Geomechanics and Engineering, *Vol. 21, No. 4 (2020) 349-356*
DOI: https://doi.org/10.12989/gae.2020.21.4.349

A plastic strain based statistical damage model for brittle to ductile behaviour of rocks

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(Received August 31, 2019, Revised April 2, 2020, Accepted April 3, 2020)

Abstract. Rock brittleness, which is closely related to the failure modes, plays a significant role in the design and construction of many rock engineering applications. However, the brittle-ductile failure transition is mostly ignored by the current statistical damage constitutive model, which may misestimate the failure strength and failure behaviours of intact rock. In this study, a new statistical damage model considering rock brittleness is proposed for brittle to ductile behaviour of rocks using brittleness index (BI). Firstly, the statistical constitutive damage model is reviewed and a new statistical damage model considering failure mode transition is developed by introducing rock brittleness parameter-BI. Then the corresponding damage distribution parameters, shape parameter m and scale parameter F_0, are expressed in terms of BI. The shape parameter m has a positive relationship with BI while the scale parameter F_0 depends on both BI and $\boldsymbol{\varepsilon_e}$. Finally, the robustness and correctness of the proposed damage model is validated using a set of experimental data with various confining pressure.

Keywords: rock brittleness; constitutive model; plastic strain; damage model; deep earth science

1. Introduction

A thorough understanding of the strength and deformation behaviours of rock is vital to the design and construction of rock engineering applications including tunnel for deep mining, storage of radioactive waste, wellbore for oil or gas production and excavation for underground space (Xie *et al.* 2020). The complete stress-strain curves of rock under compression provide us an effective way to understand the rock deformation and failure behaviours (Cao *et al.* 2010, Xue 2015), which will facilitate the design and stability of these engineering structures. Therefore, constructing a suitable constitutive model to represent the strength and deformation behaviours is an everlasting concentration for engineering applications (Martin and Chandler 1994, Cao *et al.* 2018).

Complicated mechanical responses such as strain softening, strain hardening and failure modes transition can be observed from these stress-strain curves derived from laboratory scale due to external factors (Wawersik and Fairhurst 1970, Horii and Nemat-Nasser 1986, Evans *et al.* 1990, Fredrich *et al.* 1990, Peng *et al.* 2015, 2017). The confining pressure is the main factor influencing the mechanical responses of rocks, which can lead to the failure mode transition from brittle fracturing to diffuse plastic flow (Peng 1973, Stavrogin and Tarasov 2001, Mas and Chemenda 2015), see Fig. 1. The transition behaviour of rocks may be related to the initiation and propagation of micro-cracks in rocks from a microscopic mechanism perspective (Fuenkajorn *et al.* 2012). At a relatively low confining pressure, those micro-cracks become active at low stress levels, and grow and coalesce to macroscopic fractures, leading to abrupt failures when the critical stress is reached. However, as the confining stress increases, a higher level of stress is achieved before micro-cracks coalescence occurs; thus, a greater number of micro-cracks participate in the failure process, and the failure behaviour becomes increasingly more ductile and diffuse.

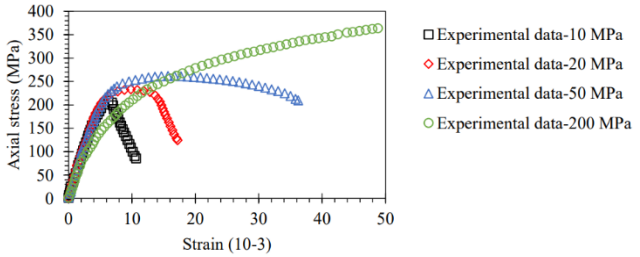

Fig. 1 A transition from brittleness to ductileness of rocks exposed to confining pressure. After Mas and Chemenda (2015)

To quantify the degree of brittleness/ductility, the concept of brittleness index (*BI*) was proposed by many researchers (Hucka and Das 1974, Andreev 1995, Holt *et al.* 2015, Akinbinu 2017, Chen *et al.* 2017). However, there is no unique definition of brittleness parameter as stated in previous studies (Hucka and Das 1974, Holt *et al.* 2015,

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ISSN: 2005-307X (Print), 2092-6219 (Online)

Akinbinu 2017). Many methods were proposed to quantify the rock brittleness based on various measurement methods, such as strain characteristics, stress characteristics, rock strength properties, rock mineral constituents, and energy balance methods (Rahimzadeh Kivi *et al.* 2018).

