Geomechanics and Engineering, *Vol. 21, No. 2 (2020) 201-206*
DOI: https://doi.org/10.12989/gae.2020.21.2.201

Quantitative assessment of depth and extent of notch brittle failure in deep tunneling using inferential statistical analysis

Kang-Hyun Lee[1a], In-Mo Lee[2b] and Young-Jin Shin[*3]

[1]Research Institute, Korea Expressway Corporation, Hwaseong-si 18489, Korea
[2]School of Civil, Environmental and Architectural Engineering, Korea University, Seoul 02841, Korea
[3]R&D Center, Hyundai Engineering & Construction, Seoul 03058, Korea

(Received December 4, 2019, Revised February 4, 2020, Accepted March 3, 2020)

Abstract. A stress-induced brittle failure in deep tunneling generates spalling and slabbing, eventually causing a v-shaped notch formation. An empirical relationship for the depth of the notch to the maximum tangential stress assuming an equivalent circular cross-section was proposed (Martin *et al.* 1999). While this empirical approach has been well recognized in the industry and used as a design guideline in many projects, its applicability to a non-circular opening is worth revisiting due to the use of equivalent circular profile. Moreover, even though the extent of the notch also contributes to notch failure, it has not been estimated to date. When the estimate of both the depth and the extent of notch are combined, a practical and economically justifiable support design can be achieved. In this study, a new methodology to assess the depth as well as the extent of notch failure is developed. Field data and numerical simulations using the Cohesion Weakening Frictional Strengthening (CWFS) model were collected and correlated with the three most commonly accepted failure criteria (σ_1/σ_3, $D_{is}=\sigma_{max}/\sigma_c$, σ_{dev}/σ_{cm}). For the numerical analyses, the D-shaped tunnel was used since most civil tunnels are built to this profile. Inferential statistical analysis is applied to predict the failure range with a 95% confidence level. Considering its accuracy and simplicity, the new correlation can be used as an enhanced version of failure assessment.

Keywords: deep tunneling; brittle failure; spalling; inferential statistical analysis

1. Introduction

Two modes of failure can be used to determine the stability of deep excavations: stress-induced failure and gravity-driven failure. When constructing a deep underground opening in hard rock, the rock mass is often subject to a stress-induced brittle failure in the form of spalling and slabbing, eventually leading to notch formation (Martin 1997, Cai *et al.* 2004, Guoqing *et al.* 2017, Shen and Barton 2018, Hamdi *et al.* 2017, Shaalan *et al.* 2018, Gong *et al.* 2018). Martin (1997) stated that cracking initiates in the region where the deviatoric stress exceeds the damage threshold. Subsequently, shearing, crushing, and extensive dilation at grain-level occur in this region, which result in the onset of thin slabbing and spalling, finally leading to notch formation. The development of the notch stops when the geometry provides sufficient confinement to stabilize the failure. The estimation of notch formation is needed to effectively support the broken ground when rock mass failure cannot be prevented because outside this notch region, the rock mass is considerably less damaged and retains its integrity (Martin *et al.* 1999). Most research carried out so far has focused on the onset of the brittle failure process or the depth of failure (Martin *et al.* 1999, Martin 1997, Fairhurst and Lin 1985, Kaiser *et al.* 2000, Lee *et al.* 2012, Lee *et al.* 2013). Martin *et al.* (1999) suggested a linear relationship between the depth of failure and the stress level for brittle rock based on field observations as follows:

$$\frac{R_f}{a} = 1.25\frac{\sigma_{max}}{\sigma_c} + 0.49(\pm 0.1) \tag{1}$$

$$d_f = R_f - a = (1.25\frac{\sigma_{max}}{\sigma_c} - 0.51)a \tag{2}$$

where the radius of failure (R_f) is normalized to the effective tunnel radius (a) and the maximum tangential stress

Table 1 Comparsion of field observation and empirical correlations (Martin *et al.* 1999)

