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Effect of two temperature on isotropic modified couple stress thermoelastic medium with and without energy dissipation

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Abstract. The objective of this paper is to study the deformation in a homogeneous isotropic modified couple stress thermoelastic medium with and without energy dissipation and with two temperatures due to thermal source and mechanical force. Laplace and Fourier transform techniques are applied to obtain the solutions of the governing equations. The displacement components, stress components, conductive temperature and couple stress are obtained in the transformed domain. Isothermal boundary and insulated boundary conditions are used to investigate the problem.The effect of two temperature and GN theory of type–II and type–III has been depicted graphically on the various components. Numerical inversion technique has been used to obtain the solutions in the physical domain. Some special cases of interest are also deduced.

Keywords: isotropic medium; two temperatures; Laplace and Fourier transform technique; modified couple stress; thermoelastic

1. Introduction

The behavior of micron-scale structures has been proven experimentally to be size dependent. Therefore, the classical continuum theory is inadequate to predict their response at nano and micro scale. Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume, in addition to the classical direct and shear forces per unit area. The classical couple stress elasticity proposed by Toupin(1962), Mindlin(1964) and Koiter (1964). This theory contains four material constants two classical and two additional for isotropic elastic materials, which are very difficult to determine by experiments. Yang *et al.* (2202) modified the classical couple stress theory and proposed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length scale parameter is needed to capture the size effect which is caused by micro-structure. This theory suffers from some inconsistencies. So, Hadjesfandiari *et al.* (2011) offered consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses. This theory was not applicable to anisotropic materials. Therefore, Chen *et al.* (2014) introduced the new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters. Lata and Kaur (2019,2019a) also established new modified couple stress model using the theory presented by Chen and Li (2014). Kumar *et al.* (2019) studied the thermoelastic thin beam in a modified couple stress with three-phase lag thermoelastic diffusion model subjected to thermal and chemical potential sources. Vibration analysis of functionally graded rectangular nano-/micro-plates was studied based on modified nonlinear coupled stress exponential and trigono-metric shear deformation plate theories by Khorshidi *et al.* (2017). Hendou and Mohammadi (2014), used an Euler–Bernoulli model for vibration analysis of micro-beams with large transverse deflection where thermoelastic damping is considered to be the main damping mechanism and displayed as imaginary stiffness into the equation of motion by evaluating the temperature profile as a function of lateral displacement. Free vibration and buckling of microbeams with the temperature change effect is presented by Ke *et al.* (2011). He *et al.* (2015) developed a new, size-dependent model for FG microplates by using the modified couple stress theory. Based on the strain gradient elasticity theory and a refined shear deformation theory, Zhang *et al.* (2015) developed an efficient, size-dependent plate model to analyze the bending, buckling, and free vibration problems of FG microplates resting on an elastic foundation. Lou *et al.* (2015) proposed a unified higher-order plate theory for FG microplates by adopting the modified couple stress theory to capture size effects and using a generalized shape function to characterize the transverse shear deformation. Thai and Kim (2013) developed a size-dependent model of the bending and free vibration of an FG Reddy plate. Ajri *et al.* (2018) analysed the non-stationary free vibration and nonlinear dynamics of the viscoelastic nano-plates in the context of consistent couple stress theory. Ghasemi, and Mohandes (2017) studied the effect of finite strain on bending of the geometrically nonlinear of micro laminated composite Euler-Bernoulli beam based on Modified Couple Stress Theory (MCST) in thermal environment. The couple stress and strain gradient theories are applied for the micro-

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scale structures by Salamat-talab *et al.* (2012), Wang *et al.* (2013), Kahrobaiyan *et al.* (2014), Shaat *et al.* (2014), Farokhi *et al.* (2015), Mohammadimehr *et al.*(2015). Abbas and Youssef discussed the two-temperature generalized thermoelasticity under ramp-type heating by finite element method. Despite of this several researchers worked on similar theory of thermoelasticity as Hassan *et al.* (2018), Marin and Nicaise (2016), Marin *et al.* (2019, 2016) Marin (1994, 2009, 2016), Abbas (2009, 2014, 2016), Abbas (2014a, b), Abbas and Youssef (2012), Abbas and Othman (2012), Abbas and Zenkour (2014), Othman and Marin (2017), Abbas and Marin (2017), Kumar *et al.* (2016), Lata and Kaur (2019), Ezzat *et al.*(2016), Sharma *et al.* (2015), Kumar *et al.* (2016), Lata (2018a, b), Othman *et al.* (2013), Arif *et al.*(2018), Fahsi *et al.* (2017).

In the present investigation, our objective is to study the deformation in a homogeneous isotropic modified couple stress thermoelastic medium with two temperatures and with and without energy dissipation. The medium is employed to the thermal and mechanical sources. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, stress components, conductive temperature and couple stress are obtained in the transformed domain and are presented graphically for different values of displacement. The effect of two temperature and GN theory of type II and type III on the resulting quantities is depicted graphically.

