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Tunnel lining load with consideration of the rheological properties of rock mass and concrete

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Abstract. Rheological processes in the rock mass for the stress-strain analysis are quite important when considering the construction of underground structures in soft rock masses, particularly in case of construction in several stages. In the analysis, it can be assumed that the reinforced concrete structure is slightly deformable in relation to the rock mass, and the rheological stress redistribution happens at the expense of the elements of rock mass. The basic elements of rheological models for certain types of rock mass and analysis of these models are presented in the first part of this paper. The second part is dedicated to the analysis of rheological processes in marl rock mass and the influence of these processes on the reinforced-concrete tunnel structure.

Keywords: rock mass; rheological models; tunnel construction; rheological processes

1. Introduction

During the design and construction of underground structures, it is of extreme importance to determine variation of the stress field from the beginning of rock mass excavation until the completion of works, depending on the technology of construction process. In addition, monitoring and measurements during construction works have demonstrated that there also exists a variation of secondary stresses over time as a result of rheological processes. Rheological processes and materials are described with mathematical relations and corresponding boundary conditions, so that they could be used within mathematical models for description of rock mass behaviour during excavation.

Matter in nature occurs as having elastic, viscous, and plastic properties. These properties, under the action of “external” influences, occur simultaneously or successively. Actual behaviour of matter is sometimes very complex, so the relations of strain and stress are also quite complex. Classical continuum mechanics, in the past, acknowledged two types of materials, i.e., two rheological models - elastic solid bodies (the Hooke’s body) and ideal fluids (the Newton’s fluid), which are considered to be the basic (simple) rheological models. Complex rheological models are composed of several simple models. They can consist of only two simple models, or of a combination of three or more basic models, which are often called three-parametric or multi-parametric models of viscoelastic materials. The more a material is complex in its structure and behaviour, the more basic models must be included in its behaviour model (Fritz 1984). Rheological elements can be connected in parallel (the described phenomena occur simultaneously) and serial (the described phenomena occur successively).

Combination of rheological elements provides rheological models, which can describe behaviour of various natural materials (rocks) over time. Behaviour of rock mass, based on the measurements in underground structures, indicates occurrence of large *„delayed“* strain, which can cause failure of rock mass. This phenomenon illustrates a long-term viscous behaviour of rocks and progressive damage occurring after redistribution of stress around the excavated cavity (Effinger and Bois 2012).

It is well known that the system of the tunnel lining and the rock mass has mechanical properties of rheological character and that the condition of stress and strain varies over time. The rheological properties of the individual components of this system have been investigated so far. However, the integral behaviour of a complex system of tunnel structure-rock mass is not sufficiently studied. The variation of stress-strain relations over time and their final values depend essentially on the rheological properties of the mutual interaction, i.e., the mutual behaviour of the tunnel-rock system as a whole. Precisely for that reason, the laws of the interaction of the tunnel-rock complex system are established and analysed in this paper. Only such setting of the problem can explain the properties of the system and emphasise the expedience of designing the optimal technological process of tunnel construction, as well as selecting the appropriate system of tunnel structure. Therefore, the focus of this paper is the development of underground pressures to the tunnel lining, taking into account the rheological properties of concrete and rock mass.

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2. Basic ideal materials

Properties of basic ideal materials are presented by elementary mechanical models for the case of plane stress state, i.e. linear and homogenous stress state. Parameters, symbols, and characteristic diagrams of stress and strain are presented in Fig. 1.

The mutual relations of strain and stress for such ideal bodies are as follows (Roylance 2001, Khoshboresh 2013):

1. $\sigma = 0$ PASCAL's fluid,
2. $\sigma = E\varepsilon$ HOOKE's elastic body (H),
3. $\sigma = \eta\dot{\varepsilon}$ NEWTON's fluid (N),
4. $\sigma = c_0$ St. VENANT's plastic body (St.V),

where:

σ - stress,
E - modulus of elasticity,
ε - strain,
η - viscosity coefficient,
$\dot{\varepsilon}$ - deformation rate,
c_0 - yielding point.

The Hookean model represents a linear relation between stress and strain. The mechanical model is symbolised by an ideal elastic spring, where the deformation, after unloading, ceases completely.

Saint Venant has proposed a model of ideally rigid-plastic material, which has the property of exhibiting no deformation until the stress value reaches a certain critical value σ_c. The mechanical model is symbolised by a slider with a rough surface, thus simulating the Coulomb friction C. By exceeding the force of friction, the body begins to deform plastically at constant stress. Materials that have such properties are called "ideally plastic", in which case permanent (irreversible) deformation after unloading remains.

Matter with ideally viscous properties is the Newtonian fluid. The element behaviour is determined by the viscosity η, which defines the resistance of matter during stress variation. This resistance is proportional to the action of internal friction. After unloading of the element, permanent (irreversible) deformation remains.

The brittle element (R) with the C_R parameter represents a rock mass failure with a change in diagram (σ, ε), as it is shown in Fig. 1.

By mutual connection of models of basic ideal bodies in a certain way, and under certain conditions, rheologicalmodels of certain complex materials can be obtained:

ELEMENT	PARAMETER	SYMBOLS	NAME	DIAGRAM σ,ε	$\dot{\varepsilon} = \frac{d\varepsilon}{dt}$
ELASTIC	E	E H	HOOKE	σ tgα = E α ε	$\dot{\varepsilon} = 0$
PLASTIC	C	St.V	St.V VENANT	σ σ_0 ε	$\dot{\varepsilon} = 0$ $\dot{\sigma} = 0$
VISCOUS	η	η N_1	NEWTON	σ $\dot{\varepsilon}$	σ α $\dot{\varepsilon}$
BRITTLE	C_R	R		σ ε	$\dot{\varepsilon} = \infty$

Fig. 1 Models of ideal basic materials

1. $\sigma = E\varepsilon + \eta\dot{\varepsilon}$ KELVIN's body (K),
2. $\sigma = \eta\dot{\varepsilon} - t_R\dot{\sigma}$ MAXWEL's body (M),
3. $\sigma = \eta\dot{\varepsilon} + c_0$ BINGHAM's viscoplastic fluid (B),
4. $\sigma = E\varepsilon + \eta\dot{\varepsilon} + t_R\dot{\sigma}$ POYNTING–THOMSON,
5. $\sigma = E\varepsilon + \eta\dot{\varepsilon} + c_0$ Elasto-viscoplastic body,
6. $\sigma = \eta\dot{\varepsilon} - t_R\dot{\sigma} + c_0$ PRANDTL–REUSS,

where:

t_R - relaxation time,
$\dot{\sigma}$ - stress rate.

