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Analytical solution for steady seepage and groundwater inflow into an underwater tunnel

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Abstract. Solutions of the water pressure and groundwater inflow distribution along the tunnel perimeter in a half-infinite aquifer were investigated considering the conditions of the constant head and constant water pressure. It is assumed that the circular tunnel is buried in a fully saturated, homogeneous, isotropic and half-infinite space. Coordinate transformation technique was adopted, the problem of solving the control equations of water pressure in the Cartesian coordinate was transformed to that in the bipolar coordinate system, which can significantly simplify the derivation procedure of the water pressure and inflow distribution. The validation results show the accuracy and advantage of the proposed approach.

Keywords: bipolar coordinate; water pressure; water inflow; half-infinite aquifer

1. Introduction

For the stability and reinforcement of a tunnel in an undrained plane, solutions are being provided and many authors have made a huge contribution (Jeffery 1921, Mindlin 1940, Atkinson and Pott 1977, Verruijt 1996, Gonzalez and Sagaset 2001, Massinas and Sakellariou 2009, Pinto and Whittle 2013, Zou and Zuo 2017, Li and Zou 2019, Qian *et al.* 2020, Huang *et al.* 2020, Li *et al.* 2020, Li and Yang 2020, Xu *et al.* 2020). But the water pressure and water inflow are indispensable topics for designing an underwater tunnel. Uncontrolled water pressure and water inflow may have a negative impact on mechanical stability around the tunnel and cause settlements of structure on the surface. With regard to the solutions of steady seepage and groundwater inflow into a circular tunnel, analytical solutions, numerical methods, and semi-analytical solutions have also been investigated over decades (Harr 1962, Schleiss 1986, Zhang and Franklin 1993, Fernandez and Alvarez 1994, Lei 1999, Bobet 2001, Bauer *et al.* 2003, Perrochet 2007, Kolymbas 2007, Park 2008, Ming *et al.* 2010, Font-Capo *et al.* 2011, Fang *et al.* 2015, Kargar *et al.* 2015, Farhadian *et al.* 2017, Aalianvari 2017). Some researchers go further who take into account the hydraulic mechanical coupling in order to deepen the understanding of surrounding rock around the tunnel (Brown 1982, Ohtsu 1999, Zou 2018).

However, the existing research results are mainly focused on the water pressure, few solutions for the distribution of groundwater inflow along the tunnel perimeter have been examined. The main reason is that for the presented analytical solutions using the complex variable method, it is difficult to obtain the explicit expression (Ming *et al.* 2010, Fahimifar and Zareifard 2013, Kargar *et al.* 2015, Fang *et al.* 2015).

The aim of this study is to develop a new approach to obtain the analytical solutions of water pressure of surrounding rock and the distribution of groundwater inflow along the tunnel perimeter under the conditions of the constant head and constant water pressure. The coordinate transformation technique was adopted during the solving process, which transforms Cartesian coordinates to the bipolar coordinate system. Validation and discussion are carried out accordingly.

2. Section title: Level 1

In order to simplify the solving procedure, it is necessary to introduce an orthogonal curvilinear coordinates system called bipolar coordinate. As shown in Fig. 1, the relation between Cartesian and bipolar coordinates is given by the invertible transformation functions and can be expressed as follows (Jeffery 1921).

$$z = x + iy = ik \coth \frac{\zeta}{2} \tag{1}$$

$$\zeta = \alpha + i\beta = \log \frac{x + i(y+k)}{x + i(y-k)} \tag{2}$$

In Fig. 1, k is the length of the pole O_1 or O_2 to the point (0, 0); $\theta_1 - \theta_2$ is the angle between the two lines joining the poles to the point Z. The curves β = constant are arcs of circles passing through O_1 and O_2. Meanwhile, r_1 or r_2 is the length of point O_1 or O_2 to the point Z, respectively. It is clear that α are constants typifying circles cutting the first set of circles orthogonally. They form a family of coaxial

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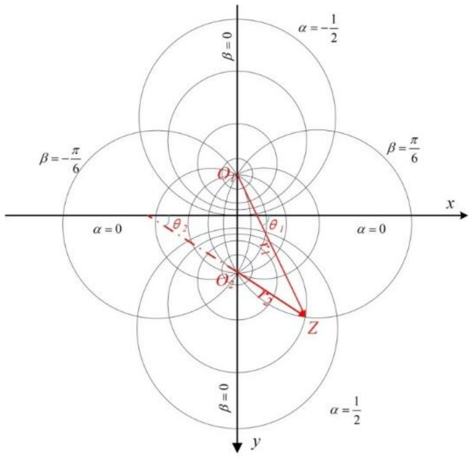