Site	Profile	Field data Depth (m)	Location	Martin *et al.*'s Depth (m)	References
Lötschberg Base Tunnel	Circular	0.45	Side	0.95	Cai *et al.*(2004), Rojat *et al.*(2009)
Kobbskaret Tunnel	D-shaped	0.3	Roof	1.18	Edelbro (2008)
Heggura Tunnel	D-shaped	0.05~0.4	Roof	1.03	Edelbro(2008)
Yeosu Oil Storage	Cavern	0.5~0.8	Roof	1.10	Lee *et al.*(2005)

*Corresponding author, Ph.D.
E-mail: iamyjshin@gmail.com
[a]Ph.D.
[b]Professor

http://www.techno-press.org/?journal=gae&subpage=7
ISSN: 2005-307X (Print), 2092-6219 (Online)

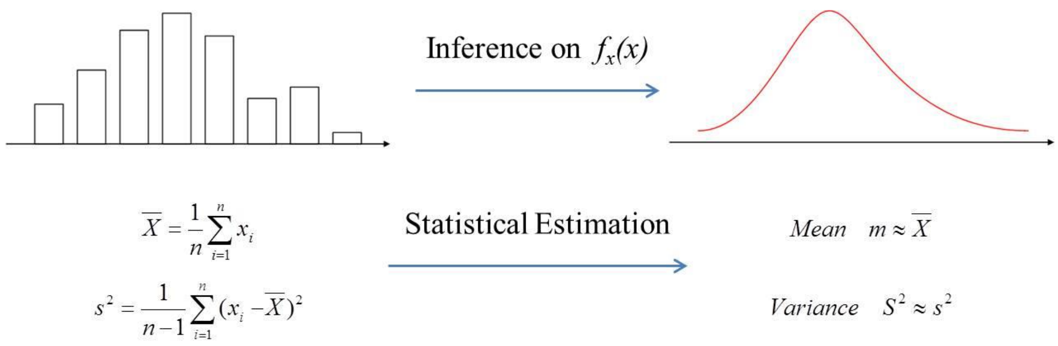

Fig. 1 Inferential statistical analysis

(σ_{max}=3σ_1-σ_3) is normalized to the laboratory uniaxial compressive strength (σ_c). σ_1 and σ_3 are the maximum and minimum far-field principal stresses, respectively, and d_f is the depth of spalling failure.

While the empirical approach has been well recognized in the industry and is used as a design guideline in many projects, some concerns have been raised about its application to non-circular openings, in particular to the D-shaped tunnel which represents most civil tunnels. Martin *et al.* (1999) therefore stated that Eqs. (1) and (2) do not allow for the distance from the excavation boundary to the effective tunnel radius (*a*) in the D-shaped tunnel. Moreover, the stress distribution around an excavation is controlled by the shape of the excavation. Openings with corners or small radii of curvature will have high compressive concentrations in these locations. In addition, 3σ_1-σ_3 is the maximum tangential stress only for circular openings, which indicates that the equation does not sufficiently account for tunnel profiles other than the circular profile. As such, the maximum tangential stress formula (D_{is}=σ_{max}=$A\sigma_1$-σ_3) in relation to the opening shapes was proposed by Lee *et al.* (2012), and is used in this study. The shape factor A used in this study was that proposed by Hoek and Brown (1980). To examine the applicability of Martin`s formula, field data were collected and compared as shown in Table 1. The results show that the formula tends to over-estimate the depth of failure, which is more than double in some cases, regardless of where the notch occurred, potentially leading to the over-design of supports. While a conservative estimate for safety would be acceptable, it would be more appropriate to have a balanced design which is economical as well as practical given that the supports are built in deep tunnels where logistics is challenging and relatively large commercial impacts are expected. Therefore, a new model to assess the depth as well as the extent of notch failure is developed. Inferential statistical analysis is applied to predict the failure range with a 95% confidence level.