3. Rheological models of rock mass

In theoretical considerations of stress state for the purpose of construction of underground structures, it is important to point out the following:

• the impact of rheological properties of rock mass on the stress state around the tunnel;

• the character of redistribution of stress in the zone around underground structure depending on the technological process of construction and static structural system.

One of the most important problems in consideration of rheological models is the analysis of strain over time. According to time-dependent strain, rock masses can be divided in two groups, wherein their behaviour is displayed through rheological models (Desai and Abel 1972). The first group represents those rock masses for which the function of time and strain under constant load has a horizontal asymptote. The Kelvin's model (K=H/N - parallel connection of a Hooke's body and a Newton's body) corresponds to this kind of rock mass (Fig. 2). On the other hand, for rock masses belonging to the second group, the time-strain function under constant load does not have a horizontal asymptote, i.e. the strain infinitely increases. The behaviour of this group of rocks is presented by the Maxwell's model (M=H-N - serial connection of a Hooke's body and a Newton's body) (Fig. 3).

The time dependence of the behaviour of rock mass can be presented by a rheological model shown in Fig. 4, where

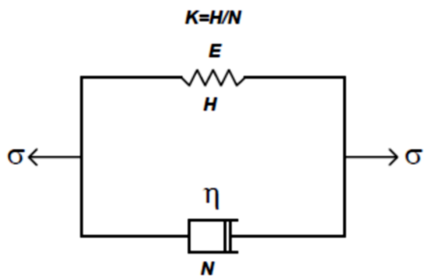

Fig. 2 Kelvin's model (Wang *et al.* 2014, 2015)

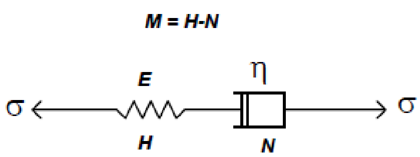

Fig. 3 Maxwell's model (Wang *et al.* 2013, 2015)

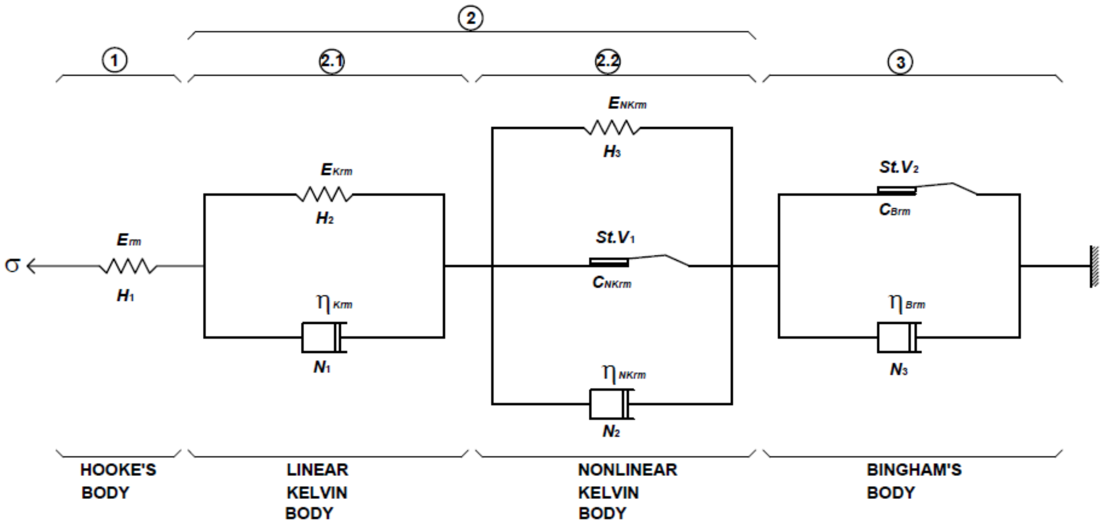

Fig. 4 Rheological model of rock mass - analysis of deformation and creep (Popović 1980)

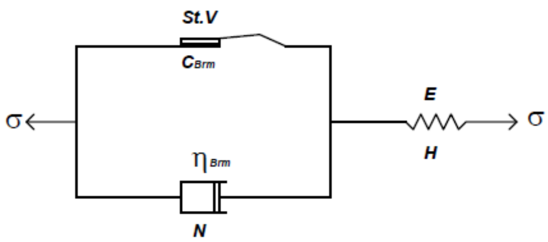

Fig. 5 Viscoelastic model with yield stress - Bingham-Hooke model (Shyshko 2013)

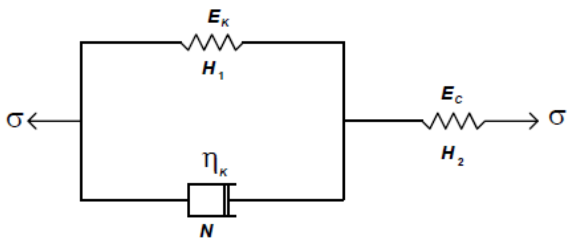

Fig. 6 Viscoelastic model - Standard Solid (Poynting) contact model (Shyshko 2013)

the variables are as follows:

