Behavior of a combined piled raft foundation in a multi-layered soil subjected to vertical loading

Srijit Bandyopadhyay^{*1}, Aniruddha Sengupta^{2a} and Y. M. Parulekar^{1b}

¹Reactor Safety Division, Bhabha Atomic Research Centre, Mumbai 400085, India ²Department of Civil Engineering, Indian Institute of technology, Kharagpur 721302, India

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Abstract. The behavior of a piled raft system in multi-layered soil subjected to vertical loading has been studied numerically using 3D finite element analysis. Initially, the 3D finite element model has been validated by analytically simulating the field experiments conducted on vertically loaded instrumented piled raft. Subsequently, a comprehensive parametric study has been conducted to assess the performance of a combined piled raft system in terms of optimum pile spacing and settlement of raft and piles, in multi-layered soil stratum subjected to vertical loading. It has been found that a combined pile raft system can significantly reduce the total settlement as well as the differential settlement of the raft in comparison to the raft alone. Two different arrangements below the piled raft with the same pile numbers show a significant amount of increase of load transfer of piled raft system, which is in line with the load transfer mechanism of a piled raft. A methodology for the factor of safety assessment of a combined pile raft foundation has been presented to improve the performance of piled raft based on its serviceability requirements. The findings of this study could be used as guidelines for achieving economical design for combined piled raft systems.

Keywords: piled raft; finite element analysis; factor of safety; pile spacing; combined piled raft

1. Introduction

With rapid urbanization, the requirement of high rise buildings along with the heavy industrial plants like thermal power plant, are increasing. Moreover, due to shortage of rock sites, some of them are being constructed at soft soil sites where the ground settlement is an issue. In a thermal power plant, the weight of the structure is very large and due to the stringent safety requirements, the allowable differential settlement of the structure is very small. Generally, the use of a raft foundation can fulfill the bearing capacity requirements, but the differential settlement might exceed the allowable limits. Hence, a pile foundation is being used in practice for many years to transmit the superstructure load to the competent foundation at a depth and to reduce the settlement to an acceptable limit. But in a piled foundation design, the contribution of the load sharing component of the raft is not considered (Katzenbach et al. 2016, De Sanctis and Russo 2008, Poulos 2001). If the complete load of the structure is transferred through the piles then the number of piles required may be large depending upon the soil condition. Hence there is a need of a combined pile raft foundation.

*Corresponding author, Scientific officer-E, M.Tech.

E-mail: yogitap@barc.gov.in

The use of a combined piled raft foundation (CPRF) has become more popular in recent years, as the CPRF system reduces the total settlement of the structure as well as the differential settlement of the raft to an acceptable limit in the most economical way. This is due to the contribution of the piles and the raft being considered together in a CPRF. The piled raft foundation is a geotechnical composite construction consisting of three elements-piles, raft and the foundation soil. CPRF may have four different kinds of interaction: pile to pile interaction, pile to soil interaction, pile to raft interaction, and raft to soil interaction (Katzenbach et al. 2005). The load transfer mechanism and the failure modes of a piled raft foundation are very complex. It has been noticed that in many developed countries, due to lack of instrumented field data on piled raft foundation and also due to the lack of expertise in the design philosophy for piled raft, many high rise buildings are constructed on pile groups (Choudhury et al. 2015).

In the early 80's, Wiesner and Brown (1980) conducted model tests on a combined piled raft system. Researchers (Butterfield and Banerjee 1971, Poulos 1994) performed a series of numerical analyses on soil pile cap interaction considering pile cap as a plate element, and piles were represented as equivalent springs. Franke (1991) demonstrated with the help of instrumentation of four buildings that a piled raft reduces settlement by about 50% as compared to a raft alone. Many researchers (Katzenbach *et al.* 2000, Mandolini *et al.* 2005, Zhang *et al.* 2010, Yamashita *et al.* 2011, Nakanishi and Takewaki 2013, Lee *et al.* 2015, Nguyen *et al.* 2014, Lee *et al.* 2010, Cho *et al.* 2012, Long and Vietnam 2010, Bourgeois *et al.* 2012, Balakumar *et al.* 2013, Nguyen *et al.* 2013, Algulin and

E-mail: srijit15@gmail.com

^aProfessor, Ph.D.

E-mail: sengupta@iitkgp.ac.in

^bScientific officer-H, Ph.D.

