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Rao-3 algorithm for the weight optimization of reinforced concrete cantilever retaining wall

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Abstract. The paper represents an optimization algorithm for reinforced concrete retaining wall design. The proposed method, called Rao-3 optimization algorithm, is a recently developed algorithm. The total weight of the steel and concrete, which are used for constructing the retaining wall, were chosen as the objective function. Building Code Requirements for Structural Concrete (ACI 318-05) and Rankine's theory for lateral earth pressure were considered for structural and geotechnical design, respectively. Number of the design variables are 12. Eight of those express the geometrical dimensions of the wall and four of those express the steel reinforcement of the wall. The safety against overturning, sliding and bearing capacity failure were regarded as the geotechnical constraints. The safety against bending and shear failure, minimum and maximum areas of reinforcement, development lengths of steel reinforcement were regarded as structural constraints. The performance of proposed algorithm was evaluated with two design examples.

Keywords: design optimization; Rao-3 algorithm; retaining wall; metaheuristic; weight optimization

1. Introduction

Especially in the last quarter of the twentieth century, new methods were developed for computational problems in scientific fields. With the help of computers, we can reach quickly to the successful results of calculations, for a long time. However, complex and nonlinear problems still need human effort for solution and take long time for its calculations. At this point, we have been using computers again as Artificial Intelligence (AI) to decrease human effort. Bellman points out in his book An Introduction to Artificial Intelligence: thinking computers can make decisions, solve problems, learn, be creative, play games and so on (Bellman 1978). AI was utilized to solve nonlinear complex problems by developing various algorithms.

The word algorithm (and the idea of studying them) comes from al-Khowarazmi, a Persian mathematician of the 9th century, whose writings also introduced Arabic numerals and algebra to Europe (Russel *et al.* 2010). Problems can be formulated as optimization algorithms.

Optimization refers to the process of finding the best possible solution(s) for a particular problem (Mirjalili 2015).

There are lots of classification in the literature for optimization techniques: exact and approximate, stochastic and deterministic, local and global, classical and advance, etc. In this paper the metaheuristic optimization techniques, which are stochastic, approximate and advanced optimization techniques, were used. Although these techniques are not fully successful at finding the global optimum solutions, they are useful and satisfying for complex optimization problems. A new metaheuristic optimization technique, Rao-3 optimization algorithm is used for this study as proposed algorithm.

There are various computer programs for every stage of civil engineering and structural design processes. Although optimization programs are relatively new among those programs, they make most of them unnecessary. Because, it is possible to make optimal designs by following the design codes and providing external and internal stability of structures at the same time by means of optimization.

Structural design optimization is a field of study for civil engineering, nowadays (Perea *et al.* 2008, Khajedzadeh *et al.* 2012, Kaveh and Khayatazad 2014 , Kim 2014, Aydın and Cakir 2015, Mirzaei 2015, Yepes *et al.* 2017, Gholizadeh *et al.* 2017, Artar 2017, Nigdeli *et al.* 2018, Kaveh and Laien 2017, Deng 2019). By virtue of the geotechnical engineering is relatively new civil engineering discipline, there is a bigger chance to make new discoveries. Nevertheless, we can encounter similar studies, which they aim to find out which optimization technique gives the best results.

Stability of earth retaining structures is a too hot to handle geotechnical problem because of being based on people. Even choosing the dimensions of reinforced concrete (RC) cantilever retaining wall, one of the most common structure types, is a time-consuming process. Because initial dimensions would be changed by the designer until the design meets the geotechnical and structural requirements. In the analysis phase, the design must satisfy safety conditions which are formulated based on safety factors against sliding, overturning and bearing stress failure modes and also bending and shear moment capacities for all the elements of structure. Elements of

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structure are; stem, heel, toe and optionally shear key. RC cantilever retaining wall is a common earth retaining structure type because of material usage for construction is less than any other retaining wall.

Uncertainties in design variables and design equations have a significant impact on the safety of geotechnical structures like retaining walls and slopes. (GuhaRay 2014)

Therefore, there is a need for shorter design process for this type of structures. It is possible to find the optimal RC retaining wall design studies with various optimization techniques in the literature. Bekdaş and Temür (2016) designed a reinforced concrete cantilever retaining wall by using TLBO (Teaching-Learning Based Optimization) algorithm. Kayebekir *et al.* (2017) also used the TLBO algorithm for design optimization of reinforced concrete retaining wall, but they evaluated the wall under both static and dynamic loads. Another study on the optimum design of a reinforced cantilever retaining wall under static and dynamic loads is belong to Kayhan and Demir (2018). They used the differential evolution algorithm as optimization algorithm. Artificial Bee Colony (ABC) algorithm, which is the one of the bio-inspired metaheuristic algorithms, was used by Dağdeviren and Kaymak (2015) for the optimum design of reinforced concrete retaining wall. ECSS (Enhanced Charge System Search) algorithm, which is inspired by the Coulomb and Gauss's laws of electrostatics in physics, used by Talatahari and Sheikholeslami (2014) for optimum design of gravity and reinforced concrete retaining wall. Yepes *et al.* (2008) proposed simulated annealing algorithm for reinforced concrete cantilever retaining wall. Democratic Particle Swarm Optimization (DPSO) and Colliding Bodies Optimization (CBO) algorithms were used by Kaveh and Soleimani. The authors compared to results with Improved Harmony Search (HIS) and Particle Swarm Optimization (PSO) algorithms.