2. Background theory

2.1 Inferential statistical analysis

Inferential statistical analysis infers the parameters of a population from a sample. The population is regarded as larger than the sample data set. When sampling from a

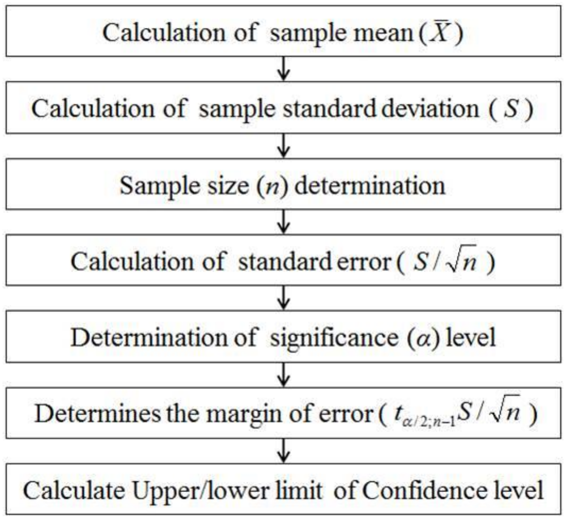

Fig. 2 Inferential statistical analysis with confidence level

population described by a probability density function $f(x:\theta)$, knowledge of parameter θ such as population mean μ or population standard deviation σ yields knowledge of the entire population. Let X_1, X_2, …, X_n be a sample of size n from a population with the probability density function (Fig. 1). An estimator T such as sample mean $\overline{X}$ or sample standard deviation $\boldsymbol{s}$ is a function of the sample as shown in Eq. (3). As such, a random sample is more likely to be representative than other types of samples.

$$T = T(X_1, X_2, ...X_n) \tag{3}$$

2.2 Interval estimation

The conclusion of a statistical inference is a statistical proposition such as point estimation or interval estimation. The point estimation of a parameter θ is a supposition of a single value, while the interval estimation provides more information about a population characteristic than does the point estimation. Interval estimation provides a confidence level for the estimate of parameter θ. Such interval estimates are called confidence intervals (Fig. 2). Assume that the confidence level is 95%; thus, 95% of all the intervals will include the true value of parameter θ. Five percent will be exceptionally far from the parameter θ. Therefore, we conclude that there is a 5% probability that the estimate can be false. This is the risk of error we are

