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A new rock brittleness index on the basis of punch penetration test data

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Abstract. Brittleness is one of the most important properties of rock which has a major impact not only on the failure process of intact rock but also on the response of rock mass to tunneling and mining projects. Due to the lack of a universally accepted definition of rock brittleness, a wide range of methods, including direct and indirect methods, have been developed for its measurement. Measuring rock brittleness by direct methods requires special equipment which may lead to financial inconveniences and is usually unavailable in most of rock mechanic laboratories. Accordingly, this study aimed to develop a new strength-based index for predicting rock brittleness based on the obtained base form. To this end, an innovative algorithm was developed in Matlab environment. The utilized algorithm finds the optimal index based on the open access dataset including the results of punch penetration test (PPT), uniaxial compressive and Brazilian tensile strength. Validation of proposed index was checked by the coefficient of determination (R^2), the root mean square error (RMSE), and also the variance for account (VAF). The results indicated that among the different brittleness indices, the suggested equation is the most accurate one, since it has the optimal R^2, RMSE and VAF as 0.912, 3.47 and 89.8%, respectively. It could finally be concluded that, using the proposed brittleness index, rock brittleness can be reliably predicted with a high level of accuracy.

Keywords: brittleness index; punch penetration test; new formulation; rock strength

1. Introduction

In the field of mining and geology engineering, brittleness is considered as one of the most crucial properties of rock, as it plays an important role not only in the failure process of intact rocks but also in the rock mass response to tunneling and mining projects. Rock brittleness has been utilized for assessment of rock burst, stability of underground, fatigue damage, penetrability, cuttability, drillability and sawability of rocks (Singh 1986, Altindag 2002, Gong and Zhao 2007, Altindag 2010, Nejati and Ghazvinian 2014, Akinbinu 2016, Mikaeil *et al.* 2017, Yagiz 2017, Haeri *et al.* 2018, Chen *et al.* 2018, Mikaeil *et al.* 2018).Various researchers have considered brittleness as a combination of rock properties, rather than any one specific property (Yagiz 2009, Meng *et al.* 2015, Khandelwal *et al.* 2016). As a consequence, there is no universally approved concept or method for precisely defining rock brittleness. As a general law, in comparison to ductile rocks, a brittle rock demonstrates very little plastic deformation at breakage phase (Yagiz 2009, Meng *et al.* 2015, Haeri and Sarfarazi 2017).

Brittleness as a property of rock has a major impact on the failure process. For instance, one of the most dominant phenomena frequently observed in deep mining and tunneling projects is rock burst, a brittle failure process which releases large amount of energy (Meng *et al.* 2015). Furthermore, in shale gas extraction by hydraulic fracturing, brittleness plays a major role in the amount of gas output, particularly in tight gas reservoirs. In fact, the degree of fracturing is controlled not only by the injection pressure but also by the brittleness of shales (Meng *et al.* 2015, Kahraman *et al.* 2018).

Over the last couple of decades, numerous studies have been conducted in order to describe rock brittleness, and also to investigate the effects of rock brittleness on different geo-engineering problems. Currently, due to the lack of a universally accepted standard for rock brittleness, a wide range of methods, including direct and indirect methods, have been developed. One of the most commonly utilized methods for direct measurement of rock brittleness has been developed by Yagiz (2009), who defined rock brittleness as the ratio of the maximum applied force on the rock sample to the corresponding penetration at that force in PPT. The proposed method by Yagiz (2009) needs specific equipment which is expensive and unavailable in most of rock mechanic laboratories. Due to the lack of access to the necessary equipment and complexity of PPT, most of researchers have utilized indirect brittleness indices in order to investigate the effects of rock brittleness in different circumstances. It also must be noted that the existing brittleness indices have been developed for a wide range of purposes and projects. For this reason, this study aimed to develop a new brittleness index based on the results of PPT.

2. Rock brittleness

2.1 Definitions

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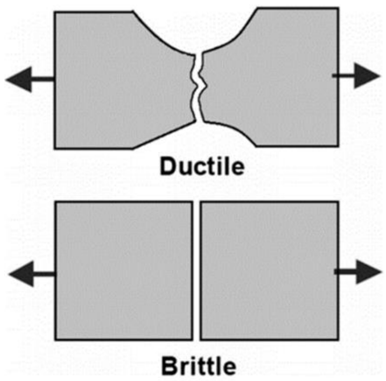

Fig. 1 The difference between ductile and brittle fracturing (Nejati and Ghazvinian 2014)

Morely (1944) defined brittleness as a parameter opposite of ductility, which was itself defined as a property of material that, under a certain amount of tension, leads to being drawn out to smaller section. In a same vein, Hetenyi (1966) described brittleness as the lack of ductility, and mentioned that brittleness is a relative concept, as it lacks a specific quantity and the amount of brittleness is contingent upon degree of reduction in the area. Obert and Duvall (1967) also mentioned that materials like cast iron, in addition to other rocks that are generally fractured in just a little higher stresses than yield stress level, demonstrate brittle failure behavior. Moreover, Ramsay (1967) defined failing tendency as the time when internal cohesiveness of rock materials deforming in the elastic range is removed. It was also added that stress condition in the moment of failure is defined in accordance to stress criteria of brittle strength. In Glossary of geology and related sciences, this parameter is described as a property of materials in which fracture and rupture occur without no or almost no plastic flow (Hucka and Das 1974). The general difference between ductile and brittle fracturing is depicted in Fig. 1.