Pedersen 2014, Basile 2015, Rabiei and Choobbasti 2016, Alnuaim et al. 2017, Ko et al. 2018, Khanmohammadi and Fakharian 2018) have also reported the effectiveness of a CPRF in reducing average and differential settlement in their studies. Reul and Randolph (2004) have performed a series of parametric studies for different pile raft combinations by means of a three-dimensional elastoplastic finite element analysis. From their study, a design strategy for an optimized design of piled raft subjected to nonuniform vertical loading was discussed. However, in their study the separation between pile and soil was neglected. Katzenbach et al. (2005) have given an overview of the theoretical and the practical development of a piled raft foundation and its use in the reduction of settlement in the high rise buildings. De Sanctis and Mandolini (2006) have considered the contribution of raft towards the overall bearing capacity of a piled raft based on experimental and 3D numerical studies. El-Garhy et al. (2013) conducted an experimental program on piled raft models in sandy soil to investigate the behavior of raft on the settlement reducing piles. The results of the tests show the effectiveness of using piles as settlement reducers. Many researchers have reported studies on combined piled raft systems applied in real life structures. Ibrahim et al. (2009) performed actual analysis and design of the 100-storey Pandemonium tower in Dubai. The foundation settlement for Burj Khalifa, Dubai has been studied by Russo et al. (2013). Kumar and Choudhury (2018) proposed a new prediction method to estimate both Ultimate Limit State (ULS) and Serviceability Limit State (SLS) bearing capacity of CPRF by evaluating interaction factors between the pile-raft and raft-pile. From the research carried out till date, it is observed that limited study has been performed on the behavior of CPRF in multi-layered soil. Moreover, there is a need to determine, the factor of safety (FOS) of CPRF which is prerequisite for economical and safe design of piled raft foundation.

In this paper, 3D finite element analysis is performed to study the nonlinear load settlement behavior of combined piled raft system located in multi-layered soil profile. A series of numerical simulations are performed to study behavior of CPRF with different pile spacing, raft thickness and different pile diameter. Thereafter, the simple procedure to predict the load sharing response and mobilized factor of safety of a combined piled raft system considering the serviceability requirement of the structure is proposed.

2. Numerical analysis of a large piled raft in a multilayered soil

In the present study, numerical simulation of square piled raft system is carried out in layered soil and its performance is investigated with parametric study. The details of the finite element modeling of the combined piled raft system is discussed henceforth.

2.1 Finite element modeling and boundary conditions

The behavior of a square piled raft (PR) is investigated using nonlinear 3D FE model in MIDAS GTS NX (MIDAS 2019). The model consists of a soil domain, a piled raft



Fig. 1 Finite element model of combined pile raft and soil

foundation and a soil structure interaction at the interface between the soil and the CPRF. Fig. 1 shows a 3D FE mesh and boundary conditions. The diameter of the piles, the spacing between the piles and thickness of the raft are varied in the parametric study. The length of the pile is considered to be 25 m and the size of the raft is assumed to be $18 \text{ m} \times 18 \text{ m}$ throughout the study. The piles and the raft are connected monolithically with each other. The pile, raft and soil are modeled using 8-node hexahedral elements (Bhowmik et al. 2013, Sinha and Hanna 2016). During finite element meshing, the raft is meshed in such a way, that along the thickness direction, the raft has minimum 3 elements, to take care of the bending effects. The model is discretized such that the soil mesh is finer near the piles and gradually becomes coarser away from the piles. The aspect ratios of the elements are kept within a reasonable limit to minimize the numerical error. In the horizontal direction, the model extends 80 m and base of the compressible layer is set 50 m below the ground level. The distance of the far field boundary from the edge of the raft is considered to be 31 m. Lee et al. (2010) have reported in their study, that the influence zone including interface was equal to the width of the raft. All degrees of freedom at the bottom of the soil domain are restrained. The side boundaries are assumed to be on rollers, that is, movements in the horizontal directions are restrained while that in the vertical direction is not restrained. A series of numerical analyses are performed on the combined piled raft system subjected to a uniform surcharge of 500 kPa. The surcharge represents the equivalent vertical loading of a typical heavy industrial structure. Since, the modeling of a pile installation process is rather complicated, the piles are assumed to be in a stressfree state at the start of the analysis (Jeong et al. 2004). The change of stress in the soil during pile installation is not part of the present analyses. The numerical analysis is carried out in three different stages. In the first stage, the gravity load acts within the foundation soil domain only and the

Soil Parameter	Layer 1	Layer 2	Layer3	Layer 4	Layer 5
Bulk Modulus (MPa)	20.8	38.9	33.3	266.7	300.0
Poisson's ratio	0.3	0.35	0.35	0.45	0.45
Young'sModulus (MPa)	24.96	35.01	29.97	80.01	90
Angle of internal friction (φ')	30.0	35.0	5.0	35.0	30.0
Cohesion (c') (kPa)	3.0	5.0	25.0	55.0	70.0
Unit weight, γ dry (kN/m ³)	20.00	20.00	17.00	21.00	21.00
Adhesion (Ca) (kPa)	3.0	5.0	15.0	45.0	65.0
Layer Thickness (m)	6	6	13	10	15

Table 1 Multi-layer soil profile

pile movement is restrained. In the second stage, the soil pile interaction is introduced and the self-weight is applied to the piles and the raft. In the last stage, after the system attains equilibrium, the vertical loads (uniform static pressures) are applied slowly on the raft (Bhowmik *et al.* 2013).

