has been made to study the effects of hall current on a magneto-thermoelastic medium under the effect of non-local parameters. The analytic expressions for the displacements, stresses, current density and temperature change have been obtained in two-dimensional transversely isotropic magneto-thermoelastic solid.

2. Basic equations

Following Eringen (2002) and Abouelregal (2019), the equation of motion for a homogeneous nonlocal magneto-thermoelastic solid rotating with a uniform angular velocity $\Omega = \Omega n$, where n is a unit vector demonstrating the direction of the rotation axis and taking into account Lorentz force is

$$\begin{aligned}(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu\ (\nabla \times \nabla \times \boldsymbol{u}) - \beta\nabla\theta \\ + (1 - \epsilon^2\nabla^2)F \\ = \rho(1 - \epsilon^2\nabla^2)[\ddot{u} + \Omega \times (\Omega \times \mathrm{u}) \\ + 2\Omega \times \dot{u}]\end{aligned} \tag{1}$$

where, $F = \mu_0\ (\vec{J} \times \overrightarrow{H_0})$ denotes the Lorentz force, $\overrightarrow{H_0}$ is the external applied magnetic field intensity vector, J is the current density vector, u is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively. The terms $\Omega \times (\Omega \times \mathrm{u})$ and $2\Omega \times \dot{u}$ are the additional centripetal acceleration due to the time-varying motion and Coriolis acceleration respectively.

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the hall current effect

$$J = \frac{\sigma_0}{1 + m^2}\left(\ E + \mu_0\left(\dot{u} \times H - \frac{1}{en_e}J \times H_0\right)\right) \tag{2}$$

The heat conduction equation with multi-dual-phase-lag and constitutive relations by Zenkour (2020) for a homogeneous non local thermoelastic solid is given as

$$K^*\mathcal{L}_v\nabla^2\theta = \ \mathcal{L}_q\frac{\partial}{\partial t}(\rho C^*\theta + \beta\theta_0 u_{i,j}), \tag{3}$$

where,

$$\mathcal{L}_v = 1 + \textstyle\sum_{r=1}^{R_1}\frac{\tau_v^r}{r!}\frac{\partial^r}{\partial t^r}, \tag{4}$$

$$\mathcal{L}_q = \varrho + \tau_0\frac{\partial}{\partial t} + \textstyle\sum_{r=2}^{R_2}\frac{\tau_q^r}{r!}\frac{\partial^r}{\partial t^r} \tag{5}$$

Here τ_v, τ_q and τ_0 are thermal memories in which τ_v is the phase lag of the temperature gradient while τ_q is the phase lag of the heat flux ($0 \le \tau_v < \tau_q$). Generally, the value of $R_1 = R_2 = R$ may reach 5 or more according as refined multi-dual-phase-lag theory required while ϱ is a non-dimension parameter (= 0 or 1 according to the thermoelasticity theory).

The constitutive relations are given by,

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) - \ \beta\theta\delta_{ij} \tag{6}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, $\boldsymbol{u} = (u, v, w)$ is the displacement vector, θ is absolute temperature and θ_0 is reference temperature, K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, $\beta = (3\lambda + 2\mu)\alpha$ where α is coefficient of linear thermal expansion, Ω is the angular velocity of the solid, e_{ij} are components of strain tensor, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation of the problem

We consider a perfectly conducting homogeneous non local isotropic magneto-thermoelastic medium, which is rotating uniformly with an angular velocity Ω initially at uniform temperature θ_0 The rectangular Cartesian coordinate system (x, y, z) is introduced, having origin on the surface (z = 0) with z-axis pointing normally downwards into the half space. The surface of the medium is subjected to normal force acting at z = 0. We restrict our analysis to two-dimensional problem with

$$\boldsymbol{u} = (u, 0, w). \tag{7}$$

We also assume that

$$E = 0, \boldsymbol{\Omega} = (0, \Omega, 0). \tag{8}$$

Now, using Eq. (7)

$$J_y = 0. \tag{9}$$

The current density components J_x and J_z using Eq. (2) are given as:

$$J_x = \frac{\sigma_0\mu_0 H_0}{1+m^2}\left(m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right), \tag{10}$$

$$J_z = \frac{\sigma_0\mu_0 H_0}{1+m^2}\left(\frac{\partial u}{\partial t} + m\frac{\partial w}{\partial t}\right). \tag{11}$$

Using Eq. (7) in Eq. (1) and Eq. (3), yields

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} + \mu \frac{\partial^2 u}{\partial z^2} - \beta\frac{\partial \theta}{\partial x} - (1 - \epsilon^2\nabla^2)\mu_0 J_z H_0 = \rho(1 - \epsilon^2\nabla^2)\left\{\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right\}, \tag{12}$$

$$(\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial z} + \mu \frac{\partial^2 w}{\partial z^2} - \beta\frac{\partial \theta}{\partial z} - (1 - \epsilon^2\nabla^2)\mu_0 J_x H_0 = \rho(1 - \epsilon^2\nabla^2)\left\{\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial w}{\partial t}\right\}, \tag{13}$$

$$K^*\mathcal{L}_v\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2}\right) = \mathcal{L}_q\frac{\partial}{\partial t}\left[\rho C^*\theta + \beta\theta_0\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right]. \tag{14}$$

we define the following dimensionless quantities

$$(x', z', u', w') = \frac{\omega_1}{c_1}(x, z, u, w),\ t'_{ij} = \frac{t_{ij}}{\beta\theta_0}, \quad t' = \omega_1 t, \quad a' = \frac{\omega_1^2}{c_1^2}a,\ \theta' = \frac{\theta}{\theta_0}, \Omega' = \frac{\Omega}{\omega_1}, \tau'_v = \omega_1\tau_v, \tau'_0 = \omega_1\tau_0, \tau'_q = \omega_1\tau_q. \tag{15}$$

where,

$$c_1^2 = \frac{\mu}{\rho} \text{ and } \omega_1 = \frac{\rho\ C^* c_1^2}{K^*}. \tag{16}$$