Different types of algorithms have been studied for cantilever retaining walls: Differential Evolution Algorithm (DEA) (Kumar and Suribabu 2017), genetic algorithm (GA) (Jasim and Al-Yaqoobi 2016), modified particle swarm optimization (MPSO) algorithm (Khajehzadeh *et al.* 2011), improved firefly algorithm with a harmony search algorithm (IFA-HS) (Sheikholeslami *et al.* 2014), ant colony optimization (ACO) algorithm (Ghazavi and Bonab 2011), harmony search based algorithm (Kaveh and Shakouri Mahmud Abadi, 2011).

The cost of the structure was selected as the objective function for the above-mentioned studies. Kaveh *et al.* (2013) studied the optimum design of reinforced concrete cantilever retaining wall by using a multi-objective algorithm, named non-dominating sorting genetic algorithm (NSGA-II). Multi- objective algorithm is used for multi-objective function optimization. They determined two objective function; economic cost and reinforcement congestion. The reinforcement congestion was one of the objective functions because the reinforcement placement in the concrete effects the integrity of the structure. Multi objective optimization is a subject for different structures too (Khalkhali 2016).

Rao-3 is a recently developed optimization algorithm. Grzywiński. and Dede (2020) used Rao algorithms for optimum design of 2D trusses, Wang *et al.* (2019) used Rao algorithms for a photovoltaic (PV) cell parameter estimation method, Rao and Pawar (2020) used Rao algorithms for optimum design of some mechanical system components.

In this study, a recently developed metaheuristic algorithm, Rao-3, was used to design the weight minimization of reinforced concrete retaining wall. by the help of MATLAB program. Two numerical examples presented to show the performance of the proposed algorithm.

2. Rao-3 Optimization algorithm

In this paper, Rao-3 optimization method is used for optimum design of reinforced concrete (RC) cantilever retaining wall with the shear key. Unlike the majority of metaheuristic algorithms, Rao-3 is not a metaphor-based metaheuristic algorithm.

Optimization techniques were classified shortly as two main topics by Feoksitov (2006) continuous and combinatorial. The continuous methods are not useful for this case, because they search the solutions in a certain continuous space. In this case we need a search area which is limited by a finite number of feasible solutions (i.e., combinatorial optimization techniques). Basically, there are two subcategories for combinatorial optimization techniques: exact and approximate methods. Exact methods can solve to small-scale problems by enumerating all sets of solutions. On the other hand, approximate methods apply a partial enumeration to attain a near-to-optimum solution, which come to a solution in a short period of time. Heuristic and metaheuristic methods are two kind of approximate method.

A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space (Osman 1995).

Metaheuristic algorithms are stochastic methods and they aim finding the statistically best solution around all candidates. Metaheuristic algorithms are mostly based on metaphors (natural phenomenon, musical instruments, planets, etc.). Rao (2020) dwelt on the complexity of metaphor-based optimization algorithms and suggested three metaphor-less simple optimization algorithm. One of them, Rao-3, was utilized for RC retaining wall with shear key design optimization. This algorithm was firstly developed by Rao (2020).

There are six main steps for implementation of Rao-3 algorithm:

1. Determine the size of population, number of variables and termination criteria. In this study, number of populations was taken as 50. The design variables are the geometrical properties of wall and reinforcement ratio for stem, hell, key and toe. This design variables are changeable according to problem. The termination criteria are determined as maximum number of generations.

2. Initialize a random population. To create initial population the random process of Matlab Programming is preferred.

3. Identify the best and worst solution in population. In

this study, minimum weight of the wall is taken as the best solution and maximum weight is taken as worst solution.

4. Modify the solutions based on the best and worst solution with a random variable coefficient. If the new one is better, use the new one. If the previous one is better, keep the previous one.

5. Repeat steps from second to fifth by the hope that to find global best solution.

6. Report the optimum solution.

3. Optimization process

The gamma process is a stochastic process with Computer-based optimization refers to using computer algorithms to search the design space of a computer model. The design variables are adjusted by an algorithm in order to achieve objectives and satisfy constraints (Parkinson *et al.* 2013).