E_{rm} - modulus of elasticity of rock for the Hooke's body (H_1),

E_{Krm} - modulus of elasticity of rock for the Hooke's body (H_2) within the linear Kelvin's body (K),

η_{Krm} - viscosity coefficient of the Newtonian fluid (N_1) within the linear Kelvin's body (K),

E_{NKrm} - modulus of elasticity of rock for the Hooke's body (H_3) within the nonlinear Kelvin's body (NK),

C_{NKrm} - yielding point of the St. Venant's body ($St.V_1$) within the nonlinear Kelvin's body (NK),

η_{NKrm} - viscosity coefficient of the Newtonian fluid (N_2) within the nonlinear Kelvin's body (NK),

C_{Brm} - yielding point of the St. Venant's body ($St.V_2$) within the Bingham's body (B),

η_{Brm} - viscosity coefficient of the Newtonian fluid (N_3) of the Bingham's body (B).

Elastic deformations are described by the Hooke's body (H_1) (1). Creep deformations, on the other hand, are perceived through two stages, whereby for the primary phase of creep (2) one linear Kelvin's body (K) (2.1) and one nonlinear Kelvin's body (NK) (2.2) are included (Jiang *et al*. 2012). The secondary stage of creep (3) is described by the Bingham's body (B). The Bingham's model consists of a Saint-Venant's element ($St.V_2$) and a dashpot in parallel (N_3) (as shown in Fig. 4). Deformation of the model is not possible before reaching the yield stress through the Saint-Venant's element. When the yield stress is reached, the model exhibits viscoplastic deformation.

With the rheological model shown above (Fig. 4), the influence of time upon the stress-strain relations in rock mass and tunnel structure can be appropriately taken into consideration in the design of underground structures. It is of paramount importance to determine the time-dependent parameters of rock mass, as this affects the rate of construction progress, and therefore, the technological process of construction.

While performing the research of deeply embedded tunnels, Quevedo and Bernaud (2018) have employed the viscoelastic law with von Mises viscoplastic criteria without hardening, in which the deformation rate is presented with the Bingham's model.

A number of recent researches have been dealing with rheological models of rock masses and their application in the construction of tunnels. Analysis of the time-dependent deformation properties of the rock mass and their impact on tunnel structures were presented in the works of Tomanović (2012, 2014), Aksoy *et al.* (2016), and Park (2017). In addition, Paraskevopoulou and Diederich (2018) investigated the total displacements of the tunnel walls in an isotropic viscoelastic environment, taking into account the tunnel construction progress and associated cumulative deformation due to the rheological behaviour of the material over time. According to these studies, it could be concluded that the researches on rheological models are still presently relevant. In the continuation of the paper, some of the models from the recent studies are briefly presented.

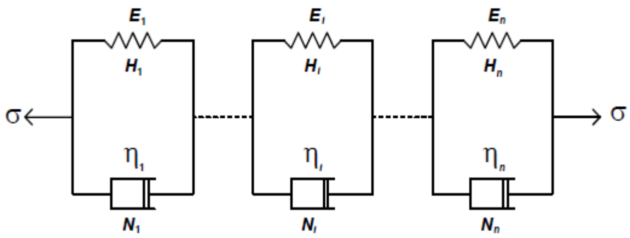

Fig. 7 General Kelvin's viscoelastic model (Wang *et al.* 2013)

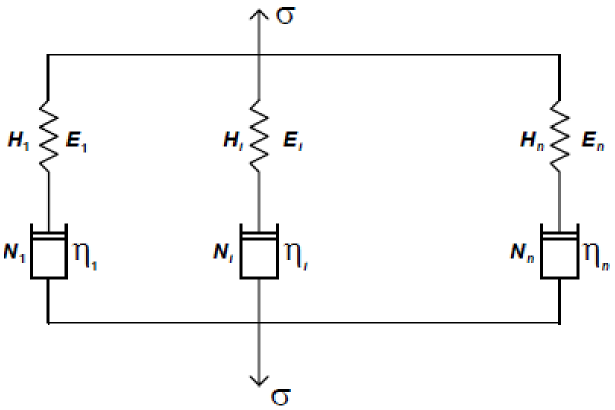

Fig. 8 General Maxwell's viscoelastic model (Wang *et al.* 2013)

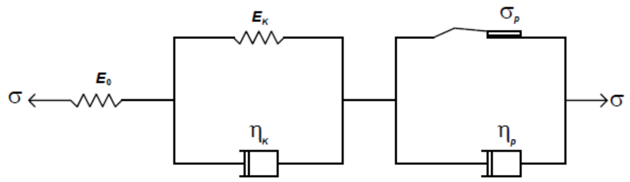

Fig. 9 Nishihara model (Zhang *et al.* 2019)

Viscoelastic model with yield stress (the Bingham-Hooke model) and viscoelastic model (the Standard Solid (Poynting) contact model) are presented in Figs. 5 and 6 (Shyshko 2013).

The Bingham–Hooke model (Fig. 5) consists of a Bingham's model (B=St.V/N) and a spring in series (H). The Bingham's material exhibits linear elasticity for stress values lower than the yield stress, as in the Saint-Venant's model, but yields linearly above that value, as in the Maxwell's model.

The Standard Solid (Poynting) contact model (Fig. 6) consists of the Kelvin's (spring and dashpot in parallel) section ($K=H_1/N$) and contains additional spring connected in series (H_2). This contact model simulates the creep and relaxation behaviour, as well as the instantaneous elasticity.

In rock mechanics, the Hookean elastic springs and Newtonian viscous dashpots are used to model a variety of rheological properties of the rock mass. To simulate more complex rheological behaviour of rocks, additional elastic springs or dashpots can be connected in parallel or in series in the general Kelvin's and Maxwell's models (Wang *et al.* 2013, 2014, 2015, 2018). The general Kelvin's and Maxwell's models are presented in Figs. 7 and 8.