Upon introducing the quantities defined by Eq. (15) in Eqs. (12)-(14), and suppressing the primes, yields

$$(1 + a_1)\frac{\partial^2 u}{\partial x^2} + a_1\frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 u}{\partial z^2} - a_2\frac{\partial \theta}{\partial x} = (1 - \epsilon^2\nabla^2)\left[\frac{M}{1+m^2}\left(\frac{\partial u}{\partial t} + m\frac{\partial w}{\partial t}\right) + \frac{\partial^2 u}{\partial t^2} - a_3\Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right] \tag{17}$$

$$(1 + a_1)\frac{\partial^2 w}{\partial z^2} + a_1\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2} - \beta\frac{\partial \theta}{\partial z} = (1 - \epsilon^2\nabla^2)\left[\frac{M}{1+m^2}\left(m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + \frac{\partial^2 w}{\partial t^2} - a_3\Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right], \tag{18}$$

$$\mathcal{L}_v a_4\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2}\right) = \mathcal{L}_q\frac{\partial}{\partial t}\left[a_5\theta + \beta\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right]. \tag{19}$$

where, $a_1 = \frac{\lambda+\mu}{\mu}$, $a_2 = \frac{\beta\theta_0}{\mu}$, $a_3 = \frac{{\omega_1}^2}{{c_1}^2}$, $a_4 = \frac{K^*\omega_1}{c_1}$, $a_5 = \rho C^*$ and $M = \frac{\sigma_0{\mu_0}^2{H_0}^2}{\rho}$.

The initial and regularity conditions are given by

$$\begin{gathered} u(x, z, 0) = 0 = \dot{u}(x, z, 0), \\ w(x, z, 0) = 0 = \dot{w}(x, z, 0), \\ \theta(x, z, 0) = 0 = \dot{\theta}(x, z, 0) \text{ for } z \geq 0, -\infty < x < \infty, \\ u(x, z, t) = w(x, z, t) = \theta(x, z, t) = 0 \ for\ t > 0\ when\ z \to \infty. \end{gathered} \tag{20}$$

Applying Laplace & Fourier Transforms defined by

$$\bar{f}(x, z, s) = \int_0^\infty f(x, z, t)\, e^{-st} dt, \tag{21}$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^{\infty} \bar{f}(x, z, s)\, e^{i\xi x} dx. \tag{22}$$

On Eqs. (17)-(19), we obtain a system of equations,

$$\left[(1 + a_1)(-\boldsymbol{\xi}^2) + D^2 - (1 + \epsilon^2\xi^2 - \epsilon^2 D^2)\left(\frac{M}{1+m^2}s + s^2 - a_3\Omega^2\right)\right]\tilde{u} + \left[\iota a_1\xi D - \left(\frac{Mm}{1+m^2}s + 2\Omega s\right)(1 + \epsilon^2\xi^2 - \epsilon^2 D^2)\right]\tilde{w} - [\iota\xi a_2]\tilde{\theta} = 0, \tag{23}$$

$$\left[\iota a_1\xi D - \left(\frac{M}{1+m^2}s + s^2 - 2\Omega s\right)(1 + \epsilon^2\xi^2 - \epsilon^2 D^2)\right]\tilde{u} + \left[(1 + a_1)D^2 - \boldsymbol{\xi}^2 + (1 + \epsilon^2\xi^2 - \epsilon^2 D^2)\left(\frac{M}{1+m^2}s + a_3\Omega^2\right)\right]\tilde{w} - [a_2 D]\tilde{\theta} = 0, \tag{24}$$

$$\left[\mathcal{L}_q s\beta(\iota\xi\tilde{u} + D\tilde{w})\right] + \left[\ \mathcal{L}_q a_5 s + \ \mathcal{L}_v\ a_4(-D^2 + \xi^2)\right]\tilde{\theta} = 0. \tag{25}$$

where,

$$\mathcal{L}_v = 1 + \sum_{r=1}^{R_1}\frac{{\tau_v}^r}{r!}s^r, \text{ and } \mathcal{L}_q = \varrho + \tau_0 s + \sum_{r=2}^{R_2}\frac{\tau_q^r}{r!}s^r. \tag{26}$$

From Eqs. (23)-(25), we obtain a set of homogeneous equations which will have a nontrivial solution if determinant of coefficient $[\tilde{u}, \tilde{w}, \tilde{\theta}]^T$ vanishes so as to give a characteristic equation as

$$[D^6 + QD^4 + RD^2 + S](\tilde{u}, \tilde{w}, \tilde{\theta}) = 0. \tag{27}$$

where,

$$Q = \frac{1}{P}\{\beta a_2\zeta_8\zeta_{10} + (\zeta_1 - \zeta_7)(\zeta_3\zeta_4 - \zeta_1\xi^2)\zeta_{11} + (\zeta_8\zeta_{11})(\zeta_3\zeta_7 - \xi^2) - \zeta_{11}(a_1{}^2\zeta_9{}^2 - 2\zeta_3\zeta_5\zeta_6)\},$$

$$R = \frac{1}{P}\{(\zeta_1 - \zeta_7)[\beta a_2\zeta_9{}^2\zeta_{10} + \zeta_{11}(\zeta_1\xi^4 + \zeta_3\zeta_4) + a_5\zeta_{10}(\zeta_1 + \zeta_3\zeta_4)] - (\zeta_3\zeta_7 - \xi^2)[\zeta_{11}(\zeta_{11}\xi^2 + \zeta_8 + \zeta_3\zeta_4) + a_5\zeta_8\zeta_{10}] + (a_1{}^2\zeta_9{}^2 - 2\zeta_3\zeta_5\zeta_6)(\zeta_{11}\xi^2 + a_6\zeta_{10}) - \zeta_3{}^2\zeta_5\zeta_6\zeta_{11}\},$$

$$S = \frac{1}{P}\{\zeta_3\zeta_6[\beta a_2\zeta_9\zeta_{10} + \zeta_3\zeta_5(\zeta_{11}\xi^2 + \zeta_{10}a_5)] + (\zeta_3\zeta_7 - \xi^2)[\beta a_2\zeta_9{}^2\zeta_{10} + (\zeta_{11}\xi^2 + \zeta_{10}a_5)(\zeta_1\xi^2 + \zeta_3\zeta_4)]\},$$