Therefore, objective function, the design variables and constraints of RC retaining wall with shear key must be determined.

3.1 The objective function

Objective function is the function which is planned to be maximized or minimized for solving the optimization problem. Therefore, the objective function can measure the performance of the optimization. Sometimes, there are more than one main goals for the optimization problem, in that case the multi objective function can be used.

Optimum retaining wall design can have many objectives. For example, Sheikholeslami *et al.* (2014) Kaveh and Behnam (2013) studied the optimum reinforced concrete retaining wall by considering the cost as the objective function.

The CO_2 emission of the retaining wall was determined as objective function by Yepes *et al.* (2012) for the optimization of the retaining wall design.

Symmetrical gravity retaining wall design optimization is examined by Sadoglu (2014). Cross-sectional area of the wall, that minimizes the cost, was selected as the objective function.

In this paper, for optimization of reinforced concrete retaining wall, the total weight of the structure was selected as the objective function. Besides, the minimum weight of the wall probably will minimize the cost, the CO_2 emission and the cross-sectional area. The objective function is:

$$f_{obj} = W_{st} + W_c \tag{1}$$

$$W_c = 100 V_C \gamma_C \tag{2}$$

where W_{st} is the weight of steel per unit length of the wall, W_c is the weight of the concrete elements, V_C is the volume of concrete per unit length of the wall and γ_C is the unit weight of concrete; and a factor of 100 is used for consistency of units.

3.2 Design variables

As it is seen in Fig. 1 the geometrical dimensions and steel reinforcement of retaining wall were chosen as design variables. The eight geometric and four structural design variables were adapted from the study originally proposed by Saribas and Erbatur (1996).

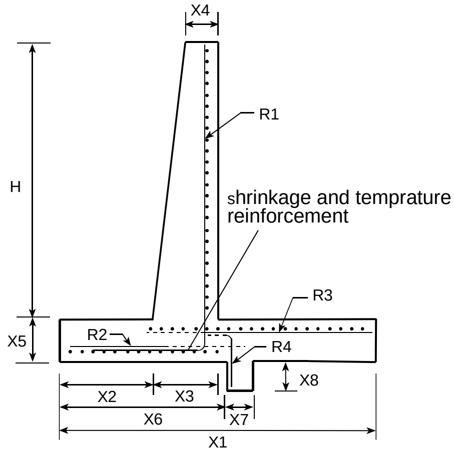

Fig. 1 Design variables for reinforced concrete retaining wall

Table 1 Reinforcement variable pool

Reinforcement			
Index No.(η)	Bars (#)	Bar Size	As (cm^2)
1	3	10	2.356
2	4	10	3.141
3	3	12	3.393
4	5	10	3.927
5	4	12	4.524
6	3	14	4.618
7	6	10	4.712
.	.	.	.
.	.	.	.
.	.	.	.
262	28	24	126.669
263	18	30	127.234
264	24	26	127.423

Geometric design variables express the geometry of the wall. The geometrical sections of the wall are noted as; X1 (the base width), X2 (the toe projection), X3 (stem thickness at the bottom of the wall) X4 (stem thickness at the top of the wall), X5 (base slab thickness), X6 (distance from the front of the toe slab to front of the shear key), X7 (width of the base shear key), X8 (height of the base shear key).

Structural design variables express reinforcement of the critical sections of the wall. The structural design variables are noted as; R1 (the vertical steel area in the stem per unit length of the wall), R2 (the horizontal steel area of the toe slab), R3 (the horizontal steel area of the heel slab), R4 (the vertical steel area of the shear key per unit length of wall).

The optimization problem was composed with continuous geometric design variables and discrete structural design variables. Reinforcement design variables represent the notation of number and diameter of bars as one variable each of them. Number of these notations are 264. The combination number of 3 to 28 numbers of the bars and 10 to 30 diameters of the bars were organized in Table 1.

3.3 Constraints

Constrains restrict the set of solutions in the search spaces of the optimization problem. In this paper, internal and external stability factors restricted the structural and geotechnical design of reinforced concrete retaining wall.

3.3.1 Geotechnical constraints

Factor of safety against slippage, overturning and bearing capacity must be compared with design factor of safety as constraints.