2. Basic equations

Following Devi *et al.* (2017), and Kumar *et al.* (2015) and the field equations for isotropic modified couple stress thermoelastic medium with and without energy dissipation and with two temperature in the absence of body forces, body couples are given by

(a) Constitutive relationships

$$t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \frac{1}{2}e_{kij}m_{lk,l} - \beta_1 T\delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha\chi_{ij}, \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \tag{3}$$

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j}. \tag{4}$$

(b) Equation of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla.\vec{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2\vec{u} - \beta_1\nabla T = \rho\ddot{\vec{u}}, \tag{5}$$

(c) Equation of heat conduction

$$K\nabla^2\varphi + K^*\nabla^2\dot{\varphi} = \rho C^*\ddot{T} + \beta_1 T_0\nabla.\ddot{\vec{u}}, \tag{6}$$

where

$$T = (1 - \alpha_1\nabla^2)\varphi. \tag{7}$$

Here $u = (u, v, w)$ is the components of displacement vector, σ_{ij} are the components of stress tensor, ε_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, α_1 is the two temperature parameter,, T is the thermodynamical temperature, φ is the conductive temperature, K^* is the coefficient of thermal conductivity, χ_{ij} is curvature, ω_i is the rotational vector, ρ is the density, K isthe materialistic constant, C^* is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/T_0 \ll 1$, G_i are the elasticity constants and, $\beta_1 = (3\lambda + 2\mu)\alpha_t$. Here α_t is the coefficients of linear thermal expansion and diffusion expansion respectively, α is the couple stress parameter, Δ is the Laplacian operator, ∇ is del operator, δ_{ij} is Kronecker's delta.

3. Formulation and solution of the problem

We consider a two dimensional homogeneous isotropic modified couple stress thermoelastic medium initially at uniform temperature T_0 occupying the region of a half space $z \geq 0$. A rectangular coordinate system (x, y, z) having origin on the surface $x_3 = 0$ has been taken. All the field quantities depend on (x, z, t). The half surface is subjected to isothermal and insulated boundary conditions.

$$\begin{aligned} u &= u(x, z, t), \\ w &= w(x, z, t), \\ \varphi &= \varphi(x, z, t). \end{aligned} \tag{8}$$

The initial and regularity conditions are given by

$$\begin{aligned} u(x, z, 0) = \ 0 \ &= \dot{u}(x, z, 0), \\ v(x, z, 0) = \ 0 \ &= \dot{v}(x, z, 0), \\ \varphi(x, z, 0) = \ 0 \ &= \dot{\varphi}(x, z, 0) for \quad z \geq 0, -\infty < x < \infty, \\ \varphi(x, z, 0) = \ 0 \ &= \dot{\varphi}(x, z, 0) for \quad z \geq 0, -\infty < x < \infty, \end{aligned}$$

Using (8) in the Eqs. (1)-(7) yields

$$(\lambda + \mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial x} - \nabla^2 u\right) - \beta_1\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$(\lambda + \mu)\frac{\partial e}{\partial z} + \mu\nabla^2 w + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial z} - \nabla^2 w\right) - \beta_1\frac{\partial T}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{10}$$

$$K\nabla^2\varphi + K^*\nabla^2\dot{\varphi} = \rho C^*(1 - \alpha_1\nabla^2)\ddot{\varphi} + \beta_1 T_0\frac{\partial^2 e}{\partial t^2}, \tag{11}$$

and

$$t_{33} = \lambda\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x}\right) + 2\mu\frac{\partial w}{\partial x} - \beta_1(1 - \nabla^2\alpha_1)\varphi, \tag{12}$$

$$t_{31} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - \frac{\alpha}{4}\nabla^2\left(-\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \tag{13}$$

$$m_{32} = \frac{\alpha}{2}\left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x\partial z}\right), \tag{14}$$

where $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2.}{\partial z^2}$

To facilitate the solution,thedimensionless quantities are defined as

$$x' = \frac{\omega^*}{c_1}x, \quad z\,' = \frac{\omega^*}{c_1}z, \;\; u' = \frac{\omega^*}{c_1}u, \;\; w' = \frac{\omega^*}{c_1}w,$$
$$t' = \omega^* t, \quad t_{ij}{}' = \frac{t_{ij}}{\beta_{1T_0}}\;, m_{ij}{}' = \frac{m_{ij}}{c_1\beta_{1T_0}}, T' = \frac{\beta_1 T}{\rho c_1^2}\;, \tag{15}$$
$$c_1^2 = \frac{\lambda + 2\mu}{\rho}\;, \quad \varphi' = \frac{\beta_1\varphi}{\rho c_1^2}, \quad \omega^{*2} = \frac{\lambda}{\mu t^2 + \rho\alpha}.$$

