Geomechanics and Engineering, *Vol. 21, No. 1 (2020) 1-9*
DOI: https://doi.org/10.12989/gae.2020.21.1.001

Effect of the rotation on the thermal stress wave propagation in non-homogeneous viscoelastic body

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(Received January 23, 2020, Revised February 25, 2020, Accepted March 1, 2020)

Abstract. In this article, an analytical solution for the effect of the rotation on thermo-viscoelastic non-homogeneous medium with a spherical cavity subjected to periodic loading is studied. The distribution of displacements, temperature, redial stress, and hoop stress in non-homogeneous medium, in the context of generalized thermo-viscoelasticity using the GL theory, is discussed and obtained. The results are displayed graphically to illustrate the effect of the rotation. Comparisons with the previous work in the absence of rotation and viscosity are made.

Keywords: rotation; relaxation times; viscoelasticity; non-homogeneous; thermoelasticity

1. Introduction

Many materials exhibit some viscoelastic response, in common metals such as steel, aluminum, copper, etc. At room temperature and small strain, the behavior does not deviate much from linear elasticity. Viscoelastic materials are those for which the relationship between stress and strain depends on time. Synthetic polymer, wood as well as metals at the high temperature display significant viscoelastic effects. With the rapid development of polymer science and the plastic industry as well as the wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic materials have become an important task for solid mechanics. In recent years the theory of magneto-thermo-elasticity dealing with mechanics aspects of advanced materials and structures, as described in the Refs. (Miara *et al.* 2007, Ezzat *et al.* 2012, Akbarzadeh and Chen 2014, Batou *et al.* 2019, Alimirzaei *et al.* 2019, Karami et 2019a). The interactions among strain, temperature and electromagnetic fields has drawn the attention of many researchers because of its extensive uses in divers fields, such as geophysics for understanding the effect of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, development of a highly sensitive superconducting magnetometer, electrical power engineering, optics, etc.

Mahmoud *et al.* (2011a, b) and Abd-Alla *et al.* (2013) investigated the effect of the rotation on plane vibrations in a transversely isotropic infinite hollow cylinder, effect of the rotation on wave motion through a cylindrical bore in a micropolar porous cubic crystal and the effect of the magnetic field and non-homogeneity on the radial vibrations in the hollow, rotating elastic cylinder. Abd-Alla *et al.* (2011a and 2013) and Abd-Alla and Mahmoud (2010a, b) investigated effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material, influences of rotation, radial vibrations in a non-homogeneous orthotropic elastic hollow sphere subjected to rotation, magneto-thermo-elastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model and they studied effect of the rotation on propagation of thermoelastic waves in a non-homogeneous infinite cylinder of isotropic material. Fahmy (2011) presented a time-stepping dual reciprocity boundary element method (DRBEM) for magneto-thermo-viscoelastic interactions in a rotating nonhomogeneous anisotropic solid. Using DRBEM, Fahmy (2012a) studied also the transient magneto-thermo-visco-elastic stresses in a non-homogeneous anisotropic solid placed in a constant primary magnetic field acting in the direction of the z-axis and rotating about it with a constant angular velocity. Mahmoud (2012) studied influence of rotation and generalized magneto-thermo-elastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. Abd-Alla and Mahmoud (2012) presented an analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media. Fahmy (2012b) employed DRBEM to investigate the transient magneto-thermoviscoelastic plane waves in a non-homogeneous anisotropic thick strip subjected to a moving heat source Abd-Alla *et al.* (2011b, c and 2012) investigated some problems as the propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under the influence of gravity field, the generalized magneto-thermoelastic Rayleigh waves in a granular medium under the effect of the gravity