Overturning failure mode results from overturning forces acting on the toe section. The ratio of driving and resisting forces for overturning is factor of safety against overturning (F_{S_O}). F_{S_O} is a geotechnical constraint for design optimization of the retaining wall:

$$g(1) = \frac{F_{S_O\ design}}{F_{S_O}} - 1 \leq 0 \tag{3}$$

There may be occur a sliding failure on the base slab due to the resultant force of the earth pressure behind the wall. The factor of safety against sliding, which is the ratio of resisting and driving forces, is a constraint for retaining wall design:

$$g(2) = \frac{F_{S_S\ design}}{F_{S_S}} - 1 \leq 0 \tag{4}$$

One of the failure modes of retaining wall (i.e. one constraint) is the bearing capacity failure. The foundation of the retaining wall was considered as a shallow foundation. The factor of safety for bearing capacity investigation is:

$$g(3) = \frac{F_{S_B\ design}}{F_{S_B}} - 1 \leq 0 \tag{5}$$

3.3.2 Structural constraints

The critical sections of the wall are examined against bending and shear failure. Design strength (M_n) must be more than (or equal to) the required strength (M_u) of the critical sections on the elements of the wall. This condition restricts the structural design with special constraints constituted according to the ACI 318-05.

$$(5-8) = \frac{M_u}{M_n} - 1 \leq 0 \tag{6}$$

M_n is the nominal flexural strength and M_u is the required flexural strength for structure.

$$g(9-12) = \frac{V_u}{V_n} - 1 \leq 0 \tag{7}$$

Similarly, V_n is the nominal shear strength. V_u is the required shear strength based on the resultant forces acting on the wall.

Required shear and moment strengths of the structure are calculated for stem, toe, heel and the shear key separately based on the code.

Area of flexural reinforcement (A_s) of stem, toe, heel and key shall not be less than $A_{s_{min}}$.

$$g(13-16) = \frac{A_{s_{min}}}{A_s} - 1 \leq 0 \tag{8}$$

Area of flexural reinforcement (A_s) of stem, toe, heel and key shall not be more than $A_{s_{max}}$.

$$g(17-20) = \frac{A_s}{A_{s_{max}}} - 1 \leq 0 \tag{9}$$

Development length (l_{db}) have importance for the bond strength between concrete and steel. Reinforcement bars may be bent as hook. Constrains based on the development length and the development length of a standard hook (from $g(23)\ to\ g(26)$) are shown for stem, toe, heel and key, respectively.

$$g(23) = \frac{l_{db_{stem}}}{X5-cc} - 1 \leq 0 \text{ or } g(23) = \frac{l_{dh_{stem}}}{X5-cc} - 1 \leq 0 \tag{10}$$

$$g(24) = \frac{l_{db_{toe}}}{X1-X2-cc} - 1 \leq 0\ or\ g(24) = \frac{l_{dh_{toe}}}{X5-cc} - 1 \leq 0 \tag{11}$$

$$g(25) = \frac{l_{db_{heel}}}{X2+X3-cc} - 1 \leq 0\ or\ g(25) = \frac{l_{dh_{heel}}}{X5-cc} - 1 \leq 0 \tag{12}$$

$$g(26) = \frac{l_{db_{key}}}{X5-cc} - 1 \leq 0\ or\ g(26) = \frac{l_{dh_{key}}}{X5-cc} - 1 \leq 0 \tag{13}$$

In this case: $g(5)$, $g(9)$, $g(13)$, $g(17)$, $g(23)$ notations are constraints for stem, $g(6)$, $g(10)$, $g(14)$, $g(18)$, $g(24)$ notations are constraints for toe, $g(7)$, $g(11)$, $g(15)$, $g(19)$, $g(25)$ notations are constraints for heel and $g(8)$, $g(12)$, $g(16)$, $g(20)$, $g(26)$ notations are constraints for key. (cc is the depth of concrete cover)

Furthermore, geometry of the wall leads us to restriction for dimensions of the wall as:

$$g(21) = \frac{X2+X3}{X1} - 1 \leq 0 \tag{14}$$

$$g(22) = \frac{X6+X7}{X1} - 1 \leq 0 \tag{15}$$

The minimum bearing stresses of shallow foundation must be greater than (or equal to) zero.

$$g(4) = q_{min} \geq 0 \tag{16}$$

3.4 Geotechnical modelling

The forces effect the stability of the wall, which are shown in Fig. 2 are; W_C: the weight of the RC cantilever retaining wall for 1 m length, Q: the surcharge load, W_F: the weight of backfill on the heel, W_t: the weight of soil on the toe, P_A: the active earth pressure, P_{P_1}: the passive earth pressure on the base shear key and P_{P_2}: the passive earth pressure on the front part of the toe section and P_B: the bearing stress force.