Using the dimensionless quantities defined by (15) in the Eqs. (9)-(14), and after suppressing the primes yields

$$a_5\frac{\partial e}{\partial x} + a_1\nabla^2 u + \frac{\alpha}{4}a_2\nabla^2\left(\frac{\partial e}{\partial x} - \nabla^2 u\right) - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{16}$$

$$a_5\frac{\partial e}{\partial z} + a_1\nabla^2 w + \frac{\alpha}{4}a_2\nabla^2\left(\frac{\partial e}{\partial z} - \nabla^2 w\right) - \frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \tag{17}$$

$$\nabla^2\varphi + a_6\nabla^2\dot{\varphi} = a_3\left(1 - \alpha_1\left(\frac{\omega^*}{c_1}\right)^2\nabla^2\right)\ddot{\varphi} + a_4\frac{\partial^2 e}{\partial t^2}, \tag{18}$$

$$t_{33} = \frac{\lambda}{\beta_1 T_0}\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x}\right) + \frac{2\mu}{\beta_1 T_0}\frac{\partial w}{\partial x} - \frac{\rho c_1^2}{\beta_1 T_0}\left(1 - \alpha_1\left(\frac{\omega^*}{c_1}\right)^2\nabla^2\right)\varphi, \tag{19}$$

$$t_{31} = \frac{\mu}{\beta_1 T_0}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - \frac{\alpha}{4\beta_1 T_0}\left(\frac{\omega^*}{c_1}\right)^2\nabla^2\left(-\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \tag{20}$$

$$m_{32} = \frac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}\left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x\partial z}\right). \tag{21}$$

where

$$a_1 = \frac{\mu}{\rho c_1^2}, \quad a_2 = \frac{\omega^{*2}}{\rho c_1^4}, \quad a_3 = \frac{\rho c_1^2 C^*}{K\omega^*}, \quad a_4 = \frac{\beta_1^2 T_0}{\rho K\omega^*},$$
$$a_5 = \frac{(\lambda + \mu)}{\rho c_1^2}, a_6 = \frac{K^*}{K},$$

The displacement components $u(x,z,t)$ and $w(x,z,t)$ are related to the scalar potentials $\Phi(\mathrm{x,z,t})$and $\Psi(\mathrm{x,z,t})$ in dimensionless form as

$$u = \frac{\partial\Phi}{\partial x} - \frac{\partial\Psi}{\partial z}, \qquad w = \frac{\partial\Phi}{\partial z} + \frac{\partial\Psi}{\partial x} \tag{22}$$

Using (22), in the Eqs. (16)-(18) , we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\Phi - T = 0, \tag{23}$$

$$(a_1\nabla^2 + \frac{\alpha}{4}a_2\nabla^4 - \frac{\partial^2}{\partial t^2})\Psi = 0, \tag{24}$$

$$a_4\frac{\partial^2}{\partial t^2}\nabla^2\Phi + \left(-\nabla^2 - a_6\frac{\partial}{\partial t}\nabla^2 + a_3\left(1 - \alpha_1\frac{\omega^*}{c_1}\nabla^2\right)\frac{\partial^2}{\partial t^2}\right)\varphi = 0, \tag{25}$$

where $\Phi = \frac{e}{\nabla^2}$, $\Psi = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$.

Applying Laplace and Fourier transformation defined by

$$\overline{f}(x_1, x_{3,}s) = \int_0^\infty f\left(x_1, x_{3,}t\right)e^{-st}dt, \tag{26}$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^{\infty}\overline{f}(x, z, s)e^{\mathrm{i}\xi x}d\,x. \tag{27}$$

On the set of Eqs. (23)-(25), we obtain system of three homogeneous equations. These resulting equations have non trivial solution if the determinant of the coefficients of $(\widehat{\Phi}, \widehat{\Psi}, \hat{\varphi},)$ vanishes, which yields the following characteristic equation

$$(PD^8 + QD^6 + RD^4 + SD^2 + T) = 0, \tag{28}$$

where

$$P = \frac{\alpha}{4}(-\;a_{11} + a_4 s^2 a_7 a_2),$$

$$\begin{aligned} Q = -(\xi^2 + s^2)\Bigg(\left(a_{10}\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) + \frac{\alpha}{4}a_{11}\xi^2\right) - \frac{\alpha}{4}a_{11}\Bigg) \\ + a_4 s^2\Bigg(-\xi^2 a_7\frac{\alpha}{4}a_2 - (1 + a_7\xi^2) \\ - a_7\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)\Bigg), \end{aligned}$$

$$\begin{aligned} R = -(\xi^2 + s^2)\left(a_{10}\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) + \frac{\alpha}{4}a_{11}\xi^2\right) + \left(-\frac{\alpha}{4}a_2\xi^4 - s^2\right)a_{10} \\ + \left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)a_3 s^2 - \frac{\alpha}{2}a_2\xi^2 a_{10}) \\ + a_4 s^2\Bigg((1 + a_7\xi^2)\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) \\ - a_7\left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right) \\ + \xi^2\left((1 + a_7\xi^2) - a_7\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)\right)\Bigg), \end{aligned}$$