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ISSN: 2005-307X (Print), 2092-6219 (Online)

field and initial stress also investigated the problem of transient coupled thermoelasticity of an annular fin. Fahmy (2012c) examined the influence of rotation and inhomogeneity on the transient magneto-thermoviscoelastic stresses in an anisotropic solid. Fahmy (2012c) discussed the transient magneto-thermo-elastic stresses in an anisotropic viscoelastic solid with and without moving heat source. The BEM is also used by Fahmy (2013a, b, c, 2014, 2018a, b, 2019a, b, 2020) to investigated different problems. Other formulations with advanced theories are also employed (Balubaid *et al.* 2019, Zaoui *et al.* 2019, Zarga *et al.* 2019, Chaabane *et al.* 2019, Boutaleb *et al.* 2019, Boukhlif *et al.* 2019, Khiloun *et al.* 2019, Abualnour *et al.* 2019, Mahmoudi et el. 2019, Medani *et al.* 2019, Hellal *et al.* 2019, Salah *et al.* 2019, Meksi *et al.* 2019, Belbachir *et al.* 2019, Addou *et al.* 2019, Bousahla *et al.* 2020, Kaddari *et al.* 2020, Tounsi *et al.* 2020, Boussoula *et al.* 2020). Othman and Fekry (2018) investigated the effect of rotation and gravity on generalized thermo-viscoelastic medium with voids. The effects of thermoelastic interactions in a rotating infinite orthotropic elastic body with a cylindrical hole and variable thermal conductivity are investigated by Mashat *et al.* (2017). The generalized fractional magneto-thermo-viscoelasticity has been investigated by Ezzat and El-Bary (2017) and Othman and Hilal (2017). Praveen Ailawalia *et al.* (2014) studied the dynamic problem in Green-Naghdi (type III) thermoelastic half-space with two temperatures.

In this paper, initial stress, rotation, and the thermo-elastic equation of the spherical cavity are decomposed into the non-homogeneous equation with boundary conditions. The effect of thermal relaxation times on the wave propagation in thermos-viscoelastic using the GL theory will be discussed. We took the material of the spherical cavity to be of Kelvin Voigt type. Thus, the exact expressions for the transient response of displacement, stresses, and temperature in the spherical cavity are obtained. The numerical calculations will be investigated for the displacement, temperature, and the components of stresses and explain the special case from this study when the rotation and non-homogeneity are neglected. Finally, numerical results are calculated and discussed.

2. Formulation of the problem

We shall Consider spherical coordinates of any represents point be $(r, 0, \phi)$ and assuming that spherical cavity is subjected to a rapid change in temperature$T(r,t)$, for the axisymmetric plane strain problem, the components of displacement $\bar{u} = \bar{u}(u_r, u_\theta, u_\phi)$ are expressed as $u_\theta = u_\phi = 0$, and $u_r = u_r(r,t)$. Let us consider an infinite non-homogeneous viscoelastic solid, and the viscoelastic nature of the material is described by the Voigt type of linear viscoelasticity. The medium is assumed to have a spherical cavity of radius a, for a spherically symmetric system the no-vanishing stresses components, is expressed as:

$$\sigma_{rr} = \tau_m(\lambda + 2\mu + P)\frac{\partial u_r}{\partial r} + (2\lambda + P)\tau_m\frac{u_r}{r} - \gamma(T + \tau_2\dot{T}) \quad (1)$$

$$\sigma_{\theta\theta} = 2\tau_m(\lambda + \mu + P)\frac{u_r}{r} + (\lambda + P)\tau_m\frac{\partial u_r}{\partial r} - \gamma(T + \tau_2\dot{T}),$$

$$\tau_{\varphi\varphi} = 2\tau_m(\lambda + \mu)\frac{u_r}{r} + \lambda\tau_m\frac{\partial u_r}{\partial r} - \gamma(T + \tau_2\dot{T}), \quad (1)$$

$$\sigma_{r\varphi} = \sigma_{r\theta} = \sigma_{\theta\varphi} = 0$$

where σ_{rr} and $\sigma_{\theta\theta}$ are radial and hoop stresses, respectively. $\tau_m = \left(1 + \tau_0\frac{\partial}{\partial t}\right)$ and τ_0 is the mechanical relaxation time due to the viscosity. The magneto-elastodynamic equation of the non-homogeneity spherical if $u_r = u_r(r,t)$, becomes:

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{2}{r}\sigma_{rr} - \frac{1}{r}\sigma_{\theta\theta} - \frac{1}{r}\sigma_{\varphi\varphi} = \rho\frac{\partial^2 u_r}{\partial t^2} - \rho\Omega^2 u_r, \quad (2)$$

where $\bar{u}$ is the displacement vector. The heat conduction equation is

$$L\left(\frac{\partial^2\theta}{\partial r^2} + \frac{2}{r}\frac{\partial\theta}{\partial r}\right) = \rho c_v\left(\frac{\partial\theta}{\partial t} + \tau_1\frac{\partial^2\theta}{\partial t^2}\right) + \gamma T_0\left[\frac{\partial}{\partial r} + \frac{2}{r}\right]\dot{u}_r. \quad (3)$$

where L is the thermal conductivity, $\gamma = \alpha_t(3\lambda + 2\mu)$, Ω is the rotation,ρ is the density of the material, c_v is the specific heat of the material per unit mass, τ_1, τ_2 are thermal relaxation parameter, α_t is the coefficient of linear thermal expansion, λ, μ are Lame elastic constants, T_1 is the absolute temperature, T_0 is reference temperature solid, θ is a temperature difference $(T_1 - T_0)$, ρ, P and μ_e are mass density, pressure, and magnetic permeability coefficient of non-homogeneous material, respectively. From equations (1) to (3) we rewrite the

$$\sigma_{rr} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)(\lambda + 2\mu + P)\frac{\partial u_r}{\partial r} + 2(\lambda + P)\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{u_r}{r} - \gamma(\theta + \tau_2\dot{\theta}),$$

$$\sigma_{\theta\theta} = 2\left(1 + \tau_0\frac{\partial}{\partial t}\right)(\lambda + \mu + P)\frac{u_r}{r} + (\lambda + P)\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial u_r}{\partial r} - \gamma(\theta + \tau_2\dot{\theta}), \quad (4)$$

$$\sigma_{\varphi\varphi} = 2\left(1 + \tau_0\frac{\partial}{\partial t}\right)(\lambda + \mu)\frac{u_r}{r} + \lambda\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial u_r}{\partial r} - \gamma(\theta + \tau_2\dot{\theta}),\ \sigma_{r\varphi} = \sigma_{r\theta} = \sigma_{\theta\varphi} = 0.$$

From Eqs. (1) and (2), we have

$$\left[\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right]\frac{\partial^2 u_r}{\partial r^2} + \left[2\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right]\frac{1}{r}\frac{\partial u_r}{\partial r} + \left[\frac{4\lambda\left(1 + \tau_0\frac{\partial}{\partial t}\right)}{(\lambda + 2\mu + P)} - 2\left(1 + \tau_0\frac{\partial}{\partial t}\right) - \right]\frac{u_r}{r^2} - \left[\frac{2}{r} + \frac{\partial}{\partial r}\right] \quad (5)$$

$$\frac{\gamma}{(\lambda + 2\mu + P)}(\theta + \tau_2\dot{\theta}) + \rho\Omega^2 u = \frac{\rho}{(\lambda + 2\mu + P)}\frac{\partial^2 u}{\partial t^2}.$$

Let $c_0 = \frac{\lambda}{(\lambda+2\mu+P)}, c_2 = \frac{\gamma}{(\lambda+2\mu+P)}, c_v = \sqrt{\frac{(\lambda+2\mu+P)}{\rho}}$

Then the visco-elastodynamic Eq. (5) becomes:

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial^2 u_r}{\partial r^2} + \left[2(n+1)\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right]\frac{1}{r}\frac{\partial u_r}{\partial r} + \left[4nc_0\left(1 + \tau_0\frac{\partial}{\partial t}\right) - 2\left(1 + \tau_0\frac{\partial}{\partial t}\right)\right]\frac{u_r}{r^2} - c_2\left[\frac{2n}{r} + \frac{\partial}{\partial r}\right](\theta + \tau_2\dot{\theta}) + \rho_0\Omega^2 u_r = \frac{1}{c_v^2}\frac{\partial^2 u_r}{\partial t^2} \quad (6)$$