To capture the complex responses of rocks, the statistical damage model (SDM) has been favoured by many researchers due to the easy implementation into computer code and application to engineering analysis (Tang *et al.* 1998, Cao *et al.* 2010, 2018, Deng and Gu 2011, Li *et al.* 2012, Liu and Yuan 2015, Wang *et al.* 2016, Zhou *et al.* 2020). In classical SDM, it is believed that the rock is a type of material with microscopic cracks, and the damage process can be regarded as the process of micro-cracks initiation, propagation and coalescence in a stochastic manner. Thus, the SDM provides us with a venue to investigate the complex responses of rocks. The softening and hardening behaviours of rocks have been explored by Cao *et al.* (2010). In this model, the damage process and mechanical responses of rocks mentioned above were believed associated with the variance of voids or pores in the natural rocks. Additionally, the strain softening behaviours can be reflected by the SDM proposed by Li *et al.* (2012) to a certain degree. Due to the emphasis of their study, it did not develop a strategy for strain softening and strain hardening of rocks. The above-mentioned SDMs based on the statistical damage mechanics have shown their own merits, however, the failure mode transition induced by external factors mentioned above is seldom presented by a constitutive model using the framework of statistical damage mechanics. Therefore, it is still necessary to develop a SDM for rocks to capture the essential mechanical response, like the failure mode transition, using a set of parameters.

Motivated by these previous studies, in this study, we propose a SDM by incorporating *BI* to capture the brittle-ductile transition. The brittleness characteristics of intact rock in previous experimental investigations are demonstrated in the following section. Then, a new damage model is proposed by incorporating the brittleness parameter based on the continuum damage theory and the statistical damage mechanics. Finally, a validation study for the proposed damage model is carried out using a set of laboratory results.

2. Brittleness index (*BI*)

As mentioned in Section 1, the brittleness of rocks can be quantified through different methods based on different measurements. In Table 1, we summarize different brittleness parameter definitions based on above-mentioned methods. Amongst these definitions of rock brittleness, the most effective and obvious way to define rock brittleness is using the stress and strain characteristics based on the stress-strain curve, see Fig. 2.

Here we briefly describe the brittleness on the basis of strain-based method and stress-based method, which will be incorporated into the proposed damage model. Hucka and Das (1974) defined brittleness as the ratio of elastic strain ε_e to the total strain or failure strain ε_f based on the stress-strain curve:

$$BI_1 = \varepsilon_e/\varepsilon_f \tag{1}$$

where ε_e and ε_f are elastic and failure strain, respectively. Instead of focusing on the pre-peak failure behaviour, Andreev (1995) used the residual strain ε_r and residual stress σ_r to define the brittleness index:

Table 1 Summary of brittleness definitions in the literature

Methods	Formulation	Comments	References
Strain	$BI_1 = \varepsilon_e/\varepsilon_f$	ε_e is the total elastic strain, ε_f is the failure strain; ε_r is the residual strain; ε_f^p is the plastic strain when frictional strength is fully mobilized; plastic strain when cohesive strength is completely degraded.	(Hucka and Das 1974)
	$BI_2 = (\varepsilon_r - \varepsilon_f)/\varepsilon_r$		(Andreev 1995)
	$BI_3 = (\varepsilon_f^p - \varepsilon_c^p)/\varepsilon_c^p$		(Hajiabdolmajid *et al.* 2002)
Stress	$BI_4 = \sigma_r/\sigma_f$	σ_r is the residual strength; σ_f is the failure strength	(Andreev 1995)
	$BI_5 = (\sigma_f - \sigma_r)/\sigma_f$		(Smoltczyk and Gartung 1979)
Rock strength	$BI_6 = \sigma_u/\sigma_t$	σ_u is the uniaxial compressive strength	(Andreev 1995)
	$BI_7 = (\sigma_u - \sigma_t)/\sigma_u + \sigma_t$	σ_t is the tensile strength	(Hucka and Das 1974)
	$BI_8 = \sigma_u * \sigma_t/2$		(Altindag 2003)
Rock mineral constituents	$BI_9 = F_{sb}/(F_{sb} + F_{wd})$	F_{sb} is the fraction of brittle minerals, F_{wd} is the fraction of ductile minerals.	(Rybacki *et al.* 2016)
Energy analysis	$BI_{10} = dW_{et}/(dW_{et} + dW_p)$	W_{et} is the total elastic energy, W_p is the plastic energy.	(Hucka and Das 1974)