Fig. 1 Principle of bipolar coordinate system

circles with the two poles as limiting points. Such circles surround the pole O_2 if $r_1/r_2>1$ ($\alpha>0$) and surround the pole O_1 if $r_1/r_2<1$ ($\alpha<0$). Obviously, $\alpha=\log(r_1/r_2)$ and $\beta=\theta_1-\theta_2$.

The coordinate β changes from - π to π when passing through the y-axis connecting the poles. If the function β is said to be periodic with period 2 π and can describe stresses and displacements, the stresses and displacements will be continuous across y-axis.

For a given bipolar coordinate system, at any point Z the infinitesimal element of arc length ds on arbitrary curve through Z can be defined as follows.

$$(ds)^2=(J_\alpha d\alpha)^2+(J_\beta d\beta)^2 \tag{3}$$

where, $J_\alpha=J_\beta=k/(\cosh\alpha-\cos\beta)$, and which can be reduced to $J=k/(\cosh\alpha-\cos\beta)$.

3. Assumptions

Several assumptions have been made to determine the specific environment in a half-infinite aquifer under the conditions of the constant head and constant water pressure: A shallow circular tunnel is in a fully saturated, continuous, isotropous, and homogeneous semi-infinite space with a horizontal water table; the axisymmetric condition is considered; the flow is steady and the fluid is incompressible.

4. Problem description

As shown in Fig. 2, the central axis of the shallow tunnel, with radius r and depth h, is parallel to the z-axis in the Cartesian coordinate system (x, y, and z). The length of the shallow tunnel is much longer than its diameter. The vertical downward direction of the y-axis is considered as the positive direction. The cross-section of the shallow tunnel and the semi-infinite space are divided into two same parts by the y-axis.

Physically, the speed of flow through natural soils for laminar flow is so slow that changes in momentum are negligible in comparison with the viscous resistance to

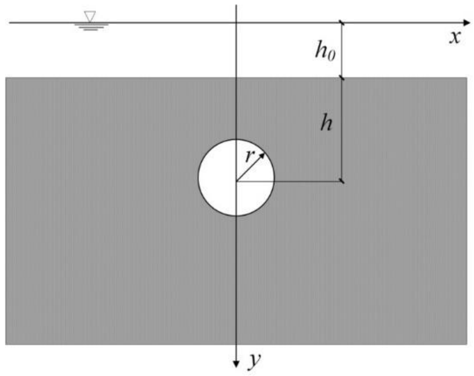


Fig. 2 Geometry of the problem

flow, which means the speed of flow can be seen as zeo. In the Cartesian coordinate system, if the non-viscous fluid and flow medium are both incompressible, according to Darcy's law and Bernoulli's equation, the Laplace equation governing two-dimensional steady flow can be expressed as follows (Kundu and Cohen 2008).

$$\nabla^2\varphi=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \tag{4}$$

where

$$\varphi(x,y)=-KH=-K(-y+p/\gamma) \tag{5}$$

in which $\varphi(x,y)$ denotes the velocity potential, H is the hydraulic head, p and γ are the water pressure and the unit weight of fluid, respectively.

In bipolar coordinate system, Eq. (5) becomes

$$\nabla^2\varphi=\frac{1}{J^2}\left[\frac{\partial}{\partial\alpha}(\frac{\partial\varphi}{\partial\alpha})+\frac{\partial}{\partial\beta}(\frac{\partial\varphi}{\partial\beta})\right] \tag{6}$$

where

$$\varphi(\alpha,\beta)=-KH=-K(-J\sinh\alpha+p/\gamma) \tag{7}$$

From the ground surface to the tunnel circumstance, the whole half plane, include a tunnel, can be portrayed by successive eccentric circles, in which α=0 denotes the ground surface and $\alpha=\alpha_i$ denotes the tunnel.