We now use the following dimensionless quantities are taken as:

$$U = \frac{u_r}{a},\quad l = \frac{L}{\rho_0 c_v},\quad t' = \frac{kc_v}{a}t,\quad T = \frac{\theta}{T_0}, \tau'_0 = \frac{c_v}{a}\tau_0,$$
$$\tau'_1 = \frac{c_v}{a}\tau_1, \tau'_2 = \frac{kc_v}{a}\tau_2,\quad r = \frac{r}{a},\quad \Omega^* = \frac{\Omega}{a} \tag{7}$$
$$\tau_{rr} = \frac{\sigma_{rr}}{(\lambda+2\mu+P)},\quad \tau_{\theta\theta} = \frac{\sigma_{\theta\theta}}{(\lambda+2\mu+P)}$$

The normal stresses relations can be right in the non-dimensional forms as:

$$U = \frac{u_r}{a},\quad l = \frac{L}{\rho_0 c_v},\quad t' = \frac{kc_v}{a}t,\quad T = \frac{\theta}{T_0}, \tau'_0 = \frac{c_v}{a}\tau_0,$$
$$\tau'_1 = \frac{c_v}{a}\tau_1, \tau'_2 = \frac{kc_v}{a}\tau_2,\quad r = \frac{r}{a},\quad \Omega^* = \frac{\Omega}{a} \tag{8}$$
$$\tau_{rr} = \frac{\sigma_{rr}}{(\lambda+2\mu+P)},\quad \tau_{\theta\theta} = \frac{\sigma_{\theta\theta}}{(\lambda+2\mu+P)}$$

Substituting of Eq. (8) into Eq. (6) gives the displacement equation in the non-dimensional form of the non-homogeneous spherical as follows:

$$\left[\left(1+\tau'_0\frac{\partial}{\partial t'}\right)\right]\frac{\partial^2 U}{\partial r^2} + \left[2(n+1)\left(1+\tau'_0\frac{\partial}{\partial t'}\right)\right]\frac{1}{r}\frac{\partial U}{\partial r}$$
$$+\left[(4nc_1-2)\left(1+\tau'_0\frac{\partial}{\partial t'}\right)\right]\frac{U}{r^2} - c_2T_0\left[1+\tau'_2\frac{\partial}{\partial t'}\right]\left(\frac{2l}{r}+\frac{1}{c_L^2}\frac{\partial}{\partial r}\right)T \tag{9}$$
$$+\rho_0 a c_1^2 \Omega^{*2} U = l^2\frac{\partial^2 U}{\partial t'^2},$$

The heat conduction equation in the non-dimensional forms is

$$\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) = l_1\left(1+\tau'_1\frac{\partial}{\partial t'}\right)\frac{\partial T}{\partial t'} + l_2\left[\frac{\partial}{\partial r}+\frac{2}{r}\right]\frac{\partial U}{\partial t'}, \tag{10}$$

where $l_1 = \frac{ac_v}{l}$, $l_2 = \frac{a\gamma_0}{\rho_0}$

3. The problem solution

We seek the general solution to the basic equations (9,10) of magneto-thermo-elastic motion as harmonic vibration in the form:

$$U(r,t') = U^*(r)e^{i\omega t'},\quad T(r,t') = T^*(r)e^{i\omega t'} \tag{11}$$

The equation of motion in Eq. (9) becomes in the form

$$\frac{d^2U^*}{dr^2} + \eta_1\frac{1}{r}\frac{dU^*}{dr} + \eta_2\frac{U^*}{r^2} + \rho_0 a c_1^2\Omega^{*2}U^* = -m_1^2U^* + \varepsilon\left(\frac{2n}{r}+\frac{d}{dr}\right)T^*, \tag{12}$$

where

$$\gamma' = (1+i\tau'_2\omega), \eta_1 = \frac{(2n+1)(1+i\omega\tau'_0)}{(1+i\omega\tau'_0)} + 1,$$
$$\eta_2 = \frac{(4nc_1-1)(1+i\omega\tau'_0)}{(1+i\omega\tau'_0)} - 1, \varepsilon = \frac{c_2T_0\gamma'}{(1+i\omega\tau'_0)},$$
$$m_1^2 = \frac{k^2\omega^2}{(1+i\omega\tau'_0)},\qquad c_1 = \sqrt{\frac{\lambda+2\mu+P}{\rho_0}}.$$