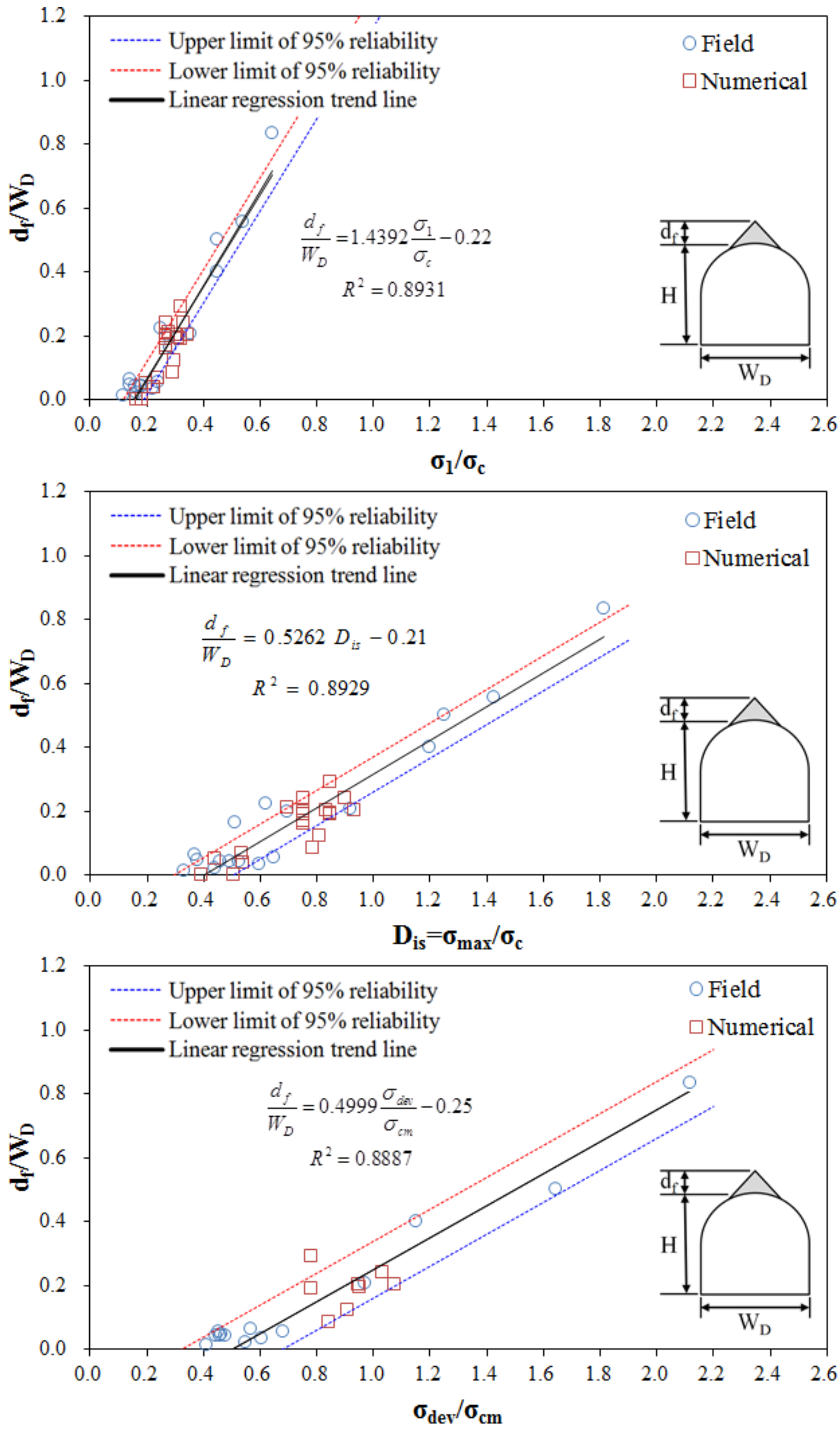

Fig. 3 Linear regression of generalized depth of notch with 95% confidence level

willing to accept. In this study, inferential statistical analysis with a 95% confidence level is applied to reasonably predict the failure range.

3. Inferential statistical model of depth and extent of notch failure

This paper proposes a new methodology to assess the depth as well as the extent of notch failure adopting Martin *et al.*'s (1999) format. The empirical observations collected by Hoek *et al.* (1995) and Martin *et al.* (1999), as well as additional field observations collected from world-wide projects and Cohesion Weakening Frictional Strengthening (CWFS) numerical simulations were all compiled as a data set. In the CWFS model, as friction is mobilized, cohesion is reduced progressively from the peak cohesion to the residual cohesion. Therefore, Hajiabdolmajid *et al.* (2002) concluded that the CWFS model is suitable for the brittle failure analysis of underground openings, in which confining stress is small and the tensile failure mechanism

