Geomechanics and Engineering, *Vol. 20, No. 3 (2020) 255-265*
DOI: https://doi.org/10.12989/gae.2020.20.3.255

Experimental and numerical study on performance of long-short combined retaining piles

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(Received March 21, 2019, Revised November 25, 2019, Accepted January 29, 2020)

Abstract. Laboratory tests are conducted to investigate the performance of retaining system with different combinations of long-short piles. Numerical analysis implemented using ABAQUS are verified by comparing numerical results with measured data. By performing numerical studies, the horizontal displacement of piles, heave of excavation bottom and bending moment of pile for various pile system with different pile lengths are investigated. Results show that long piles share higher bending moments than short piles. The increase in the number of short piles leads to a slight increase in the heave at excavation bottom for long-short pile retaining system. Retaining system with different long and short pile combinations have greater effects on the horizontal displacement of pile above the excavation bottom, compared to its counterparts below excavation bottom. For a given length of long pile, the bending moment and displacement of piles increase with the decrease in length of short piles, while the increasing rate of maximum moment of retaining pile system is insignificant. Results highlight that a reliable and economical pile retaining system can be designed by optimizing the number and length of short piles, provided that the working performance of retaining structures above excavation bottom meets the design requirement in practice.

Keywords: long-short piles; excavation; laboratory model test; numerical analysis

1. Introduction

Over the past decade, the number of underground infrastructures (e.g., basement, underground mall, metro system) has been dramatically increased in the modern cities (e.g., Shanghai, Singapore, and Tokyo) due to the high demand of space for rapid urbanization (Qian, (2016)). For the construction of these structures, a decent design and construction of deep excavations is essential. As a result, how to design an economic but safe supporting system for deep excavations has been one of the major challenges encountered in engineering practice (Frazi *et al.*, (2018)).

Row pile has been widely used as the support system for deep excavation projects over the past decades. To date, various studies have been presented on the performance (e.g., the deformation, stability, inner-force, earth pressure, pile-soil interaction and embedded ratio) of piles with bracing system. For example, Ito *et al.* (1975) proposed a model to calculate the lateral force acting on piles based on plastic theory and clarified that the model is applied under the condition of restrained pile top. Guo *et al.* (2006) investigated the responses of pile subjected to the action of lateral soil movements and indicated that the pile responses were significantly affected by the soil modulus and pile-soil relative stiffness. Amar and Bouzid (2011) proposed a semi-analytical method to study the lateral behavior of piles foundation under horizontal loading, and the lateral behavior is investigated in detail by main governing parameters analysis. Poulos *et al.* (1997) solved the responses of pile due to the lateral soil movements during excavation using two-stage analysis method and boundary-element method. Wang *et al.* (2015) combined the upper bound method with the strength reduction technique and numerical analyses software of FLAC^{3D} estimated soil stability with a vertical free-face between two adjacent supporting piles. A numerical study, which considers the interaction of soil and structure, was conducted by Jiang *et al.* (2018) to study the stress and deformation behavior of sheet pile wall structure. According to the Guangweigou floodway across the Quiren and Rende districts in Tainan, Taiwan, Chen *et al.* (2016) analyzed the reason of several sections cantilever pile filed during a heavy rainfall caused the collapse of the slope behind the row piles. Besides, many other investigations have been conducted on the behavior of pile supporting systems (e.g., Hong *et al.*(2003), Liu *et al.*(2011), Wang et el.(2011), Bisaws *et al.*(2014), Liang *et al.* (2014), Ukritchon *et al.* (2016)). However, it is noted that all the above-mentioned studies focused on the supporting system with equal-length piles, i.e., the lengths of all the retaining piles are identical.

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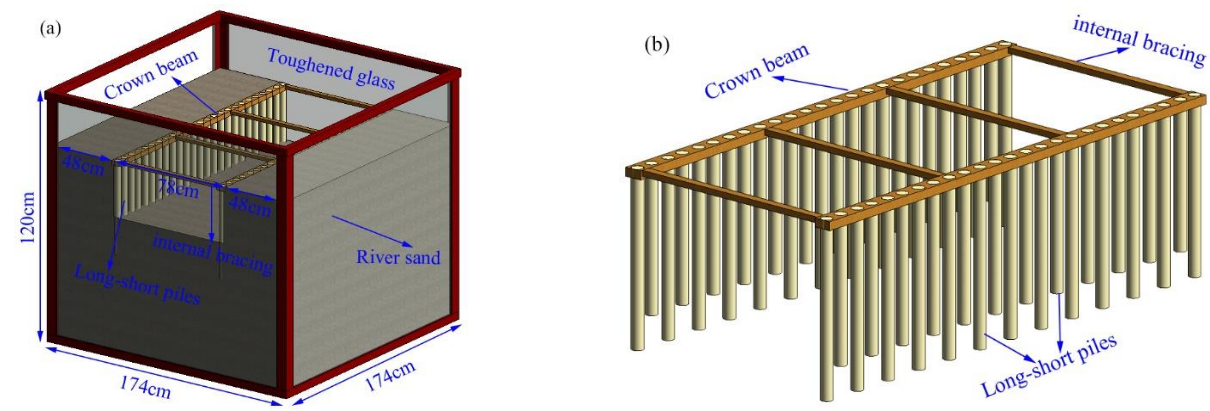
http://www.techno-press.org/?journal=gae&subpage=7
ISSN: 2005-307X (Print), 2092-6219 (Online)



(a) Model device (b) long-short piles

Fig. 1 Laboratory model test diagram

Table 1 Physical and mechanical parameters of materials

Materials name	Young modulus E (MPa)	Density ρ(g/cm^3)	Poisson ratio μ	Internal friction angle φ
Nylon rod	2272	2.2	0.23	—
rRver sand	68	1.87	0.25	32°

In contrast, over recent years, more and more engineers recognized that the bending capacity of the equal-length pile near the pile bottom is not fully utilized due to the fact that bending moment borne by the retaining structures significantly decreases with the embedded pile depth beneath the excavation bottom. That is, the current design of retaining structures with equal-length piles for deep excavation projects is significantly over-conservative, which would greatly increase the budget of project. As highlighted by Leung *et al.* (2000) that equal-length pile retaining structures were designed with consideration of the most dangerous conditions with maximum internal forces. Nevertheless, the maximum inner-force at the lower part of the pile is only about 25% of that at its upper part, leading to that the strength of the materials at lower part of the pile cannot be fully used.