$$P = \zeta_8\zeta_{11}(\zeta_1 - \zeta_7) - \zeta_5\zeta_6\zeta_{11}.$$

where, $D = \frac{d}{dz}$, $\zeta_1 = 1 + a_1$, $\zeta_2 = \frac{M}{1+m^2}s$, $\zeta_3 = 1 + \epsilon^2\xi^2$, $\zeta_4 = \zeta_2 + s^2 - a_3\Omega^2$, $\zeta_5 = (\zeta_2 m + s^2 - 2\Omega s)\epsilon^2$, $\zeta_6 = (\zeta_2 m + 2\Omega s)\epsilon^2$, $\zeta_7 = (\zeta_2 + a_3\Omega^2)\epsilon^2$, $\zeta_8 = 1 + \zeta_4\epsilon^2$, $\zeta_9 = \iota\xi$, $\zeta_{10} = \mathcal{L}_q s$, $\zeta_{11} = \mathcal{L}_v a_4$.

The roots of the Eq. (27) are $\pm\lambda_i (i = 1,2,3)$ satisfying the radiation condition that $\tilde{u}, \tilde{w}, \tilde{\theta} \to 0$ as $z \to \infty$, the solutions of equation can be written as,

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z}, \tag{28}$$

$$\tilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z}, \tag{29}$$

$$\tilde{\theta} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z}. \tag{30}$$

where,

$$d_i = \frac{P^*\lambda_i{}^4 + Q^*\lambda_i{}^2 + R^*}{T^*\lambda_i{}^4 + U^*\lambda_i{}^2 + V^*} \quad i = 1,2,3. \tag{31}$$

$$l_i = \frac{P^{**}\lambda_i{}^4 + Q^{**}\lambda_i{}^2 + R^{**}}{T^*\lambda_i{}^4 + U^*\lambda_i{}^2 + V^*} \quad i = 1,2,3.$$

$$\begin{gathered} P^* = \zeta_8\zeta_{11}, \\ Q^* = -[\zeta_{11}(\zeta_1 + \zeta_3\zeta_4) + \zeta_8(\zeta_{11}\xi^2 + \zeta_{10}a_6)], \\ R^* = \beta a_2\zeta_9{}^2\zeta_{10} + (\zeta_{11}\xi^2 + \zeta_{10}a_5)(\zeta_1\xi^2 + \zeta_3\zeta_4), \\ T^* = \zeta_{11}(\zeta_1 - \zeta_7), \\ U^* = -(\zeta_1 - \zeta_7)(\zeta_{11} + a_5\zeta_{10}) + \beta a_2\zeta_{10} + \zeta_{11}(\zeta_3\zeta_7 - \xi^2), \\ V^* = -a_5\zeta_{10}(\zeta_3\zeta_7 - \xi^2), \\ P^{**} = \zeta_8(\zeta_1 - \zeta_7) - \zeta_5\zeta_6, \\ Q^{**} = -[(\zeta_1 - \zeta_7)(\zeta_1\xi^2 + \zeta_3\zeta_4) - \zeta_8(\zeta_3\zeta_7 - \xi^2) - 2\zeta_3\zeta_5\zeta_6], \\ R^{**} = -[(\zeta_1\xi^2 + \zeta_3\zeta_4)(\zeta_3\zeta_7 - \xi^2) - \zeta_3{}^2\zeta_5\zeta_6. \end{gathered} \tag{32}$$

4. Boundary conditions

We consider on the half-surface $(z = 0)$ normal force is applied. The boundary conditions are

$$(1)\ t_{zz}(x, z, t) = G(t)\delta(x), \tag{33}$$

$$(2)\ t_{xz}(x, z, t) = 0, \tag{34}$$

$$(3)\ \theta(x, z, t) = 0, \tag{35}$$

where $\delta(x)$ is dirac delta function of x and $G(t)$ is a function defined as

$$G(t) = \begin{cases} 0; & t \le 0 \\ T_1\frac{t}{t_0}; & 0 < t \le t_0. \\ T_1; & t > t_0 \end{cases} \tag{36}$$

where t_0 indicates the length of the time to raise the heat and T_1 is a constant, this means that the boundary of the half space, which is initially at rest and has a fixed temperature t_0, is suddenly raised to a temperature equal to function $G(t)\delta(x)$ and maintained at this temperature afterwards.

Applying Laplace and Fourier transform to Eq. (33), we get,

$\tilde{\theta}(\zeta, 0, s) = \bar{G}(s)$, where $\bar{G}(s) = T_1\frac{(1-e^{-st_0})}{t_0 s^2}$.

Applying the Laplace and Fourier transform defined by Eqs. (21) and (22) on the boundary conditions (33)-(35) and then using the dimensionless quantities defined by Eq. (15) and using Eqs. (4) and (6) and substituting values of $\hat{u}, \hat{w}, \hat{\theta}$ *and* $\hat{\varphi}$ from Eqs. (28)-(30), and solving, we obtain the components of displacement, stresses and conductive temperature as

$$\tilde{u} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 M_{1i}e^{-\lambda_i z}\}, \tag{37}$$

$$\tilde{w} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 d_i M_{1i}e^{-\lambda_i z}\}, \tag{38}$$

$$\tilde{\theta} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 l_i M_{1i}e^{-\lambda_i z}\}, \tag{39}$$

$$\tilde{\varphi} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\left\{\sum_{i=1}^3 \frac{l_i}{1+a\xi^2-a\lambda_i{}^2}M_{1i}e^{-\lambda_i z}\right\}, \tag{40}$$

$$\widetilde{t_{zz}} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 R_i M_{1i}e^{-\lambda_i z}\}, \tag{41}$$

$$\widetilde{t_{zx}} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 \Delta_{2i} M_{1i}e^{-\lambda_i z}\}, \tag{42}$$