Perhaps the best definition of rock brittleness was offered by Bieniawski (1979): “brittle fracture defined as a fracture that exhibits no or little permanent (plastic) deformation”. This definition contrasts ductile fracture, in which prior to the fracture significant plastic deformations occur. Lastly, based on the brittle facture mechanism and the result of experiments, Bieniawski (1979) presented the following stages for brittle facture of the rock under multi-axial stress loading:

- Closing of cracks
- Linear elastic deformation
- Stable fracture propagation
- Unstable fracture propagation
- Forking and coalescence of cracks

Hajiabdolmajid and Kaiser (2003) defined brittleness as one of the properties of geo-materials, in which heterogeneities exist between mechanical and geometrical properties and that loading conditions lead to a non-homogeneous distribution of stress in the failing mass, which ultimately results in the potential for fracture along the plane. In addition to the above-mentioned definitions, various other definitions, more or less similar to the ones mentioned, have been provided for this property of rock.

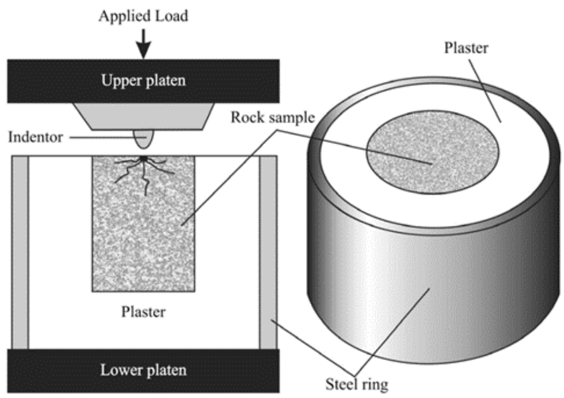

Fig. 2 Punch penetration test apparatus and sample preparation (Yagiz 2009)

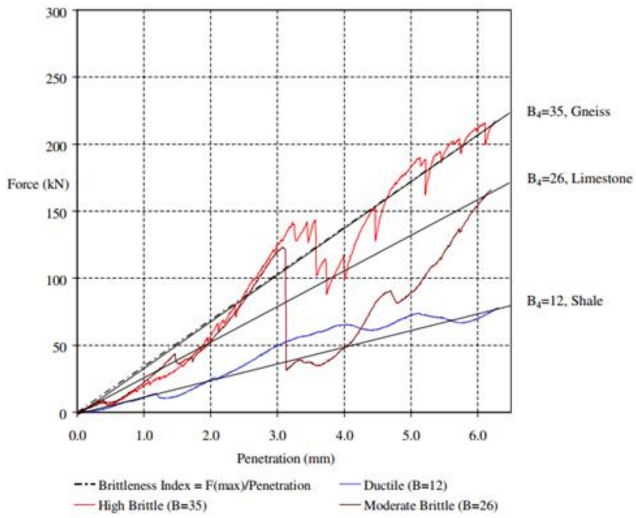

Fig. 3 Measurement of rock brittleness using force-penetration profile (Yagiz 2009)

2.2 Measurement methods

Over the years, numerous methods have been proposed by various researchers to measure rock brittleness. These methods can be categorized in two distinct groups, including direct and indirect methods. Nowadays, direct measurement of the rock brittleness can be performed using either PPT or brittleness values test. Measuring the rock brittleness by direct methods requires special devices which are expensive and generally unavailable in most of the rock mechanic laboratories.

Yagiz (2009) proposed a direct method to measure the rock brittleness using the results of PPT. The process of the PPT and sample preparation is shown in Fig. 2. For further description of the utilized apparatus with the test procedure refer to Yagiz (2009). He stated that the rock brittleness can be determined using the slope of obtained force-penetration profile in the PPT. An example of such a force-penetration profile is illustrated in Fig. 3. As it is shown, the slope was determined by drawing a line from the origin of the force-penetration profile to the maximum applied force. In other words, the measured rock brittleness is defined as the ratio of maximum applied force on specimen to the corresponding penetration (Eq. (1)).

$$BI_m = \frac{F_{max}}{P} \tag{1}$$

where BI_m is measured rock brittleness in *kN*/mm, F_{max} is maximum applied force on a rock sample in *kN* and *P* is the corresponding penetration at maximum force in *mm*.