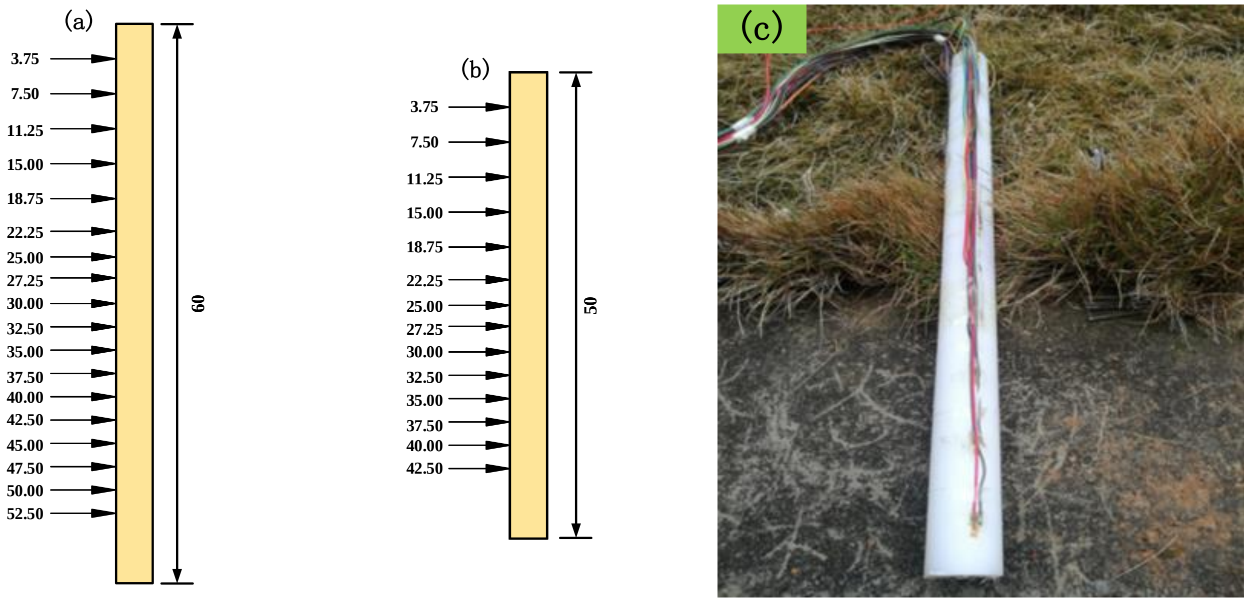
Inspired by above observations, a new idea of using long-short pile instead of equal-length pile has been adopted for the retaining structures for deep excavation over recent years, Zheng *et al.* (2008) and Li *et al.* (2010) studied the mechanical characteristics of long-short pile retaining system during excavation process by laboratory model test, and reported that the bending moment exerted at long piles is higher than its counterpart for short piles. Xu *et al.* (2015) investigated the influence of the length of short pile as well as the ratio of the number of long piles to that of short piles on the deformation and moment acting on the piles by conducting numerical analysis and proposed the optimal combination of long-short pile was proposed. Shen *et al.*(2017) analyzed the difference of the performance (e.g., bearing capacity and earth pressure) of the double row long-short pile retaining system from that of double row equal-length pile system. To date, the results of existing studies have shown that the long-short pile method as a new design idea possesses promising development prospects duet to its reliability and economy. However, the majority of existing studies were limited to the numerical studies, while laboratory studies on the performance of long-short pile retaining system are still rather limited. In particular, the existing studies mainly focused on the behavior of retaining system with one type of combination of long-short piles only, while the working performance of retaining system with different types of long-short pile combinations is still largely unknown.

To address the above issues, in this paper, a series of laboratory tests are firstly conducted to investigate the working performance of the long-short pile retaining system with different combinations of long and short piles. Then, numerical analysis implemented using ABAQUS is verified by comparing the numerical predictions with the measured results. Finally, the numerical analysis is further conducted to systematically investigate the inter-force and deformation mechanism of the long-short pile retaining system with various combinations of long and short piles. The results are presented and discussed in details.

2. Laboratory investigation

2.1 Laboratory facility and materials

To simulate the long-short pile retaining system subjected to excavation, a model device with size of 174 cm×174 cm×120 cm is designed, as shown in Fig. 1(a). The model device is a square box made of steel plate for the bottom and four toughened glass for the walls. To minimize the influence of sand wall frictions on the test results, Vaseline is painted on the internal walls of the toughened glass. The nylon rods with length of 60cm and 50cm and the diameter is 4cm are used to model the long and short piles, respectively. The pile retaining structure for the excavation is composed of long-short piles, internal bracing and crown beams as shown in Fig. 1(b). In each side of excavation, 12 long piles and 12 short piles with net pile-pile distance of 2.5cm are used for the retaining system. To mount the crown beam with the model piles, round holes are drilled corresponding to the locations of each piles, in the meanwhile, strong adhesive is applied on the connection joints to ensure that the model piles and crown beam are tightly bonded. The crown beams are made of wood with size of 174 cm×4.5 cm×3 cm. To mimic the in-situ conditions, internal bracing made of wood are also used in