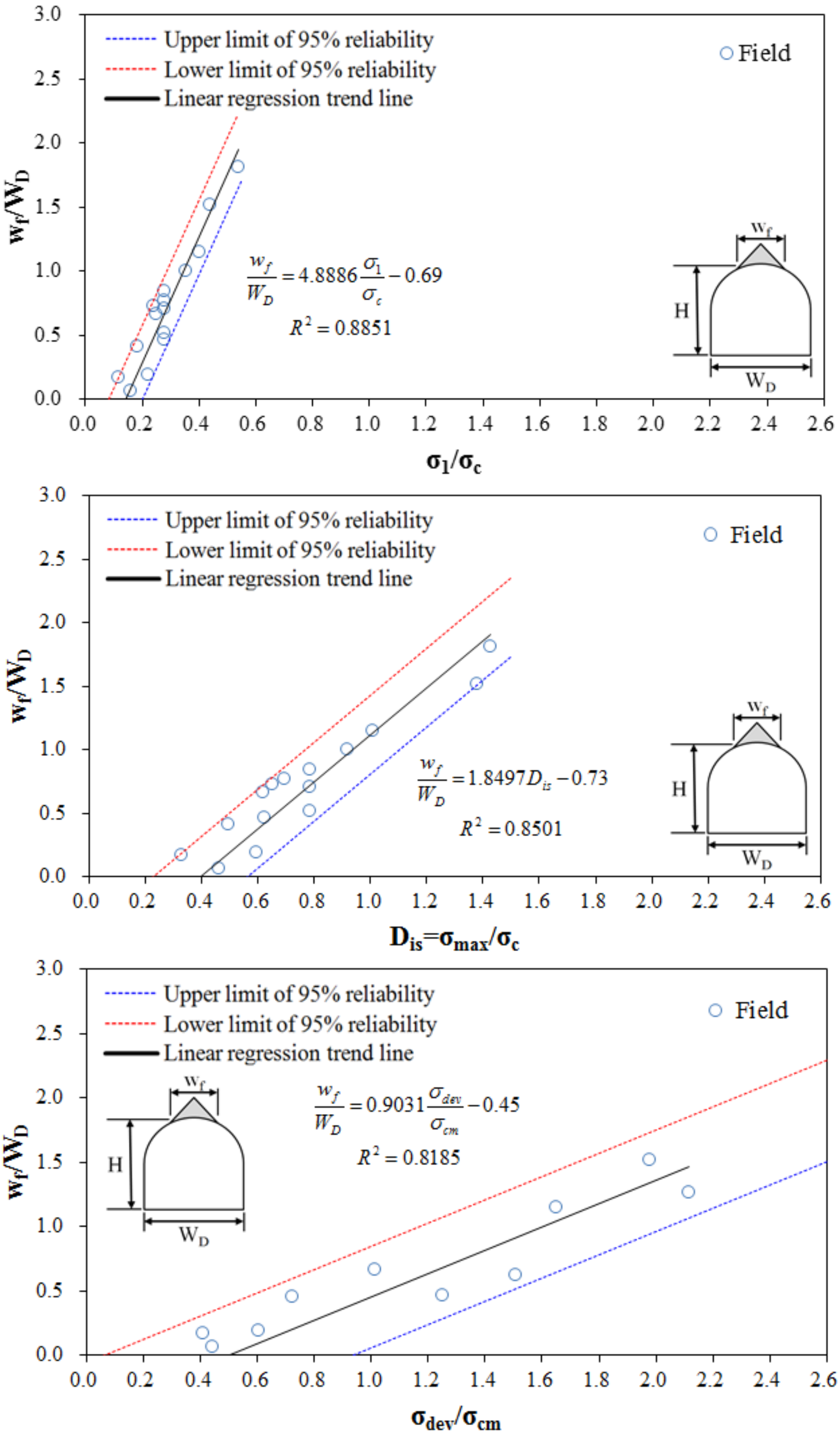

Fig. 4 Linear regression of generalized width of notch with 95% confidence level

dominates. The three most commonly accepted failure criteria (σ_1/σ_3, $D_{is}=\sigma_{max}/\sigma_c$, σ_{dev}/σ_{cm}) were used as the explanatory variables, which are together referred to as X. The depth and extent of notch (d_f, w_f) that were generalized according to the width of tunnel (W_D) were the dependent variables referred to as Y. When plotted in the Cartesian coordinate, linear regression finds the relationship between two variables by fitting a linear equation to the observed data as shown in Figs. 3 and 4. The use of interval with a 95% confidence level provides lower and upper limits to the etimate.

Changing the Equations in Figs. 3 and 4 to augmented matrices, we obtain:

$$X = [x_i]_{1\times 3} = [x_1 \quad x_2 \quad x_3] = [\sigma_1/\sigma_c \quad D_i = \sigma_{max}/\sigma_c \quad \sigma_{dev}/\sigma_{cm}], \quad i = 1, 2, 3 \tag{4}$$

$$Y = [y_i]_{1\times 3} = [y_1 \quad y_2 \quad y_3] = [LL \quad EV \quad UL], \quad j = 1, 2, 3 \tag{5}$$

Table 2 Pairwise comparison matrix considering the relative variance of linear regression of failure criteria

	σ_1/σ_c	D_i (=σ_{max}/σ_c)	σ_{dev}/σ_{cm}
σ_1/σ_c	1	5/5	5/1
D_i (=σ_{max}/σ_c)	5/5	1	5/1
σ_{dev}/σ_{cm}	1/5	1/5	1