For the sake of simplicity, a number of brittleness indices based on different concepts have been proposed for indirect measuring rock brittleness. In what follows, commons indirect methods and indices pertaining to the rock brittleness, offered by the literature, are reported. The synthesis of these methods is available in Meng *et al.* (2015). Meng *et al.* (2015) summarized the existing brittleness indices as follows:

<u>indices attained from stress – strain curve</u>

- Based on the strength
- Based on the deformation
- Based on the energy

<u>indices attained from physical – mechanical property</u>

- Based on the hardness
- Based on the fines content
- Based on the penetration test
- Based on the point load testing
- Based on the friction angle
- Based on the mineral composition

As mentioned previously, due to the lack of a standard definition and/or a universally accepted measurement method for rock brittleness, various brittleness indices have been introduced based on different concepts. Among a pool of brittleness indices, for the sake of simplicity and availability of testing equipment pertaining to strength of materials, strength-based brittleness indices have been widely utilized in a wide range of researches. Parameters in these indices include uniaxial compressive strength (UCS) and Brazilian tensile strength (BTS). Literature review revealed that four widely used strength-based brittleness indices are B_1, B_2, B_3 and B_4 (Eqs. (2)-(5)).

Hucka and Das (1974) defined B_1 as the ratio of UCS to BTS. Additionally, they stated that an increase in the difference between the compressive strength and tensile strength is associated with an increase in brittleness (Eq. (2)). By considering this fact, Hucka and Das (1974) provided another brittleness index. In this case, B_2 is defined as the ratio of UCS minus BTS to UCS plus BTS (Eq. (3)). Subsequently, Altindag (2002) introduced another strength-based brittleness index. B_3 was defined as “the area under line in relation to compressive strength and tensile strength” (Eq. (4)). More recently, Yarali and Soyer (2012) suggested B_4 based on the results of laboratory, in a study pertaining to rotary drilling (Eq. (5)).

$$B_1 = \left(\sigma_c / \sigma_t\right) \tag{2}$$

$$B_2 = \left(\sigma_c - \sigma_t\right) / \left(\sigma_c + \sigma_t\right) \tag{3}$$

$$B_3 = \left(\sigma_c \times \sigma_t\right) / 2 \tag{4}$$

$$B_4 = \left(\sigma_c \times \sigma_t\right)^{0.72} \tag{5}$$

where, σ_c and σ_t are UCS and BTS of the rock, respectively.

2.3 Applications of strength-based brittleness indices

The strength-based brittleness indices have been widely utilized by researchers for a wide range of geo-mechanical applications. In what follows, the most important and well-known studies are reviewed. Goktan (1991) investigated the relationship between the rock brittleness (B_2) and specific energy in the rock cutting process, whose results indicated no significant relationship between specific energy and B_2. It was concluded that having only one definition for brittleness is not advisable. Altindag (2002), after reviewing the factors affecting the drill-ability of rocks, performed an extensive study in order to investigate the relationship between drill-ability and rock brittleness indices. The result showed that, among the available indices (B_1 and B_3), there is a strong and significant relationship between B_3 and mechanical properties of rock like point load index, density, wave velocity, and cone indenter.

In another experimental research, Kahraman (2002) stated that drill-ability of rotary drilling and bore-ability of tunnel boring machine (TBM) can be predicted from machine parameters and rock properties. He performed statistical analysis in order to study the correlation between existing brittleness indices (B_1 and B_2) and rock drill-ability, and also bore-ability. The results of this research indicated an exponential relationship between TBM penetration rate and B_1 and B_2. It also showed that, there is no relationship between drill-ability and brittleness indices. Altindag (2003) carried out an experimental study to investigate the relationship between rock cutting specific energy and various brittleness indices include B_1, B_2 and B_3. The results suggested that specific energy has a strong and meaningful relationship with B_3.

Goktan and Yilmaz (2005) stated that rock brittleness plays an important role in the performance of drilling picks in rocks and prevents crack propagation and its spreading. They studied the correlation between normalized specific energy and B_2 as a widely used brittleness index. The results suggested a meaningful relationship between these two parameters. Atici and Ersoy (2009) performed an extensive study for examining the correlation between specific energy of cut-ability and drill-ability with different brittleness indices, including B_1, B_2, and B_3. B_3 was observed to have the most statistically significant correlation with specific energy compared with other indices. Yagiz (2009) utilized the data of 48 tunneling projects from all over the world and attempted at introducing a new index of brittleness. The study resulted in suggesting a new brittleness index based on PPT. Yagiz studied the relationship between the new Index with the existing brittleness value (B_1, B_2 and B_3) and stated that the demonstrated relationship is acceptable.