(a) Long pile (b) Short pile (c) Model pile

Fig. 2 Model piles and measuring point diagram (unit: cm)

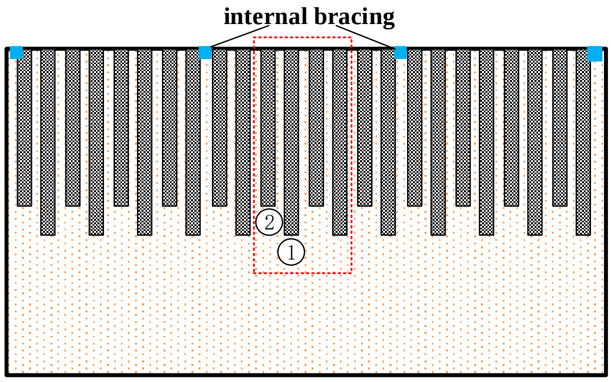
(a) Filling process (b) The observation pile of composite long-short pile retaining system

Fig. 3 Row pile retaining structure entity schematic diagram

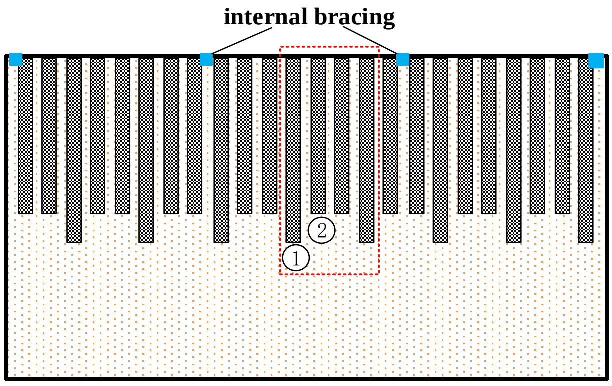
this test. For modelling the soil deposits for the excavation project, the River sand retrieved from the Gan River in Jiangxi Province, China, is used, due to that the River sand conforms to the geology condition of Nanchang, and the mechanical parameters of River sand can be obtained easily (Garhy *et al.* angle of internal friction of the river sand is measured by performing the direct sh(2013)). The ear tests. The material properties of nylon rod and river sand are shown in Table 1. Prior to mounting the strain gauges on the piles, the measuring points are marked on the piles as shown in Fig. 2. And the testing system instrumented with strain gauges is presented in Fig. 3.

2.2 Test program

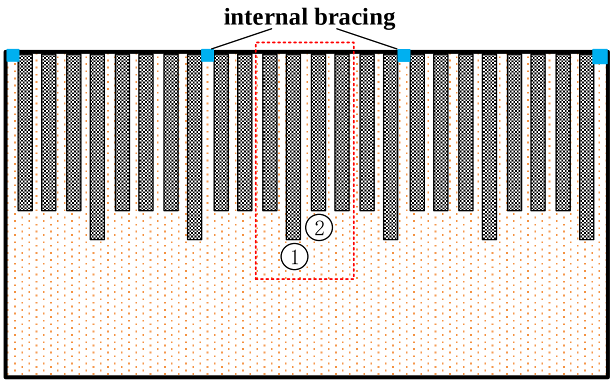
To investigate the behavior of long-short pile retaining system subjected to the excavation, the deformation of the piles is recorded after completion of each excavation step. In this work, the excavation work is completed within three steps, and the excavation depth of each step is 12 cm. That is, the total excavation depth is 36 cm. The detailed testing procedure is elaborated herewith: ① Mark length scale on the outside of toughened glass wall with the aim of visually monitoring the process of excavation work; ② Gently pour the river sand with the given height of 20cm into test device (square box) until the thickness of bottom sand layer reaches reach 60 cm; ③ Mount the strain gauges along the pile at pre-determined positions, then fix the top of piles at the crown beam as mentioned in the Section 2.1 (Fig. 2 and Fig. 3); ④ Setup four equal-length internal bracings to horizontally support the crown beams (Fig. 1(b)); ⑤ Cover the outside of the retaining structure using a membrane with aim to avoid the sand flowing through the intervals of pile into the pit during excavation process; ⑥ Again, gently pour the river sand with the given height of 20 cm on the sand deposit in test device (square box) until the top surface of sand layer reach the top plane of the crown beams; ⑦ Excavate the first layer soil to the depth of 12 cm, while recording the strain of pile subjected to the excavation until the deformation of piles are stabilized. ⑧ Repeat step ⑦ until the excavation is completed. It is



(a) Long-short composite pile retaining system



(b) Long-double-short composite pile retaining system



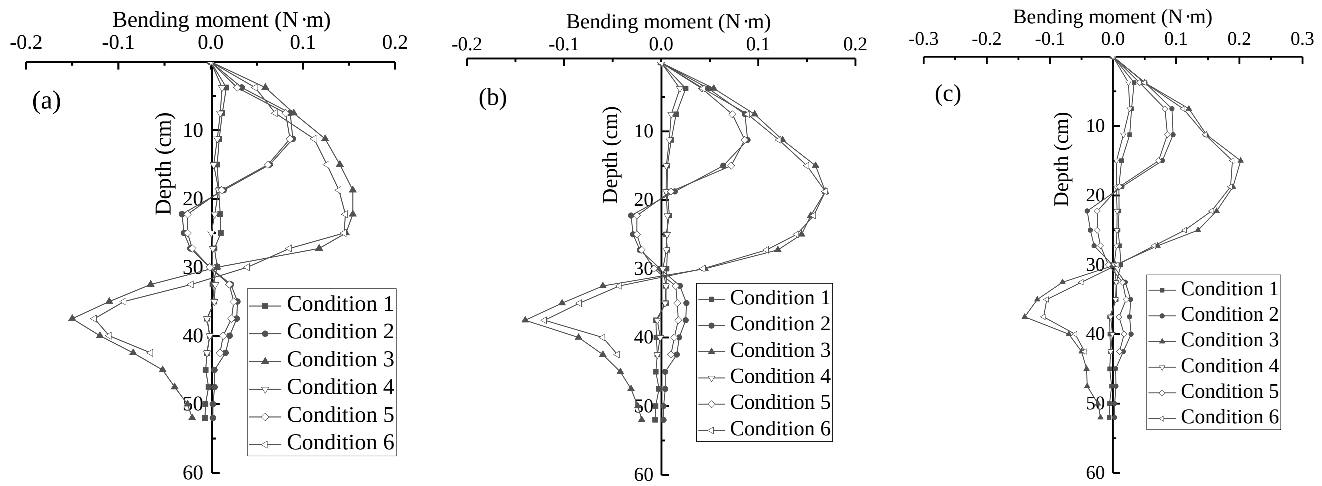
(c) Long-triple-short composite pile retaining system