According to Eqs. (4) and (5), matrices for the depth and the extent are established as shown in Eqs. (6) and (7).

$$DF = \begin{bmatrix} df_{1LL} & df_{1EV} & df_{1UL} \\ df_{2LL} & df_{2EV} & df_{2UL} \\ df_{3LL} & df_{3EV} & df_{3UL} \end{bmatrix} \tag{6}$$

$$WF = \begin{bmatrix} wf_{1LL} & wf_{1EV} & wf_{1UL} \\ wf_{2LL} & wf_{2EV} & wf_{2UL} \\ wf_{3LL} & wf_{3EV} & wf_{3UL} \end{bmatrix} \tag{7}$$

To enhance the accuracy of the models, weighting matrixes are developed using the analytic hierarchy process proposed by Saaty (1980), which is a structured technique for organizing and analyzing the element`s relative meaning and importance. In this study, the analytic hierarchy process enables the models to take into account the degree of relative importance among the three failure criteria. Thus, the hierarchy among the criteria is estimated and incorporated into the matrix; the relative variance of linear regression of failure criteria is shown in Table 2.

Each column in the pairwise comparison matrix is summed and each element in the matrix is divided by the column total. This new matrix is multiplied by itself to generate the weighting matrix as follows:

$$W = [w_i]_{1\times 3} = [w_1,\quad w_2,\quad w_3] = [0.48,\quad 0.41,\quad 0.11], \quad i = 1, 2, 3 \tag{8}$$

The consistency between the pairwise matrix and the weighting matrix is estimated by the largest eigenvalue, λ_{max}. It is based on the principle stating that for a given $n \times n$ square matrix, as λ_{max} approaches closer to n, the matrices become more consistent. The consistency index (CI) and a consistency ratio (CR) for the two matrices are given by Eqs (9) and (10). CR is employed, which is the ratio of CI and random number index (RI). When CR is less than 10%, the consistency can be verified. The CR value in this study is 0.02%.

$$CI = (\lambda_{\max} - n)/(n-1) \tag{9}$$

$$CR(\%) = (CI/RI) \times 100 \tag{10}$$

Accordingly, the assessment matrix with consideration of the weighting matrix is obtained:

$$\begin{aligned} \frac{d_f}{W_D} &= W \cdot DF = [w_i]_{1\times 3} \cdot [df_{ij}]_{3\times 3} \\ &= [\sum_{i=1,2,3\ j=1} w_i \cdot df_{ij} \quad \sum_{i=1,2,3\ j=2} w_i \cdot df_{ij} \quad \sum_{i=1,2,3\ j=3} w_i \cdot df_{ij}] \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{w_f}{W_D} &= W \cdot WF = [w_i]_{1\times 3} \cdot [wf_{ij}]_{3\times 3} \\ &= [\sum_{i=1,2,3\ j=1} w_i \cdot wf_{ij} \quad \sum_{i=1,2,3\ j=2} w_i \cdot wf_{ij} \quad \sum_{i=1,2,3\ j=3} w_i \cdot wf_{ij}] \end{aligned} \tag{12}$$

$$d_f = \frac{d_f}{W_D} \cdot W_D = [df_{LL} \quad df_{EV} \quad df_{UL}] \tag{13}$$

$$w_f = \frac{w_f}{W_D} \cdot W_D = [wf_{LL} \quad wf_{EV} \quad wf_{UL}] \tag{14}$$

Herewith, the proposed Eqs. (13) and (14) present the depth and the width of notch failure, respectively.

4. Comparison with field observations

A review of the available literature and data identified twelve and eleven cases where the depth and extent of failure around each tunnel had been measured, respectively (see Tables 3 and 4). These cases also provide descriptions of the rock type, tunnel profiles, and in situ stress states. This new inferential statistical model was applied to the cases and compared to the filed data to determine whether the model is effective. While the model shows interval estimates with a 95% confidence level, in most cases the field observation falls within the estimate interval, showing the success rates of 92% and 90%, respectively.