2.2 Material modeling

The actual site under investigation is located in the northern part of India. Geotechnical investigations of the site are performed including drilling of boreholes, collection of disturbed and undisturbed samples. Based on the results of the initial site investigations and subsequent laboratory tests, the entire soil domain under consideration is divided into 5 different layers. On the surface, there is a layer of brownish silty sand with a thickness of approximately 6 m. The loose sand is underlain by a layer of medium dense sand and the thickness of the layer is also 6 m. Below this layer, average 13 m thick dense to very dense grey sticky clay layer with N-value laying between 15 and 30 is observed. A coarse sediment consisting of silt or fine to medium sand and coarse sand with or without pebbles is encountered below 25 m. Below 35 m depth, very dense yellowish brown to yellowish grey silty fine sand is observed. The soil parameters, such as, angle of internal friction and cohesion of each soil layer are obtained from the laboratory tri-axial tests. The depth of the water table at the site is found to be considerably low, hence the effect of the water table is not considered in the analysis. The foundation soil layers are idealized as elastic perfectly plastic material with plasticity governed by the Mohr-Coulomb plasticity failure criteria. Though several advanced soil models are available, they are not utilized in this study due to the lack of test data on the unloadingreloading behavior of the soil. In the Mohr-Coulomb model, unit weight, effective cohesion, effective friction angle, angle of dilatancy and elastic modulus are specified for the soil. The angle of dilatancy is assumed to be zero for the soil. The detailed soil properties used in this study are tabulated in Table 1. Since the long term behavior of a pile raft is to be studied, the drained material properties are used in the numerical analysis (Reul and Randolph 2004). The raft and piles are made of M30 grade of concrete with modulus of elasticity $E = 5000 \sqrt{f_{ck}}$ (IS-456 2000), where f_{ck} is compressive strength of concrete in MPa. In the numerical analysis, the modified Newton-Raphson method is used for solving the nonlinear equilibrium equations.

2.3 Interaction between piled raft and soil

The interaction between the piled raft foundation and the foundation soil is incorporated by the master-slave concept, in which the foundation structure's surface is considered as a master surface and the soil surface is considered as a slave surface. Symmetric general contact modules are used in which, the separation between a pile and the soil surface is allowed. Here, zero thickness slip elements transfer shear forces across the surface in the presence of compressive forces. Hence both shear and compressive forces are transferred between pile and soil. In pile to soil interaction, the pile surface is considered as the master surface and the soil surface is considered as the slave surface. The relationship between the normal forces and the shear forces at contact surface of pile and soil are governed by the Coulomb's friction theory (Lee et al. 2010). The interface behavior between the soil and the concrete surface is not known. So, the angle of friction at the interface between a soil and a wall element is assumed to be 0.67 times of the angle of the internal friction of the soil (Bhowmik et al. 2013). Surface to surface interaction with frictional boundary is also implemented between bottom surface (master surface) of the raft and the foundation soil surface to simulate raft to soil interaction. The essential parameters which are used in the entire study are explained henceforth.

2.4 Post analysis

Vertical settlement of piled raft, which is generally determined by average settlement, is represented by Eq. (1). (Reul and Randolph 2004)

$$S_{average} = \frac{((2S_{center} + S_{corner}))}{3}$$
(1)

where, S_{center} = Settlement at raft center and S_{corner} = settlement at raft corner.

Axial load on the pile element is calculated from the stresses obtained at the integration points of the pile elements. Thus, the axial pile load P_{pile} is calculated from the summation of the vertical stress in the pile element using Eq. (2).

$$P_{pile} = \pi r^2 \sigma_v \tag{2}$$

The differential settlement of the raft is computed as given in Eq. (3) (Cho *et al.* 2012).

$$S_{diff} = S_{center} - S_{corner} \tag{3}$$

2.5 Validation of the numerical modeling

In order to validate the numerical modeling, analytical simulation of a piled raft in a multi-layered soil, is carried out. The numerical results obtained are compared with corresponding values reported by Koizumi and Ito (1967)



Fig. 2 Schematic diagram of field test on a piled raft (a) plan view and (b) section view

Table 2 Material parameters used in this study for the validation of the 3D model

Soil Parameter	Sandy silt	Silty Clay
Depth of soil (m)	(0.0-1.7)	(1.7-13.5)
Young's modulus (MPa)	13	15
Angle of internal friction (ϕ')	0.0	0.0
Poisson's ratio	0.3	0.3
Cohesion (c') (kPa)	25.00	29.64
Unit weight, y dry (kN/m ³)	18.0	18.0

Table 3 Geometry of raft and pile raft system

Type of structure involved	Raft Size $(m \times m \times m)$	Pile Diameter (m)	Pile Spacing (m)
Raft	18.0×18.0×1.5	-	-
Piled raft	18.0×18.0×1.5	0.8,1	3D, 4D, 5D and 6D