Fig. 4 Three different composite pile retaining systems

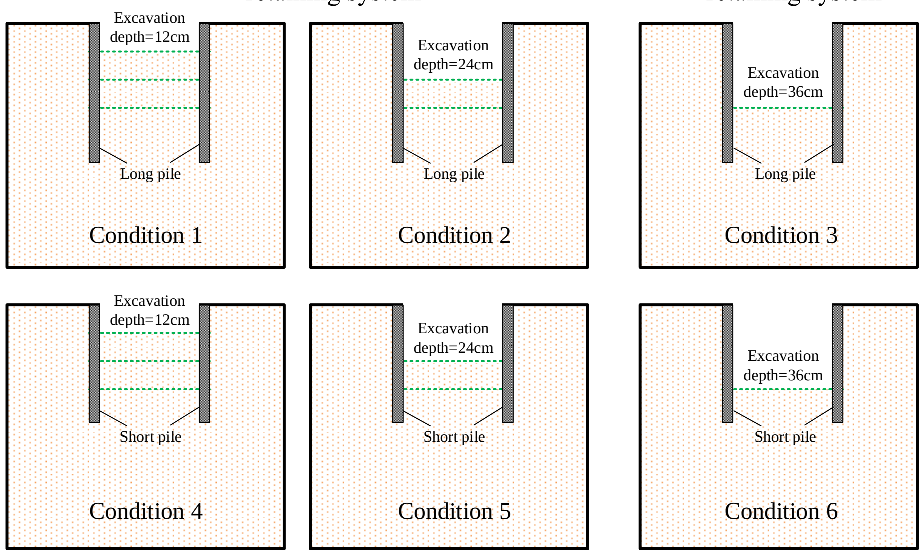
noted that, to investigate the performance of long-short pile retaining system with different combinations of long and short piles, three types of pile combinations are designed and investigated: (I) long-short pile combination, (II) long-double-short pile combination, and (III) long-triple-short pile combination, as illustrated in Fig. 4, respectively.

2.3 Experimental results and discussion

Fig. 5 shows the variation of the bending moment along the depth of the long and short piles, respectively, corresponding to the retaining systems with different combinations of long and short pile at different excavation stages. It is observed that the bending moment of both long piles and short piles (conditions 4, 5 and 6) increase significantly with the increase in the excavation depth, from 12 cm to 36 cm, for all the scenarios with different long-short pile combinations (Fig. 5(a), 5(b) and 5(c)). Interestingly, it can be seen that the bending moment at the pile beneath the excavation bottom (depth=36 cm) is positive when the excavation depth reaches 24 cm, but it subsequently changes to be negative when the excavation depth of 36 cm is completed. That is, the distribution state of the bending moment of piles changes abruptly as the excavation depth proceeds from 24 cm to 36 cm. The possible reason for such phenomenon is that the inflection point of piles moved downwards with the increase in the excavation depth, which will also cause the re-distribution of earth pressure acting on the retaining system. In addition, it is interestingly observed that the bending moment of long pile is only slightly greater than its counterpart of short pile at each excavation stage, for all the three scenarios with different long-short pile combinations. The maximum bending-moment of piles in the three scenarios appears at

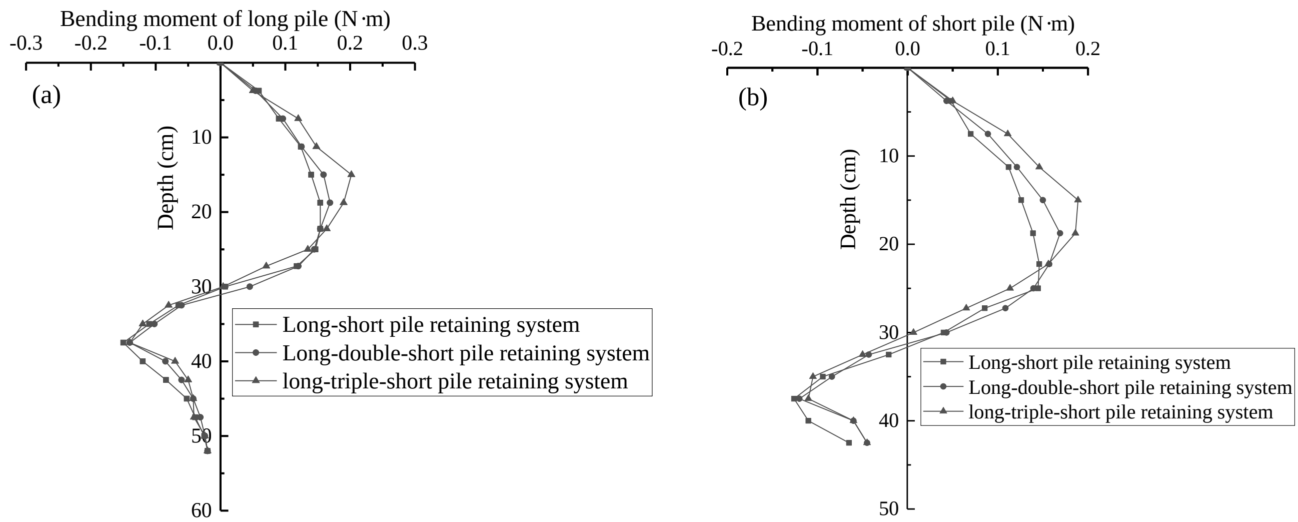


(a) Long-short composite pile retaining system (b) Long-double short composite pile retaining system (c) Long-triple-short composite pile retaining system