The result indicates that the proposed model is well suited to evaluate the notch failure. In the some locations, however the model fails to predict the depth or the width. In a Canadian mine drift tunnel, the model underestimated the depth while successfully estimated the width. HRL APSE vice versa. When looking into the two cases, they are neither sensitive to the rock type nor the tunnel profile. No clear or potential reasons for this inconsistency are suggested, indicating the necessity of further improvements to the model. Perhaps a larger data set would help to investigate how to modify the model and increase its accuracy. Nonetheless, this statistical approach has potential for the better understanding and more reasonable estimation of notch failure, thus enabling adequate engineering judgment, in particular at the design stage.

5. Conclusions

An inferential statistical model was proposed in this paper to estimate the depth and the extent of notch failure in deep tunneling. Field data were collected and compared to validate the proposed model. Based on the presented results, the following conclusions can be drawn:

(1) When constructing deep underground openings in hard rock, notch failure can occur. The reliable geometrical assessment in terms of depth and width of notch failure is important to design an adequate rock support, and therefore to control the rock stability during and after excavations.

(2) A proposed inferential statistical model was used to address this problem and when compared with field data a

good agreement is found.

(3) By using an interval estimate with a 95% confidence level, the model more reasonably estimates the failure, thus enabling adequate engineering judgment, in particular at the design stage.

(4) It should be noted that the reliability of the model from such a statistical approach is subject to providing a larger data set. This is worth revisiting to improve the model and thus to increase its accuracy when more data is available.

References

Andersson, C. (2005), "Äspö pillar stability experiment: final experiment design, monitoring results and observations", SKB Report R-05-02. http://www.skb.se/Templates/Standard____17139.aspx.

Andersson, C. (2007), "Äspö pillar stability experiment: final report: rock mass response to coupled mechanical thermal loading", SKB Technical Report TR-07-01. http://www.skb.se/ppw_en/document.asp?ppwAutnRef=PUB-1353659&id=3515&prevUrl=,.

Cai, M., Kaiser, P.K., Tasaka, Y., Maejima, T., Morioka, H. and Minami, M. (2004), "Generalized crack initiation and crack damage stress thresholds of brittle rock masses near underground excavations", *Int. J. Rock Mech. Min. Sci.*, **41**(5), 833-847. https://doi.org/10.1016/j.ijrmms.2004.02.001.

Chen, G., Li, T., Wang, W., Guo, F. and Yin, H. (2017), "Characterization of the brittleness of hard rock at different temperatures using uniaxial compression tests", *Geomech. Eng.*, **13**(1), 63-77. https://doi.org/10.12989/gae.2017.13.1.063.

Edelbro, C. (2006), "Strength of hard rock mass: a case study", Technical Report, Division of Mining and Geotechnical Engineering, Luleå University of Technology, Luleå, Sweden. http://epubl.luth.se/1402-1536/2006/13/index.html.

Edelbro, C. (2008), "Strength, fallouts and numerical modelling of hard rock masses", Ph.D. Dissertation, Luleå University of Technology, Luleå, Sweden.

Edelbro, C. (2009), "Numerical modelling of observed fallouts in hard rock masses using an instantaneous cohesion-softening friction-hardening model", *Tunn. Undergr. Sp. Technol.*, **24**(4), 398-409. https://doi.org/10.1016/j.tust.2008.11.004.

Emsley, S., Olsson, O., Stenberg, L., Alheid, H.J. and Falls, S. (1997), "ZEDEX-A study of damage and disturbance from tunnel excavation by blasting and tunnel boring", SKB Technical Report TR-97-30. http://www.skb.se/ppw_en/document.asp?ppwAutnRef=PUB-15249&id=3515&prevUrl=, 1997.

Fairhurst, C., Lin, D. (1985) "Fuzzy methodology in tunnel support design", *Proceedings of the 26th US symposium on rock Mechanics*, Rapid City, South Dakota, U.S.A., June.

Gong, F.Q., Luo, Y., Li, X.B., Si, X.F. and Tao, M. (2018), "Experimental simulation investigation on rockburst induced by spalling failure in deep circular tunnels", *Tunn. Undergr. Sp. Technol.*, **81**, 413-427. https://doi.org/10.1016/j.tust.2018.07.035.