Altindag (2010), by performing simple regression analyses on raw data obtained from the literature, investigated the relationship between penetration rate (PR) of rock drilling and different brittleness indices. It was firstly concluded that a low correlation coefficient, ranging from 0 to 0.66, exists between PR and B_1 and B_2, and that correlation coefficient of PR with B_3 and B_4 is higher, ranging from 0.64 to 0.92. Subsequently, this researcher used PR_n index (normalized penetration rate) instead of PR, which resulted in finding no meaningful relationship between PR_n index and B_1 and B_2. Nevertheless, a stronger relationship was observed between B_4 and PR_n compared with B_4 and PR.

Heidari *et al.* (2014) claimed that brittleness indices and porosity are related to one another, as brittleness indices are related to UCS and BTS, two parameters which are in fact affected by porosity. The utilized brittleness indices were B_1, B_2, and B_3. In this research, statistical analysis method was utilized in order to investigate the relationship between brittleness indices and rock porosity in dry and saturated conditions. The results showed that the utilized brittleness indices had a weak relationship with rock porosity in both dry and saturated conditions.

Mikaeil *et al.* (2014) conducted a study in order to investigate the relationship between system vibration as an important parameter in measurement of rock sawing process and different brittleness indices. The utilized brittleness indices in this research were B_1, B_2, B_3 and B_4. The researchers claimed that B_3 and B_4 are the most appropriate brittleness indices for estimating vibration system in rock sawing process. Nejati and Moosavi (2016) studied the relationship between fracture toughness related to mode I and II (K_{IC} and K_{IIC}, respectively) and different brittleness indices. Statistical Analysis showed that B_4 among different brittleness indices has the highest correlation with both K_{IC} and K_{IIC}.

Ko *et al.* (2016) investigated the relationship between Cerchar Abrasiveness Index (CAI) and different brittleness indices of igneous and metamorphic rocks. The results showed that B_1 and B_3 have the highest impact on CAI in igneous and metamorphic rocks, respectively. Mikaeil *et al.* (2017) conducted extensive researches in order to investigate the effect of brittleness on the amount of energy used in the rock sawing process. The utilized data were categorized into two groups of hard and soft rocks, with accordance to the nature of the fracture process in hard and soft rocks. The results indicated a strong relationship between B_3 index, in both categories, with the amount of specific energy.

3. Development of the new brittleness index

3.1 Derivation of the base form

The strength-based brittleness indices, as the most common ones, have been developed based on diverse assumptions for different purposes. As discussed in the previous section, based on the different combinations of uniaxial compressive and tensile strength, four different brittleness indices were developed. By considering these equations, it is revealed that, among the four indices, three of them have the same form. In fact, Eqs. (2),(3) and (5) can be rewritten as follow:

$$B_1 = (\sigma_c / \sigma_t) = \sigma_c^{1} \times \sigma_t^{-1} \tag{6}$$

$$B_3 = \frac{(\sigma_c \times \sigma_t)}{2} = \frac{1}{2}\sigma_c^{1} \times \sigma_t^{1} \tag{7}$$

$$B_4 = (\sigma_c \times \sigma_t)^{0.72} = \sigma_c^{0.72} \times \sigma_t^{0.72} \tag{8}$$

Form Eqs. (6)-(8), it can be inferred that B_1, B_3, and B_4 have an identical base form as follows:

$$B = \sigma_c^{x} \times \sigma_t^{y} \tag{9}$$

In fact, it can be stated that the main difference between Eqs. (2),(3) and (5) is related to the powers of UCS and BTS in these equations. In this research, Eq. (9) is selected as the base form of the new brittleness index. In this regard, this study aimed to find the optimal **x** and **y** for Eq. (9).

3.2 Materials and methods

In this study, an open access dataset published by Yagiz (2009) was utilized for calculating the coefficients of suggested equation. Dataset was composed of various rocks including sedimentary, metamorphic and igneous rocks. Rock types pertaining to the dataset are categorized by percentage in Fig. 4. The first part of dataset was established by performing UCS and BTS tests in accordance with ASTM (1995) standard. The second part of dataset is the measured values of rock brittleness which was established by carrying out PPT according to Yagiz (2009). The tests were conducted at the Earth Mechanics Institute of Colorado School of mines, in the USA. The basic descriptive statistics of the open access database, including 48 cases, are given in Table 1. It should be mentioned that in this research, the open access database was divided into two groups randomly: one group for calculating the coefficients of Eq. (9) including 75% of the datasets (i.e., 36 datasets) and the other group including the rest of the datasets (i.e., 12 datasets) for testing the performance of proposed model.