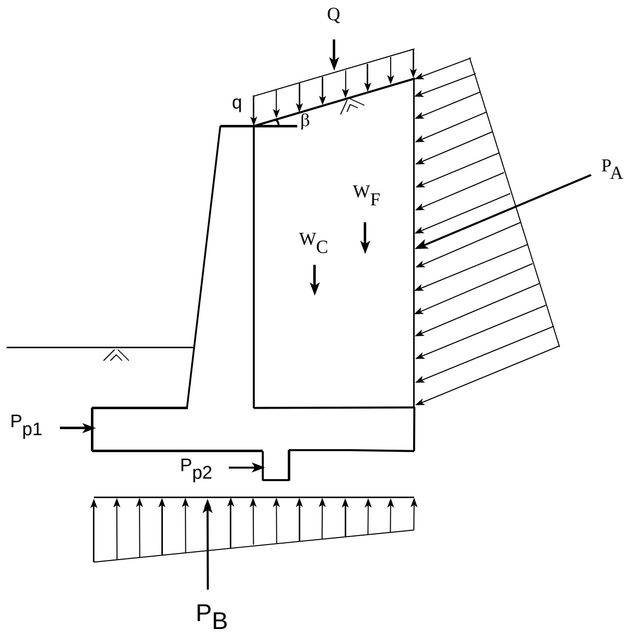

Fig. 2 The forces acting on reinforced concrete retaining wall

Rankine's earth pressure theory was used for geotechnical modelling of the optimization problem. Passive and active earth pressures, P_A and P_P, can be calculated with the passive and active earth pressure coefficients according to the Rankine's earth pressure theory. The active earth pressure coefficient is:

$$K_a = \cos\beta * \left(\cos\beta - \sqrt{(\cos\beta)^2 - (\cos\emptyset)^2}\right) / \left(\cos\beta + \sqrt{(\cos\beta)^2 - (\cos\emptyset)^2}\right) \quad (17)$$

The passive earth pressure coefficient is:

$$K_p = \left[tan\left(45 + \frac{\emptyset}{2}\right)\right]^2 \quad (18)$$

$\emptyset$ is the friction angle of backfill and β is backfill slope angle. There are two different soils in the system: one is the backfill and the other is the base soil. φ_{base}, c_{base} and γ_{base} are the friction angle, the cohesion and the unit weight of the base soil, respectively. φ, c and γ_{fill} are the friction angle, the cohesion and the unit weight of the backfill soil, respectively.

Two different soils have two friction angles and two pressure coefficients: K_{p_1} (the passive earth pressure coefficient of the backfill soil) and K_{p_2} (the passive earth pressure coefficient of the base soil).

The safety factor for overturning, the ratio between the sum of the moments of resisting forces ($\sum M_R$) and the sum of the moments of driving forces ($\sum M_O$). Passive earth pressure was not included to the resisting moments.

$$F_{S_O} = \frac{\sum M_R}{\sum M_O} \quad (19)$$

Another safety factor is for sliding failure mode. The ratio of resisting and driving forces is the factor of safety:

$$F_{S_S} = \frac{\sum F_R}{\sum F_D} \quad (20)$$

F_R (resisting force) can be expressed as:

$$\sum F_R = \left(\sum N\right) * \tan\left(\frac{2}{3} * \varphi_{base}\right) + \frac{2}{3} * B * c_{base} + \sum P_P \quad (21)$$

$\sum N$ is the expression of :

$$\sum N = W_c + W_F + Q + P_A \sin\beta \quad (22)$$

$\sum P_P$ is the sum of the passive forces P_{p_1} and P_{p_2}. Passive forces can be calculated as:

$$P_{p_1} = \frac{1}{2}\gamma_{fill} {D_1}^2 K_{P_1} + 2c{D_1}^2\sqrt{K_{P_1}} \quad (23)$$

$$P_{p_2} = \frac{1}{2}\gamma_{base} {h_{sk}}^2 K_{P_2} + 2c_{base}{h_{sk}}^2\sqrt{K_{P_2}} \quad (24)$$

h_{sk} is the depth of the base soil in front of the shear key and D_1 is the total depth of the retained soil causing passive earth pressure.

F_D (the driving force) is the sum of the horizontal components of active force:

$$F_D = P_A \cos\beta \quad (25)$$

Therefore, the factor of safety against to bearing capacity failure is the ratio of the ultimate bearing capacity (q_u) and the maximum applied bearing stress (q_{max}).

$$F_{S_B} = \frac{q_u}{q_{max}} \quad (26)$$

According to the Terzaghi's bearing capacity theory, the ultimate bearing capacity:

$$q_u = c_{base}N_c + \gamma_{fill}D_1N_q + \frac{1}{2}\gamma_{base}N_g(B - 2e) \quad (27)$$

B is the width of the base slab (X1). N_c, N_q and N_g are Terzaghi coefficients.

The eccentricity (e) is expressed as:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \quad (28)$$

The minimum and maximum bearing stresses of shallow foundation are:

$$q_{min,max} = \frac{\sum V}{B}\left(1 \mp \frac{6e}{B}\right) \quad (29)$$

3.5 Structural modelling

Structural properties of the wall must be adequate against bending and shear failure. Critical sections of the wall (heel, toe, stem and shear key) are investigated with regard to shear, and moment capacity conditions. The shear key acts during the sliding, it is not considered as a safety factor under other conditions.