Hajiabdolmajid, V., Kaiser, P.K. and Martin, C.D. (2002), "Modelling brittle failure of rock", *Int. J. Rock Mech. Min. Sci.*, **39**(6), 731-741. https://doi.org/10.1016/S1365-1609(02)00051-5.

Hamdi, P., Stead, D. and Elmo, D. (2017), "A review of the application of numerical modelling in the prediction of depth of spalling damage around underground openings", *Proceedings of the 51st US Rock Mechanics/Geomechanics Symposium*, San Francisco, California, U.S.A., June.

Hoek, E. and Brown, E.T. (1980), *Underground Excavation in Rock*, The Institution of Mining and Metallurgy, London, U.K.

Hoek, E., Kaiser, P.K. and Bawden, W.F. (1995), *Support of Underground Excavations in Hard Rock*, Balkema, Rotterdam, The Netherlands.

Kaiser, P.K., Diederichs, M.S., Martin, C.D., Sharp, J. and Steiner, W. (2000), "Underground works in hard rock tunneling and mining", *Proceedings of the GeoEng2000: An International Conference on Geotechnical & Geological Engineering*, Melbourne. Australia, November.

Lee, H.S., Lee, D.H., Kim, H.Y., Hong, J.S., Choi ,Y.T., Kim, S.J. and Park, Y.J. (2005) "Stability evaluation and microseismic monitoring around large underground oil storage cavern in over-stresses rock mass", *Proceedings of the KTA 2005 Annual Conference*, Seoul, Korea.

Lee, K.H., Bang, J.H., Lee, I.M. and Shin, Y.J. (2013), "Use of fuzzy probability theory to assess spalling occurrence in underground openings", *Int. J. Rock Mech. Min. Sci.*, **64**, 60-67. https://doi.org/10.1016/j.ijrmms.2013.08.024.

Lee, K.H., Lee, I.M. and Shin, Y.J. (2012), "Brittle rock property and damage index assessment for predicting brittle failure in excavations", *Rock Mech. Rock Eng.*, **45**, 251-257. https://doi.org/10.1007/s00603-011-0189-7.

Martin, C.D. (1997), "The effect of cohesion loss and stress path on brittle rock strength", *Can. Geotech. J.*, **34**(5), 698-725. https://doi.org/10.1139/t97-030.

Martin, C.D. (2005), "Preliminary assessment of potential underground stability (wedge and spalling) at Forsmark, Simpevarp and Laxemar sites", SKB Report R-05-71. http://www.skb.se/Templates/Standard____17139.asp x.

Martin, C.D., Kaiser, P.K., McCreath, D.R. (1999), "Hoek–Brown parameters for predicting the depth of brittle failure around tunnels", *Can. Geotech. J.*, **36**(1), 136-151. https://doi.org/10.1139/t98-072.

Pelli, F., Kaiser, P.K. and Morgenstern, N.R. (1991), "An interpretation of ground movements recorded during construction of the Donkien-Morien tunnel", *Can. Geotech. J.*, **28**(2), 239-254. https://doi.org/10.1139/t91-030.

Rojat, F., Labiouse, V., Kaiser, P.K. and Descoeudres, F. (2009), "Brittle rock failure in the Steg lateral adit of the Lötschberg base tunnel", *Rock Mech. Rock Eng.*, **42**(2), 341-359. https://doi.org/10.1007/s00603-008-0015-z.

Saaty, T.L. (1980), *The Analytic Hierarchy Process*, McGraw Hill, New York, U.S.A.

Shaalan, H.H., Ismail, M.A.M. and Azit, R. (2018), "Brittle failure simulation of Pahang Selangor raw water transfer tunnel under rock overstressing", *Proceedings of the ISRM International Symposium-10th Asian Rock Mechanics Symposium*, Singapore, October- November.

Shen, B. and Barton, N. (2018) "Rock fracturing mechanisms around underground openings", *Geomech. Eng.*, **16**(1), 35-47. https://doi.org/10.12989/gae.2018.16.1.035.

CC