Also, the heat conduction Eq. (12) becomes in the form

$$(\nabla^2+\beta_1)T^* = \beta_2\left[\frac{d}{dr}+\frac{2}{r}\right]U^*, \tag{13}$$

where $\nabla^2 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}$, $\beta_1 = l_1(\omega^2\tau'_1 - i\omega)$, $\beta_2 = i\omega\ l_2$,

To solve Eqs. (12) and (13), let

$$U^*(r) = \frac{d\xi(r)}{dr}, \tag{14}$$

$$\frac{d}{dr}\left[\frac{d^2\xi(r)}{dr^2} + \frac{\eta_1}{r}\frac{d\xi(r)}{dr} + \frac{\eta_2}{r^2}\xi(r)\right] + f_1\Omega^{*2}\frac{d\xi(r)}{dr} = -m_1^2\frac{d\xi(r)}{dr} + \varepsilon\left(\frac{2n}{r}+\frac{d}{dr}\right)T^*, \tag{15}$$

By comparing the coefficient of $\frac{d}{dr}$ in Eq. (14), we get

$$\frac{d^2\xi(r)}{dr^2} + \frac{\eta_1}{r}\frac{d\xi(r)}{dr} + \left[\frac{\eta_2}{r^2} + m_1^2\right]\xi(r) = \varepsilon T^* \tag{16}$$

The heat conduction Eq. (13) becomes in the form

$$(\nabla^2+\beta_1)T^* = \beta_2\nabla^2\xi(r). \tag{17}$$

From Eqs. (14), (15) and (16), we have

$$\frac{d^4\xi(r)}{dr^4} + [\eta_1+1]\frac{1}{r}\frac{d^3\xi(r)}{dr^3} + \left[\Gamma_1 + \frac{\eta_2-\eta_1}{r^2}\right]\frac{d^2\xi(r)}{dr^2}$$
$$+\left[\frac{\eta_2+\eta_1}{r^3} + \frac{\Gamma_2}{r}\right]\frac{d\xi(r)}{dr} + \beta_1N^2\xi + f_1\Omega^{*2}\frac{d\xi(r)}{dr} = 0 \tag{18}$$

where $\Gamma_1 = m_1^2 + \beta_1 - \varepsilon\beta_2$, $\Gamma_2 = m_1^2 + \beta_1\eta_1 - \varepsilon\beta_2$.

Decoupling Eqs. (17) and (18), we obtain:

$$(\nabla^2+\chi_1{}^2)(\nabla^2+\chi_2{}^2)(\xi) = 0, \tag{19}$$

$$(\nabla^2+\chi_1{}^2)(\nabla^2+\chi_2{}^2)(T^*) = 0, \tag{20}$$

where $\beta_4 = \frac{\beta_1}{l}$, χ_1^2 and χ_2^2 are the roots with positive real parts of the biquadratic Eqs. (19) and (20) in the form:

$$\chi^4 + (m_1{}^2 + \beta_4{}^2 - \eta_1\eta_2)\chi^2 + m_1{}^2\beta_4{}^2 = 0. \tag{21}$$

Assuming the regularity conditions for ξ and T^*, in Eqs. (19) and (20) we can determine $\xi(r)$, and from this, we can determine T^*, in the form

$$\xi = D_1h_0{}^{(2)}(\chi_1r) + D_2h_0{}^{(2)}(\chi_2r), \tag{22}$$

$$T^* = D_1h_0{}^{(2)}(\chi_1r) + D_2h_0{}^{(2)}(\chi_2r), \tag{23}$$

The solutions (21) and (22)are obtained in terms of spherical Hankel's function.

WhereD_1 and D_2 are arbitrary constants and $h_0{}^{(2)}$ is the Hankel's function of its order zero and second kind.