Pile Spacing (D=0.8 m)	Pile arrangement	Number of piles	Pile Spacing (D=1m)	Pile arrangement	Number of piles
E_3D	7×7 3D Spacing	49	O_3D	6×6 3D Spacing	36
E_4D	6×6 4D Spacing	36	O_4D	5×5 4D Spacing	25
E_5D	5×5 5D Spacing	25	O_5D	4×4 5D Spacing	16
E_6D	4×4 6D Spacing	16	O_4_6D	4×4 (mixture of 4D and 6D Spacing)	16

Table 4 configuration of piled raft system

for vertically loaded instrumented piled raft. The fully instrumented piled raft is installed in the soil near the city of Tokyo. The foundation soil consists of sandy silt with gravel and organic silty clay. The soil properties are summarized in Table 2, and are taken from Jeong and Cho (2014). All the test piles are 300 mm in diameter and 5.5 m in length. The raft is 1.9 m thick, which includes 0.6 m thick plate loading platform made of concrete. The spacing between the two piles is '3D' or 0.9 m, where D is the diameter of the pile. The raft is made of concrete and the Young's modulus and Poisson's ratio for the raft used in the



Fig. 3 Comparison of numerical and Experimental results

FE analysis are 30 GPa and 0.2, respectively. The piles are made of steel of Young's modulus of 210 GPa and Poisson's ratio of 0.2. The soil below the foundation is made up of two layers, a 1.7 m thick silty sand layer over 11.8 m thick silty clay layer. In the FE model, the piled raft system is surrounded by a square soil mass of 30 m in plan and up to a depth of 13.5 m. The schematic diagram of the field test on the piled raft is shown in Fig. 2. The pile and the raft is modeled with 2448 numbers 8-node hexahedral brick element with 4447 numbers of nodes. The soil is discretized by using 15120 numbers of 8-node hexahedral elements along with 17290 numbers of nodes. A vertical load is applied at the top of the raft. The numerically predicted load-settlement curve for the piled raft is in reasonable agreement with the test results as shown in Fig. 3. Some discrepancies between the experimental and the numerical results are observed. These discrepancies are arising due to the change in stresses in the soil during the pile installation which is not modeled numerically (Jeong et al. 2004). In addition to this, small differences in test and numerical result also arise as the nonlinear soil behavior is modeled with Mohr-Coulomb model and this model does not account for consolidation and plastic volume changes in a soil.

3. Parametric study

The behavior of a combined piled raft due to application of vertical load depends on the pile spacing, pile diameter(D) and raft thickness. To study the behavior of a piled raft for vertical loads, a series of numerical studies are performed with different pile spacing, such as, 3D, 4D, 5D and 6D, where D is the diameter of piles, with different raft thickness, such as, 1 m, 1.5 m, and 2 m, and with two different pile diameters of 0.8 m and 1 m. The size of the raft and the pile length are considered to be 18 m × 18 m and 25 m, respectively throughout the analysis. The assumed geometry of the piled raft is presented in Table 3. Various arrangements of pile spacing considered in the analysis are shown in Fig. 4. The behavior of a raft without any pile is also studied for different raft thickness. The various configurations of piles underneath the raft considered in this study are given in Table 4.



Fig. 4 Configurations of piled raft (PR) used in parametric study

3.1 Effect of pile spacing effect in piled raft behavior

Figs. 5 and 6 show the load settlement curves for the piled raft system with the different pile spacing for two different pile diameter, 0.8 m, and 1.0 m, respectively. The load carrying capacity of the foundation with pile spacing at 3D, 4D, 5D and 6D corresponding to 80 mm of settlement (corresponding to 10% of D, where, the pile diameter is 0.8 m) is 400, 335, 305 and 255 kPa respectively. Similar behavior is also observed for a combined piled raft system



Fig. 5 Load settlement behavior of UPR and PR system with 3D pile spacing for 0.8 m pile diameter, Raft thickness = 1.5 m



Fig. 6 Load settlement behavior of UPR and PR systems with 3D pile spacing for 1 m pile diameter, raft thickness=1.5 m

with 1 m pile diameter and pile diameter 1 m with different pile spacing. It is observed that the load carrying capacity of the piled raft decreases with increasing pile spacing. Similar findings is also noticed by Chow *et al.* (2001) and Sinha and Hanna (2016). The settlement of raft without piles and for various piled raft configurations with 300 kPa loading is reported in Table 5. The load transfer through a pile depends on two factors: one is mobilization of skin friction, and load transfer due to end bearing. It is observed that due to a decrease in the pile spacing, which increases the number of piles, there is more mobilization of skin friction