$$\begin{aligned} S = -(\xi^2 + s^2)\left(-\frac{\alpha}{4}a_2\xi^4 - s^2\right)a_{10} + \left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)a_3 s^2 \\ - \frac{\alpha}{2}a_2\xi^2 a_{10}) + \left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)(-a_{10}\xi^2 \\ + a_3 s^2) \\ + a_4 s^2\Bigg((1 + a_7\xi^2)\left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right) \\ - \xi^2\Bigg((1 + a_7\xi^2)\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) \\ - a_7\left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)\Bigg)\Bigg), \end{aligned}$$

$$\begin{aligned} T = -(\xi^2 + s^2)\left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)(-a_{10}\xi^2 + a_3 s^2) \\ - a_4 s^2\xi^2(1 + a_7\xi^2)\left(a_1\xi^2 - \frac{\alpha}{4}a_2\xi^4 - s^2\right). \end{aligned}$$

The roots of the Eq. (28) are $\pm\lambda_i(i = \;\; 1,2,3,4,5)$, using the radiation condition that $\widehat{\Phi}, \widehat{\Psi}, \widehat{\varphi} \to 0$ as $z \to \infty$, the solution of Eq. (30) may be written as

$$(\widetilde{\Phi}, \widetilde{\Psi}, \tilde{\varphi}) = \sum_{i=1}^{4}(1, R_i, S_i)A_i e^{-\lambda_i z}, \tag{29}$$

where

$$R_i = \frac{-(\xi^2 a_{10} + s^2)(-\xi^2 a_{10} + s^2 a_3) + (-2\xi^2 a_{10} + s^2 a_{12})\lambda_i^2 + a_{10}\lambda_i^4}{\begin{gathered}\left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)(-\xi^2 a_{10} + s^2 a_3) + \left(\left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)a_{10} + \left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)(-\xi^2 a_{10} + s^2 a_3)\right)\lambda_i^2 \\ + \left(\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)a_{10} - \frac{\alpha}{4}a_2(-\xi^2 a_{10} + s^2 a_3)\right)\lambda_i^4 - \frac{\alpha}{4}a_{11}\lambda_i^6\end{gathered}},$$

$$S_i = \frac{-(\xi^2 + s^2)\left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right) + \left(-(\xi^2 + s^2)\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) + \left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)\right)\lambda_i^2 + \left(\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right) + (\xi^2 + s^2)\frac{\alpha}{4}a_2\right)\lambda_i^4 - \frac{\alpha}{4}a_2\lambda_i^6}{\begin{gathered}\left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)(-\xi^2 a_{10} + s^2 a_3) + \left(\left(-\xi^2 a_1 - \frac{\alpha}{4}a_2\xi^4 - s^2\right)a_{10} + \left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)(-\xi^2 a_{10} + s^2 a_3)\right)\lambda_i^2 + \\ \left(\left(-a_1 + \frac{\alpha}{2}a_2\xi^2\right)a_{10} - \frac{\alpha}{4}a_2(-\xi^2 a_{10} + s^2 a_3)\right)\lambda_i^4 - \frac{\alpha}{4}a_{11}\lambda_i^6\end{gathered}},$$

and $\quad a_7 = \alpha_1\left(\frac{\omega^*}{c_1}\right)^2, a_8 = a_7 a_3, a_9 = a_6 + a_8, a_{10} = 1 - a_9 s^2, a_{11} = a_2 a_{10},\; a_{12} = a_3 + a_{10}$.

4. Boundary conditions

The appropriate mechanical and thermal boundary conditions are defined by

$$t_{33}(x,z,t) = -F_1\psi_1(x)\delta(t), \tag{30}$$

$$t_{31}(x,z,t) = 0, \tag{31}$$

$$m_{32}(x,z,t) = 0, \tag{32}$$

$$h_1\frac{\partial\phi}{\partial z}(x,z,t) + h_2\phi(x,z,t) = 0. \tag{33}$$

where $\delta(t)$ is the Dirac delta function, F_1 is the magnitude of the force applied, $\psi_1(x)$ specify the source distribution function along x-axis, $h_1 \to 0$ corresponds to isothermal boundaries and $h_2 \to 0$ corresponds to insulated boundary.