5. Solution for water pressure

5.1 Considering constant water pressure at tunnel circumstance

For the incompressible fluid, the pressure function also satisfies the Laplace equation which could be expressed as follows.

$$\nabla^2 p=\frac{1}{J^2}(\frac{\partial^2 p}{\partial\alpha^2}+\frac{\partial^2 p}{\partial\beta^2})=0 \tag{8}$$

Without considering elevation head around the tunnel circumference, on the basis of the symmetry of the present problem and there is the discontinuity of 2π in β on passing

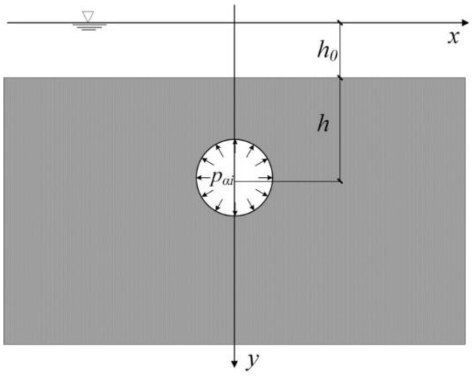


Fig. 3 Drained tunnel with constant water pressure at tunnel circumstance

between the y-axis connecting the poles, the solution must be even in β and periodic in β, of period 2π. Therefore, a solution of water pressure can be expressed in the form of

$$p' = f(\alpha)\cos n\beta \tag{9}$$

Substituting Eq. (7) in Eq. (6), it yields to

$$(\frac{d^2}{d\alpha^2} - n^2) f(\alpha) = 0 \tag{10}$$

Eq. (10) is an ordinary differential equation whose solution can be expressed as follows.

For n=0,

$$f_0(\alpha) = A_0 + B_0\alpha \tag{11}$$

For $n \geq 1$,

$$f_n(\alpha) = A_n e^{n\alpha} + B_n e^{-n\alpha} \tag{12}$$

Therefore, Eq. (9) can be written as follows

$$p' = \sum_{n=1}^{\infty}[(A_n e^{n\alpha} + B_n e^{-n\alpha})\cos n\beta] + A_0 + B_0\alpha \tag{13}$$

The elevation head around the tunnel circumference in bipolar system is

$$y = \frac{k\sinh\alpha}{\cosh\alpha - \cos\beta} \tag{14}$$

The term β in the denominator can be expanded in infinite series, which is shown as follows.

$$\frac{\sinh\alpha}{\cosh\alpha - \cos\beta} = 2\left\{\sum_{n=1}^{\infty}\left[\left(\cosh\alpha - \sinh\alpha\right)^n \cos n\beta\right]\right\} + 1 \tag{15}$$

Combining Eq. (15) with Eq. (13), the solution to Eq. (8) is

$$\begin{aligned} p = {} & \sum_{n=1}^{\infty}[(A_n e^{n\alpha} + B_n e^{-n\alpha})\cos n\beta] + A_0 + B_0\alpha \\ & + 2k\gamma\{\sum_{n=1}^{\infty}[(\cosh\alpha - \sinh\alpha)^n \cos n\beta]\} + k\gamma \end{aligned} \tag{16}$$

which will be valid if $\alpha > 0$.

Boundary conditions of Eq. (8) are

$$\begin{cases} p|_{\alpha=0} = h_0\gamma \\ p|_{\alpha=\alpha_i} = p_{\alpha_i} \end{cases} \tag{17}$$

where, h_0 is the hydraulic head at the ground surface, and p_{α_i} is water pressure at the tunnel perimeter.

Substituting Eq. (17) into Eq. (16), it results in

$$\begin{cases} A_0 = h_0\gamma \\ B_0 = \dfrac{p_{\alpha_i} - h_0\gamma - k\gamma}{\alpha_i} \\ A_n = -\dfrac{2k\gamma(\cosh\alpha_i - \sinh\alpha_i)^n}{e^{n\alpha_i} - e^{-n\alpha_i}} \\ B_n = -A_n \end{cases} \tag{18}$$

So that Eq. (16) is found to be

$$\begin{aligned} p = {} & \sum_{n=2}^{\infty}[2A_n \sinh n\alpha \cos n\beta] \\ & + \frac{k\gamma\sinh\alpha}{\cosh\alpha - \cos\beta} + h_0\gamma + \frac{p_{\alpha_i} - h_0\gamma - k\gamma}{\alpha_i}\alpha \end{aligned} \tag{19}$$

where,

$$A_n = -\frac{k\gamma(\cosh\alpha_i - \sinh\alpha_i)^n}{\sinh n\alpha_i} \tag{20}$$