From Eqs. (11), (14), (22) and (23) one can determine the stress-strain relations, the solution for the displacement, temperature, and the radial and hoop stresses are found to have the forms:

$$U = \{A_1h_1{}^{(2)}(\chi_1r) + A_2h_1{}^{(2)}(\chi_2r)\}e^{i\omega t'}, \tag{24}$$

$$T = \{D_1h_0{}^{(2)}(\chi_1r) + D_2h_0{}^{(2)}(\chi_2r)\}e^{i\omega t'}, \tag{25}$$

$$\tau_{rr} = \left\{L_1h_0{}^{(2)}(\chi_1r) + \frac{L_2}{r}h_1{}^{(2)}(\chi_1r)\right\}A_1e^{i\omega t'} + \left\{L_3h_0{}^{(2)}(\chi_2r) + \frac{L_2}{r}h_1{}^{(2)}(\chi_2r)\right\}A_2e^{i\omega t'}, \tag{26}$$

$$\tau_{\theta\theta} = \left\{L_4 h_0{}^{(2)}(\chi_1 r) + \frac{L_5}{r} h_1{}^{(2)}(\chi_1 r)\right\} A_1 e^{i\omega t'} + + \left\{L_6 h_0{}^{(2)}(\chi_2 r) + \frac{L_5}{r} h_1{}^{(2)}(\chi_2 r)\right\} A_2 e^{i\omega t'}, \quad (27)$$

4. Boundary conditions

The boundary of the cavity is assumed to be subjected to the magnetic field and a periodic loading of frequency *r=a*

$$u(r,t) = 0, \qquad r = a \quad (28a)$$

$$\tau_{rr} = -\sigma_0 e^{i\omega t}, \quad r = a \quad (28b)$$

where σ_0 is a constant, represent the periodic loading. Also, it is assumed that there is no temperature change on the boundary of thecavity, which implies that the surface temperature of the cavity constantly maintainsthe reference temperature, T_0, that is,

$$T = 0, \qquad \frac{\partial T}{\partial r} + T = 0, \qquad r = a \quad (28c)$$

Also, the tangential component of the electric field is assumed to be continuous across the boundary of the cavity.

We get the arbitrary constants of the solution of the current problem as:

$$z_i = \frac{\chi_i{}^2 - m_1{}^2}{t'\chi_i}, \; D_i = z_i A_i, \qquad i = 1,2.$$

$$A_1 = -\frac{\sigma'_0 \quad h_0 \quad {}^{(2)} \; (\chi_2 \;)}{d_1},$$

$$A_2 = \frac{\sigma'_0 \quad z_1 h_0 \quad {}^{(2)} \; (\chi_1 \;)}{d_1}, \sigma_0{}' = \frac{\sigma_0}{\gamma T_0}.$$

$$d_1 = z_2 h_0{}^{(2)}(\chi_2)\{L_1 h_0{}^{(2)}(\chi_1) + L_2 h_1{}^{(2)}(\chi_1)\} - z_1 h_0{}^{(2)}(\chi_1)\{L_3 h_0{}^{(2)}(\chi_2) + L_2 h_1{}^{(2)}(\chi_2)\},$$

$$L_1 = (1 + i\omega t_0{}' + r_H{}^2)\chi_1 - (1 + i\omega t_2{}')z_1, \quad (29)$$

$$L_2 = (1 + i\omega t_0{}' + r_H{}^2)(2\lambda_e - 2),$$

$$L_3 = (1 + i\omega t_0{}' + r_H{}^2)\chi_2 - (1 + i\omega t_2{}')z_2,$$

$$L_4 = (1 + i\omega t_0{}' + r_H{}^2)\chi_1 - (1 + i\omega t_2{}')z_1,$$

$$L_5 = (1 + i\omega t_0{}' + r_H{}^2)(1 - \lambda_e),$$

$$L_6 = \lambda_e(1 + i\omega t_0{}' + r_H{}^2)\chi_2 - (1 + i\omega t_2{}')z_2$$

$$\lambda_e = \frac{\lambda}{\lambda + 2\mu + P}$$

This is the solution of the current problem for the case of the non-homogeneous isotropic viscoelastic unbounded body with a spherical cavity with the effect of the rotation, that coincides with previously published.

5. Discussion and numerical results

The results presented in this paper should prove useful for researchers in material science, designers of new materials, low-temperature physicists as well as for those working on the development of a theory magneto-thermo-visco-elastic.