The Nishihara model (Song *et al.* 2016 and Zhang *et al.*

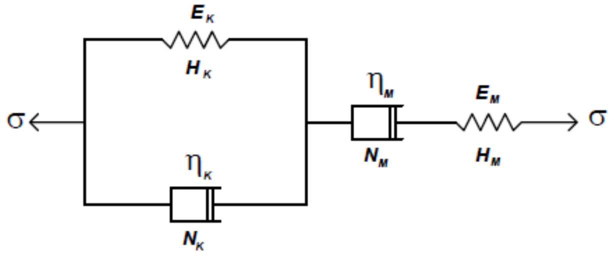

Fig. 10 Viscoelastic Burgers' model (Wang *et al.* 2018)

2019), presented in Fig. 9, can be used to effectively describe the yielding properties of the rock mass. A visco-elastoplastic model, termed the Nishihara body (Fig. 9), comprises a Kelvin's body (K) and a Bingham–Hooke body (B-H), with σ_p being a critical stress level (long-term strength). The creep curve for the Nishihara body can be used to describe decay, steady, and unstable creep behaviour. The Nishihara model describes the variation in the attenuation period and steady period fairly well.

Based on the study of rock salt, Ma *et al.* (2017) concluded that the development of axial creep deformation can be described by the Burgers' viscoelastic model (Fig. 10). The Burgers' creep model is composed of a Maxwell's model ($M=H_M$-N_M) and a Kelvin's model ($K=H_K/N_K$) connected in series. The viscoelastic Burgers' model was used in a number of studies considering the creep phenomenon (Debernardi 2008, Wang *et al.* 2017, Wang *et al.* 2018). Wang *et al.* (2018) pointed out that when the Burgers' viscoelastic model is introduced, with increasing time, the displacements accelerate rapidly, and then slow down after $t = 1.5T_K$ (where $T_K=\eta_K/E_K$ - see the model).

In the study of the behaviour of chlorite schist, Yang *et al.* (2017) introduced the visco-elastoplastic rheological mechanical model CVISC to describe its behaviour. This model has also been implemented in the FLAC 3D software. The model consists of the Burger's rheological component (K-M) and the Mohr–Coulomb friction component, as shown in Fig. 11, where:

c - cohesion,
ϕ - friction angle,
ψ - parameter of the Lacerda and Houston's relaxation model,
σt - tensile strength.

The volumetric behaviour is of elastoplastic nature only and is governed by the linear elastic law and the plastic flow rule (Fig. 11(a)), whereas the deviatoric behaviour is of visco-elastoplastic character and is driven by the Burgers' model and the same plastic flow rule (Fig. 11(b)). This means that the viscoelastic strains are deviatoric and depend only on the deviatoric stress. On the other hand, the plastic strains are both deviatoric and volumetric, and depend on the total stress in accordance with the chosen flow rule.

An example of analogous model that gives a reasonable approximation of the behaviour of some of the rock masses under certain loading conditions is shown in Fig. 12. That is the Cividini and Gioda's simplified model. The Cividini and Gioda's simplified model comprises a Kelvin's element (K) in series with a Bingham–Hooke element (B-H).

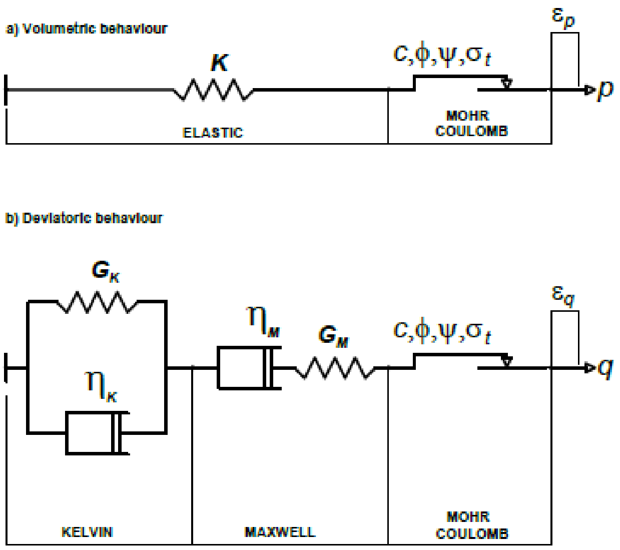

Fig. 11 Sketch of the CVISC model (Debernardi 2008)

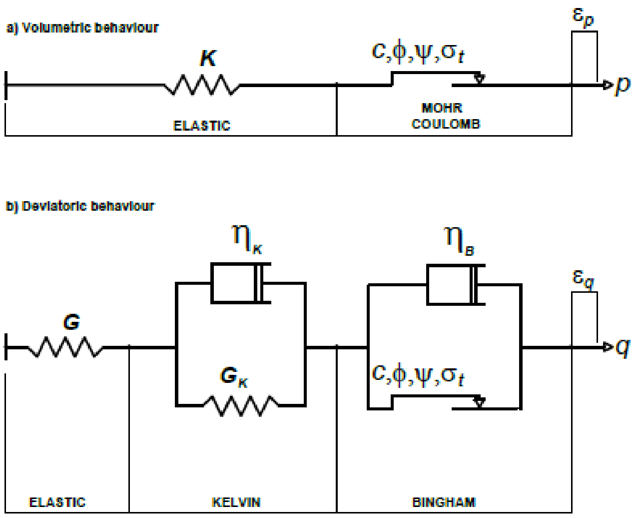

Fig. 12 Sketch of the Cividini and Gioda's simplified model (Debernardi 2008)