According to Eq. (5), the hydraulic head around the tunnel circumstance is

$$\begin{aligned} \mathrm{H}(\alpha,\beta) = {} & \sum_{n=2}^{\infty}\frac{2A_n \sinh n\alpha \cos n\beta}{\gamma} \\ & + h_0 + \frac{p_{\alpha_i}/\gamma - h_0 - k}{\alpha_i}\alpha \end{aligned} \tag{21}$$

5.2 Considering constant hydraulic head at the tunnel perimeter

According to Eq. (21), hydraulic head also satisfies the Laplace equation which is found to be the form

$$\nabla^2 H = \frac{1}{J^2}(\frac{\partial^2 H}{\partial\alpha^2} + \frac{\partial^2 H}{\partial\beta^2}) = 0 \tag{22}$$

In bipolar coordinate, similar to Eq. (13), the solution of Eq. (22) is

$$H = \sum_{n=1}^{\infty}[(A_n' e^{n\alpha} + B_n' e^{-n\alpha})\cos n\beta] + A_0' + B_0'\alpha \tag{23}$$

For Eq. (23), boundary conditions are as follows

$$\begin{cases} H|_{\alpha=0} = h_0 \\ H_{\alpha=\alpha_i} = H_{\alpha_i} \end{cases} \tag{24}$$

Then the constants values in Eq. (23) are

$$\begin{cases} A_n' = B_n' = 0 \\ A_0' = h_0 \\ B_0' = \dfrac{H_{\alpha_i} - h_0}{\alpha_i} \end{cases} \tag{25}$$

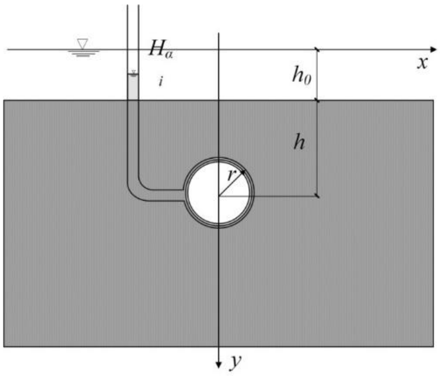


Fig. 4 Drained tunnel with constant hydraulic head at the tunnel circumstance

Therefore, solution of Eq. (23) can be written as follows.

$$H = h_0 + \frac{H_{a_i} - h_0}{\alpha_i}\alpha \tag{26}$$

Substituting Eq. (26) into Eq. (5), water pressure surrounding the tunnel can be expressed as follows

$$p = h_0\gamma + \frac{H_{a_i} - h_0}{\alpha_i}\gamma\alpha + J\sinh\alpha\gamma \tag{27}$$

Note that Eq. (26) and (27) are in accordance with the solution in Ming *et al.* (2010), if both solutions are transformed to Cartesian coordinate.

6. Solution for groundwater inflow

The discharge is the product of the stream's cross-sectional area and its mean velocity, which could be written as follows (Kundu and Cohen 2008)

$$q = -\int_{\partial\Omega}\mathbf{v}\cdot\mathbf{n}ds \tag{28}$$

where, $\partial\Omega$ represents the tunnel surface, n outward unit normal vector, *ds* arc length along the surface and $\mathbf{v} = -K\nabla H$ ($\nabla H = \frac{1}{J}\frac{\partial H}{\partial\alpha}\mathbf{e}_\alpha + \frac{1}{J}\frac{\partial H}{\partial\beta}\mathbf{e}_\beta$. $\mathbf{e}_\alpha$ and $\mathbf{e}_\beta$ are unit vectors tangent to the bipolar coordinate curve which α and β varies, respectively).

By means of the divergence theorem of Gauss, the derivation surface integrals could be converted to volume integrals, which means

$$q = -\int_{\partial\Omega}\mathbf{v}\cdot\mathbf{n}ds = K\int_{\Omega}\Delta H d\Omega \tag{29}$$

where Ω is the cross-section of the tunnel and $\Delta H = \nabla^2 H$ which could be expressed by

$$\nabla^2 H = \frac{1}{J^2}\left[\frac{\partial}{\partial\alpha}(\frac{\partial H}{\partial\alpha}) + \frac{\partial}{\partial\beta}(\frac{\partial H}{\partial\beta})\right] \tag{30}$$

For constant hydraulic head condition, in terms of Eq. (29), seepage flow can be expressed as follows.