$$\widetilde{t_{xx}} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 S_i M_{1i}e^{-\lambda_i z}\}, \tag{43}$$

$$\widetilde{J_x} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 U_i M_{1i}e^{-\lambda_i z}\}, \tag{44}$$

$$\widetilde{J_z} = T_1\frac{(1-e^{-st_0})}{\Delta t_0 s^2}\{\textstyle\sum_{i=1}^3 V_i M_{1i}e^{-\lambda_i z}\}, \tag{45}$$

$$\Delta = \textstyle\sum_{i=1}^3 M_{3i}\mathrm{N}_i. \tag{46}$$

where,

$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}$, $M_{12} = \Delta_{21}\Delta_{33} - \Delta_{31}\Delta_{23}$, $M_{13} = \Delta_{32}\Delta_{21} - \Delta_{31}\Delta_{22}$, $M_{31} = \Delta_{22}\Delta_{33} + \Delta_{32}\Delta_{23}$, $M_{32} = \Delta_{21}\Delta_{33} + \Delta_{31}\Delta_{23}$, $M_{33} = \Delta_{32}\Delta_{21} + \Delta_{31}\Delta_{22}$,

$\Delta_{2i} = \iota\xi d_i - \lambda_i$, $\Delta_{3i} = l_i\lambda_i$, $\mathrm{N}_i = \lambda_i d_i(\lambda + 2\mu) + \beta l_i$, $\mathrm{R}_i = \lambda_i d_i(\lambda + 2\mu) + \beta\theta_0 l_i$, $\mathrm{U}_i = (m - d_i)\frac{\sigma_0\mu_0 H_0 s}{1+m^2}$,

$\mathrm{V}_i = (1 + m d_i)\frac{\sigma_0\mu_0 H_0 s}{1+m^2}$, $\mathrm{S}_i = \iota\xi(\lambda + 2\mu) - \beta\theta_0 l_i$; $i = 1,2,3$.

5. Particular cases

(i) If $a = 0$, then from Eqs. (37)-(45), the corresponding expressions for displacements, stresses, current density and conductive temperature for nonlocal isotropic solid without two temperature are obtained.

(ii) If $\epsilon = 0$, then from Eqs. (37)-(45), the corresponding expressions for displacements, stresses, current density and conductive temperature for local isotropic solid with hall current and two temperature are obtained.

(iii) If $\epsilon = a = 0$, then from Eqs. (37)-(45), the corresponding expressions for displacements, stresses, current density and conductive temperature for isotropic local thermoelastic solid are obtained.

(iv) If $m = \epsilon = 0$, then from Eqs. (37)-(45), the corresponding expressions for displacements, stresses, current density and conductive temperature for local isotropic solid without hall current are obtained.

6. Inversion of the transformation

For obtaining the solution of the problem in physical domain, the transforms in Eqs. (37)-(45) need to be inverted. Here all the displacement components, stress components and conductive temperature are of the form $f(\xi, z, s)$, being a function of z and the parameters of Laplace and Fourier transforms s and ξ. For obtaining the function $f(x, z, t)$ in the physical domain, we first invert the Fourier transform as used by Sharma *et al.* (2008), using

$$\tilde{f}(x, z, s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x) f_e - i \sin(\xi x) f_0| \, d\xi. \tag{47}$$

where, f_e and f_0 are respectively the even and odd parts of $\hat{f}(\xi, z, s)$. Thus the expression gives the Laplace transform $\tilde{f}(x, z, s)$ of the function $f(x, z, t)$, which can be inverted Following Honig and Hirdes (1984).

The Last step is to calculate the integral in Eq. (47), which is evaluated by the method as described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and discussion

Magnesium material has been selected for the purpose of numerical calculation which is transversely isotropic and according to Dhaliwal and Singh (1980), physical data for which is given as

$$\lambda = 9.4 \times 10^{10} Nm^{-2},\ \mu = 3.278 \times 10^{10} Nm^{-2}, K^* = 1.7 \times 10^2 Wm^{-1}K^{-1},\ \rho = 1.74 \times\ 10^3 Kgm^{-3}, \theta_0 = 298\ K, C^* = 10.4 \times 10^2 JKg^{-1}deg^{-1},\ \mu_0 = 4\pi \times 10^{-7} Hm^{-1}, \sigma_0 = \frac{10^{-9}}{36\pi} Fm^{-1}, H_0 = 1\ Jm^{-1}n\ b^{-1},\ a = 0.05.$$

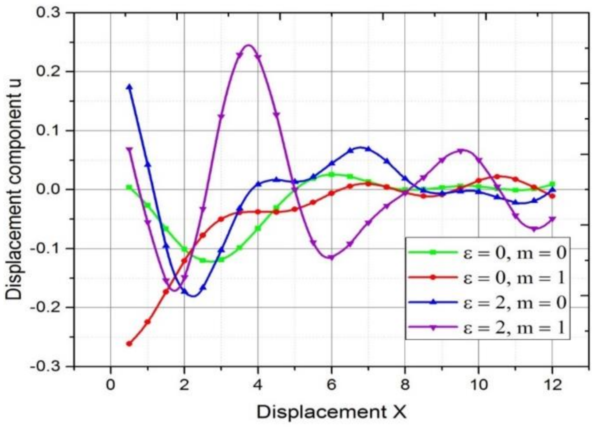

Fig. 1 Variation of displacement component u with displacement x

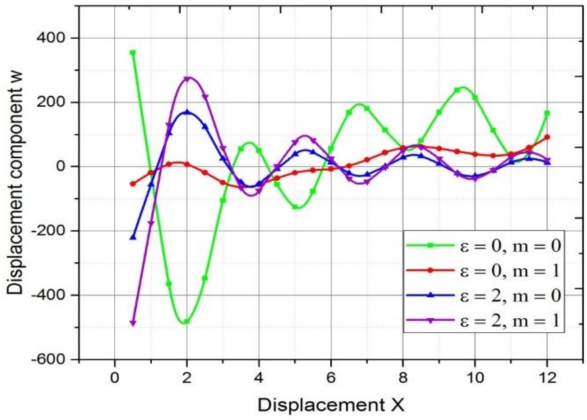

Fig. 2 Variation of displacement component w with displacement x

Using the above values, a comparison of values of displacement components u and w, stress components t_{zz}, t_{xx}, t_{zx}, current density components J_z, J_x and conductive temperature φ for a transversely isotropic nonlocal magneto-thermoelastic solid with distance x has been made and the effects of hall current and nonlocality have been studied.