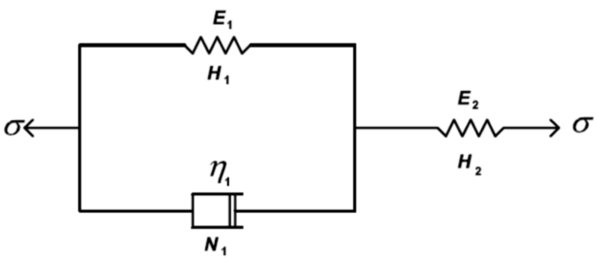

Fig. 13 Extended elasticity model (Wang *et al.* 2017)

The plastic yield criterion of the slider of the Bingham's element is defined according to the Mohr–Coulomb yield criterion. As for the model, the main assumption is to split the mechanical behaviour of the rock mass into a volumetric and a deviatoric component. Also, the model assumes that volumetric behaviour is of elastoplastic character only and is governed by the linear elastic law and the plastic flow rule, whereas the deviatoric behaviour is visco-elastoplastic.

This paper presents and analyses a complex rheological model, the so-called extended elasticity model, presented in Fig. 13. This rheological model includes, as the limiting cases, the aforementioned rheological models, the parallel and the serial connections of the Hooke's body (H_1 and H_2) and the Newton's body (N_1). The model describes phenomena that can occur in a variety of geotechnical problems in sufficiently accurately qualitative terms. At constant stress, strain increases up to a certain value, which is finite in a general case. It becomes infinite for the boundary transition $E_1 \rightarrow 0$, where E_1 stands for the modulus of elasticity of the parallel connection of the Hooke's body (H_1) (Fig. 13). The relaxation of stress at constant strain is partial in general case. For the boundary transition $E_1 \rightarrow 0$ the relaxation becomes total, whereas for the boundary transition $E_2 \rightarrow 0$ there is no relaxation, whereby E_2 designates the modulus of elasticity of the serial connection of the Hooke's body (H_2) (Fig. 13).

4. Load transmission under the influence of rheological processes

Based on the rheological model, primarily of the environment (rock mass), adequate design concept of tunnel construction technology must be considered (both during excavation and construction of lining structure of a tunnel, when it is required), so that correlated interaction of rheological properties of the structure and the environment, i.e., transmission of load, could be provided.

4.1 Load transmission - theoretical bases

By employing the model presented in Fig. 13, the load to the tunnel lining can be determined by taking into consideration the rheological properties of concrete and rock mass. The solution is presented for the circular ring (Fig. 14), where:

a - external tunnel radius;
d - thickness of the concrete lining;
p_0 - constant pressure in the undisturbed rock mass;
p_i - interactive pressure between the lining and the rock;
w - displacement;
ν - Poisson's coefficient;
E_{rm} - modulus of elasticity of rock mass;
E_c - modulus of elasticity of concrete.

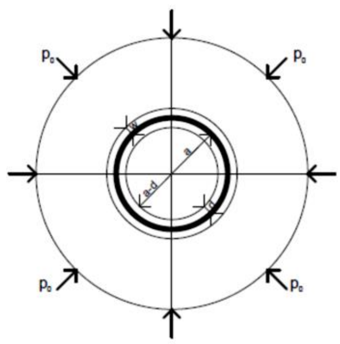

Fig. 14 Load to the tunnel structure accounting for the rheological properties of rock mass and concrete

For the constant pressure p_0 it is assumed that the Heim's hypothesis is valid and the tunnel is sufficiently deeply embedded.

Heim's hypothesis is based on the assumption that the structure is exposed to equal stresses in all directions. Heim developed this hypothesis during the excavation of alpine tunnels, where it was found that the hydrostatic state of stress exists in the rock mass, i.e. $\sigma_v=\sigma_h=p_0$, where σ_v is a vertical component of normal stress ($\sigma_v=\gamma_{rm}h$, γ_{rm} is unit weight of rock mass, h is depth of observation), σ_h is a horizontal component of normal stress. The lateral pressure coefficient for this hypothesis is defined as $\lambda=\sigma_h/\sigma_v=1.0$ (Selimović 2003).

For the rotationally symmetrical stress and strain state, the following is obtained:

$$w=\frac{a\,(1+\nu)}{E_{rm}}(p_0-p_i) \tag{1}$$

On the other hand, lining displacement is given as:

$$w=p_i\frac{a^2}{E_c d} \tag{2}$$

Eqs. (1) and (2) result in the interactive pressure:

$$p_i=p_0\frac{1}{1+\frac{a}{d}\frac{E_{rm}}{E_c}\frac{1}{1+\nu}} \tag{3}$$

When at time t the excavation is executed, at the location of the future contact of the tunnel lining with the rock mass there is an onset of elastic strain and displacement:

$$w_E=\frac{a(1+\nu)}{E_{rm}}p_0 \tag{4}$$

It is also assumed that ν = const with time. Based on this, it can be concluded that stress state in rock mass is invariable up to the moment the tunnel lining is installed.

Considering the rheological model (Fig. 13), for the constant stresses, it can be written:

$$\varepsilon=\sigma\left[\frac{1}{E_2}+\frac{1}{E_1}\left(1-e^{-t\frac{E_1}{\eta_1}}\right)\right] \tag{5}$$

$$\varepsilon_0=\frac{\sigma}{E_2}\qquad t=0;\quad E_2=E_{rm} \tag{6}$$

That is:

$$\varepsilon=\varepsilon_0\left[1+\frac{E_2}{E_1}\left(1-e^{-t\frac{E_1}{\eta_1}}\right)\right]$$
$$\varepsilon=\varepsilon_0\left[1+\frac{E_2}{E_1}\left(1-e^{-t\eta_{rm}}\right)\right] \tag{7}$$

where:

E_1 - modulus of elasticity of rock mass for the Hooke's body (H_1) (Fig. 13),

E_2 - modulus of elasticity of rock mass for the Hooke's body (H_2) (Fig. 13),

η_1 - viscosity coefficient of the Newton's body (N_1) (Fig. 13),

η_{rm} - coefficient of viscosity of rock mass,

ε - strain,

ε_0 - initial strain.