$$q = K\int_{\Omega}\Delta H d\Omega = 2\pi K\frac{H_{a_i} - h_0}{\alpha_i} \tag{31}$$

As for the distribution of inflow along the tunnel circumference, in the condition of inflow along the angular sector $-\pi < \beta_0 < \beta < \beta_1 \le \pi$ of the tunnel circumstance, the expression for discharge can be expressed by

$$q_{\beta_0\beta_1} = (\beta_1 - \beta_0)K\frac{H_{a_i} - h_0}{\alpha_i} \tag{32}$$

If β changes from -π to π when crossing the segment of y-axis, then,

$$q_{\beta_0\beta_1} = (\beta_0 - \beta_1 + 2\pi)K\frac{H_{a_i} - h_0}{\alpha_i} \tag{33}$$

Similarly, for constant water pressure condition, seepage flow is

$$q = 2\pi K\frac{p_{\alpha_i}/\gamma - h_0 - k}{\alpha_i} \tag{34}$$

Distribution of inflow along the angular section $-\pi < \beta_0 < \beta < \beta_1 \le \pi$ of the tunnel circumstance is

$$q_{\beta_0\beta_1} = K\int_{\beta_0}^{\beta_1}\left.\frac{\partial H}{\partial\alpha}\right|_{\alpha_i}d\beta \tag{35}$$

If β changes from -π to π when crossing the segment of y-axis, then,

$$q_{\beta_0\beta_1} = K\int_{-\pi}^{\beta_0}\left.\frac{\partial H}{\partial\alpha}\right|_{\alpha_i}d\beta + \int_{\beta_1}^{\pi}\left.\frac{\partial H}{\partial\alpha}\right|_{\alpha_i}d\beta \tag{36}$$

where,

$$\frac{\partial H}{\partial\alpha} = -\sum_{n=2}^{\infty}[2n\frac{k(\cosh\alpha_i - \sinh\alpha_i)^n}{\sinh n\alpha_i}\cosh n\alpha_i\cos n\beta] + \frac{p_{\alpha_i}/\gamma - h_0 - k}{\alpha_i} \tag{37}$$

It is noted that Eq.(31) and (34) are the same solutions as Kolymbas and Wagner's (2007) and the differences between them are clear which are owing to the different boundary conditions.

7. Validation

To validate the proposed solution in this study, the results by the proposed solution and FLAC3d are compared with those in Ming *et al.* (2010). The parameters of the numerical and theoretical calculations are all selected from Ming *et al.* (2010) The tunnel perimeter is subjected to zero water pressure (i.e., $p_{\alpha_i} = 0$), while the soil's permeability is k=10^{-5} m/s. Buried depth of circular tunnel is h=20 m and