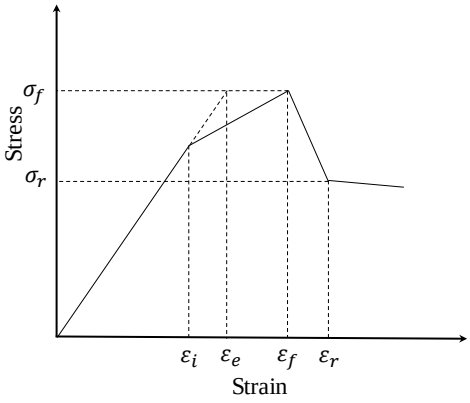

Fig. 2 Typical stress-strain curve of rock material. ε_i, ε_e, ε_f and ε_r are the strain for the onset of dilatancy, elastic strain, failure strain and residual strain; σ_f and σ_r are failure strength and residual strength (after Rybacki *et al.* 2016)

$$BI_2 = (\varepsilon_r - \varepsilon_f)/\varepsilon_r \quad (2)$$

$$BI_4 = \sigma_r/\sigma_f \quad (3)$$

where ε_r is the residual strain; σ_r and σ_f are residual stress and failure stress, respectively. Smoltczyk and Gartung (1979) modifid Eq. (3) to the following expression:

$$BI_5 = (\sigma_f - \sigma_r)/\sigma_f \quad (4)$$

In this study, for simplicity, the brittleness using the elastic strain and failure strain, see Eq. (1), is implemented into the statistical damage model to capture the brittleness/ductility mechanical responses.

3. The classical statistical damage model (SDM)

In this section, the fundamentals of the classical SDM developed by previous studies are firstly reviewed and presented.

3.1 Damage evolution law

In nature, intact rock contains many microscopic elements such as microscopic cracks and deflects. Therefore, the damage process can be regarded as the process of micro-cracks initiation, propagation and coalescence (Cao *et al.* 2010). Assuming the strength of microscopic element is randomly distributed, a SDM can be established based on statistical damage mechanics. Two steps are needed to establish a SDM (Liu and Yuan 2015): choosing a proper failure criterion for micro-cracks, such as the maximum strain criterion, Mohr-Coulomb criterion and Drucker-Prager criterion; and then determining a certain distribution of these micro-cracks such as Power- and Weibull distribution. In this study, the Weibull distribution is adopted for the strength distribution of microscopic elements.

Applying the Weibull distribution to the microscopic strength distribution of microscopic elements, the probability density function is (Weibull 1951):

$$P(F) = \frac{m}{F_0}\left(\frac{F}{F_0}\right)^{m-1} exp\left[-\left(\frac{F}{F_0}\right)^m\right] \quad (5)$$

where F is an elemental strength parameter depending on the strength criterion used; m is the shape parameter or a homogeneous index of Weibull distribution; F_0 is the scale parameter of Weibull distribution.

Adopting the maximum strain theory (Tang *et al.* 1998) as the failure criterion of microscopic element strength, F can be replaced by the strain ε. Then the probability density function becomes:

$$P(\varepsilon) = \frac{m}{F_0}\left(\frac{\varepsilon}{F_0}\right)^{m-1} exp\left[-\left(\frac{\varepsilon}{F_0}\right)^m\right] \quad (6)$$

Assuming the total number of microscopic elements is N and the number of failed microscopic elements is n, we define the damage variable D as the ratio of the number of failed elements to the total microscopic elements:

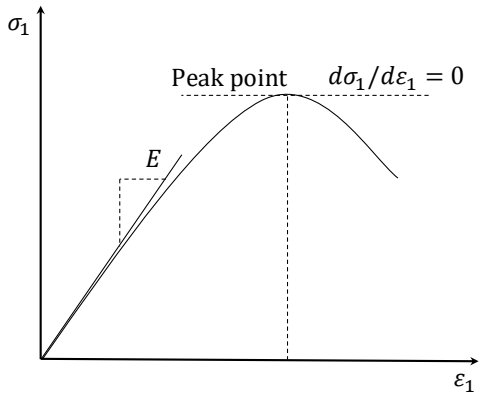

Fig. 3 Determination of damage distribution parameters using the Extremum method. After Li *et al.* (2012)

$$D = \frac{n}{N} \quad (7)$$

where damage variable D falls in the range between 0 and 1. If local strain increases from 0 to F, the number of failed microscopic elements n is:

$$n = N\int_0^F P(\varepsilon)dF = N\left\{1 - exp\left[-\left(\frac{\varepsilon}{F_0}\right)^m\right]\right\} \quad (8)$$