For displacement w the following is obtained:

$$w=w_E\left[1+\frac{E_2}{E_1}\left(1-e^{-t\eta_{rm}}\right)\right] \tag{8}$$

The radial displacement of the excavated contour, until the moment the concrete lining is activated, is the function of time in the following form:

$$w(t)=w_E\left[1+\varphi_{rm}\left(1-e^{-t\eta_{rm}}\right)\right] \tag{9}$$

where $\varphi_{rm}=E_2/E_1$ (for rock mass).

The concrete lining becomes activated at the moment $t=T$, and from that time on, pressure $p=p(t)$ starts to develop at the contact of concrete and rock mass. For $t=T$ the following is obtained:

$$w(T)=w_E\left[1+\varphi_{rm}\left(1-e^{-T\eta_{rm}}\right)\right] \tag{10}$$

Due to the gradual increase of pressure $p(t)$, displacement at the contact of concrete and rock mass starts to develop in the following way:

$$\begin{aligned} w(t)&=w_E\left[1+\varphi_{rm}\left(1-e^{-t\eta_{rm}}\right)\right]-\\ &-\frac{a(1+\nu)}{E_{rm}}\sum_{j=0}^{n}\Delta p_j\left[1+\varphi_{rm}\left(1-e^{-j\Delta t\eta_{rm}}\right)\right]=\\ &=w_E\left[1+\varphi_{rm}\left(1-e^{-t\eta_{rm}}\right)\right]-\\ &-\frac{a(1+\nu)}{E_{rm}}\int_T^t\left[1+\varphi_{rm}\left(1-e^{-(t-\tau)\eta_{rm}}\right)\right]dp=\\ &=w_E\left[1+\varphi_{rm}\left(1-e^{-t\eta_{rm}}\right)\right]-\\ &-\frac{a(1+\nu)}{E_{rm}}\int_T^t\left[1+\varphi_{rm}\left(1-e^{-(t-\tau)\eta_{rm}}\right)\right]\dot{p}\,d\tau \end{aligned} \tag{11}$$

$$\begin{aligned} w(t)-w(T)&=w_E\,\varphi_{rm}\left[e^{-T\eta_{rm}}-e^{-t\eta_{rm}}\right]\\ &-\frac{a(1+\nu)}{E_{rm}}\int_T^t\left[1+\varphi_{rm}\left(1-e^{-(t-\tau)\eta_{rm}}\right)\right]\dot{p}\,\mathrm{d}\tau \end{aligned} \tag{12}$$

Displacement of the concrete lining in radial direction is:

$$w_c(t)=\frac{a^2}{E_c d}\int_T^t\left[1+\varphi_c\left(1-e^{-(t-\tau)\eta_c}\right)\right]\dot{p}\,d\tau \tag{13}$$

where:

η_c - coefficient of viscosity of concrete;

$\varphi_c=E_2/E_1$ (for concrete).

By equating $w_c(t)=w(t)-w(T)$, the following is obtained:

$$w_E\ \varphi_{rm}\left[e^{-T\eta_{rm}}-e^{-t\eta_{rm}}\right]=\int_T^t\left\{\begin{array}{l}\frac{a(1+\nu)}{E_{rm}}\left[1+\varphi_{rm}\left(1-e^{-(t-\tau)\eta_{rm}}\right)\right]\\+\frac{a^2}{E_c d}\left[1+\varphi_c\left(1-e^{-(t-\tau)\eta_c}\right)\right]\end{array}\right\}\dot{p}\,\mathrm{d}\tau \quad (14)$$

The expression on the right side can be written as:

$$\int_T^t K(t,\tau)\mathrm{d}p = p(\tau)k(\tau,t)\Big|_T^t-\int_T^t\frac{\partial K(t,\tau)}{\partial\tau}p(\tau)\,\mathrm{d}\tau \quad (15)$$

The right side of Eq. (15) changes the form into:

$$p(t)K(t,t)-p(T)K(T,t)-\int_T^t\frac{\partial K(t,\tau)}{\partial\tau}p(\tau)\,\mathrm{d}\tau \quad (16)$$

where

$$K(t,t)=\text{const}=\frac{a(1+\nu)}{E_{rm}}+\frac{a^2}{E_c d}=K \quad (17)$$

$$p(T)=0$$

$$\frac{\partial K(t,\tau)}{\partial\tau}=-\frac{a(1+\nu)}{E_{rm}}\varphi_{rm}\eta_{rm}e^{-(t-\tau)\eta_{rm}}-\frac{a^2}{E_c d}e^{-(t-\tau)\eta_c} \quad (18)$$

The following designations are introduced:

$$P=\varphi_{rm}\eta_{rm}\frac{a(1+\nu)}{KE_{rm}};\quad Q=\varphi_c\eta_c\frac{a^2}{KE_c d}; \quad (19)$$

$$G(t,\tau)=Pe^{-(t-\tau)\eta_{rm}}+Qe^{-(t-\tau)\eta_c} \quad (20)$$

$$f(t)=\frac{w_E\ \varphi_{rm}}{K}\left[e^{-T\eta_{rm}}-e^{-t\eta_{rm}}\right] \quad (21)$$

Eq. (14), with the transformation in Eq. (15), changes the form as presented in Eq. (22), which represents the Volterra's integral equation of the second order:

$$f(t)=p(t)+\int_T^t G(t,\tau)p(\tau)\,\mathrm{d}\tau \quad (22)$$

By differentiating Eq. (22), the following is obtained:

$$\dot{f}(t)=\dot{p}(t)+G(t,t)p(t)+\int_T^t\frac{\partial G(t,\tau)}{\partial t}p(\tau)\,\mathrm{d}\tau \quad (23)$$