1) The solid green colored line with center symbol square corresponds to local parameter $(\epsilon = 0)$ and $m = 0$.

2) The solid reddish colored line with center symbol circle represents local parameter $(\epsilon = 0)$ and $m = 1$.

3) The solid blue colored line with center symbol upward triangle corresponds to nonlocal parameter $(\epsilon = 2)$ and $m = 0$.

4) The solid purplish colored line with center symbol downward triangle represents nonlocal parameter $(\epsilon = 2)$ and $m = 1$.

Fig. 1 shows the variations of the displacement component u for isotropic magneto-thermoelastic nonlocal medium with hall effects. It is clear that the values of u follow oscillatory pattern. For $\epsilon = 0$ and $m = 1$, the variations are increasing rapidly for $0 < x < 2$ while later on it follows oscillatory path but the oscillations are less as compared to other values. All other values for different ϵ

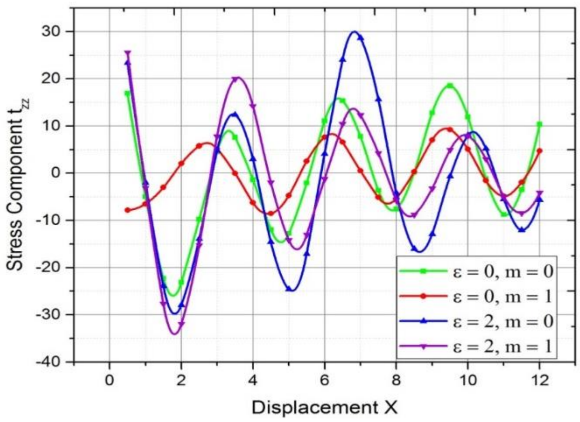

Fig. 3 Variation of stress component t_{zz} with displacement x

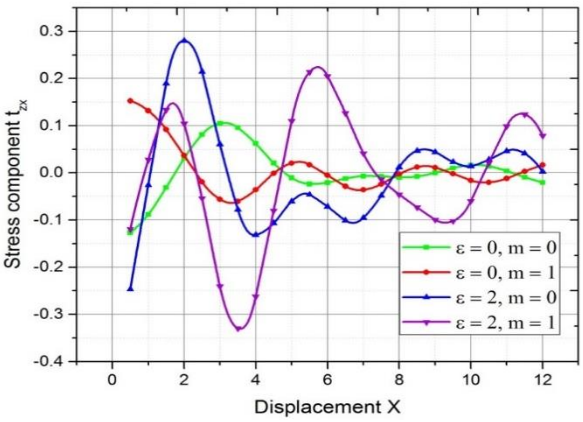

Fig. 4 Variation of stress component t_{zx} with displacement x

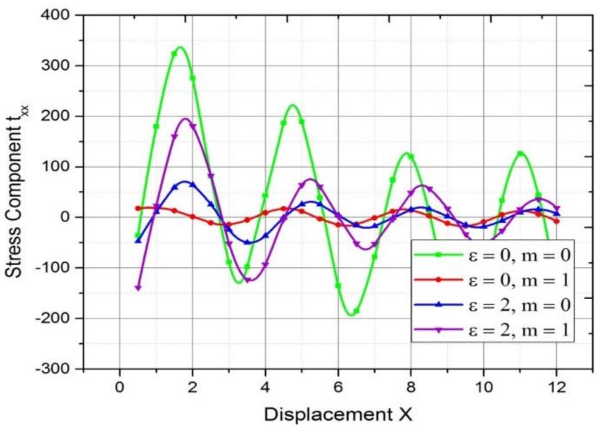

Fig. 5 Variation of stress component t_{xx} with displacement x

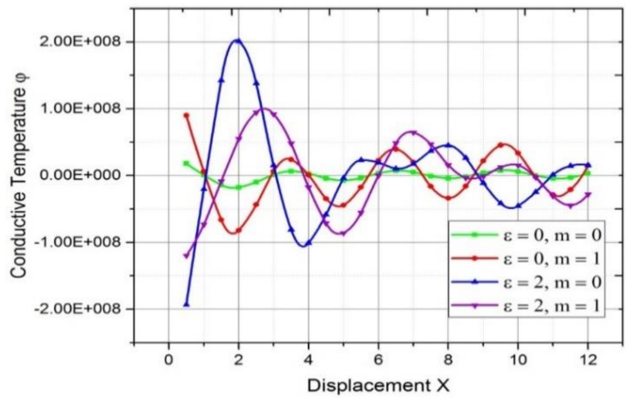

Fig. 6 Variation of conductive temperature φ with displacement x

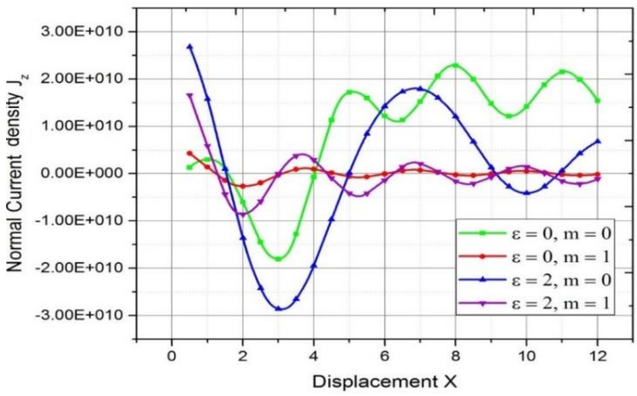

Fig. 7 Variation of normal current density J_z with displacement x

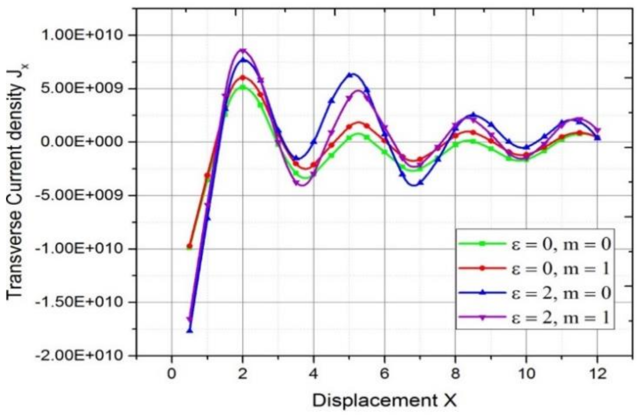

Fig. 8 Variation of transverse current density J_x with displacement x

and m follows perfectly oscillatory path from beginning to end. Fig. 2 depicts the variation of values of displacement component w w.r.t displacement. The pattern is oscillatory with a clear difference between values for local and non-local parameters. For all the values w follows oscillatory pattern. For $\epsilon = 0$ and $m = 0$, the variations are following oscillatory path with highest magnitude of variations. Fig. 3 describes the variations of the stress component t_{zz}. Here too the behavior followed is oscillatory while the variations are perfectly oscillatory for all the values but the effects of hall effect and nonlocality are clearly visible. Fig. 4 shows the variation of stress component t_{zx}. Here too the behavior followed is oscillatory with nonlocality effects clearly having more magnitude of oscillations for both vaues of m. Fig. 5 shows the variation of stress component t_{xx}. The behavior followed is oscillatory with more magnitude of oscillations for$\epsilon = 0,\ m = 0$ and $\epsilon = 2,\ \mathrm{m} = 0$. Fig. 6 shows the variation of conductive temperature φ. The pattern followed is oscillatory for all the values with the magnitude of variations less for $\epsilon = 0,\ m = 0$ and maximum for $\epsilon = 2,\ m = 0$. Fig. 7 shows the variation of normal current density vector J_z w.r.t. x. The pattern followed is oscillatory with the maximum magnitude of variations for $\epsilon = 2,\ m = 0$ and minimum for $\epsilon = 0,\ m = 1$. Also, the effects for nonlocal parameter and hall current are clearly visible. Fig. 8 shows the variation of transverse current density J_x w.r.t. x. The variation for all values is oscillatory with almost same type of trends followed. The difference for all values shows the effects of nonlocality and hall current.

8. Conclusions