Fig. 7 Load settlement of raft on different pile group in a PR system

Table 5 Settlements corresponding to various pile configurations

Pile configuration	Settlement corresponding to 300 kPa vertical load (mm)
Raft Only (UPR)	168
3D Spacing	61
4D Spacing	75
5D Spacing	94
6D Spacing	121

resulting in the increase in the load carrying capacity of a piled raft. Fig. 7(a) presents load settlement curve of arrangement E_3D, O_3D, E_4D and O_4D. E_3D arrangement has a total of 49 piles with a pile diameter of 0.8 m. O 3D has 36 piles with a pile diameter of 1 m. In the numerical analyses, all other parameters are kept same in both cases. The surface area for mobilization of skin friction of E 3D pile arrangement is 3077.2 m^2 with the pile tip area of 24.61 m². The corresponding surface area and pile tip area of O 3D arrangement are 2826.0 m² and 28.26 m² respectively. As shown in Fig. 7(a), the load carrying capacity of E 3D is higher than O 3D. All the piles are floating piles, so the pile tip area has less participation in the initial portion of the load settlement curve. Similar behavior is observed for pile arrangements E_4D and O 4D. Fig. 7(b) shows the load settlement curve of two different pile arrangements, E 5D and O 4D with 25 numbers of pile in both cases. In the arrangement corresponding to O 4D, the piles are of 1.0 m in diameter and the skin friction force is much more due to a larger surface area as compared to other arrangements. Thus, the piled raft with O 4D pile arrangement has more stiffness as compared to the piled raft of E_5D pile arrangement. The influence of the stiffness increases with the increase in the pile diameter.

3.2 Load sharing between the piles and the raft in a piled raft

The load transfer mechanism in a combined piled raft

system is different from any other conventional system. The mobilized stresses and the displacement fields of the piles and the raft overlap within the soil and produce a complex pile-raft and pile to pile interactions in a piled raft foundation (Katzenbach and Moormann 2002). The load transfer for piled raft is represented by Eq. (4) (Reul and Randolph 2004).

$$Q_{total} = Q_{raft} + Q_{Pile} \tag{4}$$

where, Q_{total} = Total load transferred to the piled raft system, Q_{raft} = Total load transferred through the raft, and Q_{pile} = Total load transfer through the pile. Q_{raft} is different from the ultimate load carrying capacity of the raft without piles (Q_{UR}) and Q_{pile} is different from the group pile capacity of the piles (Q_{GP}). Incorporating all the interaction effects, Eq. (4) can be rewritten as Eq. (6) (Katzenbach *et al.* 2000).

$$Q_{GP} = \alpha_{pp} \cdot \Sigma \cdot Q_{SP} \tag{5}$$

$$Q_{total} = \alpha_{rp}.Q_{UR} + \alpha_{pr}.\alpha_{pp}.\Sigma.Q_{SP}$$
(6)

qhere, Q_{SP} is load carrying capacity single pile. α_{rp} , α_{pr} and α_{pp} are the raft-pile, pile-raft and pile to pile interaction factors. Many researchers (Long 1993, Poulos 2001, Park and Lee 2014) suggested the value of α_{pp} to be 1.0. The ultimate load carrying capacity of a raft without any pile, Q_{UR} can be obtained from the bearing capacity equation given in Eq. (7).

$$Q_{UR} = \frac{2}{3} c_{weighted} N_c s_c d_c i_c + q(N_q - 1) s_q d_q i_q \tag{7}$$

where, Q_{UR} is the ultimate load carrying capacity of a raft without any pile, $c_{weighted}$ is weighted cohesion of soil upto the depth equal to the width of footing. N_c , N_γ and N_q are the bearing capacity factors. s_c , s_γ and s_q are bearing capacity correction factors for shape, d_c , d_γ and d_q are bearing capacity correction factors for depth and i_c , i_γ and i_q are bearing capacity correction factors for inclination. $c_{weighted}$ and $\gamma_{weighted}$ are calculated up to the depth equal to the width of footing. In this way soil layered effects are incorporated in the calculation of ultimate capacity of raft.

The ultimate loading carrying capacity of single pile is calculated from Eq. (8)

$$Q_{SP} = \Sigma(\alpha_i c_i + K_i \sigma_i \tan \delta) A_s + (cN_c + q_d N_q) A_p \quad (8)$$

where, Q_{SP} is the ultimate load carrying capacity of a single pile, α_i is the adhesion factor of i^{th} layer, δ is soil pile friction angle, generally considered as 0.67 times of angle of internal friction of soil φ . c_i is the average cohesion of i^{th} layer, K_i is the coefficient of active earth pressure in i^{th} layer. σ'_i is effective overburden pressure of i^{th} layer. q_d is the effective over burden pressure at pile tip. N_c and N_q are the bearing capacity factors. A_p is the area of the pile base and A_s is the surface area of the pile. Kumar and Choudhury (2018) also suggested the same procedure for calculating the load carrying capacity of a single pile and a raft without any pile.