(d) The diagrammatic drawing of 6 different conditions

Fig. 5 Variation of the bending moment along the depth of the long and short piles, respectively, corresponding to the retaining systems with different combinations of long and short pile at different excavation stages



(a) Long pile (b) Short pile

Fig. 6 Distribution curves of bending moment of piles with different long-short piles combination around 8~12 cm away from the excavation bottom.

To further investigate the performance of retaining system with different long-short pile combinations, Fig. 6 presents the comparison of bending moment distribution along the depth of both short and long piles for different retaining system subjected to the excavation with depth of 36cm. It is observed that, for both short and long piles, the maximum bending moment above the excavation bottom increase gradually when the retaining system changes from long-short pile combination to long-triple-short pile combination. Specifically, it is observed that the maximum bending moment of long pile increases from 0.154N·m to 0.202N·m, with an increasing rate of 31.2%, while the counterpart of short piles grows from 0.146N·m to 0.189N·m, with an increasing rate of 29.5%. This indicates that the increasing rate of bending moment for long piles is slightly greater than its counterpart for short piles when the retaining system changes from long-short pile combination to long-triple-short pile combination. Also, it is found that for all above scenarios, the long piles generally share slightly higher bending moment than short piles, especially when higher excavation depth is reached.

3. Numerical investigation

In order to investigate the effect of different design factors on the performance of the long-short pile retaining system, numerical investigations are performed by employing the finite element software ABAQUS in following section.

3.1 Numerical model and verification

The model, developed in Abaqus V6.14, for long-double-short pile retaining system is presented in Fig. 7. To simulate the soil, the numerical model with size of 174 cm×174 cm×120 cm by employing the modified cam-clay model is used (Schofield *et al.* 1968, Tang *et al.* 2018). The model parameters are experimentally determined as shown in Table 2. Eight-node continuum reduced integration (C3D8R) elements are chosen to numerical the soil with an approximate global seed size of 1.2 cm (Gupta *et al.* 2017). Roller boundary conditions are applied to the four vertical sides of the model and the bottom boundary is fixed. The element types of C3D8R with elastic constitutive model is used to simulate the long-short piles and crown beam. Interaction between the soil and piles are modelled by applying the surface-to-surface contract, and the tangential and normal behavior of the surface-to-face contract are assumed to be coulomb friction (the friction coefficient is 0.25) and hard contact, respectively (Liu *et al.* (2014)). Then the tie constraints are applied to connect the bottom face of piles and soil to ensure that there is no slippage or separation between piles and soil. Besides, beam elements (B31) are used to represent the internal bracing support (Burd *et al.* 2016), and the tie constraints are used between internal bracing support and crown beam and piles to ensure that there is no relative displacement generated between the joint of pile and crown beam or piles. Finally, the simulation steps applied in this analysis are shown in Table 3.

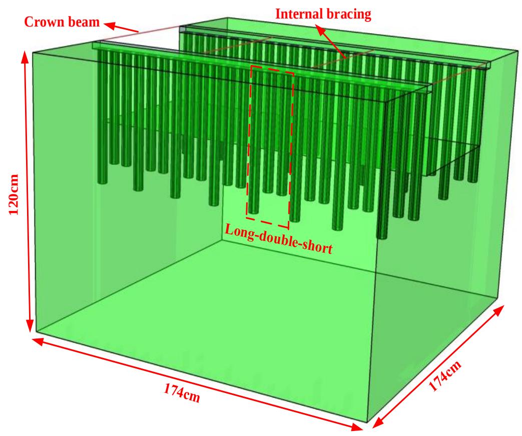


Fig. 7 Long-double-short finite element model schematic diagram

Table 2 Soil physical and mechanical parameters

Physical parameters	ρ (kg/m^3)	Poisson ratio v	Log volume λ	bulk modulus κ	Stress ratio M	e_0
River sand	1.87	0.25	0.0887	0.0045	1.25	1.362

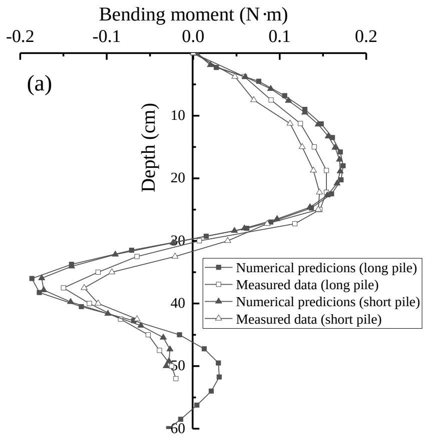
Table 3 Simulation steps

Step number	Step name	Step number	Step name
Step1	Geostress balance with soil and long-short piles	Step4	Excavate the first layer soil to the depth of 24 cm
Step2	Setup four internal bracings	Step5	Excavate the first layer soil to the depth of 36 cm
Step3	Excavate the first layer soil to the depth of 12 cm		