From above discussion, it is clear that there is a great impact of nonlocal parameter and hall current on the components of displacements, stress components, current density and conductive temperature in an isotropic magneto-thermoelastic medium. It is observed from the figures (1-8) that nonlocality is playing a significant effect. Under the combined effects of nonlocality and hall current; all the components are following an oscillatory path with respect to variations in x. This paper gives an inspiration to study the effects of nonlocalty further in magneto-thermoelastic materials. The results obtained in this paper can be useful for the people interested in the fields of nonlocal thermoelasticity, nonlocal material sciences and material designing. The results provide a motivation to investigate conducting thermoelectric materials as a new class of applicable thermoelectric solids. The research has more importance as the interaction of hall current, rotation and nonlocality together has not been studied yet and so can found various applications especially for the researchers working in the field of optics, acoustics, geomagnetic, and oil prospecting, geophysics, marine engineering, acoustics etc.

References

Abbas, I.A. (2014), "Eigenvalue approach for an unbounded medium with a spherical cavity based upon two-temperature generalized thermoelastic theory", *J. Mech. Sci. Technol.*, **28**, 4193-4198. https://doi.org/10.1007/s12206-014-0932-6.

Abbas, I.A. and Othman, M.I. (2012), "Generalized thermoelastic interaction in a fiber-reinforced anisotropic half-space under hydrostatic initial stress", *J. Vib. Control*, **18**(2), 175-182. https://doi.org/10.1177/1077546311402529.

Abdelmalek A., Bouazza, M., Zidour, M. and Benseddiq, N. (2017), "Hygrothermal effects on the free vibration behavior of composite plate using nth-order shear deformation theory: A micromechanical approach", *Iran. J. Sci. Technol. Trans. Mech. Eng.*, 1-13. https://doi.org/10.1007/s40997-017-0140-y.

Abouelregal, A.E. (2019), "Rotating magneto-thermoelastic rod with finite length due to moving heat sources via Eringen's nonlocal model", *J. Comput. Appl. Mech.*, **50**(1), 118-126, https:/doi.org/10.22059/jcamech. 2019.275893.360.

Abualnour, M., Chikh, A., Hebali, H., Kaci, A., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2019), "Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory", *Comput. Concrete*, **24**(6), 489-498. https://doi.org/10.12989/cac.2019.24.6.489.

Atwa, S.Y. and Jahangir, A. (2014), "Two temperature effects on plane waves in generalized thermo-microstretch elastic solid", *Int. J. Thermophys.*, **35**(1), 175-193. https://doi.org/10.1007/s10765-013-1541-9.

Balubaid, M.,Tounsi, A., Dakhel, B. and Mahmoud, S.R. (2019), "Free vibration investigation of FG nanoscale plate using nonlocal two variables integral refined plate theory", *Comput. Concrete*, **24**(6), 579-586. https://doi.org/10.12989/cac.2019.24.6.579.

Belbachir, N., Draich, K., Bousahla, A.A., Bourada, M., Tounsi, A. and Mohammadimehr, M. (2019), "Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings", *Steel Compos. Struct.*, **33**(1), 81-92. https://doi.org/10.12989/scs.2019.33.1.081.

Belkacem, A., Tahar, H.D., Abderrezak, R., Amine, BM., Mohamed, Z and Boussad, A. (2018), "Mechanical buckling analysis of hybrid laminated composite plates under different boundary conditions", *Struct. Eng. Mech.*, **66**(6), 761-769. https://doi.org/10.12989/sem.2018.66.6.761.