Nominal flexural strength (M_n) is the strength which is calculated by using assumptions and equations of a cross section. There are nominal axial load strength (P_n) and nominal shear strength (V_n) also.

A safer design strength (ϕM_n) can be reached by multiplying nominal strength by a reduction factor (ϕ). Flexural strength reduction factor is 0.9 according to the ACI 318-05 for tension-controlled sections. According to ACI 318-05, nominal strength must be greater than (or equal to) required strength. Required flexural strength (M_u), required axial load strength (P_u) and required shear strength (V_u) can be calculated by the help of the factored loads and forces in load combinations acting on the element.

Nominal flexural strength can be described as:

$$M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \tag{30}$$

where A_s is area of reinforcement and f_y is the yield strength of reinforcement, d is the effective depth of the stress block, and a is the depth of equivalent rectangular stress block. The equivalent rectangular compressive stress block is used to provide convenience.

A safer design strength (ϕV_n) can be reached by multiplying nominal strength by a reduction factor (ϕ). Shear strength reduction factor is 0.75 for shear and torsion according to the code. According to ACI 318-05, design strength must be greater than (or equal to) required strength.

$$V_n = \Phi 0.17\sqrt{f_c} bd \tag{31}$$

A_s, minimum area of flexural reinforcement, shall not be less than that given by;

$$A_{s_{min}} = 0.25\frac{\sqrt{f_c}}{f_y} bd \tag{32}$$

and not less than $\left(1.4/_{f_y bd}\right)$ according to ACI 318-05 chapter 10.

The minimum steel reinforcement, ρ_{min}, can be calculated with given formula:

$$\rho_{min} = \frac{A_s}{bd} \tag{33}$$

The reinforcement ratio, ρ_b which produces balanced strain conditions under flexure, is the limit for maximum steel reinforcement ratio ρ_{max}.

$$\rho_b = \left(\frac{0.85\beta_1 f_c}{f_y}\right)\left(\frac{600}{600 + f_y}\right) \tag{34}$$

Development length for the bars, whose diameter is smaller than 19 mm, is described by ACI 318-05 as:

$$l_d = \left(\frac{12 f_y \psi_t \psi_e \lambda}{25\sqrt{f_c}.}\right) d_b \tag{35}$$

where d_b is the diameter of the bar.

Development length for the bars, whose diameter is bigger than 19 mm, is described by ACI 318-05 as:

$$l_d = \left(\frac{12 f_y \psi_t \psi_e \lambda}{20\sqrt{f_c}.}\right) d_b \tag{36}$$

Also, l_d shall not be less than 300 mm. According to ACI 318-05 for the concrete beams, modification factor for casting location (ψ_t), modification factor for development length (ψ_e) and factor base on reinforcement coating (λ) are 1.0.

When the conditions are unfavorable for the development length of straight bars, one can use hooks for bonding of the reinforcing bars to the concrete. According to the ACI 318-05 code l_{dh} can be calculated with given formula:

$$l_{dh} = \left(\frac{0.24 f_y}{\sqrt{f_c}}\right) d_b \tag{37}$$

where l_{dh} is the development length of a standard hook. l_{dh} shall not be less than the smaller $8d_b$ and 150 mm.

4. Numerical Optimization Problems

In this section, two numerical optimization problems were exampled. The efficiency of the proposed algorithm was evaluated by using these examples.

To reach the global optimum, which is the best solution around the candidate solutions, is the goal of the optimization process. The iterations are tried many times because of the will of avoiding the local optimum. In this study, each example was run 30 times to reach a better solution. Results were stated as the best and mean values. The population size and numbers of iterations were selected as 50 and 1000, respectively.

4.1 Example 1

The RC retaining wall without shear key was subjected

Table 2 Input parameters for example 1 (Camp and Akin 2012)

Input Parameters	Symbol	Value	Unit
Stem height	H	3	m
Reinforcing steel yield strength	f_y	400	MPa
Concrete compressive strength	f_c	21	MPa
Concrete cover	cc	7	cm
Shrinkage and temperature reinforcement percentage	ρ_{st}	0,002	-
Surcharge load	Q	20	kPa
Backfill slope	B	10	°
Internal friction angle of base soil	Φ_{base}	0	°
Internal friction angle of retained soil	Φ	36	°
Unit weight of retained soil	γ_{fill}	17,5	kN/m^3
Unit weight of base soil	γ_{base}	18,5	kN/m^3
Unit weight of steel	Gs	78,5	kN/m^3
Unit weight of concrete	γ_c	23,5	kN/m^3
Depth of soil in front of wall	D	0,5	m
Factor of safety for overturning stability	$F_{SO_{design}}$	1,5	-
Factor of safety for sliding	$F_{SS_{design}}$	1,5	-
Factor of safety for bearing capacity	$F_{SB_{design}}$	1,5	-
Cohesion of the base soil	c_{base}	125	kPa