Substituting Eq. (8) into Eq. (7), the damage variable can be expressed:

$$D = 1 - exp\left[-\left(\frac{\varepsilon}{F_0}\right)^m\right] \quad (9)$$

Assuming the stress acting on the effective area of the damaged material is the effective stress σ_{ij}^*, based on the Lemaitre's strain-equivalent principle (Lemaitre 1984), the effective stress σ_{ij}^* can be expressed using nominal stress σ_{ij}:

$$\sigma_{ij}^* = \sigma_{ij}/(1 - D) \quad (10)$$

According to the generalized Hooke's law, the strain can be expressed in the following equation on the principal stress direction:

$$\varepsilon_i = \frac{1}{E}[\sigma_i^* - \mu(\sigma_j^* + \sigma_k^*)] \quad (11)$$

where μ is the Poisson's ratio. Substituting Eq. (10) into Eq. (11), one can obtain the stress-strain relationship:

$$\varepsilon_i = \frac{1}{E(1-D)}[\sigma_i - \mu(\sigma_j + \sigma_k)] \quad (12)$$

Then the stress-strain relationship is obtained on the major principal stress direction:

$$\sigma_i = E\varepsilon_i exp\left[-\left(\frac{\varepsilon_i}{F_0}\right)^m\right] + \mu(\sigma_j + \sigma_k) \quad (13)$$

3.2 Determination of damage parameters m and F_0

In the SDM, two Weibull distribution parameters (m and F_0) can be determined through 'Extremum method' (Cao *et al.* 2010, Deng and Gu 2011). In the stress-strain curve, the

peak point is quite close to the yield point which can be used to determine the model parameters m and F_0. At the peak point, the derivative of σ_1 with corresponding ε_1 should be zero, see, Fig. 3, i.e.,:

$$\varepsilon_1 = \varepsilon_{1f}, \sigma_1 = \sigma_{1f} \tag{14}$$

$$\varepsilon_1 = \varepsilon_{1f}, \frac{d\sigma_1}{d\varepsilon_1} = 0 \tag{15}$$

where σ_{1f} and ε_{1f} are stress and strain corresponding to the peak point. Substituting (14) and (15) into differentiated Eq. (13), the distribution parameter m and F_0 can be calculated through the following equations:

$$m = \frac{1}{\ln\{E\varepsilon_{1f}/[\sigma_{1f} - \mu(\sigma_2 + \sigma_3)]\}} \tag{16}$$

$$F_0 = \sqrt[m]{m}\varepsilon_{1f} \tag{17}$$

3.3 Incorporating rock brittleness index-BI

To derive the rock response of intact rock, the damage distribution parameters can be calculated with determined failure strength, failure strain and Young's modulus of intact rock, which can be obtained through laboratory tests. However, it is hard to determine the failure strain of rocks, and Li *et al.* (2012) suggested to obtain the failure strain through the laboratory tests. Based on the definition of strain-based brittleness, the failure strain can be determined through Eq. (1) with determined Young's modulus and *BI*.

A variety of *BI* definitions have been proposed to obtain the rock brittleness as stated in Section 2. Invoking Eq. (1) and introducing the plastic strain, the rock brittleness can be expressed:

$$BI = \frac{\varepsilon_e}{\varepsilon_e + \varepsilon_p} = \frac{\varepsilon_e}{\varepsilon_{1f}} \tag{18}$$

where ε_e and ε_p are the elastic and plastic strain, respectively, and $\varepsilon_e + \varepsilon_p$ equals to the total failure strain ε_{1f} on the major principal stress direction. Then the damage distribution parameter m becomes:

$$m = \frac{1}{\ln\{E\varepsilon_{1f}/[\sigma_{1f} - \mu(\sigma_2 + \sigma_3)]\}} = \frac{1}{\ln\{\sigma_{1f}/BI[\sigma_{1f} - \mu(\sigma_2 + \sigma_3)]\}} \tag{19}$$

From the expression of damage distribution parameter m, we can conclude that it is independent of the failure strength and deformation modulus and is only related to the rock brittleness parameter. The damage distribution parameter F_0 becomes:

$$F_0 = \sqrt[m]{m}\frac{\varepsilon_e}{BI} = \sqrt[m]{m}\frac{\sigma_{1f}}{E * BI} \tag{20}$$

Then the stress-strain relationship on the principal stress direction becomes:

$$\sigma_1 = E\varepsilon_1 exp\left[\left(\frac{E * BI}{\sqrt[m]{m} * \sigma_{1f}}\varepsilon_1\right)^{\frac{1}{\ln\{\sigma_{1f}/BI[\sigma_{1f} - \mu(\sigma_2 + \sigma_3)]\}}}\right] + \mu(\sigma_2 + \sigma_3) \tag{21}$$

4. Model validation

To assess the capability of the proposed damage model in predicting the brittleness/ductility responses on rocks, the experimental results mentioned in Section 1 with various confining pressure are compared with the theoretical predictions derived from our proposed damage model i.e. Eq. (21). In this section, the model parameters determination process is first presented and then we discuss the comparison between experimental and theoretical results.