With the aid of Eqs. (8),(19)-(22), (26)-(27), (29) and the boundary conditions (30)-(33), the expressions for the components of displacements, stress, couple stress and conductive temperature are obtained in the transformed domain as

$$\tilde{u} = \frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4}(\mathrm{i}\xi + \lambda_i R_i)B_{1i}\,e^{-\lambda_i z}, \tag{34}$$

$$\tilde{w} = \frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4}(-\lambda_i + \mathrm{i}\xi R_i)B_{1i}\,e^{-\lambda_i z}, \tag{35}$$

$$\tilde{\varphi} = \frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{4}S_i B_{i1}\,e^{-\lambda_i z}, \tag{36}$$

$$\widetilde{t_{33}} = \frac{-F_1\widehat{\psi_1}(\xi)}{\beta_1 T_0\Delta}\sum_{i=1}^{4}\lambda(-\xi^2+\lambda_i^2) + 2\mu\lambda_i(\lambda_i - \mathrm{i}\xi R_i) - \rho c_1^2\left(\left(1-\alpha_1\frac{\omega^*}{c_1}(-\xi^2+\lambda_i^2)\right)S_i\right)B_{i1}\,e^{-\lambda_i z}, \tag{37}$$

$$\widetilde{t_{31}} = \frac{-F_1\widehat{\psi_1}(\xi)}{\beta_1 T_0\Delta}\sum_{i=1}^{4}\left(\mu(-2\mathrm{i}\xi\lambda_i - (-\xi^2+\lambda_i^2)) - \frac{\alpha}{4}\left(\frac{\omega^*}{c_1}\right)^2(-\xi^4-\lambda_i^4+2\xi^2\lambda_i^2)R_i\right)B_{i1}\,e^{-\lambda_i z}, \tag{38}$$

$$\widetilde{m_{32}} = \frac{-F_1\widehat{\psi_1}(\xi)\alpha\omega^*}{2\beta_1 T_0 c_1^2}\sum_{i=1}^{4}(\lambda_i^3 - \xi^2\lambda_i)R_i\,B_{i1}e^{-\lambda_i z}. \tag{39}$$

where

$$B_{11} = \Delta_1/A_{11},$$

$$B_{21} = -\Delta_2/A_{12},$$

$$B_{31} = \Delta_3/A_{13},$$

$$B_{41} = -\Delta_4/A_{14},$$

$$A_{1i} = \frac{\lambda(-\xi^2+\lambda_i^2)}{\beta_1 T_0} + \frac{2\mu(\lambda_i + \mathrm{i}\xi R_i)}{\beta_1 T_0} - \frac{\rho c_1^2}{\beta_1 T_0}\left(\left(1-\alpha_1\frac{\omega^*}{c_1}(-\xi^2+\lambda_i^2)\right)S_i\right)$$

$$A_{2i} = \frac{1}{\beta_1 T_0}\left(\mu(-2\mathrm{i}\xi\lambda_i - (-\xi^2+\lambda_i^2)) - \frac{\alpha}{4}\left(\frac{\omega^*}{c_1}\right)^2(-\xi^4-\lambda_i^4+2\xi^2\lambda_i^2)R_i\right),$$

$$A_{3i} = \frac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}(\lambda_i^3 - \xi^2\lambda_i)R_i,$$

$$A_{4i} = (-h_1\lambda_i + h_2)S_i,$$

$$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4,$$

$$\Delta_1 = A_{11}A_{22}(A_{33}A_{44} - A_{43}A_{34}) - A_{11}A_{23}(A_{32}A_{44} - A_{42}A_{34}) + A_{11}A_{24}(A_{32}A_{43} - A_{42}A_{33}),$$

$$\Delta_2 = A_{12}A_{21}(A_{33}A_{44} - A_{43}A_{34}) - A_{12}A_{23}(A_{31}A_{44} - A_{41}A_{34}) + A_{24}A_{12}(A_{31}A_{43} - A_{41}A_{33}),$$

$$\Delta_3 = A_{13}A_{21}(A_{32}A_{44} - A_{42}A_{34}) - A_{22}A_{13}(A_{31}A_{44} - A_{41}A_{34}) + A_{13}A_{24}(A_{31}A_{42} - A_{41}A_{32}),$$

$$\Delta_4 = A_{14}A_{21}(A_{32}A_{43} - A_{42}A_{33}) - A_{22}A_{14}(A_{31}A_{43} - A_{41}A_{33}) + A_{14}A_{23}(A_{31}A_{42} - A_{41}A_{32}),$$

and $A_i = -\frac{1}{\Delta}B_{i1}F_1\widehat{\psi_1}(\xi)$.

5. Particular cases

(i)If $\alpha_1 = 0$ in the Eqs. (34)-(39), we obtain the components of displacements, stress, conductive temperature and couple stress with and without energy dissipation and without two temperature.

(ii) If $K^* = 0$ in the Eqs. (34)-(39), we obtain the components of displacements, stress, conductive temperature and couple stress for GN theory of type II.

(iii) If $K^* \neq 0$ in the Eqs. (34)-(39), we obtain the components of displacements, stress, conductive temperature and couple stress for GN theory of type III.

(iv) If $K = 0$ in the Eqs. (34)-(39), we obtain the components of displacements, stress, conductive temperature and couple stress for classical coupled thermoelasticity.