The load sharing between the pile and the raft in a CPRF is quantified by a factor "coefficient of the piled raft (α_{CPRF})", which is defined as the sum of the total load carried by the piles to the total load acting on the CPRF and is calculated from Eq. (9) (Katzenbach and Choudhury 2013).

$$\alpha_{CPRF} = \frac{\Sigma R_{pile}}{Q_{total}} \tag{9}$$

where, ΣR_{pile} is the load carried by piles in a piled raft and R_{pile} is the load carried by a single pile in a piled raft. R_{pile} is obtained from the FE analysis considering the stresses obtained in the pile elements and using Eq. (2) described in the aforementioned section. If the value of α_{CPRF} is zero, it signifies the full contribution of a raft in the load sharing. If it is one, it signifies a free standing pile group only. Katzenbach et al. (2005) have proposed a design philosophy based on the piled raft coefficient. Fig. 8 presents the values of load sharing coefficient α_{CPRF} of a piled raft for different settlement ratio of S_{CPRF}/S_{RF}. The S_{CPRF} and S_{RF} are the settlement of a piled raft and a raft foundation respectively, due to an applied load. In Fig. 8 values are shown for pile spacing of 3D, 4D and 5D, for three different superstructure loading of 50, 260 and 450 KPa. In these cases, the raft size is $18 \text{ m} \times 18 \text{ m}$ and the diameter of a pile is assumed to be 0.8 m. The lower bound and the upper bound curves of the CPRF coefficient, as proposed by Katzenbach et al. (2005), are also shown for comparison. For 3D pile spacing, the settlement of a piled raft (S_{CPRF}) for 50, 260 and 450 kPa loading are 7 mm, 46 mm and 125 mm and for a raft without any pile, the values of (S_{RF}) are 20 mm, 137 mm and 305 mm, respectively. The load shared by the piles to the total load, α_{CPRF} in the case



Fig. 9 Variation coefficient of piled raft with piled raft settlement

of a piled raft with 3D pile spacing is 0.937, 0.909 and 0.871 for the superstructure loading of 50, 260 and 450 kPa respectively, as shown in Fig. 8. Similar behavior is observed by other researchers in their study (Lee et al. 2014, Park et al. 2016). The measured higher values of $\alpha_{\rm CPRF}$ indicate a conservative design of a CPRF. The same procedure has been followed for the piled rafts with pile spacing of 4D and 5D. From Fig. 8, it is observed that the value of $\alpha_{\rm CPRF}$ decreases as the spacing of piles under a raft increases. This is due to an increase in the load sharing contribution of the raft in a combined piled raft system. The load sharing between the raft and the piles in a piled raft is represented by the piled raft coefficient α_{CPRF} , which depends on the magnitude of the structural load and the settlement of a piled raft. Fig. 9 presents the variation of α_{CPRF} with the piled raft settlement. In Fig. 9, three piled rafts, with a different number of piles, have been considered for the numerical study. The O 3D pile configuration has 36 numbers of piles with a pile spacing of 3D. The pile configuration O 5D has 16 piles with pile spacing of 5D. It is observed that during the initial portion of the loading, the value of α_{CPRF} is close to 1, which indicates that the contribution of a raft is close to zero. When the loading



Fig. 10 Load settlement of piled raft on different raft thickness

increases, the contribution of raft increases. Under an increasing total load on the piled raft, the value of this coefficient for O 3D pile configuration remains within 95% to 87% with the settlement. This means that the resistance of the pile remains approximately in same range of average 90% with increase in load. For the case with O 5D pile configuration, an increasing total load and settlement of the piled raft leads to a significant increase of the resistance of the raft as the piles can mobilize only a small resistance. A permissible settlement criterion of 0.35% of the width (B) of a piled raft foundation is suggested by Lee et al. (2010). It gives a permissible settlement of 63 mm. The settlement criteria of 0.1D given by IS code 2911 (IS-2911 2010) suggests a permissible settlement of 100 mm. It is observed from Fig. 9 that for a settlement of 63 mm, in the case of 3D pile spacing, the contribution of the piles in a piled raft is 90%. In the case of 5D pile spacing for the same settlement, the contribution of the piles in a piled raft is 65%. Similarly for the settlement of 100 mm (for 0.1D), in the case of 3D pile spacing, the contribution of the piles in a piled raft is 88%. In the case of 5D pile spacing for the same settlement value, the contribution of the piles in a piled raft is 55%. Thus, for the 5D pile spacing, the contribution of a raft in load sharing is significant.