That is:

$$\begin{gathered}\frac{w_E\varphi_{rm}\eta_{rm}}{K}e^{-t\eta_{rm}}=\dot{p}(t)+Rp(t)-\eta\left[f(t)-p(t)\right]\\ \int_T^t G(t,\tau)p(\tau)\,\mathrm{d}\tau=f(t)-p(t)\end{gathered} \quad (24)$$

$$\dot{p}(t)+(R+\eta)p(t)-\eta\ f(t)-\frac{w_E\varphi_{rm}\eta_{rm}}{K}e^{-t\eta_{rm}}=0 \quad (25)$$

$$\begin{gathered}\dot{p}(t)+(R+\eta)p(t)-\frac{w_E\varphi_{rm}\eta_{rm}}{K}\left(e^{-t\eta_{rm}}-e^{-t\eta_{rm}}\right)\\-\frac{w_E\varphi_{rm}\eta_{rm}}{K}e^{-t\eta_{rm}}=0\end{gathered} \quad (26)$$

If $\eta_{rm}=\eta_c=\eta$ and $R=P+Q$:

$$\dot{p}(t)+(R+\eta)p(t)=\frac{w_E\varphi_{rm}\eta}{K}e^{-T\eta} \quad (27)$$

$$p_0(t)=\frac{w_E\varphi_{rm}\eta}{K(R+\eta)}e^{-T\eta} \quad (28)$$

By solving differential equation in Eq. (27):

$$p(t)=Ce^{-(R+\eta)t}+\frac{w_E\varphi_{rm}\eta}{K(R+\eta)}e^{-T\eta} \quad (29)$$

$$p(T)=0\quad\Rightarrow\quad C=-\frac{w_E\varphi_{rm}\eta}{K(R+\eta)}\frac{e^{-T\eta}}{e^{-(R+\eta)T}} \quad (30)$$

The final solution of the equation is obtained in the following form:

$$p(t)=\frac{w_E\varphi_{rm}\eta}{K(R+\eta)}e^{-T\eta}\left[1-e^{-(R+\eta)(t-T)}\right] \quad (31)$$

When $t\rightarrow\infty$:

$$p(\infty)=\frac{w_E\varphi_{rm}\eta}{K(R+\eta)}e^{-T\eta} \quad (32)$$

A more complex load condition of tunnel lining will be considered when the creep and shrinkage effects of concrete are also taken into consideration.

Relations of stress and strain, which vary over time t, are presented in the form of the Volterra's integral equations:

$$\varepsilon(t)=\frac{\sigma(t)}{E_c(t)}\int_{t_1}^t K(t,\tau)\frac{\sigma(\tau)}{E_c(\tau)}\,\mathrm{d}\tau+\varepsilon_S(t,t_1) \quad (33)$$

$$\begin{gathered}\frac{\sigma(t)}{E_c(t)}=\varepsilon(t)-\int_{t_1}^t(R(t,\tau)\varepsilon(\tau)-\varepsilon_S(t,\tau_1))\,\mathrm{d}\tau+\\+\int_{t_1}^t R(t,\tau)\varepsilon_S(t,\tau)\,\mathrm{d}\tau\end{gathered} \quad (34)$$

where:

$E_c(t)$ - modulus of elasticity of concrete,

$\varepsilon_s(t)$ - strain that occurred due to the shrinkage of concrete.

Kernel *K(t,τ)* of integral equation and its resolvent *R(t,τ)* depend primarily on the chosen rheological model. These relations are valid in the area of the linear theory of concrete creep, when stresses do not exceed 50-60% of the concrete strength.

For the purpose of simplification of calculation, it is most often considered that the modulus of elasticity of concrete $E_c(t)$ does not change over time. In that case:

$$K(t,\tau) = -\frac{\partial}{\partial \tau}\varphi(t,\tau) \quad \varphi(t,\tau) = E \times C(t,\tau) \tag{35}$$

where:

$\varphi(t, \tau)$ - coefficient of concrete creep,
$C(t, \tau)$ - specific creep,
E - modulus of elasticity of concrete considering shrinkage and creep effects.

In order to facilitate consideration of the impact of the creep and shrinkage of concrete in the design of structures in practice, various algebraic proposals were made with an aim to simplify these relations.

Fritz (1984) presents the relations between stresses and strain in the following form:

$$\varepsilon(t) = \frac{\sigma(t)}{E_\varphi(t)} \quad ; \quad E_\varphi^c(t) = \frac{E}{1+\varphi(t,t_1)} \tag{36}$$

where:

$E_\varphi(t)$ - ideal modulus of elasticity,
$E_\varphi^c(t)$ - ideal modulus of elasticity of concrete.

If this relation is employed in Eqs. (17) and (19), the following is obtained:

$$K(t,t_1) = \frac{a(1+\nu)}{E_{rm}} + \frac{a^2}{E_\varphi^c(t)d} = \frac{a(1+\nu)}{E_{rm}} + \frac{a^2}{\frac{E\times d}{1+\varphi(t,t_1)}} \tag{37}$$

$$Q(t,t_1) = \varphi_c\eta_c \frac{a^2}{K(t,t_1)\frac{E}{1+\varphi(t,t_1)}d} \tag{38}$$

Then:

$$R(t,t_1) = P(t,t_1) + Q(t,t_1) = \varphi_{rm}\eta_{rm}\frac{a(1+\nu)}{K(t,t_1)E_{rm}} + \\ + \frac{\varphi_c\eta_c a^2}{\frac{K(t,t_1)E}{1+\varphi(t,t_1)}d} \tag{39}$$

The concrete creep coefficient $\varphi(t,t_1)$ consists of the return segments $\varphi_r(t,t_1)$ and the non-return segments $\varphi_i(t,t_1)$:

$$\varphi(t,t_1) = \varphi_r(t,t_1) + \varphi_i(t,t_1) \\ \varphi_i(t,t_1) = \bar{\varphi}\,[\,f_2(t) - f_1(t_1)] \tag{40}$$

Function $\bar{\varphi}_i(t,t_1)$ depends on the theoretical thickness of concrete element and the relative humidity of the environment in which it is situated. These functions are provided in graphical and numerical terms (Ivković 1965, Praščević 1973).