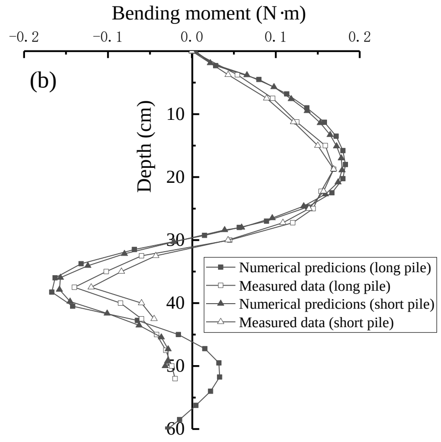
Fig. 8 shows the comparison of numerical predictions of bending moment along the depth of both long and short piles with the corresponding test results presented in this work. Noted that, the predicted and measured results of bending moment are corresponding to the same position of pile (i.e, No.1 and No.2 in Fig. 4). It can be observed that, for both short and long piles, the numerical predictions are consistent in an acceptable manner with measured results on the variation of bending moment along the depth of whole pile, highlighting that the numerical analysis are well validated by the measured results.

3.2 Horizontal displacement of pile subjected to retaining system with different long-short pile combinations

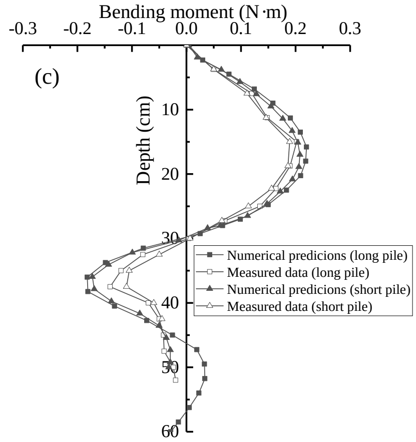
Fig. 9 shows the horizontal displacement contours of the pile retaining system with different short and long pile combinations when the excavation is completed (excavation depth=36 cm). It can be observed that the distribution patterns of horizontal displacement along the depth of piles for different retaining systems are similar. For all cases, the horizontal displacements of the piles above the excavation bottom are significantly greater than that of the piles beneath the excavation bottom. It is also observed that the maximum horizontal displacement of pile above the



(a) Long-short

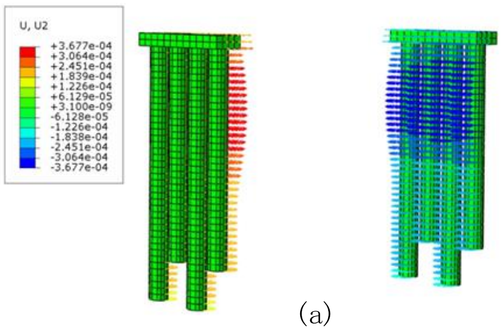


(b) Long-double-pile



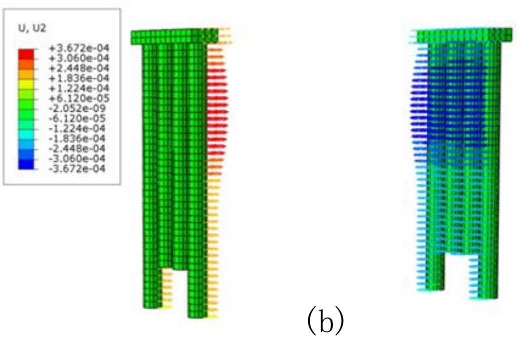
(c) Long-triple-pile

Fig. 8 Comparison between numerical and experimental results



(a)

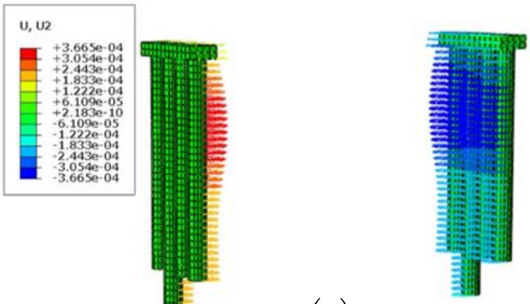
(a) Long-short retaining system



(b)

(b) Long-double-short retaining system

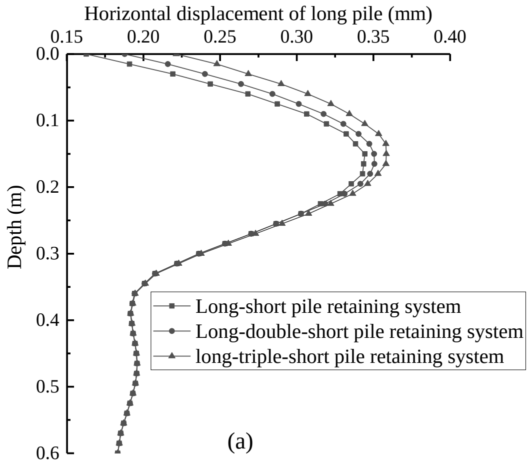
Fig. 9 Cloud picture of displacement and various curve of depth horizontal displacement

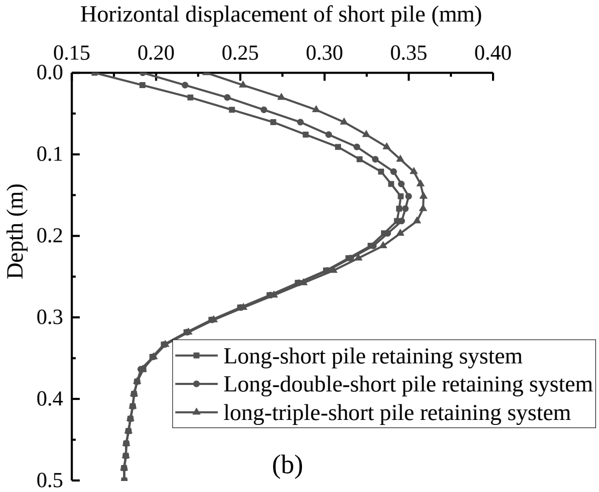


(c)