6. Inversion of the transformations

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (34)-(39). Here the displacement components, normal and tangential stresses ,conductive temperature and couple stress are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f\ (\xi\ ,z,s)$. To obtain the function $f(x,z,t)$ in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-\mathrm{i}\xi x}\hat{f}(\xi\ ,z\ ,s)\,d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty}|cos\ (\xi\ x)f_e - \ isin(\xi\ x)f_0|d\xi\ . \tag{40}$$

where f_e and f_0 are respectively the odd and even parts of $\hat{f}(\xi\ ,z\ ,s)$. Thus the expression (40) gives the Laplace transform $\bar{f}(\xi\ ,z,s)$ of the function $f(x,z,t)$. Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(\xi\ ,z,s)$ can be inverted to $f(x,z,t)$.

The last step is to calculate the integral in Eq. (41). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

7. Numerical results and disscussions

For numerical computations, following Sherief and Saleh (2005), we take the copper material as: $\lambda = 7.76 \times 10^{10} Kgm^{-1}s^{-1}$, $\mu = 3.86 \times 10^{10} Kgm^{-1}s^{-1}$, $T_0 = 293K, C^* = 3831 \times 10^3 JKg^{-1}K^{-1}, \alpha_t = 1.78 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}, K = 1,, \alpha = .05 Kgms^{-2}$,and F_1 is the force of constant magnitude of 1N.

Software GNU octave has been used to determine the components of displacements,conductive temperature, normal stress, tangential stress and couple stress for homogeneous isotropic thermoelastic medium with distance x for two different values $K^* = 0$ and $K^* = 2\ W\ m^{-1}$ with two temperature graphically.

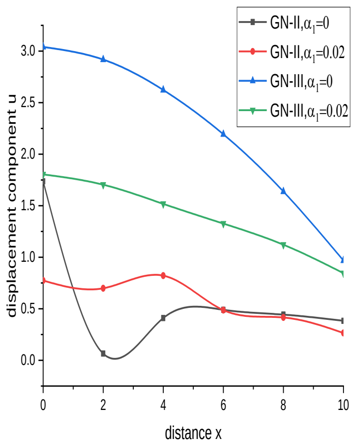

Fig. 1 Variation of displacement u with the distance x(isothermal boundary)

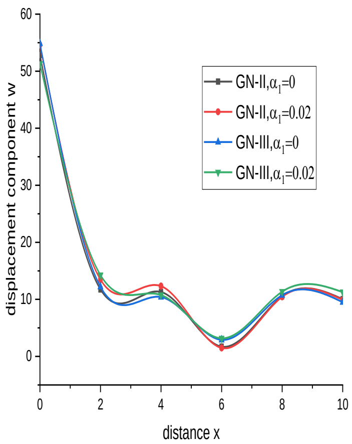

Fig. 2 Variation of displacement w with the distance x(isothermal boundary)

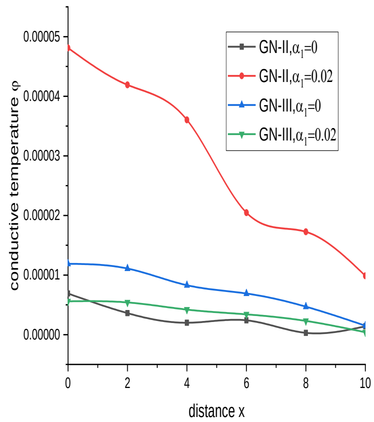

Fig. 3 Variation of conductive temperature φ with the distance x (isothermal boundary)

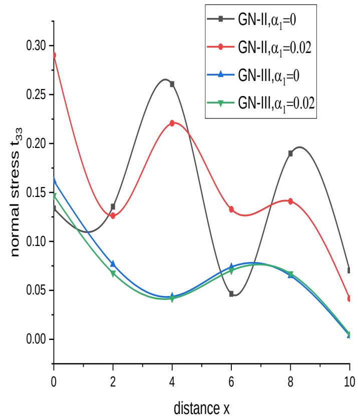

Fig. 4 Variation of normal stress t_{33} with the distance x (isothermal boundary)

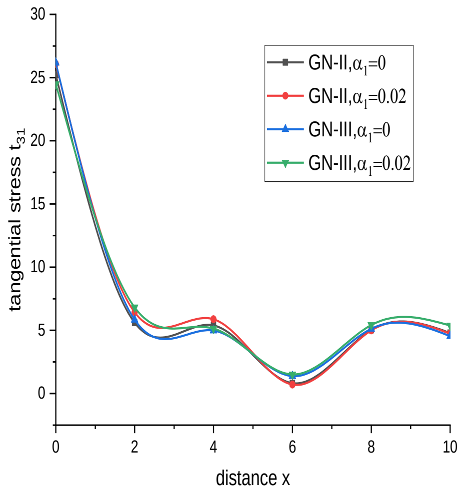

Fig. 5 Variation of tangential stress t_{31} with the distance x (isothermal boundary)