Table 1 Basic descriptive statistics of the open access dataset

Parameter (unit)	Min.	Max.	Ave.	SD	Var.
UCS (MPa)	9.5	327	126.38	70.25	4936.33
BTS (MPa)	2.3	17.8	7.81	3.41	11.63
BIm (kN/mm)	10	45	27.45	9.41	88.59

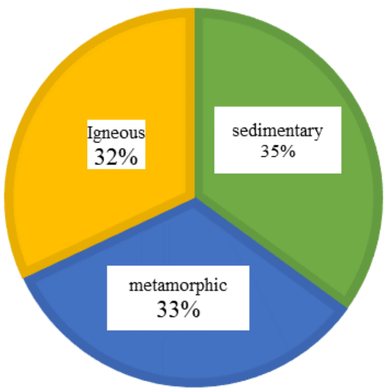

Fig. 4 Distribution of rock types pertaining to the open access dataset

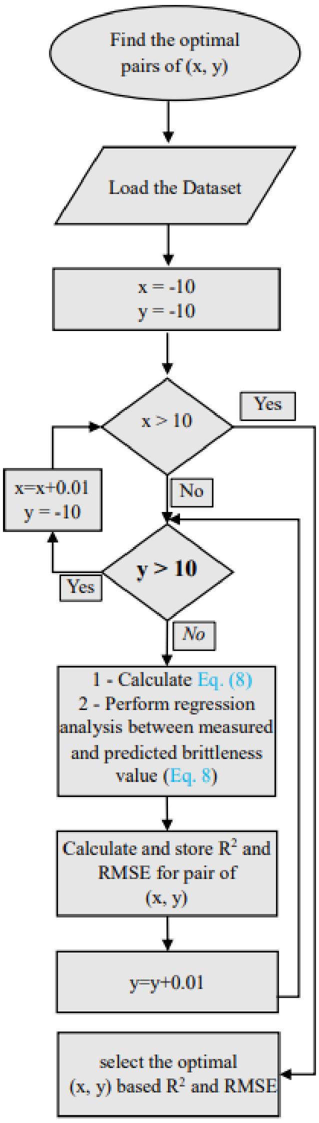

Fig. 5 Flowchart of utilized algorithm in Matlab

3.3 Calculation of x and y

In order to calculate the coefficients of suggested equation (Eq. (9)), an innovative algorithm was developed in Matlab environment. All possible scenarios for **x** and **y**, ranging from -10 to 10 with 0.01 incremental step, are checked by proposed algorithm, and subsequently, the best scenario of (**x, y**) with the highest value of R^2 and the lowest value of RMSE, are selected. The flowchart of utilized algorithm is illustrated in Fig. 5. In the following, the general process of the algorithm is discussed.

In the first step, the open access dataset provided by Yagiz (2009) is loaded. In the following, algorithm considers both of **x** and **y** equal to -10. At this time, Algorithm calculates Eq. (9) and stores the result in a vector column. Subsequently, the simple regression analysis is performed between the results of previous step and measured values of rock brittleness. The results of simple regression including R^2 and RMES are stored in two different matrices. In the next step, **y** is increased by 0.01 and the process is repeated until **y** reaches 10. By reaching **y** to 10, **x** is increased by 0.01 and **y** is reset to -10 and the whole process are repeated until **x** reaches to 10.

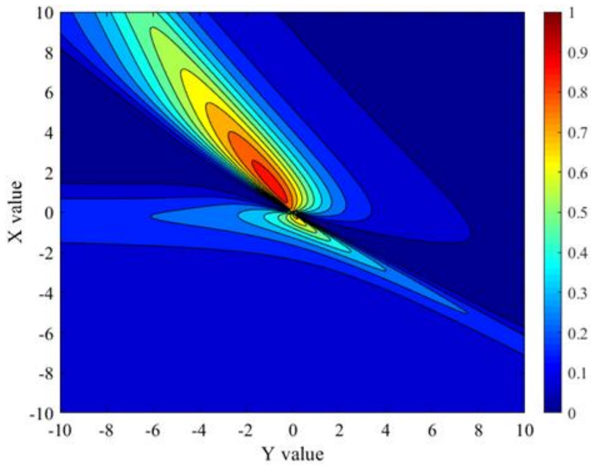

Fig. 6 Graphical representation of R^2 for all scenarios

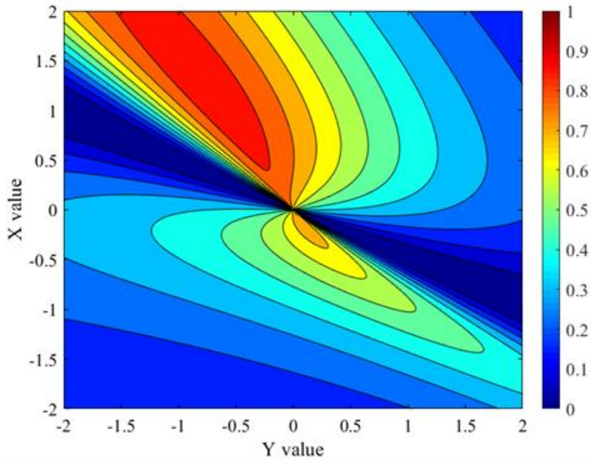

Fig. 7 Graphical representation of R^2 for interested areas

In this study, in order to find the optimal pair of **(x, y)**, 4004001 scenarios were checked in total. The results of this study are depicted in Fig. 6, which represents the filled contour plot of R2 for all pairs of **(x, y)**. The x-axis and y-axis are related to **y** and **x** values related to Eq. (9), respectively. As it can be seen from the figure, the central areas show higher values of R^2 than the rest of areas. Therefore, a detailed investigation is performed with regards to the central areas. To this end, the interest area is limited to $-2<x<2$, and $-2<y<2$. The obtained results for the interest area are shown in Fig. 7.