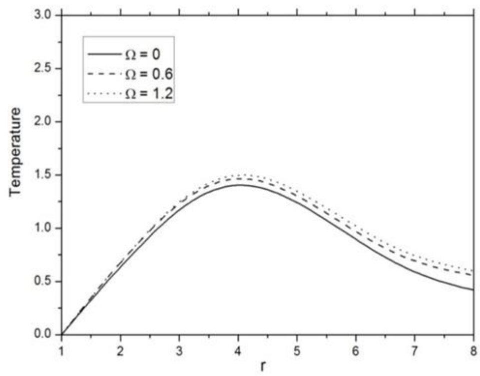

Fig. 1 Variation of temperature versus the radius r at varies values of rotation when $\tau_1 = 0.5, \tau_0 = 0.4, w = 2x10^3$, $P = 1.5$

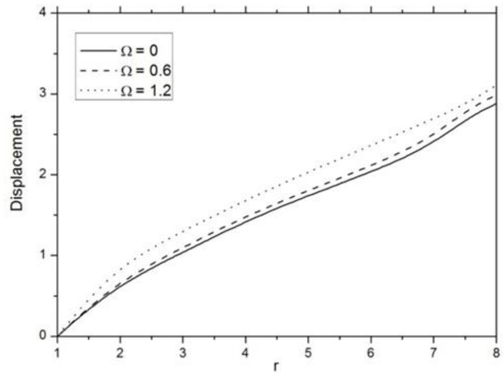

Fig. 2 Variation of radial displacement versus the radius r at varies values of rotation when $\tau_1 = 0.5, \tau_0 = 0.4$, $w = 2x10^3$, $P = 1.5$

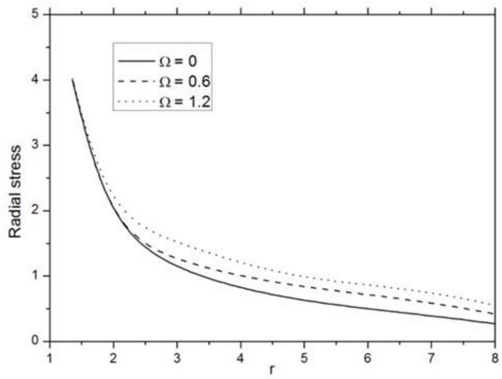

Fig. 3 Variation of radial stress versus the radius r at varies values of rotation when $\tau_1 = 0.5, \tau_0 = 0.4, w = 2x10^3$, $P = 1.5$

The copper material was used chosen for purposes of numerical evaluations. The constants of the problem are given by Kumar *et al.* (2016). The numerical technique outlined above was used to obtain the temperature, radial displacement, radial stress, and hope stress inside the sphere. These distributions are shown in the figures. (1-2), respectively. Important phenomena are observed in all these computations (Othman and Song 2008, Ezzat and Atef 2011): It was found that for large values of time, the coupled and the generalized give close results. The case is

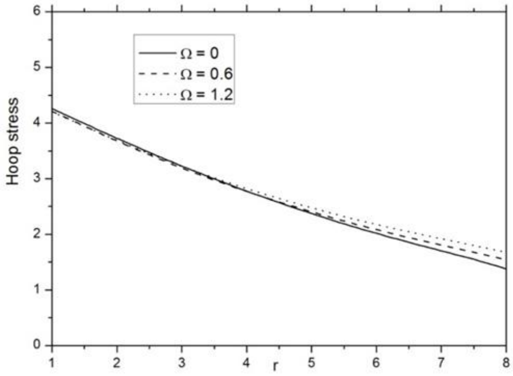

Fig. 4 Variation of hoop stress versus the radius rat varies values of rotation when $\tau_1 = 0.5, \tau_0 = 0.4, w = 2x10^3$

quite different when we consider the small value of time. The coupled theory predicts infinite speeds of wave propagation. This is evident from the fact that the obtained solutions are not identically zero for any values of time but fade gradually very small values at points for removed from the surface. The solutions obtained in the context of GL theory, however, exhibit the behavior of finite speeds of wave propagation. The computational were carried out for the value of thermal relaxation time, namely $\tau_1 = 0.5$, mechanical relaxation time, namely $\tau_0 = 0.4$, and the frequency, namely, $\omega = 2x10^2$. For the sake of brevity, some computational results are not being presented here. Figs. 1-4 show the solution corresponding to the use of the generalized visco-thermoelastic and non-homogeneous medium subjected to initial stress, the rotation, thermal relaxation times. Fig. 1 shows the temperature distribution, represents the solution corresponding to the use of the effect of the rotation.