(a) In Figs. 1-6, solid line with centre symbol square corresponds to GN theory of type- II, $\alpha_1 = 0$, solid line with centre symbol circle corresponds to GN theory of type- II, $\alpha_1 = 0.02$,solid line with centre symbol triangle corresponds to GN theory of type-III, $\alpha_1 = 0$ and solid line with centre symbol inverted triangle corresponds to GN theory of type-III, $\alpha_1 = 0.02$ for mechanical force

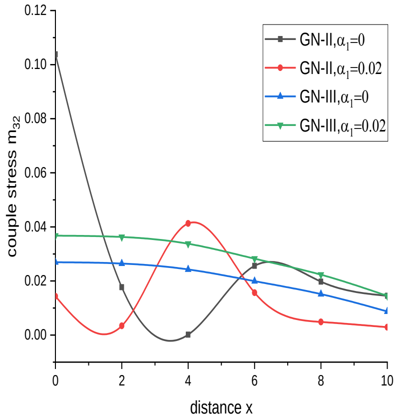

Fig. 6 Variation of couple stress m_{32} with the distance x (isothermal boundary)

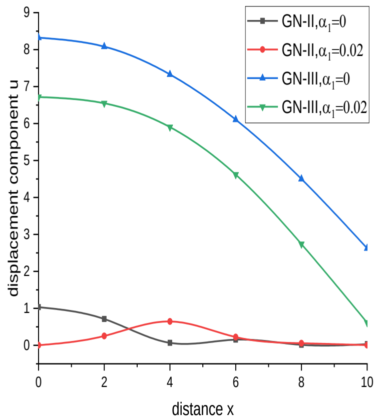

Fig. 7 Variation of displacement component u with the distance x (isothermal boundary)

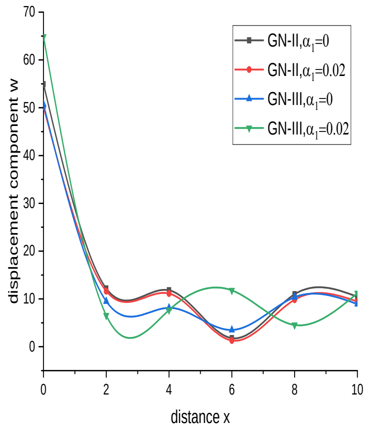

Fig. 8 Variation of displacement component w with the distance x(isothermal boundary)

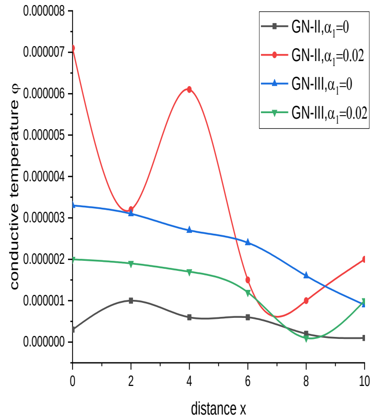

Fig. 9 Variation of conductive temperature φ with the distance x (insulated boundary)

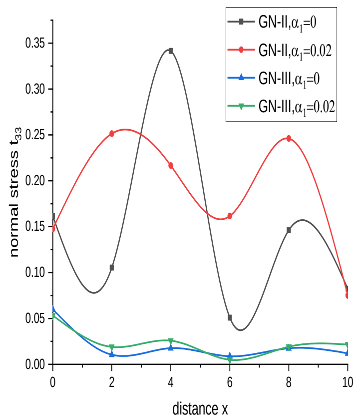

Fig. 10 Variation of normal stress t_{33} with the distance x(insulated boundary)

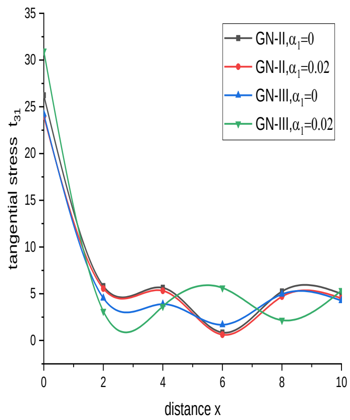

Fig. 11 Variation of tangential stress t_{31} with the distance x (insulated boundary)

with isothermal boundaries.

(b) In Figs. 7-12, solid line with centre symbol square corresponds to GN theory of type-II, $\alpha_1 = 0$, solid line with centre symbol circle corresponds to GN theory of type-II, $\alpha_1 = 0.02$, solid line with centre symbol triangle corresponds to GN theory of type-III, $\alpha_1 = 0$ and solid line with centre symbol inverted triangle corresponds to GN theory of type-III, $\alpha_1 = 0.02$ for mechanical force with insulated boundaries.