In case when $\eta_{rm} \equiv \eta_c \approx \eta$, the load to the tunnel lining in function of time t and time of activation of concrete lining T is:

$$p(t) = \frac{w_E\varphi_{rm}\eta}{K(t,t_1)[R(t,t_1)+\eta]}e^{-T\eta}\left[1 - e^{-[R(t,t_1)+\eta](t-T)}\right] \tag{41}$$

When $t \to \infty$:

$$p(\infty) = \frac{w_E\varphi_{rm}\eta}{K(t,t_1)[R(t,t_1)+\eta]}e^{-T\eta} \tag{42}$$

Eqs. (41) and (42) are of significance from the theoretical aspect and can be applied when all the properties of the rock mass and concrete structure are known. The calculation is very complex, so this paper analyses the pressure ratio in a simpler way, using Eqs. (31)-(32). This approach has also been used in the numerical example in the subsequent part of the paper.

On the basis of the rheological model, presented in Fig. 13, and derived Eqs. (31)-(32), the modulus of elasticity of rock mass depending on time $E_{rm}=E_{rm}(t)$ and ratio $p(\infty)/p$ are obtained (Fig. 15).

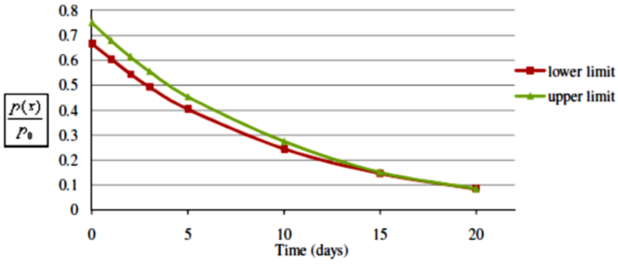

Fig. 15 Distribution of pressures over time

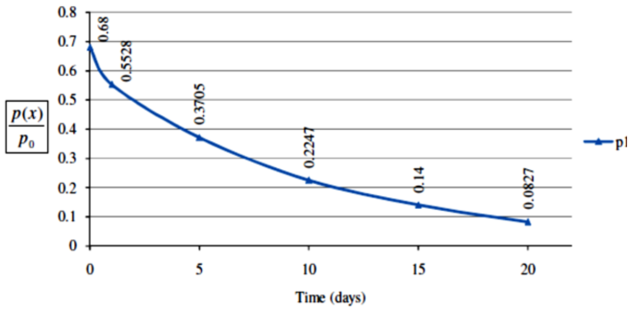

Fig. 16 Pressure diagram as a function of time of activation of concrete lining *T*

The diagram depicted in Fig. 15 is given for the Belgrade marls, with limiting values φ_{rm}=2 (lower limit) and φ_{rm}=3 (upper limit).

4.2 Numerical example

A tunnel structure, with a radius a=3.60 m and a lining thickness d=0.25 m, is constructed in the grey marl stone having the properties E_{rm}=785.00 MPa, ν_{rm}=0.35, γ_{rm}=20 kN/m^3, φ_{rm}=2, and η_{rm}=0.1. The tunnel is built at a depth of h=35 m, with the following characteristics of the concrete lining E_c=31.5 GPa, ν_c=0.1, and φ_c=2.2 (concrete of a C25/30 class).

The dependence of the final pressure at the contact of rock mass and tunnel structure on the time T is presented in the following diagram (Fig. 16).

By using Eq. (3) for interactive pressure and by calculating $p(\infty)$ for time intervals T, the relation for the rock mass modulus is obtained in the following form:

$$E_{rm} = \frac{d}{a} E_c (1+\nu_{rm}) \left[\frac{p_0}{p(\infty)} - 1 \right] \tag{43}$$

That is, when the effects of concrete creep and shrinkage are taken into consideration:

$$E_{rm} = \frac{d}{a} \frac{E(1+\nu_{rm})}{1+\varphi(t,t_1)} \left[\frac{p_0}{p(\infty)} - 1 \right] \tag{44}$$

From these expressions, time dependence of the modulus of elasticity of rock mass can be observed.

Depending on the technological process of construction, it is necessary to complete the concrete lining and to activate its interaction with the surrounding rock mass within the time T. For that reason, in the design of tunnel structures, it is necessary to determine the time difference from the moment the excavation starts until the moment the concrete lining is installed. Using this time cycle and a rheological model, the elasticity modulus of rock mass $E_{rm}=E(t-T)$ is determined.

On the basis of this model, it can be concluded that for the design of tunnel structures, it is necessary to determine E_{rm} and to calculate static impact with the value obtained in this way. The assumption of the mutual dependence of the static structural system, technological construction process, and basic properties and characteristics of rock masses is confirmed by the previous statements.

5. Conclusions

Implementation of rheological models and determination of rock mass load transmission to the tunnel structure over time makes it possible to determine the relation of stress and strain in the rock mass and tunnel structure (Manojlović 1987). For the analysis of mutual relation of technological construction process, static structural system, and geotechnical parameters, it is necessary to determine development of deformation processes in the rock mass over time using the chosen rheological model prior to the calculation. By this, the time component of transmission of the load to the tunnel structure is defined. The most important thing is that, when using the rheological models, actually measured mechanical characteristics can be entered: moduli of elasticity and strain, uniaxial compressive strength, functional relations of stress, strain, and strain rates obtained from the triaxial test, as well as strength parameters obtained from the shear test (friction angle and cohesion).

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