3.3 Raft thickness effect in combined piled raft behavior (CPRF)

The load settlement behavior of a CPRF with pile spacing of 3D and raft thickness of 1 m, 1.5 m and 2 m, is presented in Fig. 10(a). The size of the raft is 18 m \times 18 m and the diameter of a pile is 1.0 m. No significant improvement of load settlement behavior of a piled raft is observed for increasing overall stiffness. The stiffness of a raft plays an important role from the serviceability point of view of the structure. The differential settlement of a piled raft is obtained from Eq. (3). The differences in the settlements between the corner and the center points of a raft are presented in Fig. 10(b) for various raft thicknesses. An increase in raft thickness causes an increase in the rigidity of the raft, which in turn causes a decrease in the differential settlement. But, it is observed that, for the initial portion of a loading, up to 200 KPa, the differential settlement of a raft is nearly the same for all the thicknesses. However, with the increasing loads, the differential settlement of a raft decreases with the increase of the raft thickness. It is also noticed that a major part of the initial loading is carried by the piles with the help of skin friction and a little portion of the load is shared by the raft. When the full skin friction is mobilized in the piles, a major part of the load is transferred to the soil by the raft. During this scenario, the stiffness of a raft plays an important role in reducing the differential settlements of a piled raft.

4. An optimum arrangement of piles in a piled raft for differential settlement reduction

The behavior of the combined piled raft system depends on the pile spacing and location of piles below the raft. In this study, two different pile configurations with the same numbers of pile is considered. Pile arrangements of O 5D and O 4 6D are shown in Fig. 4. In both the cases, the numbers of pile are 16. In the case of O 5D, spacing of piles is 5D distributed uniformly throughout the raft, but in the case of O_4_6D, the arrangement (spacing) of piles is a mixture of 4D (at the center) and 6D (away from the center). Fig. 11 shows the load-settlement curve of two different CPRFs of different pile arrangements below the raft. It shows that CPRF with O_5D configuration gives a higher load carrying capacity than CPRF with O 4 6D configuration. In the O 4 6D arrangement, more number of piles are located at the central location, for which there is no additional effect in piled raft interactions. Rabiei (2010) also observed similar kind of behavior of piled raft system in his study. From the above study it is observed that, for uniform loading condition, uniform arrangement of piles give improved results than non-uniform pile arrangement below the raft. However, as per the study of Rabiei and Choobbasti (2016), economic design of piled raft foundation subjected to non-uniform loading can be obtained by placing the piles in a more dense configuration beneath the maximum load positions.



Fig. 11 Load settlement curve of piled raft with different pile arrangement

5. The factor of safety (FOS) assessment for a Piled Raft

The factor of safety (FOS) assessment of a combined piled raft system is a relevant topic from the safety and serviceability point of view. A simplified approach for calculating the factor of safety of a combined piled raft system is described here. Here, FOS of a combined piled raft is obtained in terms of factor of safety of pile group and factor of safety of raft without any pile (De Sanctis and Mandolini 2006). The factor of safety of a raft alone for any loading can be expressed by Eq. (10).

$$FS_{UR} = \frac{Q_{UR}}{Q} \tag{10}$$

where, Q_{UR} is the ultimate load carrying capacity of raft alone, and 'Q' is the applied load on a piled raft. The FOS of group pile can be expressed as given in Eq. (11).

$$FS_{GP} = \frac{Q_{GP}}{Q} \tag{11}$$

where, Q_{GP} is the ultimate load carrying capacity of the pile group. The ultimate load carrying capacity of a raft without any pile (Q_{UR}) and the ultimate load carrying capacity of a pile group (Q_{GP}), are obtained from Eqs. (7) and (8).

The FOS of a combined piled raft is obtained similarly from Eq. (12)

$$FS_{PR} = \frac{Q_{PR}}{Q} \tag{12}$$

The load carrying capacity of piled raft can be calculated by Eq. (13)

$$Q_{PR} = \eta \times (Q_{UR} + Q_{GP}) \tag{13}$$

where η is piled raft efficiency factor. The value of η varies between 0.7 to 1. Here, the value of η is assumed to be 0.8. Details regarding the value of η are described in Kumar and

Choudhury (2018). Hence, the FOS of a piled raft can be rewritten by combining Eqs. (11), (12) and (13).

$$FS_{PR} = \eta \times (FS_{UR} + FS_{GP}) \tag{14}$$

The FOS of a CPRF is calculated with respect to the average settlement of the raft in a piled raft foundation.