The following steps are used to derive the damage distribution parameters, damage variable and corresponding stress-strain curves (see Fig. 4).

(1) Identify the basic material parameters including brittleness parameter *BI*, failure strength σ_{1f} and Young's modulus E or elastic strain ε_e based on the experimental results;
(2) Calculate the damage distribution parameters m and F_0 through Eqs. (19) and (20), respectively;
(3) Derive the damage variable D expression from Eq. (9);
(4) Substitute damage distribution parameters m and F_0 into Eq. (21) to obtain the rock response.

The experimental results from Mas and Chemenda (2015) on Tavel limestone under the confining pressure of 10, 20, 50 and 200 MPa are considered for validation. From the experimental results, as the confining stress increases from 10 MPa to 200 MPa, a clear transition from brittle failure behaviours, strain-softening to strain hardening behaviours is observed. Following the implementation procedure mentioned in Section 4.1, the damage parameters were calculated from the presented results and summarized in Table 2.

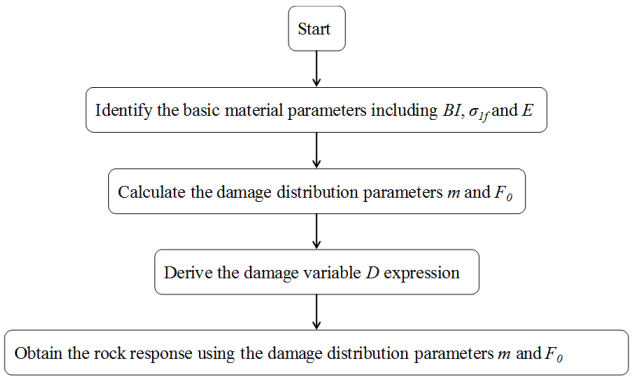

Fig. 4 Flow chart of the implementation of the proposed damage model

Table 2 Damage parameters for the Tavel Limestone with various confining pressure

Confining pressure (MPa)	*BI*	*m*	F_0
10	0.76	3.64	392.51
20	0.55	1.67	576.22
50	0.35	0.95	710.21
200	0.13	0.49	653.56

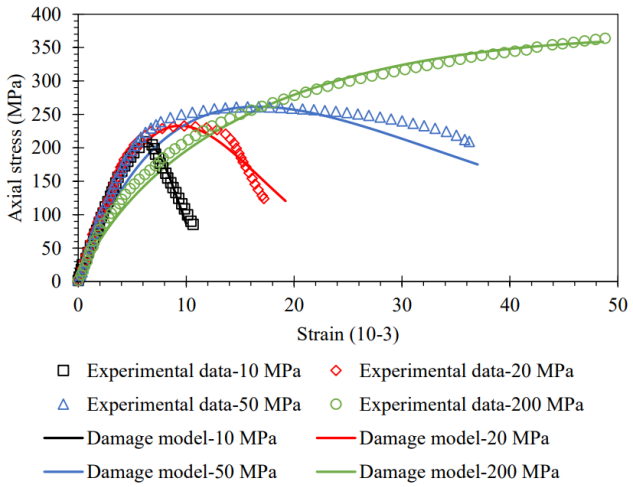

Fig. 5 Comparison between the proposed damage model and experimental results (Mas and Chemenda 2015) under various confining stress

It indicates that the *BI* negatively relates to the confining pressure. Then the corresponding damage parameters including the shape parameter m and the scale parameter F_0 are calculated through Eqs. (19) and (20), respectively, and listed in Table 2. The shape parameter m decreases from 3.64 to 0.49 as the confining pressure increases, showing a similar tendency as *BI*. On the other hand, the scale parameter increases from 392.51 to 710.22 firstly as the confining pressure increases from 10 MPa to 50 MPa, however, decreases to 653.56 when the confining pressure increases up to 200 MPa. It may lie in that the scale parameter depends on a set of factors including *BI*, σ_{1f} and *E*.

Based on the derived damage parameters and *BI,* the corresponding stress-strain curves for Tavel limestone can be derived, as plotted in Fig. 5 together with experimental results.

The theoretical results obtained from our proposed model are in good agreement with the experimental results. Therefore, the proposed statistical damage model can predict the main experimental results both pre- and post-peak failure behaviours, and correctly capture the brittle/ductile mechanical behaviours, including the failure strength, brittle behaviour, strain-softening behaviour, strain hardening behaviour and failure mode as the confining pressure increases.