Belmahi, S., Zidour, M. and Meradjah, M. (2019), "Small-scale effect on the forced vibration of a nano beam embedded an elastic medium using nonlocal elasticity theory", *Adv. Aircraft Spacecraft Sci.*, **6**(1), 1-18. https://doi.org/10.12989/aas.2019.6.1.001.

Belmahi, S., Zidour, M., Meradjah, M., Bensattalah, T. and Dihaj, A. (2018), "Analysis of boundary conditions effects on vibration of nanobeam in a polymeric matrix", *Struct. Eng. Mech.*, **67**(5), 517-525. https://doi.org/10.12989/sem.2018.67.5.517.

Bensattalah, T., Hamidi, A., Bouakkaz, K., Zidour, M. and Daouadji, T.H. (2020), "Critical buckling load of triple-walled carbon nanotube based on nonlocal elasticity theory", *J. Nano Res.*, **62**, 108-119. https://doi.org/10.4028/www.scientific.net/JNanoR.62.108.

Bouazza, M., Amara, K., Zidour, M., Tounsi, A. and Adda-Bedia, E.A. (2015), "Postbuckling analysis of functionally graded beams using hyperbolic shear deformation theory", *Rev. Inform. Eng. Appl.*, **2**(1), 1-14. https://doi.org/10.18488/journal.79/2015.2.1/79.1.1.14.

Bousahla, A.A., Bourada, F., Mahmoud, S.R., Tounsi, A., Algarni, A., Adda Bedia, E.A. and Tounsi, A. (2020), "Buckling and dynamic behavior of the simply supported CNT-RC beams using an integral-first shear deformation theory", *Comput. Concrete*, **25**(2), 155-166. https://doi.org/10.12989/cac.2020.25.2.155.

Chen, P.J. and Gurtin, M.E. (1968), "On a theory of heat conduction involving two temperatures", *J. Appl. Math. Phys.*, **19**, 614-627. https://doi.org/10.1007/BF01594969.

Chikr, S.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, S.R. and Tounsi, A. (2020), "A novel four-unknown integral model for buckling response of FG sandwich plates resting on elastic foundations under various boundary conditions using Galerkin's approach", *Geomech. Eng.*, **21**(5), 471-487. https://doi.org/10.12989/gae.2020.21.5.471.

Dhaliwal R.S. and Singh A. (1980), *Dynamic Coupled Thermoelasticity*, Hindustan Publisher Corporation, New Delhi, India.

Dihaj, A., Zidour, M., Meradjah, M., Rakrak, K., Heireche, H. and Chemi, A. (2018), "Free vibration analysis of chiral double-walled carbon nanotube embedded in an elastic medium using non-local elasticity theory and Euler Bernoulli beam model", *Struct. Eng. Mech.*, **65**(3), 335-342. https://doi.org/10.12989/sem.2018.65.3.335.

Ebrahimi, F and Shafiei, N. (2016), "Application of Eringen's nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams", *Smart Struct. Syst.*, **17**(5), 837-857. https://doi.org/10.12989/sss.2016.17.5.837.

Edelen, D.G.B, Green, A.E. and Laws, N. (1971), "Nonlocal continuum mechanics", *Arch. Rational Mech. Anal.*, **43**(1), 36-44. https://doi.org/10.1007/BF00251543.

Edelen, D.G.B. and Laws, N. (1971), "On the thermodynamics of systems with nonlocality", *Arch. Rational Mech. Anal.*, **43**(1), 24-35. https://doi.org/10.1007/BF00251543.

Eringen, A.C. (2002), *Nonlocal Continuum Field Theories*, Springer, New York, U.S.A.

Ezzat, M.A. and El-Bary, A.A. (2017), "A functionally graded magneto-thermoelastic half space with memory-dependent derivatives heat transfer", *Steel Compos. Struct.*, **25**(2), 177-186. https://doi.org/10.12989/scs.2017.25.2.177.

Gafour, Y., Hamidi, A., Benahmed, A., Zidour, M. and Bensattalah, T (2020), "Porosity-dependent free vibration analysis of FG nanobeam using non-local shear deformation and energy principle", *Adv. Nano Res.*, **8**(1), 37-47. https://doi.org/10.12989/anr.2020.8.1.037.

Hassan, M., Marin, M., Ellahi, R. and Alamri, S.Z. (2018), "Exploration of convective heat transfer and flow characteristics synthesis by Cu-Ag/water hybrid nano-fluids", *Heat Transfer Res.*, **49**(18), 1837-1848. http://doi.org/10.1615/HeatTransRes.2018025569.

Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of laplace transform", *J. Comput. Appl. Math.*, **10**(1), 113-132. https://doi.org/10.1016/0377-0427(84)90075-X.

Hosseini, S.M. (2020), "A GN-based modified model for size-dependent coupled thermoelasticity analysis in nano scale, considering nonlocality in heat conduction and elasticity: An analytical solution for a nano beam with energy dissipation", *Struct. Eng. Mech.*, **73**(3), 287-302, https://doi.org/10.12989/sem.2020.73.3.287.

Kaddari, M., Kaci, A., Bousahla, A.A., Tounsi, A., Bourada, F., Tounsi, A., Adda Bedia, E.A. and Al-Osta, M.A. (2020), "A study on the structural behaviour of functionally graded porous plates on elastic foundation using a new quasi-3D model: Bending and free vibration analysis", *Comput. Concrete*, **25**(1), 37-57. https://doi.org/10.12989/cac.2020.25.1.037.

Kumar, R., Sharma, N. and Lata, P. (2016), "Effect of hall current in a transversely isotropic magnetothermoelastic two temperature medium with rotation and with and without energy dissipation due to normal force", *Struct. Eng. Mech.*, **57**(1), 1225-4568. https://doi.org/10.12989/sem.2016.57.1.091.

Lata, P. and Singh, S. (2019), "Effect of nonlocal parameter on nonlocal thermoelastic solid due to inclined load", *Steel Compos. Struct.*, **33**(1), 123-131. https://doi.org/10.12989/scs.2019.33.1.123.