In Fig. 1 displacement u corresponding to the $\alpha_1 = 0$ decreases for the range $0 < x < 2$ and increases for $2 < x < 4$ and assumes constant value in the remaining range, corresponding to the $\alpha_1 = 0.02$ variation of u is oscillatory with the decrease in amplitude as x increases for GN theory of type –II and displacement u decreases for $0 < x < 10$ corresponding to the $\alpha_1 = 0$ and $\alpha_1 = 0.02$ in case of GN theory of type–III. In Fig. 2

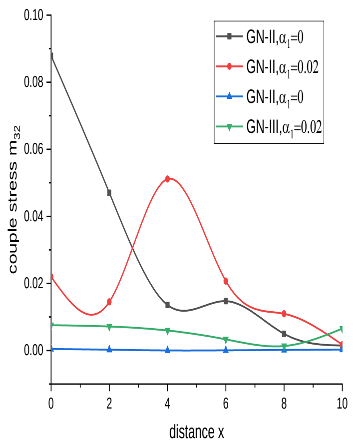

Fig. 12 Variation of couple stress m_{32} with the distance x (insulated boundary)

displacement w follows oscillatory behavior for both the $\alpha_1 = 0$ and $\alpha_1 = 0.02$ with almost equal amplitude of the variationfor both GN theory of type –II and GN theory of type–III. In Fig. 3 conductive temperature φ depicts oscillatory behavior corresponding to the $\alpha_1 = 0$ and $\alpha_1 = 0.02$ for both GN theory of type–II and GN theory of type –III. And variation is descending oscillatory in case of$\alpha_1 = 0.02$,GN theory of type–III with highest amplitude amongst all cases. In Fig. 4 normal stress t_{33} follows smooth oscillatory behavior corresponding to both the $\alpha_1 =$ and $\alpha_1 = 0.02$ except the amplitude and magnitude/value of the variation for GN theory of type –II.In case of GN theory of type–III, normal stress t_{33} corresponding to both $\alpha_1 = 0$ and $\alpha_1 = 0.02$ decreases smoothly for $0 < x < 4$ and $8 < x < 10$,increases in the remaining range. In Fig. 5 variation tangential stress t_{31} is similar to the variation of displacement w except the magnitude of variation for all the four curves. In Fig. 6 curve for the variation of couple stress m_{32} corresponding to the $\alpha_1 = 0$ decreases in the range $0 < x < 4$, increases for $4 < x < 6$ and assumes constant magnitude in rest and corresponding to the $\alpha_1 = 0.02$ decreases in the range $0 < x < 2$, $4 < x < 6$ and increases for $2 < x < 4$ assumes constant magnitude in rest for GN theory of type –II. For GN theory of type–III, couple stressm_{32} decreases for the entire range of x for both the values of two temperature parameters with the difference in the amplitude of both the curves.

In Fig. 7 displacement u shows descending oscillatory behavior for $\alpha_1 = 0$ and curve corresponding to $\alpha_1 = 0.02$ decreases for $0 < x < 4$ and increases for the rest of the range for GN theory of type –II and decreases with the increase in x for GN theory of type–III. In Figs. 8 and 10 characteristic curves for the displacement w and tangential stresst_{31} resp. follow oscillatory trend. In Fig. 9 conductive temperature φ follow oscillatory trend but with difference in the magnitude for both the curves for $\alpha_1 = 0$ and $\alpha_1 = 0.02$ corresponding to the GN theory of type–II. And corresponding to the GN theory of type–III conductive temperature decreases with the increase in distance x corresponding to $\alpha_1 = 0$, and for $\alpha_1 = 0.02$it decreases for $0 < x < 8$ and increases in the rest. In Fig. 11 normal stress t_{33}follow oscillatory trend for all the cases except that magnitude is smaller in case of GN theory of type–III. In Fig. 12 couple stress m_{32} shows descending oscillatory trend for GN theory of type–II,$\alpha_1 = 0$. For GN theory of type–II, $\alpha_1 = 0.02$curve assumes constant magnitude. And corresponding to the GN theory of type–III,$\alpha_1 = 0$ decreases for $0 < x < 4$ and corresponding to the GN theory of type–III,$\alpha_1 = 0.02$ decreases for $0 < x < 8$, and both curves increases for the rest.

8. Conclusions

From the graphs, it is clear that there is a significant impact of two temperature , GN theory of type–II and GN theory of type–III on the deformation of various components of stresses, displacement, conductive temperature, couple stress. The effect of two temperature in isotropic modified couple stress thermoelastic for insulated and isothermal boundaries, with and without energy dissipation has an imperative impact in the investigation of the deformation of the body. As distance x varied from the point of application of the source, the components of displacements, conductive temperature, normal stress, tangential stress and couple stress pursue an oscillatory pattern or smoothly decreasing behaviour . It is seen that as the disturbances travel through different constituents of the medium, the variations of displacement components, normal stress t_{33}, tangential stress t_{31} and conductive temperature φ, it experiences changes, resulting in an varying/ non- uniform pattern of curves. The results of this problem exceptionally valuable in the two dimensional problem of homogeneous isotropic thermoelastic solid with two temperature which has various geophysical and industrialised applications and helpful for designers of new materials.

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