5. Discussion

5.1 Sensitivity analysis

In the derived SDM, the damage distribution parameters including the shape parameter m and the scale parameter F_0 and intact rock response are brittleness parameter *BI* related. The influence of *BI* on damage distribution parameters and rock response is investigated in this section.

As stated in the previous section, the damage distribution parameter m only relates to the brittleness parameter *BI*, indicating that m reflects the brittleness of the rocks. The relationship between these two parameters is plotted in Fig. 6 from Eq. (19). The shape parameter is nonlinearly and positively related to the brittleness parameter *BI,* which confirms the conclusion by previous studies (Liu *et al.* 2017).

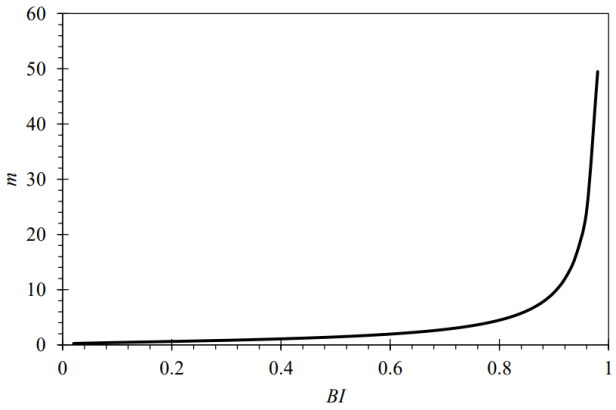

Fig. 6 Damage distribution parameter m versus rock brittleness parameter *BI*

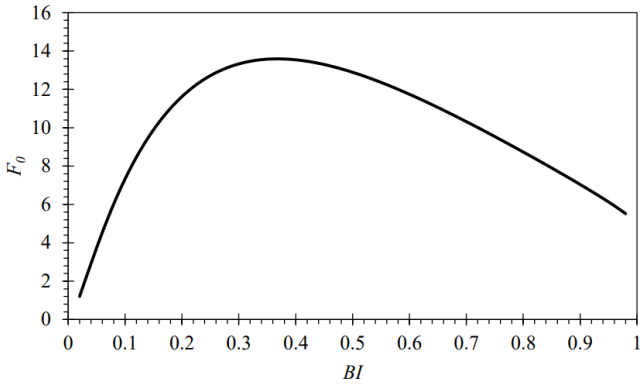

Fig. 7 Damage distribution parameter F_0 versus rock brittleness parameter *BI*

Table 3 The damage distribution parameters m and F_0 with corresponding brittleness *BI*

BI	*m*	F_0
0.40	1.09	162.51
0.60	1.96	140.91
0.80	4.48	104.81

The scale parameter F_0 is also plotted against the rock brittleness parameter *BI*, where the elastic strain is set to 5 mm, see Fig. 7. The result shows that there is a Parabola relationship between the scale parameter F_0 and the brittleness parameter *BI*. The F_0 increases at the initial stage, reaches to the maximum value when *BI* equals to 0.36 and then decreases to 5 mm as *BI* approaches 1.

To investigate the effect of *BI* on the rock response, the failure strength and elastic strain are set to 60.00 MPa and 5.00 mm, respectively, while the brittleness parameter *BI* increases from 0.40 to 0.80. The corresponding damage distribution parameters m and F_0 are listed in Table 3. As the brittleness *BI* increases from 0.40 to 0.80, the damage variable D, damage evolution rate D_r and stress-strain