(c) Long-triple-short retaining system

Fig. 9 Continued

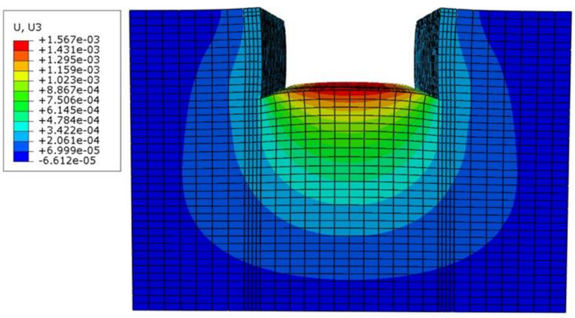




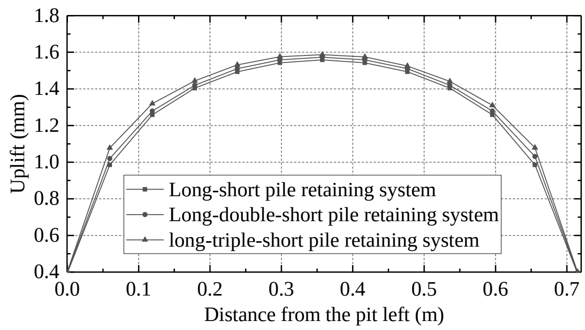
(a) The horizontal displacement of long pile

(b) The horizontal displacement of short pile

Fig. 10 Horizontal displacement distribution curves of pile retaining system with different long and short pile combinations along the depth



(a)



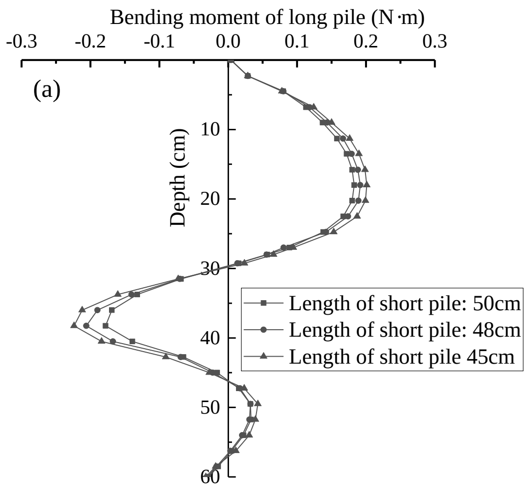
(b)

(a) Heave cloud picture of excavation bottom corresponding to the long-triple-short pile retaining system

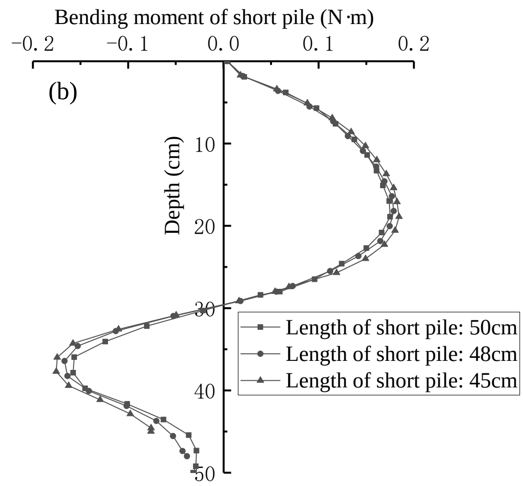
(b) Heave curve at excavation bottom of long-short pile, long-double-short pile and long-triple-short pile retaining system

Fig. 11 Bottom heave cloud picture and heave distribution curves corresponding to the different combinations of long-short pile retaining system

excavation bottom for the retaining system with long-triple-short pile combination is greater than its counterparts of other two retaining systems. Specifically, the maximum horizontal displacement of long pile at the retaining system with long-short, long-double-short and long-triple-short pile combinations are 0.345 mm, 0.350 mm and 0.359 mm, respectively, and the corresponding horizontal displacement of short pile are 0.344 mm, 0.350 mm and 0.358 mm, respectively. That is, the effects of the retaining system with different long and short pile combinations on the horizontal displacement of the pile above the excavation bottom is

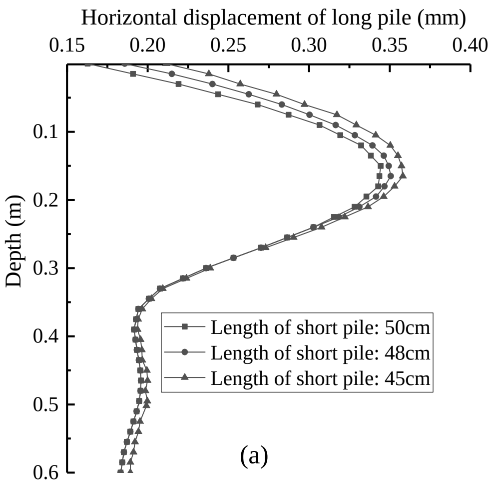


(a) Long pile

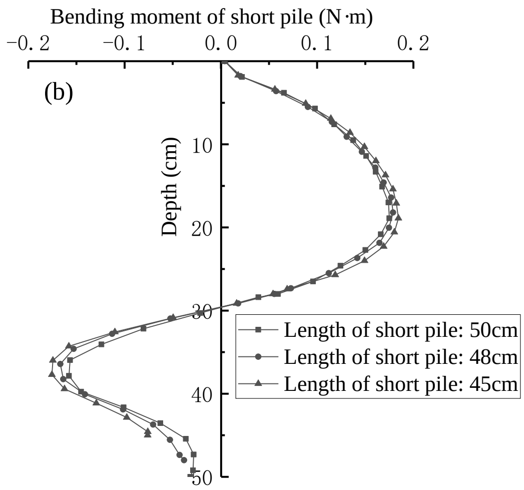


(b) Short pile

Fig. 12 Bending moment distribution curves of long and short piles along different depths corresponding to the retaining system combined with short piles with different lengths and long piles with a constant length of 60 cm