For an economical design, the contribution of the raft needs to be considered for the load carrying capacity of a piled raft system. Fig. 12(a) shows the variation of factor of safety of piles in a piled raft with average settlement of piled raft. It is observed that the FOS of piles in a piled raft decreases rapidly with settlement. The value of FS_{GP} is calculated for two different settlement of 63 mm and 100 mm respectively. As stated before, the 63 mm settlement corresponds to the settlement criterion of 0.35% of B as suggested by Lee et al. (2010). The 100 mm settlement corresponds to the settlement criterion of 0.1D, where B and D are the width of foundation and diameter of pile, respectively. For the present analysis, B = 18 and diameter of pile, here D = 1m is considered. It may be noted that, for the settlement of 63 mm the values of FS_{GP} are 2.6, 2.4 and 1.8 for the pile spacing of 3D, 4D and 5D, respectively. Similarly for the settlement of 100 mm the values of FS_{GP} are 1.85, 1.8 and 1.3 for the pile spacing of 3D, 4D and 5D respectively. Fig. 12(b) shows the variation of FOS of the raft with average settlement of piled raft. The higher value of FS_{UR} indicate that the contribution of the raft is less in a piled raft system. The variation of FS_{PR} with the settlement is shown in Fig. 12(c). It is observed that for the settlement of 63 mm, the values of FS_{PR} are 30, 20 and 15 for the pile spacing of 3D, 4D and 5D respectively. Similarly, for the settlement of 100 mm, the values of FS_{GP} are 17.5, 12.5 and 8 for the pile spacing of 3D, 4D and 5D, respectively. It is thus noticed, that the value of FOS of a piled raft decreases with the increase in pile spacing in a piled raft system.

The methodology for arriving at an economical design of a piled raft foundation is an iterative process and is described henceforth using the previously explained concept of factor of safety of the pile, raft and the piled raft. Initially, the allowable average settlement or differential settlement of a piled raft foundation is fixed as per codal provisions. Subsequently, the piled raft foundation design parameters like, pile layout and spacing of piles below the raft, diameter of the pile, length of the pile and thickness of the raft are determined. In the next step, the ultimate load capacity of the raft without any pile and that for the pile group is obtained by using Eqs. (7) and (8) respectively. The piled raft system is then analyzed using FEM and the load-settlement curve of the system is obtained. The load carried by the piled raft at the particular allowable settlement is obtained from the load settlement curve. The FOS of the raft and the pile group in a piled raft foundation is obtained using Eqs. (11) and (12). If the FOS of the raft is greater than 4 then the design is very conservative. Hence, to make the design more economical, the spacing of the piles is increased and the length of the piles is decreased. If the FOS of the piles in a piled raft system is less than 2,



Fig. 12 Factor of safety of combined piled raft system: (a) FOS of pile, (b) FOS in raft and (c) FOS in overall piled raft system



Fig. 13 Flow chart for determination of the factor of safety of a combined piled raft system

then the design is critical and has to be made more conservative. Hence, the spacing of the piles is decreased and the length of the piles is increased and all the steps are repeated till any one or both of the governing criteria (FOS of the raft less or equal to 4 in a piled raft and FOS of piles in a piled raft greater or equal to 2) is achieved. Once this is achieved, the iteration is stopped and the parameters, like pile layout, spacing of piles below the raft, diameter of the piles and length of the pile and the thickness of the raft are finalized. Fig. 13 illustrates the general steps required for calculating the FS_{PR} of a pile raft system in an economic way.

6. Conclusions

A series of parametric studies on a combined piled raft foundation in a multi-layered soil are performed by varying the pile spacing and the raft thickness using a threedimensional finite element method. For the validation of the numerical models, field tests performed by KoIzUMI and ITo (1967) for a vertically loaded piled raft were numerically simulated.

• In a combined piled raft system, the initial stiffness is provided by the pile group by mobilizing the skin frictional resistance. But as the settlement of a CPRF increases, the load sharing by the raft also increases until the failure point is reached. The pile spacing and its diameter play a major role in optimizing the behavior of a piled raft. To achieve an economical design of a combined piled raft system, these parameters should be judiciously considered. It is also noticed that a large diameter pile provides more initial stiffness than a small diameter pile, if all the other parameters are kept constant. The corner pile of a piled raft provides high initial stiffness than a center pile. Hence, in the same line two different arrangement of piled raft has been studied in which the number of pile required is very less but performance of the piled raft is better in terms of reducing its differential settlement.

• In a combined piled raft system, the piles are used to reduce foundation settlement. A simplified method has been explained to calculate the FOS of a piled raft system. The FOS of a CPRF is obtained from the ultimate load carrying capacity of the piles and the raft. In CPRF system, the sensitive parameters such as, pile spacing, diameter should be carefully chosen by a practicing engineer to enhance the performance of a piled raft in terms of its serviceability requirements. In the current design practice, the spacing, depth and the diameter of piles in a piled raft foundation is fixed based upon the numerical analysis of the system and verification of settlement criteria for the design load. The evaluation of the FOS of a raft in a piled raft and that for the piles in a piled raft are seldom considered in the design. In this study, a design methodology is suggested by creating a balance between the FOS of piles in a piled raft, assuming it to be not less than 2 and that for the raft in a piled raft not greater than 4. If the FOS of piles in a piled raft is less than 2 and that of raft is greater than 4, the design is revised in iterative way until one of the two conditions is fulfilled. The design methodology proposed in this study is an improvement over the current design practice and this methodology can serve as a guideline to produce a safer and economical design for a combined piled raft foundation.

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