Table 3 Upper and lower bounds of design variables for Example 1 (Camp and Akin 2012)

Design Variable	Unit	Lower Bound	Upper Bound
X1	m	1,3090	2,3333
X2	m	0,4363	0,7777
X3	m	0,2000	0,3333
X4	m	0,2000	0,3333
X5	m	0,2722	0,3333
R1	-	1	264
R2	-	1	264
R3	-	1	264

Table 4 Optimum design variables for example 1

	X1	X2	X3	X4	X5	R1	R2	R3
Rao-3	1.83	0.74	0.21	0.20	0.28	82 28*10	20 11*10	26 4*18

Table 5 Final results of optimum design for example 1

	Steel	Concrete	Design Objective (kg/m)	
	kg/m	m^3/m	Best	Mean
Rao-3	96.6160	1.1267	2744.2	2892.6

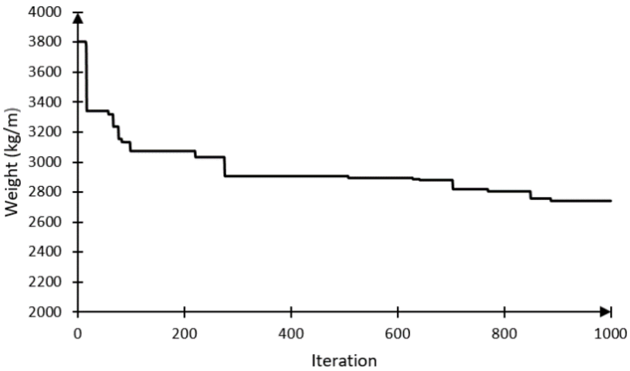

Fig. 3 Convergence history for optimum design for Example 1

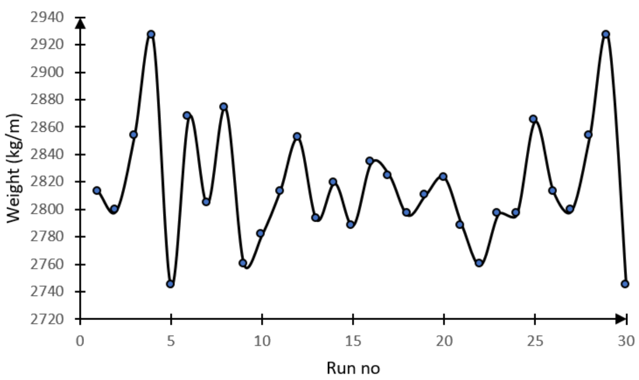

Fig. 4 The optimum design for each run for Example 1

to this example. The objective function is selected as weight of the retaining wall. The design parameters i.e., properties of the retaining wall system are shown in the Table 2.

There are five geometrical dimensions of the wall (X1-X5), which are continuous variables, and three structural variables related to reinforcement (R1-R3), which are discrete variables, as indicated in Table 3. The upper and lower boundaries of design variables were shown in the Table 3.

The optimum design variables according to the proposed algorithm are shown in Table 4.

The best and mean objective functions, the total weight of steel and the total weight of the concrete for the design optimization problem are shown in Table 5.

The best and mean objective function values are seen in Table 5 for each algorithm. According to the results of Rao-3 algorithm, the best objective function is 2,744.2 kg/m.

Table 6 Input parameters for example 2 (Camp and Akin 2012)

Input Parameters	Symbol	Value	Unit
Stem height	H	4,5	m
Reinforcing steel yield strength	f_y	400	MPa
Concrete compressive strength	f_c	21	MPa
Concrete cover	cc	7	cm
Shrinkage and temperature reinforcement percentage	ρ_{st}	0,002	-
Surcharge load	q	30	kPa
Backfill slope	β	0	°
Internal friction angle of retained soil	φ	28	°
Unit weight of retained soil	γ_{fill}	18,5	kN/m^3
Internal friction angle of base soil	φ_{base}	34	°
Unit weight of concrete	γ_c	23,5	kN/m^3
Unit weight of base soil	γ_{base}	17	kN/m^3
Unit weight of steel	Gs	78,5	kN/m^3
Depth of soil in front of wall	D	0,3	m
Factor of safety for overturning stability	$F_{S_{O_{design}}}$	1,5	-
Factor of safety for sliding	$F_{S_{S_{design}}}$	1,5	-
Factor of safety for bearing capacity	$F_{S_{B_{design}}}$	1,5	-
Cohesion of the base soil	c_{base}	0	kPa