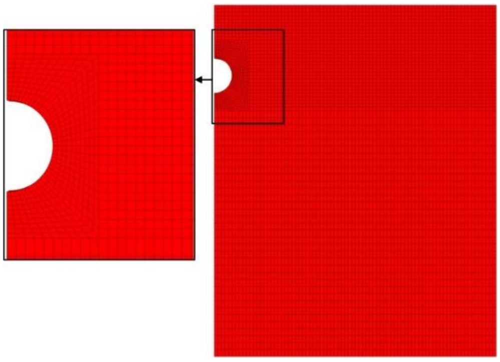


Fig. 5 Numerical model

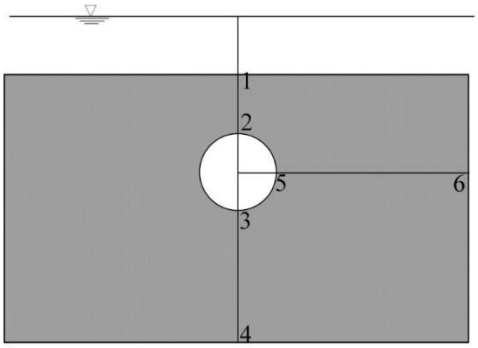
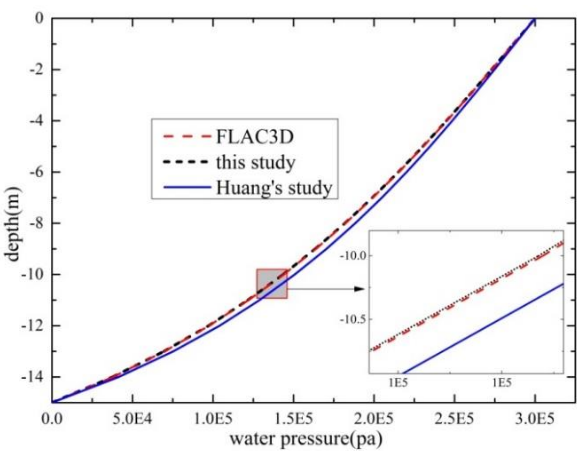
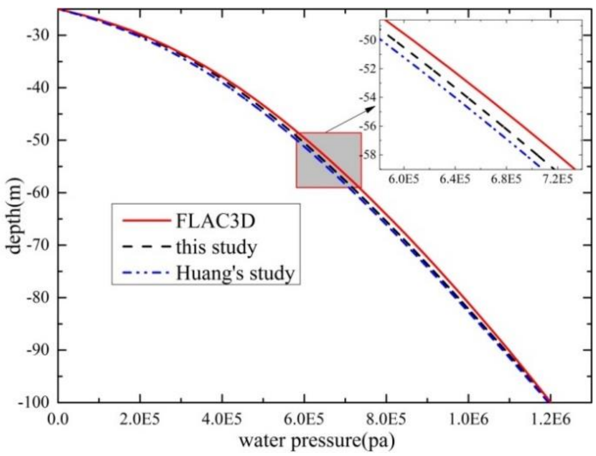


Fig. 6 Calculation points in the aquifer



(a) Water pressure in line (1-2)



(b) Water pressure in line (3-4)

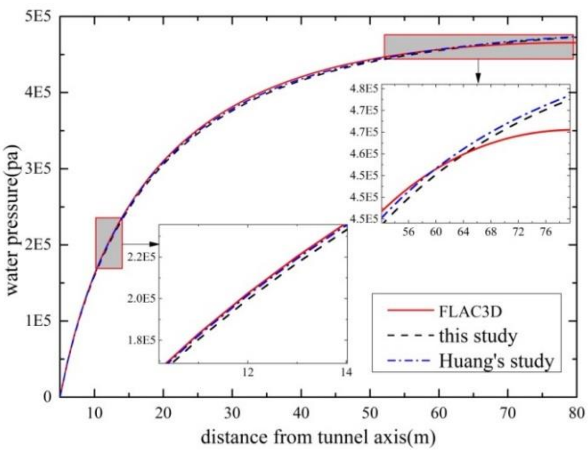


Fig. 7 Results of water pressure between the proposed solutions and those in Ming *et al.* (2010).

radius of the tunnel is r=5 m. Water depth above the ground surface is h_0=30 m. The size of the numerical model is 80 in the x-direction and 100 in the y-direction (see Fig. 5), which is similar to that in Ming *et al.* (2010). Corresponding points are chosen from three lines (1-2), (3-4) and (5-6) as shown in Fig.6 in order to carry out the validation.

Fig. 7(a) and 7(b) show that the results of the proposed solutions are agreed better with those of FLAC3d than those in Ming *et al.* (2010) do in the y-direction. As in the x-direction, Fig.7(c) reveals that the proposed solutions are more exact than Ming's when the points are far from the tunnel. However, there are still some minor differences between the proposed solutions and FLAC3d. The reason is that the proposed solutions are derived by assuming a half-infinite space while the extent of the numerical model is limited. For the points at infinity, water pressure is only associated with the elevation head, and the numerical results of water pressure seem to get close to the boundary condition faster than the theoretical results.

8. Conclusions

This study proposed a new approach to obtain the closed-form analytical solutions for the water pressure surrounding the tunnel and steady-state groundwater inflow into a drained circular tunnel in a half-infinite aquifer. Compared with the previous analytical solutions, the following improvements have been achieved.

(1) By using the bipolar coordinate system, consider the two different boundary conditions (i.e., constant total head and constant water pressure), the derivation process is very simple. Furthermore, the proposed approach can be easy to obtain the explicit expression of the distribution equations of groundwater inflow along the tunnel perimeter.

(2) In comparison with the existing solutions, the proposed solution is more exact for fully drained shallow tunnels and enables readers to derive the distribution of inflow along the tunnel circumference under the condition of constant water pressure for the first time.

The new analytical solution could form a theoretical basis for the interpretation of the problems of the seepage into a shallow circular tunnel.

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