Fig. 2 shows the radial displacement in generalized visco-thermoelastic and non-homogeneous medium subjected to initial stress, the rotation, thermal relaxation times. Both Figs. 1 and 2 indicate that the medium along the radius r undergoes expansion deformation because of these effects. The radial displacement and temperature increase with increasing initial stress, and it increases with increasing the radius r.

Figs. 3 and 4 show the radial stress and hoop stress in generalized visco-thermoelastic and non-homogeneous medium subjected to initial stress, the rotation, thermal relaxation times. In the figure (3) the radial stress decreases with increasing the radius r and initial stress, but it decreases with increasing rotation when the small value of the radius r less than 2.

Fig. 4 shows the hoop stress in generalized visco-thermoelastic and non-homogeneous medium subjected to initial stress, the rotation, thermal relaxation times. From both figures, the redial stress and hoop stress decreases with increasing the radius r where the figures (3,4) represent the solution of redial stress and hoop stress corresponding to the use of the effect of initial stress and effect of rotation, it was found that near the surface cavity where the boundary conditions domain the Coupled and the generalized theories give very close results. Inside the sphere, the solution is markedly different. This is because thermal waves in the coupled theory travel are not identically zero (though it may be very small) for any small of time. By comparing with results in Refs. (Othman and Song 2008, Ezzat and Atef 2011, Kumar *et al.* 2016) it was found that u have the same behavior in both media. However, the values of u in the generalized thermoelastic medium are larger in comparison with those in the thermoelastic medium. The same remark for σ_{rr} in comparing figures. This is due to the influence of relaxation time, magnetic field, and frequency. These results are specific for the example considered, other cases may have different trends because of the dependence of the results on themechanical properties of the material as is demonstrated in Refs. (Bhattacharyya *et al.* 2007, Sharma and Kumar 2013) that have many applications in scientific and technical disciplines and materials science.

6. Conclusions

The elasto-dynamic equations for the generalized thermo-viscoelasticity theory under the effect of initial stress, the rotation, relaxation times, have complicated nature. The method used in this study provides a quite successful approach in dealing with such problems. The displacement, temperature, and stress components have been obtained in analytical form. This approach gives an exact solution in the Hankel's transform domain that appears in the governing equations of the problem considered. Numerical results are calculated, discussed and illustrated graphically. This work can be extended in the future work for other type of materials (Panda and Katariya 2015, Daouadji 2017, Lal *et al.* 2017, Behera and Kumari 2018, Ayat *et al.* 2018, Narwariya *et al.* 2018,Rezaiee-Pajand *et al.* 2018, Younsi *et al.* 2018, Panjehpour *et al.* 2018, Hirwani *et al.* 2018ab, Hirwani and Panda 2018, Sahoo *et al.* 2018, Ahmed *et al.* 2019, Tlidji *et al.* 2019, Boulefrakh *et al.* 2019, Hussain *et al.* 2019, Avcar 2019, Sahla *et al.* 2019, Bourada *et al.* 2019, Semmah *et al.* 2019, Pandey *et al.* 2019, Mehar and Panda 2019, Hirwani and Panda 2019abc, Mehar *et al.* 2019, Dash *et al.* 2019, Berghouti *et al.* 2019, Draiche *et al.* 2019, Karami et 2019bc, Kunche *et al.* 2019, AddaBedia *et al.* 2019, Draoui *et al.* 2019, Karami *et al.* 2019de, Ramteke *et al.* 2019, Katariya and Panda 2019ab and 2020, Mehar *et al.* 2020, Asghar *et al.* 2020, Matouk *et al.* 2020, Bellal *et al.* 2020, Khosravi *et al.* 2020, Rahmani *et al.* 2020, Hussain *et al.* 2020a, b, Karami *et al.* 2020).

Acknowledgments

This project was funded by the Deanship of Scientific Research (DSR) at King Abdulaziz University, Jeddah, under grant No. (G: 449-130-1440). The authors, therefore, acknowledge with thanks DSR for technical and financial support.

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