Table 7 Upper and lower bounds of design variables for Example 2 (Camp and Akin 2012)

Design Variable	Unit	Lower Bound	Upper Bound
X1	m	1,96	5,5
X2	m	0,65	1,16
X3	m	0,25	0,5
X4	m	0,25	0,5
X5	m	0,4	0,5
X6	m	1,96	5,5
X7	m	0,20	0,5
X8	m	0,20	0,5
R1	-	1	264
R2	-	1	264
R3	-	1	264
R4	-	1	264

Table 8 Optimum design variables for example 2

	X1	X2	X3	X4	X5	X6	X7	X8	R1	R2	R3	R4
Rao-3	3.22	0.65	0.39	0.25	0.48	2.13	0.20	0.50	142 26*14 mm	60 7*18 mm	94 22*12 mm	41 12*12 mm

Table 9 Final results of optimum design for example 2

	Steel	Concrete	Design Objective (kg/m)	
Rao-3	kg/m	m^3/m	Best	Mean
	287.6356	3.0975	7566.6	7627.84

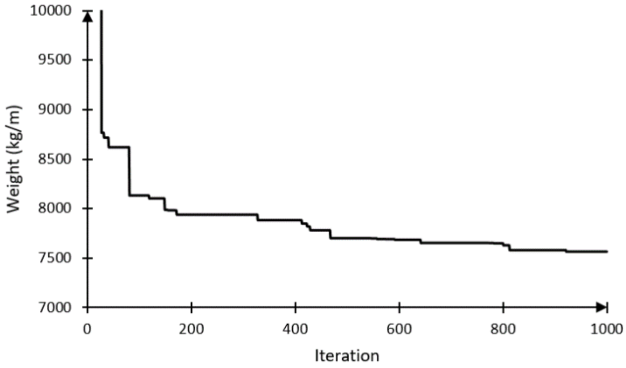

Fig. 5 Convergence history for optimum design for Example 2

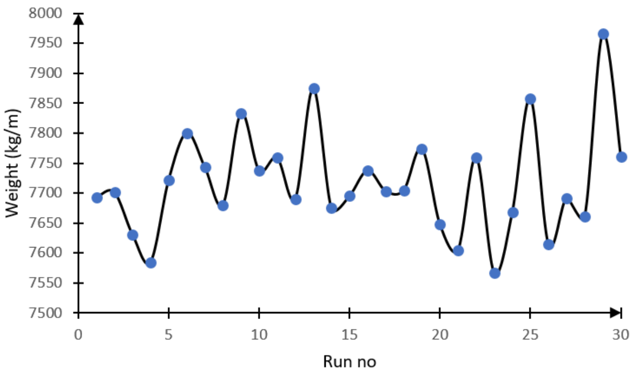

Fig. 6 The optimum design for Example 2

Convergence history in Fig. 3 shows the designs through the iterations. It seems that better results can be obtained by increasing the value of iteration number.

Fig. 4 shows the optimum design for each run. Elapsed time for run of optimal result in this example is 137.554646 seconds.

4.2 Example 2

The RC retaining wall without shear key was subjected to this example for the same objective function. Four new variables were arisen with the shear key: three geometric variables for the geometrical dimensions of the wall (X6-X8) and one for reinforcement of the shear key (R4). Cohesionless soil conceived for this example to design a better structure for long-term. The design parameters i.e., properties of the retaining wall system can be seen in the Table 6.

The upper and lower boundaries were shown in the Table 7.

In Table 9, the best and mean design objectives and the weight of steel and concrete of the proposed algorithm was shown. The best design objective of Rao-3 algorithm is 7,566.6 kg/m.

In Fig. 5, convergence history for optimum design is shown. As it is mentioned before, increasing the value of iteration number may help obtaining better results in this case too.

Fig. 6 shows the optimum design for each run. Elapsed time for run of optimal result in this example is 179.239671 seconds.

5. Conclusions

Reinforced concrete cantilever retaining wall design by using Rao-3 optimization algorithm is the main purpose of this study. In accordance with this purpose, the cantilever retaining wall was modelled as an optimization problem by determining the design variables, constraints and objective function.

• Stated optimization problem is more complicated than most structural design optimization problems due to the exhausting checking process. Nevertheless, the retaining wall weight is minimized, easily and quickly.

• It is possible that proposed method did not reach its best solution among the candidate solutions. Because during the optimization process, each example was run 30 times. One can reach to better results by running the program a several times.

The computer processor, which the program run, is Intel (R) Core (TM) i3-3227U CPU 1.90 GHz.

The elapsed time for these design examples were 179.239671 seconds and 137.554646 seconds, respectively. These elapsed times shows the performance of the proposed algorithm.

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