The result showed that the optimal value for pair of **(x, y)** is equal to (1.26, -0.76). In this case, not only R^2 is the highest value but also the RMSE is the lowest value among all the scenarios. After substituting this pair in Eq. (9), the final form of suggested brittleness index was achieved as (Eq. (10)):

$$B_{new} = \sigma_c^{1.26} \times \sigma_t^{-0.76} = \frac{\sigma_c^{1.26}}{\sigma_t^{0.76}} \tag{10}$$

If the equipment pertaining to PPT which is expensive and generally uncommon laboratory system in present is not

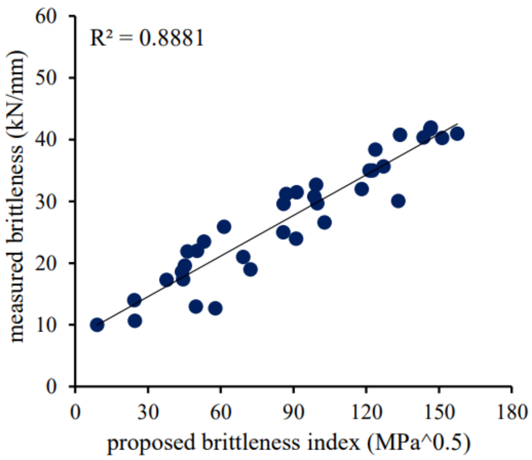

Fig. 8 Relationship between measured brittleness and B_{new} (train data)

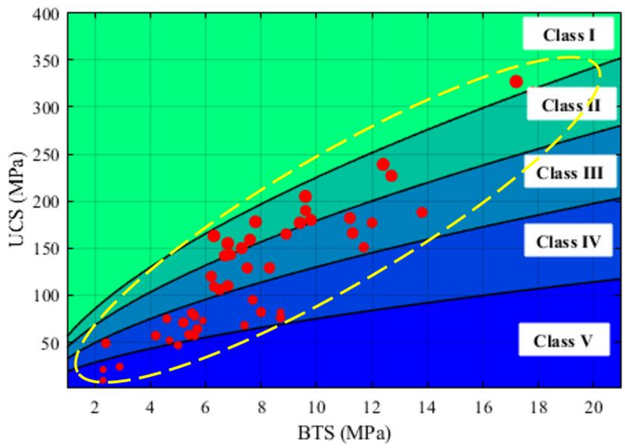

Fig. 9 Graphical representation of the suggested classification

Table 2 Suggested rock brittleness classification

Class	Brittleness index (MPa^0.5)	Brittleness description
I	$B_{new} \geq 160$	Very High
II	$120 \leq B_{new} < 160$	High
III	$80 \leq B_{new} < 120$	Medium
IV	$40 \leq B_{new} < 80$	Moderate
V	$B_{new} < 40$	Low

available, the proposed brittleness index (B_{new}) can be used to estimate the rock brittleness. The relationship between measured brittleness and B_{new} is shown in Fig. 8.

In addition, investigation showed that there is a singularity in accordance to the pair of (0,0). In this point, R^2 dramatically decreases to zero. The logical explanation for the occurrence of this singularity is that replacing (0,0) in Eq. (9) leads to Eq. (9) returns 1 for all values of σ_c and σ_t, as given in Eq. (11).

$$B = \sigma_c^{\,0} \times \sigma_t^{\,0} = 1 \tag{11}$$

In this condition, there is no relationship between measured and predicted brittleness, and therefore the value of R^2 for the pair of (0,0) decreases to zero.

Finally, based on the proposed index, a classification system is suggested in order to categorize rock brittleness into five distinct classes (Table 2). The graphical representation of the suggested classification is illustrated in Fig. 9. In fact, using Fig. 9, it is possible to determine the degree of the rock brittleness directly based on BTS and UCS.

In addition, the results of PPT is scattered along with the classification system, as it is shown in Fig. 9. The red points correspond to the PPT data and their size corresponds to their value, the bigger the higher value. Also, the applicable range of the proposed classification system is marked by a yellow dashed ellipse. It should be point out that the proposed classification tries to assign a class for a given rock sample with arbitrary pair of (σ_c, σ_t). There is maybe no rock, for instance, with σ_c=400 and σ_t= 1 MPa, however, it is too hard to define a precise relationship between these two parameters. Hence, it was tried to consider all possible conditions for σ_c and σ_t, as it was previously considered for **x** and **y** in Eq. (9).