(a) Long pile



(b) Short pile

Fig. 13 Horizontal displacement distribution curves of long and short piles along different depths corresponding to the retaining system combined with short pile with different lengths and long piles with a constant length of 60 cm

relatively significant, and such effects on horizontal displacement of both long and short piles are almost same. However, the effects of the retaining system with different long and short pile combinations on the horizontal displacement of pile beneath the excavation bottom is insignificant (Fig. 10), which indicating that the selection of long and short pile combination is mainly governed by the tolerance of the performance of the pile above the excavation bottom.

3.3 Heave of excavation bottom subjected to pile retaining system with different short and long pile combinations

Fig. 11 shows the heave contours of excavation bottom corresponding to the retaining system with different long and short combinations. It is observed that the maximum heave occurs at the excavation center (i.e., center of the pit) for all the cases with different long and short pile combinations. Also, it is found that the heave at excavation center is 1.56 mm and 1.59 mm respectively, corresponding to the retaining system with combinations of long-short and long-triple-short, with an increasing rate of 1.9%. That is, the increase in the number of short piles used in the studied long-short pile retaining system would not lead to a significant increase in the heave at excavation bottom.

3.4 The influence of short piles length

Fig. 12 shows the bending moment distribution curves of long and short piles in the retaining system along the depth, where the long pile with length of 0.6m and short piles with 0.5m, 0.48m and 0.45m, are used. It is observed that, the maximum bending moment of both long and short piles increases slightly at the both positions beneath and above the excavation bottom as the length of short pile in the given long-short pile composite retaining system decreases from 0.50 m to 0.45 m. For example, the decrease in the length of short pile from 0.50 m to 0.45 m only leads to a slight increase in the bending moment of short pile from 0.175 N·m to 0.184N·m. Also, it can be found that the bending moment above the excavation bottom is larger than its counterpart beneath excavation bottom, especially for the short piles. This again indicates that the proper selection of long and short pile combination is mainly governed by the tolerance of the performance of the pile above the excavation bottom.

Fig. 13 shows the horizontal displacement distribution curves of long and short piles in the retaining system along the depth, where the long pile with length of 0.6 m and short piles with 0.5 m, 0.48 m and 0.45 m, are used. It is observed that, the horizontal displacements of both long and short pile increase slightly with the decrease in the length of short piles, especially for the horizontal displacement above the excavation bottom. By comparing the observations in Figs. 13(a) and 13(b), it is interestingly found that the horizontal displacements of both long and short piles are nearly equal. Moreover, the distribution patterns of horizontal displacement along depth are similar for all the scenarios for short piles with different lengths.

4. Conclusions

To investigate the performance of long-short pile retaining system subjected to the excavation, a series of laboratory tests are firstly conducted. The retaining systems with different long and short pile combinations (i.e., long-short pile retaining system, long-double-short pile retaining system and long-triple-short pile retaining system) are studied. Then, a three-dimensional finite element analysis is performed and verified by comparing the numerical results with the measured data presented in current work. By performing the numerical studies using the experimentally validated finite element method, the horizontal displacement of piles, the heave of excavation bottom and the bending moment of pile for various scenarios with different short pile lengths are comprehensively investigated. The major conclusions can be drawn as follows:

• It is found that the long pile shares more bending moment than short pile in the long-short pile composite retaining system, especially when higher excavation depth is reached. Also, the bending moment of both of long and short piles increases with the increasing number of short piles (i.e., the long and short pile combinations change from long-short pile retaining system to long-triple-short pile retaining system).

• For both long and short piles, the numerical analysis is capable of reproducing the predictions that are well consistent with measured results on the variation of bending moment along the depth of whole pile, highlighting that the numerical analysis is well validated by the measured results.

• The effects of the retaining system with different long and short pile combinations and different length of short pile on the horizontal displacement of the pile above the excavation bottom is much more significant compared to its counterpart beneath the excavation bottom. Also, such effects on the horizontal displacement are almost same for both long and short piles. These observations highlight that the selection of long and short pile combination is mainly governed by the tolerance of the performance of the pile above the excavation bottom.

• The maximum heave occurs at the excavation center for all the cases of retaining system with different long and short pile combinations. In particular, the increase in the number of short piles used in the long-short pile retaining system would not lead to a significant increase in the heave at excavation bottom.

• Similarly, the increments of the maximum bending moment of both long and short piles at the position either beneath or above the excavation bottom are insignificant as the length of short pile in the long-short pile retaining system decreases. The observations highlight that a reliable and economical pile retaining system can be designed by optimizing the short-pile length and the number of short piles, provided that the working performance of retaining structures above excavation bottom meets the design requirement in practice.

It should be noted that the above conclusions are drawn based on the preliminary results from the laboratory tests and numerical analysis presented in current work only. It is of immensely important for further studies on the performance of pile retaining system with different long-short pile combinations in terms of the laboratory and filed tests, for developing the design specification of long-short pile retaining system in engineering practice in the future.

Acknowledgments

Thanks are given to people and organizations who contribute to the success of this work. The financial support from National Natural Science Foundation of China (NSFC Grant No. 51878276) the China National Funds for Distinguished Young Scholar (NSFC Grant No.51725802) and Jiangxi Key Laboratory of Infrastructure Safety Control in Geotechnical Engineering (No. 20161BCD40010) is gratefully acknowledged.

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