Lata, P. and Singh, S. (2020), "Time harmonic interactions in non local thermoelastic solid with two temperatures", *Struct. Eng. Mech.*, **74**(3), 341-350. https://doi.org/10.12989/sem.2020.74.3.341.

Lata, P. and Kaur, I. (2019), "Effect of hall current in transversely isotropic magneto thermoelastic solid rotating medium with fractional order heat transfer due to normal force", *Adv. Mater. Res.*, **7**(3), 203-220. https://doi.org/10.12989/amr.2018.7.3.203.

Mahmoudi, A., Benyoucef, S., Tounsi, A., Benachour, A., Adda Bedia, E.A. and Mahmoud, S.R. (2019), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **21**(6), 1906-1926. https://doi.org/10.1177/1099636217727577.

Marin, M. (2010), "Some estimates on vibrations in thermoelasticity of dipolar bodies", *J. Vib. Control*, **16**(1), 33-47. https://doi.org/10.1177%2F1077546309103419.

Marin, M. and Florea, O. (2014), "On temporal behaviour of solutions in thermoelasticity of porous micropolar bodies", *Analele Universitatii Ovidius Constanta Seria Matematica,* **22**(1), 169-188. https://doi.org/10.2478/auom-2014-0014.

Marin, M. and Nicaise, S. (2016), "Existence and stability results for thermoelastic dipolar bodies with double porosity", *Continuum Mech. Thermodyn.*, **28**(6), 1645-1657. https://doi.org/10.1007/s00161-016-0503-4.

Marin, M., Vlase, S., Ellahi, R. and Bhatti, M.M. (2019), "On the partition of energies for the backward in time problem of thermoelastic materials with a dipolar structure", *Symmetry*, **11**(7), 1-16. https://doi.org/10.3390/sym11070863.

Medani, M.,Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A. A., Tounsi, A. and Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate using energy principle", *Steel Compos. Struct.*, **32**(5), 595-610. https://doi.org/10.12989/scs.2019.32.5.595.

Othman, M.I.A. and Abbas, I.A. (2012), "Generalized thermoelasticity of thermal-shock problem in a non-homogeneous isotropic hollow cylinder with energy dissipation", *Int. J. Thermophys.*, **33**, 913-923. https://doi.org/10.1007/s10765-012-1202-4.

Press W.H., Teukolshy S.A., Vellerling W.T. and Flannery B.P. (1986), *Numerical Recipes in Fortran*, Cambridge University Press, Cambridge, U.K.

Rahmani, M.C., Kaci, A., Bousahla, A.A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H. and Tounsi, A. (2020), "Influence of boundary conditions on the bending and free vibration behavior of FGM sandwich plates using a four-unknown refined integral plate theory", *Comput. Concrete*, **25**(3), 225-244. https://doi.org/10.12989/cac.2020.25.3.225.

Rakrak, K., Zidour, M., Heireche, H., Bousahla, A.A. and Chemi, A. (2016), "Free vibration analysis of chiral double-walled carbon nanotube using non-local elasticity theory", *Adv. Nano Res.*, **4**(1), 31-44. https://doi.org/10.12989/anr.2016.4.1.031.

Refrafi, S., Bousahla, A.A., Bouhadra, A., Menasria, A., Bourada, F., Tounsi, A., Adda Bedia, E.A., Mahmoud, S.R., Benrahou, K.H. and Tounsi, A. (2020), "Effects of hygro-thermo-mechanical conditions on the buckling of FG sandwich plates resting on elastic foundations", *Comput. Concrete*, **25**(4), 311-325. https://doi.org/10.12989/cac.2020.25.4.311.

Sharma, N., Kumar, R. and Lata, P. (2016), "Thermomechanical interactions in transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipiation with combined effects of rotation, vacuum and two temperatures", *Appl. Math. Model.*, **40**(13-14), 6560-6575. https://doi.org/10.1016/j.apm.2016.01.061.

Soleimani, A., Dastani, K., Hadi, A. and Naei, M.H. (2019), "Effect of out of plane defects on the postbuckling behaviour of graphene sheets based on nonlocal elasticity theory", *Steel Compos. Struct.*, **30**(6), 517-534. https://doi.org/10.12989/scs.2019.30.6.517.

Tlidji, Y., Zidour, M., Draiche, K., Safa, A., Bourada, M., Tounsi, A., Bousahla, A.A. and Mahmoud S.R. (2019), "Vibration analysis of different material distributions of functionally graded microbeam", *Struct. Eng. Mech.*, **69**(6), 637-649. https://doi.org/10.12989/sem.2019.69.6.637.

Tounsi, A., Al-Dulaijan, S.U., Al-Osta, M.A., Chikh, A., Al-Zahrani, M.M., Sharif, A. and Tounsi, A. (2020), "A four variable trigonometric integral plate theory for hygro-thermo-mechanical bending analysis of AFG ceramic-metal plates resting on a two-parameter elastic foundation", *Steel Compos. Struct.*, **34**(4), 511-524. https://doi.org/10.12989/scs.2020.34.4.511.

Youssef, H.M. (2005), "Theory of two-temperature-generalized thermoelasticity", *IMA J. Appl. Math.*, **71**, 383-390. https://doi.org/10.1093/imamat/hxh101.

Youssef, H.M. and Al-Lehaibi, E.A. (2007), "State space approach of two-temperature generalized thermoelasticity of onedimensional problem", *Int. J. Solids Struct.*, **44**, 1550-1562. https://doi.org/10.1016/j.ijsolstr.2006.06.035.

Zenkour, A.M. (2018), "A generalized thermoelastic dual-phase lagging response of thick beams subjected to harmonically varying heat and pressure", *J. Theor. Appl. Mech.*, **56**(1), 15-30. http://doi.org 10.15632/jtam-pl.56.1.15.

Zenkour, A.M. (2020), "Magneto-thermal shock for a fiber-reinforced anisotropic half-space studied with a refined multi-dual-phase-lag model", *J. Phys. Chem. Solids*, **137**, 109213. https://doi.org/10.1016/j.jpcs.2019.109213.

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