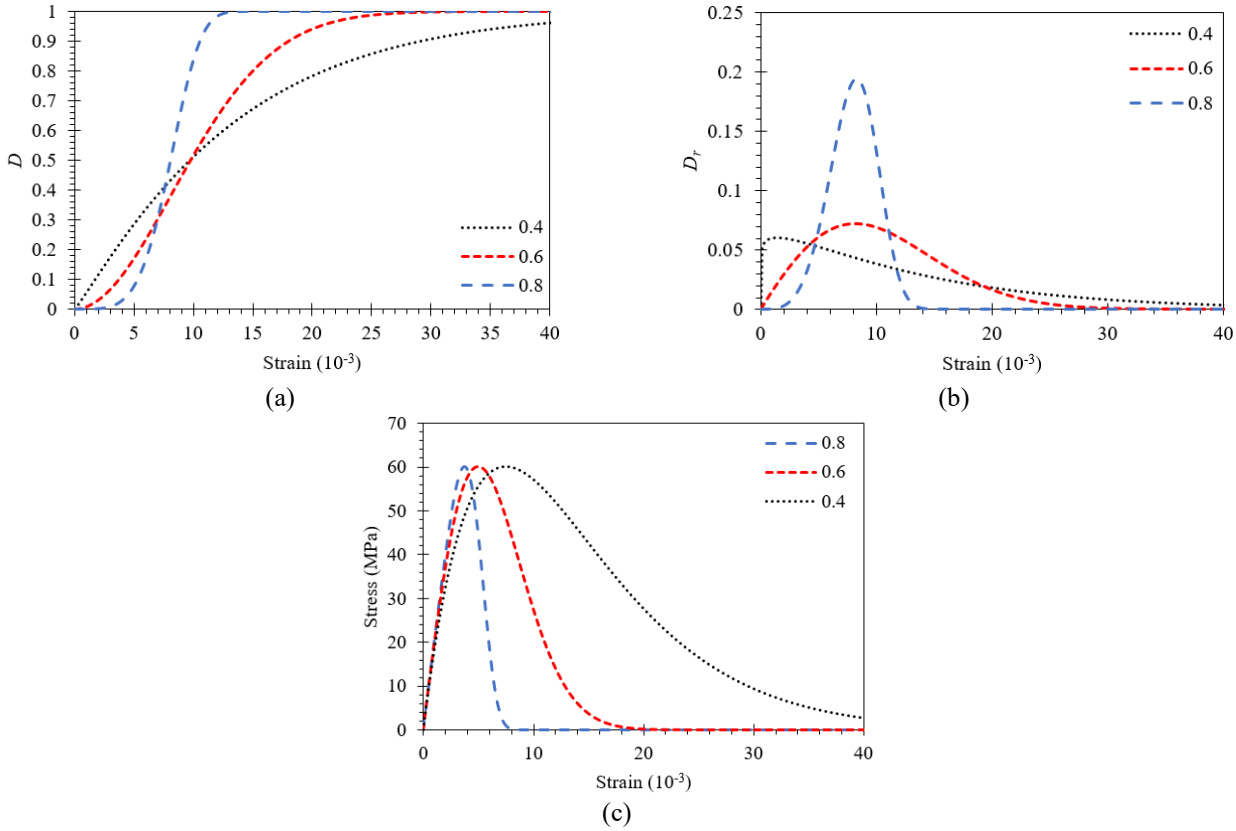

Fig. 8 Damage variable D (a) Damage evolution rate D_r, (b) and Intact rock responses and (c) versus the strain with various brittleness parameter BI

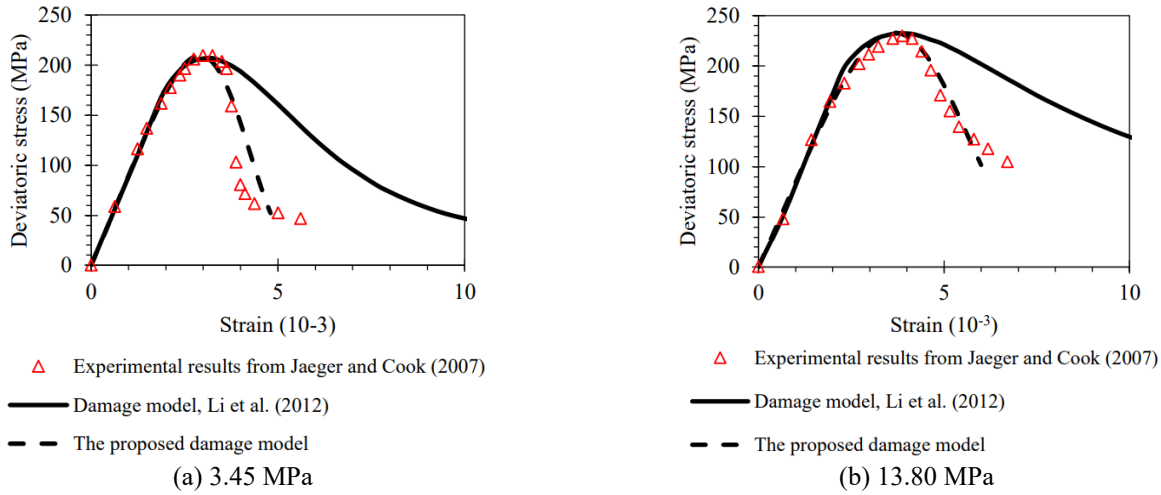

Fig. 9 Comparison between experimental results and theoretical results from previous damage model (Li *et al.* 2012) and the proposed damage model at various confining pressures

relation are plotted Fig. 8, respectively.