4. Comparison of the proposed brittleness index with pervious indices

In this section, the performance of the suggested brittleness index (Eq. (10)) is compared with Eqs. (2)-(5). For this purpose, 12 datasets, which were not incorporated in the development of the suggested index, were used for model testing and validating. In order to evaluate the performance of predictive models, predicted values from each model were compared with the measured brittleness using PPT. The comparison between measured rock brittleness and the predicted ones using Eqs. (2)-(5) and (10) can be found in Table 3. Moreover, the relationship between measured brittleness index and predicted values form suggested and existing brittleness indices are depicted in Figs. 10-14. Furthermore, the relative error of predictive models (Eqs. (2)-(5) and (10)) from measured values was calculated and is presented graphically in Fig. 15. The obtained results showed that the relative errors range of the predicted values for proposed index (-0.18 to +0.17) is smaller than the relative errors range of B_1 (-0.26 to +0.27), B_2(-0.25 to +0.23), B_3 (-0.21 to +0.30) and B_4 (-0.21 to

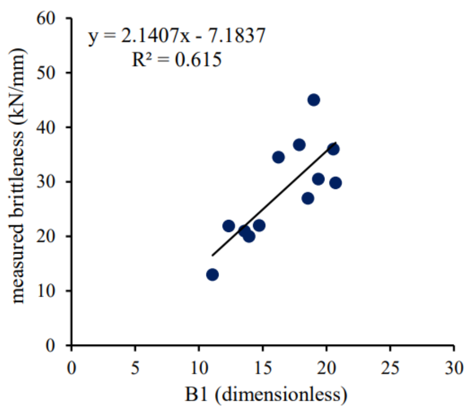

Fig. 10 Relationship between measured brittleness and B_1 (test data)

Table 3 The comparison between measured rock brittleness and predicted brittleness using different indices

No	Rock type	UCS (MPa)	BTS (MPa)	BIm (kN/mm)	Predicted Rock brittleness using					Brittleness Class
					B_1	B_2	B_3	B_4	B_{new}	
1	Sedimentary	120	6.2	30.5	34.25	34.20	26.39	25.07	27.27	III
2	Metamorphic	227	12.7	36.8	31.08	31.20	38.00	38.47	33.25	II
3	Sedimentary	143	6.9	29.8	37.18	36.81	28.81	27.41	30.40	III
4	Sedimentary	150	7.3	36	36.80	36.48	29.71	28.33	30.81	II
5	Metamorphic	95	7.7	21.9	19.23	18.16	26.25	24.94	19.41	IV
6	Igneous	165	8.9	27	32.50	32.57	32.22	31.08	30.01	III
7	Igneous	57	4.2	21	21.87	21.29	16.68	17.52	17.24	IV
8	Metamorphic	182	11.2	34.5	27.60	27.66	35.03	34.48	28.76	III
9	Sedimentary	78	5.6	20	22.63	22.17	21.83	21.18	19.38	IV
10	Igneous	81	5.5	22	24.34	24.11	22.00	21.32	20.23	IV
11	Metamorphic	52	4.7	13	16.50	14.84	16.85	17.63	15.17	IV
12	Igneous	327	17.2	45	33.51	33.52	43.73	47.53	40.09	I

Table 4 performance indices values for each brittleness index

BI	Number	Type	Equation	R2	VAF(%)	RMSE
B_1	(1)	Linear	$BI_m = 2.14B_1 - 7.18$	0.615	61.5	5.36
B_2	(2)	Power	$BI_m = 103.72B_2^{10.73}$	0.747	63.2	5.25
B_3	(3)	Log	$BI_m = 8.57\ln(B_3) - 24.33$	0.839	83.9	3.47
B_4	(4)	Power	$BI_m = 3.098B_4^{0.439}$	0.788	81.7	3.70
B_{new}	(9)	Linear	$BI_m = 0.227B_{new} + 5.750$	0.912	89.8	3.47

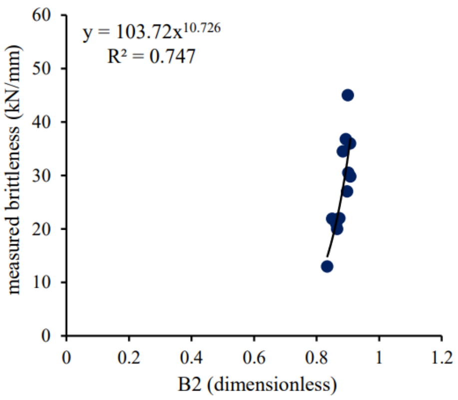

Fig. 11 Relationship between measured brittleness and B_2 (test data)