As shown in Fig. 8(a), the damage variable D curve is influenced by the brittleness parameter BI. As brittleness parameter BI increases, the damage variable-strain relation becomes steeper, and the strain needed for the onset of damage dramatically increases. The damage evolution increases gradually when BI is small, i.e., BI= 0.40. On the other hand, the damage evolution increases quickly when the brittleness parameter BI is large, i.e., BI=0.80. Consequently, the strain needed for the full damage, where D=1.00, decreases as BI increases. Overall, the damage model with larger BI demonstrates more homogeneity characteristics.

In Fig. 8(b), we demonstrate the damage evolution rate D_r with various brittleness parameter versus strain. As BI increases, the strain at the maximum damage evolution rate D_r decreases, which concise with the dramatic increase in the strain of damage evolution.

The intact rock responses, stress-strain curves, with various BI, are presented in Fig. 8(c). The results

demonstrate that the peak strength of the damage models is independent of *BI*. Additionally, Young's modulus of damage models is also independent of *BI* and only depends on the elastic strain. As the brittleness parameter *BI* increases, the percentage of the plastic strain out of the total strain is increased. We can conclude that the damage model becomes more ductile as the brittleness parameter *BI* decreases. When the brittleness parameter *BI* is small, the damage model can tolerate a large inelastic deformation without losing the load-carrying capacity, this is in agreement with previous work (Hajiabdolmajid and Kaiser 2002).

5.2 Comparison with the previous damage model

As stated in Section 1, a SDM proposed by (Li *et al.* 2012) to deal with strain softening behaviours for intact rocks. To better evaluate the proposed damage model in this study, the experimental data used for validation in the previous work (Li *et al.* 2012), and theoretical predictions from the previous damage model and the proposed damage model in this study are plotted in Fig. 9 for comparison.

The results showed that theoretical predictions from both damage models have a good agreement with experimental results in terms of pre-peak failure behaviours. However, a clear discrepancy between theoretical prediction based on Li's model and experimental results is observed in the post-peak region. They argued the reason behind this may relate to the failure modes, which can be captured by the proposed damage model in this study. However, our proposed damage model can better capture the post-peak failure behaviours under different confining pressure with less discrepancy. In our damage model, *BI* equals to 0.80 and 0.75 with corresponding confining stress of 3.45 and 13.80 MPa. The proposed damage model can better capture the brittle post-failure behaviour compared with Li's damage model.

6. Conclusions

As a fundamental problem in rock engineering, rock brittleness is an important property of rocks and is concerned with both pre- and post- peak failure behaviours. In the framework of statistical damage mechanics, we developed a plastic strain based SDM by incorporating the brittleness index (*BI*) to capture the transition from brittle fracture to ductile flow.

The proposed damage model only contains three macroscopic parameters, namely BI, E and σ_{1f}, which can be derived from experimental data. The comparison between theoretical predictions from the proposed damage model and experimental results under various confining pressure indicates that the proposed damage model is able to capture the transition from brittle fracture to ductile flow. The shape parameter m only relates nonlinearly to the brittleness index (*BI*) of the rocks. However, the scale parameter F_0, which can be regarded as a strength parameter, depends on both the brittleness parameter-*BI* and the failure strength adopted.

In further works, the crack closure effect on the mechanical behaviours of rocks at the initial stage should be taken into consideration to derive a complete stress-strain relation. It may also be more realistic and perhaps more robust for damage models to incorporate stochastic properties of rocks because of the existence of inherent heterogeneity in nature.

Acknowledgments

The PhD scholarship provided by the China Scholarship Council (CSC) to the first author is gratefully acknowledged. This research is financially supported by the Department of Science and Technology of Guangdong Province (No. 2019ZT08G315) and the Natural Science Foundation of China (No. 51827901).

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List of symbols

BI	Brittleness index
D	Damage variable
E (GPa)	Young's modulus of rocks
F	The element strength parameter depending on the strength criterion used
F_0	Scale parameter of Weibull distribution
K	Compaction coefficient
m	Shape parameter or a homogeneous index of Weibull distribution
N	The total number of all microscopic elements
n	The number of all failed microscopic elements under a certain loading
$P(F)$	The percentage of damaged one out of the total number of microscopic elements in the rock
ε_1	Strain on the major principal stress direction
ε_e	Elastic strain
ε_f	Failure strain or total strain at failure
ε_r	Residual strain
μ	Poisson's ratio of the material
σ_f (MPa)	Failure stress
σ_r (MPa)	Residual stress
σ_i (MPa)	The nominal stress, i = 1, 3
σ_i^* (MPa)	The effective stress, i = 1, 3