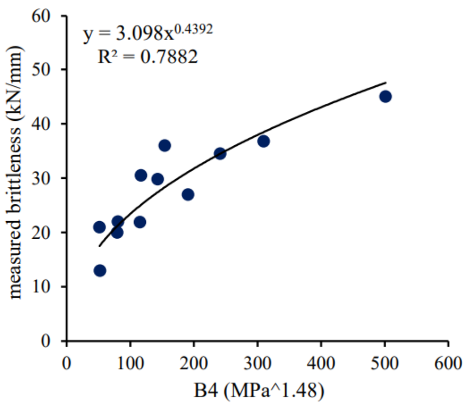

Fig. 13 Relationship between measured brittleness and B_4 (test data)

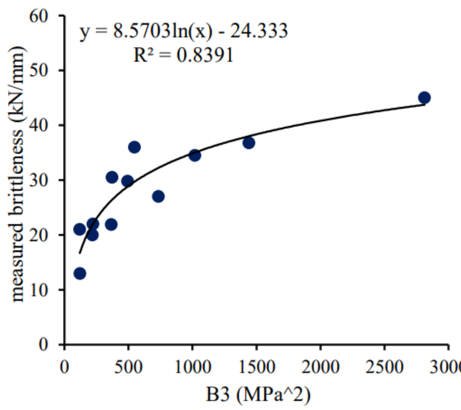

Fig. 12 Relationship between measured brittleness and B_3 (test data)

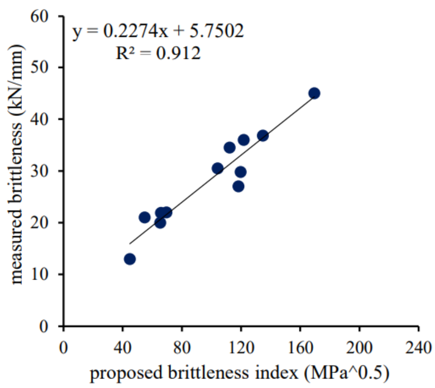

Fig. 14 Relationship between measured brittleness and B_{new} (test data)

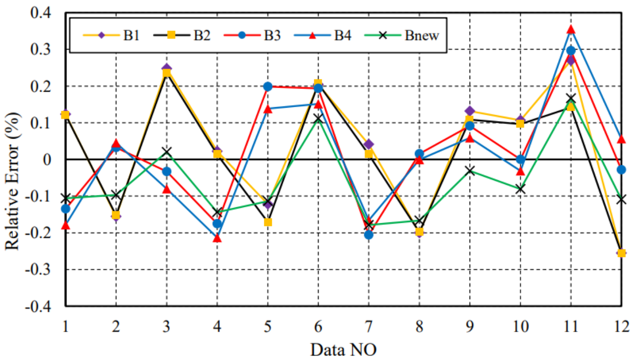

Fig. 15 Comparing relative errors of each strength-based brittleness indices

+0.36). Hence, it can be stated that the most accurate predictor is the suggested index in this study.

In addition, in this study three indices including R^2, RMSE, and variance account for (VAF) between measured and predicted values of rock brittleness were used to evaluate the performance of suggested index. A model is considered to be properly developed when R^2 is 1, VAF is 100% and RMSE is 0. Eqs. (12) and (13) were utilized to calculate the RMSE and VAF, respectively.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(M_i - P_i)^2} \tag{12}$$

$$VAF(\%) = \left[1 - \frac{\mathrm{var}(M_i - P_i)}{\mathrm{var}(M_i)}\right] \times 100 \tag{13}$$

where M_i and P_i are correspondingly measured and predicted values of brittleness and N is the number of testing sample. The values of performance indices for all predictive models were listed in Table 4. The comparison between the suggested brittleness index in this study and existing strength-based brittleness indices (Eqs. (2)-(5)) bears witness to the better prediction performance of the former in terms of the performance indices.

5. Conclusions

The main aim of this study was to develop a new strength-based brittleness index for the prediction of rock brittleness. By considering the most common strength-based indices, it was revealed that, among the four indices, three of them have the same base form, and the main difference between these indices is related to UCS and BTS coefficients. Hence, the obtained base form was chosen as the base of the new brittleness and it was tried to find the optimal coefficients for UCS and BTS. To this end, an innovative algorithm in Matlab environment was developed in order to calculate the coefficients of suggested equation. In this study, 4004001 different pairs for coefficients of UCS and BTS were checked by the proposed algorithm. The utilized algorithm finds the optimal coefficients of the suggested brittleness index based on the open access dataset including the results of PPT, UCS and BTS of 48 different rock samples collected from 48 tunnel cases. The result showed that the optimal values for coefficients of UCS and BTS are **1.26**, **-0.76**, respectively. The performance of proposed model was compared to existing strength-based brittleness indices using three performance indices including R^2, RMSE and VAF (%). The results showed, among the different brittleness indices, the suggested equation is the most accurate one, since it has the optimal R^2, RMSE and VAF (%) as 0.912, 3.47 and 89.8%, respectively. It could finally be concluded that, using the suggested brittleness index, rock brittleness can be predicted with a high level of accuracy. It is evident that the proposed models are developed using a limited series of experimental data and should be utilized in the same conditions.

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